Title: Modular representations and perverse sheaves on affine flag varieties

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Collection: Geometric Representation Theory

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Abstract: I will give an overview of a joint project with Simon Riche and Laura Rider and another one with Dima Arinkin aimed at a modular version of the equivalence between two geometric realization of the affine Hecke algebra and derived Satake equivalence respectively. As a byproduct we obtain a proof of the Finkelberg-Mirkovic conjecture and a possible approach to understanding cohomology of higher Frobenius kernels with coefficients in a G-module.

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Modular representations and and perverse sheaves on affine flag varieties.

joint w Riche, Rider.

G - reductive$/k$, $F = k((t))$, $G_F(k) = G(F)$, $k = \bar{k}$ - char 0 field

$$G_F \supset G_O \supset I., \quad \mathcal{F}\ell = G_F / I.$$

$\underline{D_I(\mathcal{F}\ell) \supset Perv_I(\mathcal{F}\ell)}$ with coeff-nts in a field $K = \bar{K}$

of char $(K) = \ell > 0$.

$$G_O(k) = G(k[[t]]), \quad O = k[[t]].$$

affine Hecke category

1) recap of Laura's talk $\qquad Perv_I(\mathcal{Fl}) = P_I$

Consider the quotient $P_I \longrightarrow P_I^\circ$, kill all irreducibles whose support has dim > 0.

$$\mathrm{Irr}(P_I) = \{ IC_w \mid w \in W_{aff} \}$$

extended affine Weyl group.

$$P_I^\circ = P_I / \langle IC_w \mid \ell(w) > 0 \rangle.$$

Remk. $D_I(\mathcal{Fl})$ is monoidal, under convolution

$*$ induces a t-exact convolution of $D_I(\mathcal{Fl}) / \langle IC_w \rangle_{\ell(w)>0}$

$/\!/$

$D_I^\circ(\mathcal{Fl})$.

P_I° is monoidal abelian.

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Thm $P_I^\circ \simeq \mathrm{Rep}(Z_{G^\vee}(u))$. $G^\vee$ - dual group.

and u - regular unipotent. (ℓ is not too small)

Proof is based on $\underbrace{\text{Gaitsgory's}}_{\text{Satake equivalences}}$ central functors & monodromy automorphism.

a descr-n of the fiber f-2 is $\Phi_{\mathcal{F}\ell_1}$ - generic van. cycles. (G - s.c.)

$\Downarrow$ Tannakian formalism

$P_I^\circ = \mathrm{Rep}(H)$, $H \subset G^\vee$ commuting with $u \in G^\vee$ a unipotent element.

Need to check 1) u - regular.

2.

Rmk 1) in char 0 has been checked using Frobenius wts. ← doesn't work in char ℓ.

Instead, using Wakimoto, produce filtration indexed wts of $T^\vee$, pass to subquotient corr-g to ν, $\nu+\alpha$, α – simple root. Check that monodromy is nontrivial. $\Rightarrow$ regularity.

2) $H \subset Z_{G^\vee}(u)$. Use that if we project

$$\begin{array}{ccc} Z(M) \in P_I(\mathcal{Fl}) & \longrightarrow & P_{IW}(\mathcal{Fl}) \\ & & \Downarrow \\ & & Z_{IW}(M) \end{array} \qquad \begin{array}{c} \cap \\ D_{(I_-,\psi)}(G/I). \end{array}$$

Lemma $Z_{IW}(M)$ is tilting if M is tilting.

$I, I_- \subset G(K)$

For T_1, T_2 - tilting in $\mathcal{P}_{IW}(\mathcal{F}\ell)$,

$$\begin{tikzcd}
P_I(\mathcal{F}\ell) \arrow[r] \arrow[dr] & \mathcal{P}_{IW}(\mathcal{F}\ell). \arrow[d] \\
& \mathcal{P}_I^0(\mathcal{F}\ell)
\end{tikzcd}$$

$$\mathrm{Hom}(T_1, T_2) \xrightarrow{\sim}$$

$$\xrightarrow{\sim} \mathrm{Hom}_{P_I^0}(\overline{T_1}, \overline{T_2})$$

For T_1, T_2 - tilting in $\mathcal{P}_{IW}(\mathcal{F}\ell)$,

$$\begin{tikzcd} P_I(\mathcal{F}\ell) \arrow[r] \arrow[dr] & \tilde{P}_{IW}(\mathcal{F}\ell) \arrow[d] \\ & P_I^0(\mathcal{F}\ell) \end{tikzcd}$$

$\mathrm{Hom}(T_1, T_2) \xrightarrow{\sim}$

$\xrightarrow{\sim} \mathrm{Hom}_{P_I^0}(\overline{T}_1, \overline{T}_2)$

Using this + dim counting conclude $H = Z_{G^\vee}(u)$.

Applications.

- (unipotent) biWhittaker category & derived Satake (w. Arinkin)
- equivalence $D_I(\mathcal{F}\ell) \simeq D\,Coh^{G^\vee}(T^*G^\vee/B^\vee \times_{\mathfrak{g}^\vee} T^*G^\vee/B^\vee)$
  & modular HC bimodules
  $\Downarrow$
  RM conjecture.
- Questions on p-cells & $H^*(G_1, T)$

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$$\mathrm{Hom}(Z_{IW}(T_1), Z_{IW}(T_2)) \simeq \mathrm{Hom}(\overline{Z_{IW}(T_1)}, \overline{Z_{IW}(T_2)})$$

$T_1, T_2 \in G^\vee$-mod, tilting.

$$\uparrow$$
$$P_I^\circ$$

$$\{T_1 \otimes \mathcal{O}, T_2 \otimes \mathcal{O}\}$$

$$\simeq \mathrm{Hom}_{Z_{G^\vee}(U)}(T_1, T_2) = \mathrm{Hom}_{\mathrm{Coh}^{G^\vee}(G^\vee(U))}$$

$$= \mathrm{Hom}_{\mathrm{Coh}^{G^\vee}(\mathcal{U})}(T_1 \otimes \mathcal{O}, T_2 \otimes \mathcal{O}).$$

$\mathcal{U} \subset G^\vee$ - set of unipotent elements.

$$\mathrm{Ext}(Z_{IW}(T_1), Z_{IW}(T_2)).$$

$$\langle Z_{IW}(T) \mid \mathrm{Tilt}\rangle \subset D^b \mathrm{Coh}^{G^\vee}(\mathcal{U})$$

Can consider the averaging functor.

$$D_{IW}(\mathcal{F}\ell) \xrightarrow{Av_{I^-,\psi}} D\left({}_{(I^-,\psi)}\backslash G_F/_{(I^-,\psi)}\right).$$

Claim $\Xi_{IW}(T)$ generate orthogonal to the kernel of this.

Cor / relying on monodromic version of the $P_I^0 = \mathrm{Rep}(Z_{G^\vee}(u))$.

$$D\left({}_{I^-,\psi}\backslash G/_{I^-,\psi}\right) \supset \mathrm{Wh} \simeq D^b\mathrm{Coh}^{G^\vee}_{u}(G^\vee).$$

Cor (Arinkin) $D\left({}_{G_O}\backslash G_F/_{G_O}\right) \simeq D\,\mathrm{Coh}^{G^\vee}\left(\{1\}\times_{G^\vee}\{1\}\right).$

Answer suggested by Drinfeld, GMRR.

$$D\left({}_{G_O}\backslash G_F/G_O\right) \xrightarrow{\sim} \mathrm{End}_{Wh}\left(D\left({}_{I,\psi}\backslash G_F/G_O\right)\right) \simeq D\,\mathrm{Rep}\,G^\vee$$

Using this + Soergel method. can deduce.

$(\widehat{\mathfrak{g}^\vee_{\mathcal{N}}} \simeq \widehat{G^\vee_u})$
as $G^\vee$-varieties.

Thm $D_I(\mathcal{Fl}) \simeq D\, Coh^{G^\vee}(T^*(G^\vee/B^\vee) \times_{\mathfrak{g}^\vee} T^*(G^\vee/B^\vee))$.

$\wr|$

$D^b((\mathfrak{g} \oplus \mathfrak{g}, G)\text{-mod}_{0,0})$

fixed HC central char.

t-exact up to shift by $\frac{1}{2}$ codim of
on orbit on
subquotient corr-g to
an orbit $O \in \mathcal{N}$

(a quantum analogue should -

Cor $\quad G^\vee\text{-mod}_{\hat{0}} \simeq \mathrm{Perv}_{I^0}(\mathcal{H})$

regular block $\quad$ radical of Iwahori.

Cor $\quad D_{C(0)}(\mathcal{H}) \curvearrowright D(G^\vee\text{-mod}_0)$

A similar action in the quantum relates to $H^*(u_q) \simeq \mathrm{Ext}_{\mathrm{Perv}(\mathcal{H})}(\delta_e, \mathrm{Satake}(\mathbb{C}[\,.\,]))$

$\mathcal{O}(\mathcal{N}_e)$ $\quad$ $\cong$ $\quad$ [ABG].

$\mathcal{O}(\mathcal{N})$

modularity sufficient $\quad$ 2-tuples of commuting nilp.

can construct a map based on $c_1(\det)$

$H^*(G_2) \Longleftrightarrow \mathrm{Ext}(\delta_e, \mathrm{Satake}\ \mathcal{O}(G^{(2)}))$

$\mathcal{O}\ c_1(\det) \Rightarrow c_1 \quad c^{[p]}, \quad c^{[p^2]} \ldots c^{[p^{k-1}]}$ $\quad$ Yun Zhu

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A, $D^b(A) \simeq D^b\,\mathrm{Coh}(\tilde{g})$

A' $\tilde{\mathcal{N}}$.

$(g \oplus g, G)\text{-mod}_{\hat{0},\hat{0}} \simeq A \otimes_{\mathcal{O}(g)} A^{op}\text{-mod}_{\mathcal{N}}$.

$A = \mathrm{End}(\mathcal{E})$

tilting bdle,

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Cor $\quad G^\vee\text{-mod}_{\hat{0}} \simeq \mathrm{Perv}_{I^0}(\mathcal{F}\ell)$

regular block

radical of Iwahori:

Cor $\quad D_{C(0)}(\mathcal{G}r) \circlearrowright D(G^\vee\text{-mod}_0)$

A similar action in the quantum

relates to $H^*(u_q) \simeq \underbrace{\mathrm{Ext}(\delta_e,\ \mathrm{Satake}(\Theta(\mathcal{L})))}_{\mathrm{Perv}(\mathcal{G}r)}$

$\mathcal{O}(\mathcal{N}_e)$ $\qquad \wr\!\!\mid$ $\qquad$ [ABG].

$\mathcal{O}(\mathcal{N})$

modulo nilpotent ... $\quad$ 2-tuples of commuting nilp.

can construct a map based on $c_1(\det)$

$H^*(G_2) \Longleftrightarrow \mathrm{Ext}(\delta_e,\ \mathrm{Satake}\ \mathcal{O}(G^{(2)}))$

$c_1(\det) \Rightarrow \zeta_1 \quad \zeta^{[p]}, \quad \zeta^{[p^2]} \ldots \zeta^{[p^{k-1}]}$