Title: Modular representations and perverse sheaves on affine flag varieties

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Collection: Geometric Representation Theory

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Abstract: I will give an overview of a joint project with Simon Riche and Laura Rider and another one with Dima Arinkin aimed at a modular version of the equivalence between two geometric realization of the affine Hecke algebra and derived Satake equivalence respectively. As a byproduct we obtain a proof of the Finkelberg-Mirkovic conjecture and a possible approach to understanding cohomology of higher Frobenius kernels with coefficients in a G-module.

Modular representations and
and perverse sheaves on affine flag varieties.

joint w Riche, Rider.

G - reductive$/_k$, $F = k((t))$, $G_F(k) = G(F)$, $k = \bar{k}$ - char 0 field

$G_F \supset G_0 \supset I$. , $\mathcal{F}\ell = G_F / I$.

$\underline{D_I(\mathcal{F}\ell) \supset Perv_I(\mathcal{F}\ell)}$ with coeff-nts in a field $K = \overline{K}$ of char $(K) = \ell > 0$.

$G_0(k) = G(k[[t]])$, $O = k[[t]]$.

affine Hecke category.

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1) recap of Laura's talk $\quad Perv_I(\mathcal{F}\ell) = P_I$

Consider the quotient $P_I \twoheadrightarrow P_I^\circ$, kill all irreducibles whose support has dim > 0.

$$\mathrm{Irr}(P_I) = \{ IC_w \mid w \in W_{aff} \}$$

extended affine Weyl group.

$$P_I^\circ = P_I / \langle IC_w \mid \ell(w) > 0 \rangle.$$

Remk, $D_I(\mathcal{F}\ell)$ is monoidal, under convolution

$*$. induces a t-exact convolution of $D_I(\mathcal{F}\ell) / \langle IC_w \rangle_{\ell(w)>0}$ $= D_I^\circ(\mathcal{F}\ell)$.

P_I° is monoidal abelian.

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Thm $P^\circ_I \simeq \mathrm{Rep}(Z_{G^\vee}(u))$, $G^\vee$ - dual group.

and u - regular unipotent. (ℓ is not too small)

Proof is based on $\underbrace{\text{Gaitsgory's}}_{\text{Satake equivalences}}$ central functors & monodromy automorphism.

$\Downarrow$ Tannakian formalism

$P^\circ_I = \mathrm{Rep}(H)$, $H \subset G^\vee$ commuting with $u \in G^\vee$ a unipotent element.

a descr-n of the fiber f-r is $\Phi_{\mathcal{Fl}_1}$ - generic van. cycles. (G-s.c.)

Need to check 1) u - regular.

2.

Rmk 1) in char 0 has been checked using Frobenius wts. ← doesn't work in char ℓ.

Instead, using Wakimoto, produce filtration indexed wts of $T^\vee$, pass to subquotient corr-g to ν, $\nu+\alpha$, α – simple root. check that monodromy is nontrivial. $\Rightarrow$ regularity.

2) $H \subset Z_{G^\vee}(u)$. Use that if we project

$$\begin{tikzcd} Z(M) \in P_I(\mathcal{F}\ell) \arrow[r] & P_{IW}(\mathcal{F}\ell) \\ & Z_{IW}(M) \arrow[u, phantom, "\cup"] \end{tikzcd} \qquad \begin{array}{c} \cap \\ D_{(I_-,\psi)}(G/I). \end{array}$$

$$Z(M) \longleftrightarrow Z_{IW}(M)$$

<u>Lemma</u> $Z_{IW}(M)$ is tilting iff M is tilting.

$I, I_- \subset G(\mathcal{K})$

For T_1, T_2 - tilting in $\mathcal{P}_{IW}(\mathcal{F}\ell)$,

$$\begin{tikzcd} \mathcal{P}_I(\mathcal{F}\ell) \arrow[r] \arrow[dr] & \mathcal{P}_{IW}(\mathcal{F}\ell). \arrow[d] \\ & \mathcal{P}_I^0(\mathcal{F}\ell) \end{tikzcd}$$

$$\mathrm{Hom}(T_1, T_2) \xrightarrow{\sim} \mathrm{Hom}_{\mathcal{P}_I^0}(\overline{T}_1, \overline{T}_2)$$

For T_1, T_2 - tilting in $P_{IW}(\mathcal{F}l)$,

$$P_I(\mathcal{F}l) \longrightarrow P_{IW}(\mathcal{F}l).$$
$$\downarrow \quad \mathcal{F} \mapsto \overline{\mathcal{F}}$$
$$P_I^0(\mathcal{F}l)$$

$\mathrm{Hom}(T_1, T_2) \xrightarrow{\sim}$

$\xrightarrow{\sim} \mathrm{Hom}_{P_I^0}(\overline{T}_1, \overline{T}_2)$

Using this + dim counting conclude $H = Z_{G^\vee}(u)$.

Applications

- (unipotent) bi Whittaker category & derived Satake (w. Arinkin)

- equivalence $D_I(\mathcal{F}l) \simeq D\,\mathrm{Coh}^{G^\vee}(T^\vee G^\vee/B^\vee \times_{\mathfrak{g}^\vee} T^* G^\vee/B^\vee)$
  & modular HC bimodules
  $\Downarrow$
  RM conjecture.

- Questions on p-cells & $H^*(G_2, T)$

$$\mathrm{Hom}\,(Z_{Iw}(T_1), Z_{Iw}(T_2)) \simeq \mathrm{Hom}\,(\overline{Z_{Iw}(T_1)}, \overline{Z_{Iw}(T_2)})$$

$$\boxed{T_1, T_2 \in G^\vee\text{-mod, tilting.}} \qquad \begin{matrix}\uparrow \\ P_I^\circ\end{matrix}$$

$$\simeq \mathrm{Hom}_{Z_{G^\vee}(u)}(T_1, T_2) = \mathrm{Hom}_{Coh^{G^\vee}(G^\vee(u))} (T_1 \otimes \mathcal{O}, T_2 \otimes \mathcal{O})$$

$$= \mathrm{Hom}_{Coh^{G^\vee}(\mathcal{U})}(T_1 \otimes \mathcal{O}, T_2 \otimes \mathcal{O}).$$

$\mathcal{U} \subset G^\vee$ - set of unipotent elements.

$$\mathrm{Ext}(Z_{Iw}(T_1), Z_{Iw}(T_2)).$$

$$\langle Z_{Iw}(T_1) \mid Tilt \rangle \subset D^b Coh^{G^\vee}(\mathcal{U})$$

Can consider the averaging functor.

$$D_{IW}(\mathcal{F}\ell) \xrightarrow[Av_{I^-,\psi}]{} D\left({}_{(I^-,\psi)}\backslash G_F/_{(I^-,\psi)}\right).$$

Claim $\Xi_{IW}(T)$ generate orthogonal to the kernel of this.

Cor | relying on monodromic version of the

$$P_I^0 = \mathrm{Rep}\,(Z_{G^\vee}(u)).$$

$$D\left({}_{I,\psi}\backslash G/_{I^-,\psi}\right) \supset Wh \simeq D^! Coh^{G^\vee}_{\mathcal{U}}(G^\vee).$$

Cor (Arinkin) $\quad D\left({}_{G_O}\backslash G_F/_{G_O}\right) \simeq D\,Coh^{G^\vee}\left(\{1\} \times_{G^\vee} \{1\}\right).$

Answer is suggested by Drinfeld, GMRR.

$$D\left({}_{G_O}\backslash G_F/G_O\right) \xrightarrow{\sim} End_{Wh}\left(D\left({}_{I,\psi}\backslash G_F/G_O\right)\right) \simeq D\,Rep\,G^\vee$$

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Using this + Soergel method. can deduce.

($\widehat{\mathfrak{g}^\vee_{\mathcal{N}}} \simeq \widehat{G^\vee_u}$)
as $G^\vee$-varieties.

Thm $D_I(\mathcal{F}\ell) \simeq D\,Coh^{G^\vee}(T^*(G^\vee/B^\vee) \times_{\mathfrak{g}^\vee} T^*(G^\vee/B^\vee))$.

$\wr\!\!\wr$

$D^b((\mathfrak{g}\oplus\mathfrak{g}, G)-mod_{0,0})$

fixed HC central char.

t-exact up to shift by $\frac{1}{2}$ codim of an orbit on subquotient corr-g to an orbit $O \in \mathcal{N}$

(a quantum analogue should -

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Cor $\quad G^\vee\text{-mod}_{\hat{0}} \simeq \mathrm{Perv}_{I^0}(\mathcal{F}\ell)$

regular block

readical of Iwahori:

Cor $\quad D_{C(0)}(\mathcal{G}r) \circlearrowleft D(G^\vee\text{-mod}_0)$

A similar action in the quantum

relates to $H^*(u_q) \simeq \mathrm{Ext}_{\mathrm{Perv}(\mathcal{G}r)}(\delta_e, \mathrm{Satake}(C|.)$

$\mathcal{O}(\mathcal{N}_e)$ $\qquad \cong \qquad$ [ABG].

modulaly shifting

$\imath$-tuples of commuting nilp.

$\mathcal{O}(\mathcal{N})$

can construct a map based on $c_1(\det)$

$H^*(G_2) \Longleftrightarrow \mathrm{Ext}(\delta_e, \mathrm{Satake}\, \mathcal{O}(G^{(1)}))$

$\mathcal{O}\; c_1(\det) = c_1 \quad c^{[p]}, \quad c^{[p^2]} \ldots c^{[p^{k-1}]}$ Yun Zhu

$\boxed{A}$, $D^b(A) \cong D^b\,Coh(\tilde{\mathfrak{g}})$

A' $\tilde{\mathcal{N}}$.

$(\mathfrak{g}\oplus\mathfrak{g}, G)\text{-mod}_{\hat{0},\hat{0}} \simeq A \underset{\mathcal{O}(\mathfrak{g})}{\otimes} A^{op}\text{-mod}_{\mathcal{N}}$

$A = End(\mathcal{E})$

tilting bdle,

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Cor $\quad G^\vee\text{-mod}_{\hat{0}} \simeq \mathrm{Perv}_{I^0}(\mathcal{H})$

regular block

readical of Iwahori:

Cor $\quad D_{C(0)}(\mathcal{H}) \circlearrowright D(G^\vee\text{-mod}_0)$

A similar action in the quantum relates to $H^*(u_q) \simeq \underbrace{\mathrm{Ext}(\delta_e, \mathrm{Satake}(\mathcal{O}(G)))}_{\mathrm{Perv}(\mathcal{H})}$

$\mathcal{O}(N_e)$ $\quad \mathfrak{I}|$ $\quad$ [ABG].

modulary suffix $\quad$ 2-tuples of commuting nilp.

$\mathcal{O}(N)$

can construct a map based on $c_1(\det)$

$H^*(G_2) \longleftrightarrow \mathrm{Ext}(\delta_e, \mathrm{Satake}\ \mathcal{O}(G^{(2)}))$

$c_1(\det) \to S_1 \quad C^{[p]}, \quad C^{[p^2]} \ldots C^{[p^{k-1}]}$