Title: Parabolic restriction for Harish-Chandra bimodules and dynamical R-matrices

Speakers: Pavel Safronov

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Abstract: The category of Harish-Chandra bimodules is ubiquitous in representation theory. In this talk I will explain their relationship to the theory of dynamical R-matrices (going back to the works of Donin and Mudrov) and quantum moment maps. I will also relate the monoidal properties of the parabolic restriction functor for Harish-Chandra bimodules to the so-called standard dynamical R-matrix. This is a report on work in progress, joint with Artem Kalmykov.

Parabolic restriction for Harish-Chandra bimodules and dynamical R-matrices

Pavel Safronov

University of Zurich

June 2020

(joint work with Artem Kalmykov)

Questions:

1. An R-matrix can be encoded into the data of a braided monoidal category $\mathcal{C}$ together with a monoidal functor $F\colon \mathcal{C} \to \mathrm{Vect}$. How to encode R-matrices that depend on dynamical parameters?
2. Is there a categorification of the Harish-Chandra isomorphism $\mathrm{Z}(\mathrm{U}\mathfrak{g}) \xrightarrow{\sim} \mathrm{Z}(\mathrm{U}\mathfrak{h})^W$?
3. Symplectic implosion is a way to turn Hamiltonian G-spaces into Hamiltonian H-spaces. What is its quantization?

Hamiltonian reduction

Suppose X is a smooth variety with a Hamiltonian G-action and a moment map $\mu\colon X \to \mathfrak{g}^*$. We get an induced quotient map

$$\mu\colon [X/G] \longrightarrow [\mathfrak{g}^*/G].$$

Observe: can write $[\mathfrak{g}^*/G] = \mathrm{T}^*[1](\mathrm{B}G)$. In particular, $[\mathfrak{g}^*/G]$ has a natural 1-shifted symplectic structure (Pantev–Toën–Vaquié–Vezzosi).

Theorem (Calaque)

A 1-shifted Lagrangian in $[\mathfrak{g}^*/G]$ is the same as a Hamiltonian G-variety.

Given such a Lagrangian $L \to [\mathfrak{g}^*/G]$:

- $L \times_{[\mathfrak{g}^*/G]} \mathfrak{g}^* \cong X$ is the original variety.
- $L \times_{[\mathfrak{g}^*/G]} \mathrm{B}G \cong X /\!\!/ G$ is the Hamiltonian reduction.

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Harish-Chandra bimodules

Deformation quantization:

- Symplectic variety X: deform $\mathrm{QCoh}(X)$ to a plain category.
- 1-shifted symplectic space X: deform $\mathrm{QCoh}(X)$ to a monoidal category.

We have:

$$\mathrm{QCoh}([\mathfrak{g}^*/G]) \cong \mathrm{Mod}_{\mathrm{Sym}(\mathfrak{g})}(\mathrm{Rep}(G)).$$

Quantum version:

$$\mathrm{HC}(G) \cong \mathrm{Mod}_{\mathrm{U}\mathfrak{g}}(\mathrm{Rep}(G)) \subset {}_{\mathrm{U}\mathfrak{g}}\mathrm{BiMod}_{\mathrm{U}\mathfrak{g}},$$

the category of Harish-Chandra bimodules: $\mathrm{U}\mathfrak{g}$-bimodules whose diagonal action is integrable.

Example

H is a torus with character lattice Λ. Then $\mathrm{HC}(H)$ is the category of Λ-graded sheaves $M = \oplus_{\lambda\in\Lambda} M(\lambda)$ on $\mathfrak{h}^*$ with tensor structure

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$$M \otimes^{\mathrm{HC}} N = \bigoplus_{\lambda\in\Lambda} \lambda^* M \otimes N(\lambda).$$

Quantum Hamiltonian reduction

Definition

Let A be an algebra with a G-action. A *quantum moment map* $\mu\colon \mathrm{U}\mathfrak{g} \to A$ is a G-equivariant algebra map, such that $[\mu(x), -]\colon A \to A$ coincides with the $\mathfrak{g}$-action on A for any $x \in \mathfrak{g}$.

Proposition (Ben-Zvi–Brochier–Jordan, S)

An algebra in $\mathrm{HC}(G)$ *is a* G*-equivariant algebra with a quantum moment map* $\mu\colon \mathrm{U}\mathfrak{g} \to A$.

Equivalently: given a category $\mathcal{C}$ with a *strong* G-action (a $\mathcal{D}(G)$-module category), $\mathcal{C}^G$ is an $\mathrm{HC}(G)$-module category.

- $\mathcal{C}^G \otimes_{\mathrm{HC}(G)} \mathrm{Mod}_{\mathrm{U}\mathfrak{g}} \cong \mathcal{C}$ is the original category.
- $\mathcal{C}^G \otimes_{\mathrm{HC}(G)} \mathrm{Rep}(G)$ is the quantum Hamiltonian reduction.

5 (5 of 15)

Symplectic implosion

Fix G, a complex semisimple group, $H \subset B \subset G$ a maximal torus and a Borel subgroup.
Symplectic implosion was introduced by Guillemin–Jeffrey–Sjamaar (for compact Lie groups) and Dancer–Kirwan–Swann (for complex Lie groups) as a way to obtain H-Hamiltonian spaces from G-Hamiltonian spaces.

Theorem (S)

$$[\mathfrak{g}^*/G] \leftarrow [\mathfrak{b}/B] \rightarrow [\mathfrak{h}^*/H]$$

is a 1-shifted Lagrangian correspondence. Given a G-Hamiltonian space given by a Lagrangian $L \to [\mathfrak{g}^*/G]$, its symplectic implosion is the Lagrangian $L \times_{[\mathfrak{g}^*/G]} [\mathfrak{b}/B]$.

Symplectic implosion

The correspondence

$$\begin{array}{ccc} & [\mathfrak{b}/B] & \\ \swarrow & & \searrow \\ [\mathfrak{g}^*/G] & & [\mathfrak{h}^*/H] \end{array}$$

has the following interpretation. There is an isomorphism

$$[G\backslash \mathrm{T}^*(G/N)/H] \cong [\mathfrak{b}/B].$$

Then:

- $\mathrm{T}^*(G/N)/H \to \mathfrak{g}^*$ is the Grothendieck–Springer resolution (moment map for the G-action).
- $\mathrm{T}^*(G/N)/H \to \mathfrak{h}^*$ is a family of twisted cotangent bundles of G/B.

Key fact: $[\mathfrak{b}/B] \to [\mathfrak{h}^*/H]$ is an isomorphism over the locus of regular weights.

The correspondence

Grothendieck-Springer resolution

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Intermezzo

classical	quantum
$[\mathfrak{g}^*/G]$	$\mathrm{HC}(G) \subset {}_{\mathrm{U}\mathfrak{g}}\mathrm{BiMod}_{\mathrm{U}\mathfrak{g}}$
$X = L \times_{[\mathfrak{g}^*/G]} \mathfrak{g}^*$	$\mathcal{C}^G \otimes_{\mathrm{HC}(G)} \mathrm{Mod}_{\mathrm{U}\mathfrak{g}} \cong \mathcal{C}$
$X /\!\!/ G = L \times_{[\mathfrak{g}^*/G]} \mathrm{B}G$	$\mathcal{C}^G \otimes_{\mathrm{HC}(G)} \mathrm{Rep}(G)$
$[\mathfrak{g}^*/G] \leftarrow [\mathfrak{b}/B] \rightarrow [\mathfrak{h}^*/H]$	$\mathrm{res}\colon \mathrm{HC}(G) \to \mathrm{HC}(H)$, $\mathrm{ind}\colon \mathrm{HC}(H) \to \mathrm{HC}(G)$

Questions:

1. Categorical meaning of dynamical R-matrices and dynamical Weyl group.
2. Categorical analog of $\mathrm{Z}(\mathrm{U}\mathfrak{g}) \cong \mathrm{Z}(\mathrm{U}\mathfrak{h})^W$.
3. Quantization of $[\mathfrak{g}^*/G] \leftarrow [\mathfrak{b}/B] \rightarrow [\mathfrak{h}^*/H]$.

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$$\mathrm{T}^*(G/N)/H \to \mathfrak{h}^*, \quad \downarrow G/B$$

Parabolic restriction and induction

Quantization of $[\mathfrak{g}^*/G] \leftarrow [\mathfrak{b}/B] \rightarrow [\mathfrak{h}^*/H]$ is

$$\mathrm{HC}(G) \curvearrowright \mathrm{Mod}^N_{\mathrm{U}\mathfrak{g}}(\mathrm{Rep}(H)) \curvearrowleft \mathrm{HC}(H).$$

Equivalently:

$${}_{\mathrm{U}\mathfrak{g}}\mathrm{BiMod}^G_{\mathrm{U}\mathfrak{g}} \curvearrowright {}_{\mathrm{U}\mathfrak{g}}\mathrm{BiMod}^B_{\mathrm{U}\mathfrak{h}} \curvearrowleft {}_{\mathrm{U}\mathfrak{h}}\mathrm{BiMod}^H_{\mathrm{U}\mathfrak{h}}.$$

Let

$$M^{\mathrm{univ}} = \mathrm{U}\mathfrak{g} \otimes_{\mathrm{U}\mathfrak{b}} \mathrm{U}\mathfrak{h} \in \mathrm{Mod}^N_{\mathrm{U}\mathfrak{g}}(\mathrm{Rep}(H))$$

be the universal Verma module. Taking adjoints we obtain functors $\mathrm{res}\colon \mathrm{HC}(G) \to \mathrm{HC}(H)$, $\mathrm{ind}\colon \mathrm{HC}(H) \to \mathrm{HC}(G)$.

- **Parabolic restriction** is

$$\mathrm{res}(X) = (X \otimes_{\mathrm{U}\mathfrak{g}} M^{\mathrm{univ}})^N, \qquad X \in \mathrm{HC}(G).$$

- **Parabolic induction** is

$$\mathrm{ind}(Y) = \mathrm{coind}^G_P(M^{\mathrm{univ}} \otimes_{\mathrm{U}\mathfrak{h}} Y), \qquad Y \in \mathrm{HC}(H).$$

  Equivalently,

$$\mathrm{ind}(Y) = \Gamma(G/B, (\pi_* \mathcal{D}_{G/N})^H \otimes_{\mathrm{U}\mathfrak{h}} \underline{Y}).$$

Parabolic restriction and induction

$\mathfrak{b}/B = \mathfrak{g}^*/\!/N/H$

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Parabolic restriction

Proposition (Kalmykov–S)

res *is naturally lax monoidal. Moreover,* $\mathrm{U}\mathfrak{h} \to \mathrm{res}(\mathrm{U}\mathfrak{g})$ *is an isomorphism.*

- res sends algebras in $\mathrm{HC}(G)$ to algebras in $\mathrm{HC}(H)$, i.e. G quantum moment maps to H quantum moment maps.
- $\mathrm{res}\colon \mathrm{HC}(G) \to \mathrm{HC}(H)$ induces the map

$$\mathrm{Z}(\mathrm{U}\mathfrak{g}) \longrightarrow \mathrm{Z}(\mathrm{U}\mathfrak{h})$$

  on endomorphisms of units. So, res categorifies the Harish-Chandra homomorphism.

Remark: ind is also lax monoidal. Moreover, $\mathrm{ind}(\mathrm{U}\mathfrak{h}) = \mathrm{U}\mathfrak{g} \otimes_{\mathrm{Z}(\mathrm{U}\mathfrak{g})} \mathrm{Z}(\mathrm{U}\mathfrak{h})$ (Miličić).

Parabolic restriction

Consider $\mathrm{HC}(H)^{gen} \subset \mathrm{HC}(H)$ obtained by replacing $\mathrm{QCoh}(\mathfrak{h}^*)$ by $\mathrm{QCoh}(\mathfrak{h}^* \setminus \Lambda)$.

Theorem (Kalmykov–S)

Generically ${}_{\mathrm{U}\mathfrak{g}}\mathrm{BiMod}^{B}_{\mathrm{U}\mathfrak{h}}$ *is a free rank 1 module over* ${}_{\mathrm{U}\mathfrak{h}}\mathrm{BiMod}^{H}_{\mathrm{U}\mathfrak{h}}$.

Corollary

The functor $\mathrm{res}\colon \mathrm{HC}(G) \to \mathrm{HC}(H)^{gen}$ *is strongly monoidal and exact.*

Idea. Asherova–Smirnov–Tolstoy and Zhelobenko have introduced an *extremal projector* $P \in \hat{\mathrm{U}}\mathfrak{g}$ satisfying

$$P^2 = P, \qquad e_\alpha P = P f_\alpha = 0.$$

It is well-defined on $\mathrm{U}\mathfrak{g}$-modules with a locally nilpotent action of $\mathfrak{n}$ and a non-integral action of $\mathfrak{h}$ and it projects onto highest-weight vectors.

$$\mathrm{HC}(H) \rightleftarrows {}_{\mathrm{U}\mathfrak{g}}\mathrm{BiMod}^{B}_{\mathrm{U}\mathfrak{h}} \circlearrowleft P$$

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Parabolic restriction

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There is a natural monoidal functor

$$\mathrm{free}\colon \mathrm{Rep}(G) \longrightarrow \mathrm{HC}(G)$$

given by $V \mapsto \mathrm{U}\mathfrak{g} \otimes V$.

Theorem (Kalmykov–S)

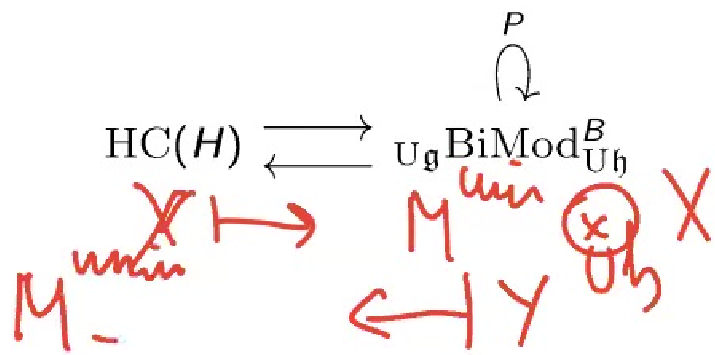
The diagram

$$\begin{array}{ccc} \mathrm{Rep}(G) & \xrightarrow{\mathrm{free}_G} & \mathrm{HC}(G) \\ \downarrow & & \downarrow \mathrm{res} \\ \mathrm{Rep}(H) & \xrightarrow{\mathrm{free}_H} & \mathrm{HC}(H)^{gen} \end{array}$$

is commutative.

Proof again uses extremal projectors.

It is well-defined on $\mathrm{U}\mathfrak{g}$-modules with a locally nilpotent action of $\mathfrak{n}$ and a non-integral action of $\mathfrak{h}$ and it projects onto highest-weight vectors.



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Theorem (Kalmykov–S)

Dynamical R-matrices

Let $\mathcal{C}$ be a braided monoidal category and $F\colon \mathcal{C} \to \mathrm{Vect}$ a monoidal functor. The *R-matrix*

$$R\colon F(x) \otimes F(y) \xrightarrow{F(\sigma_{x,y})} F(y) \otimes F(x) \xrightarrow{\sigma^{-1}_{F(x),F(y)}} F(x) \otimes F(y)$$

satisfies the *Yang–Baxter equation*

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12} \in \mathrm{End}(F(x) \otimes F(y) \otimes F(z)).$$

Suppose

$$F\colon \mathcal{C} \xrightarrow{G} \mathrm{Rep}(H) \xrightarrow{\mathrm{free}} \mathrm{HC}(H)$$

has a monoidal structure. Then one can extract a *dynamical R-matrix* $R\colon \mathfrak{h}^* \to \mathrm{End}(G(x) \otimes G(y))$ which satisfies the *dynamical Yang–Baxter equation*

$$R_{12}(\lambda - h^{(3)})R_{13}(\lambda)R_{23}(\lambda - h^{(1)}) = R_{23}(\lambda)R_{13}(\lambda - h^{(2)})R_{12}(\lambda).$$

Related: Donin–Mudrov describe dynamical R-matrices in terms of dynamical categories.

Example

The functor $\mathrm{Rep}(G) \xrightarrow{\mathrm{free}} \mathrm{HC}(G) \xrightarrow{\mathrm{res}} \mathrm{HC}(H)^{gen}$ is monoidal, which gives rise to the

$$R\colon F(x)\otimes F(y)\xrightarrow{F(\sigma_{x,y})} F(y)\otimes F(x)\xrightarrow{\sigma^{-1}_{F(x),F(y)}} F(x)\otimes F(y)$$

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Example

The functor $\mathrm{Rep}(G)\xrightarrow{\text{free}}\mathrm{HC}(G)\xrightarrow{\text{res}}\mathrm{HC}(H)^{gen}$ is monoidal, which gives rise to the *standard rational dynamical R-matrix.*

Dynamical Weyl group

Dynamical Weyl group

Question: what is the Weyl group action on $\mathrm{HC}(H)$ categorifying $\mathrm{Z}(\mathrm{U}\mathfrak{h})^W$?
Recall the correspondence $[\mathfrak{g}^*/G] \leftarrow [\mathfrak{b}/B] \rightarrow [\mathfrak{h}^*/H]$. In particular, we obtain

$$H\backslash T^*(N\backslash G/N)/H$$

$$[\mathfrak{h}^*/H] \swarrow \qquad \searrow [\mathfrak{h}^*/H]$$

Quantization:

$$\mathrm{HC}(H) \curvearrowright \mathcal{D}_{\mathsf{bimon}}(N\backslash G/N) \curvearrowleft \mathrm{HC}(H).$$

Dynamical Weyl group

For any $w \in W$ we obtain a lax monoidal functor $\mathrm{HC}(H) \to \mathrm{HC}(H)$. After localization, get a monoidal functor

$$\mathrm{res}\colon \mathrm{HC}(G) \to \mathrm{HC}(H)^{\hat{W}, gen}.$$

For free Harish-Chandra bimodules $\mathrm{U}\mathfrak{g} \otimes V$ we obtain rational functions

$$A_w\colon \mathfrak{h}^* \to \mathrm{End}(V).$$

Theorem (Kalmykov–S, in progress)

This recovers the dynamical Weyl group action of Etingof–Tarasov–Varchenko.

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