Title: Yangians and cohomological Hall algebras of Higgs sheaves on curves

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Collection: Geometric Representation Theory

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Abstract: We will review a set of conjectures related to the structure of cohomological Hall algebras (COHA) of categories of Higgs sheaves on curves. We then focus on the case of P^1, and relate its COHA to the affine Yangian of sl_2.

$(M'', M') \longleftarrow (0 \to M' \to M \to M'' \to 0) \longmapsto M$

($\rightsquigarrow$ p is proper)

- q is "1-truncated" Ext-stack
  - $d=1$: q is smooth
  - $d=2$: q is "quasi-smooth"

For "nice" invariants / cohomology theories $S \to F(S)$ on stacks, $(*)$ induces an algebra structure on $F(\mathcal{M})$ via $p_! \, q^* : F(\mathcal{M} \times \mathcal{M}) \to F(\mathcal{M})$.

Ex: F : (constructible) functions $\rightsquigarrow$ (constructible) Hall algebra (Ringel) } "Hall algebras"

$D^b_{constr}(\cdot)$ $\rightsquigarrow$ "geometric" Hall algebra (Lusztig)

H^{BM}_* $\rightsquigarrow$ COHA

A_* $\rightsquigarrow$ AHA

K_0 $\rightsquigarrow$ KHA

OBM $\rightsquigarrow$ "...."HA

$\vdots$

} "Cohomological Hall algebras"

Kontsevich-Soibelman
S-Vasserot
Yang-Zhao
Minets
S-Sala
Y. Zhao, Kapranov-Vasserot
Joyce
$\vdots$

One can also work directly at the level of categories (Porta-Sala, Dyckerhoff-Kapranov, ... , KLR b

- Some guiding principles (especially if $d=1$)

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- Some guiding principles (especially if d=1)

1) Hall algebras look like one half of a (quantum) envelopping algebras

2) Hall algebras of derived equivalent categories correspond to the same quantum group (but different hal

3) If $\mathcal{B}$ is abelian category s.t. $\mathcal{M}_{\mathcal{B}}$ "=" $T^*\mathcal{M}_{\mathcal{A}}$ then COHAs of $\mathcal{B}$ are affinizations of Hall alg

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ation of 3):

Q quiver : $\mathcal{A} = \mathrm{Rep}(Q)$ $\quad d=1$

$A_{Q,\alpha}(t) =$ Kac pol. counting abs. indec. reps of Q of dim α. $\qquad A_{Q,\alpha}(t) \in \mathbb{N}[t]$ (Hausel-Letellier-Rodriguez-Villegas)

$\rightsquigarrow$ Hall algebra of functions on $\mathcal{M}(\mathbb{F}_q)$

$$\begin{cases} H_{Q/\mathbb{F}_q} \simeq U_q^+(\tilde{\mathfrak{g}}_Q) & \mathrm{ch}(\tilde{\mathfrak{g}}_Q) = \sum_\alpha A_{Q,\alpha}(q)\, e^\alpha \\ \quad\uparrow \qquad\qquad \uparrow & \\ H^{sph}_{Q/\mathbb{F}_q} \simeq U_q^+(\mathfrak{g}_Q) & \mathrm{ch}(\mathfrak{g}_Q) = \sum_\alpha A_{Q,\alpha}(0)\, e^\alpha \end{cases}$$

(Sevenhant - Van den Berg

(Kac conj = Hausel's thm)

Here, $\mathfrak{g}_Q$ is Kac-Moody algebra assoc. to Q.

$\tilde{\mathfrak{g}}_Q$ is a Borcherds algebra (Cartan matrix: cuspidal count polynomial (Deng-Xiao, Bo

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Conj: $\tilde{\mathfrak{g}}_Q$ = Maulik-Okounkov algebra assoc. to Q.

$\leadsto$ CoHAs / KHAs of $\mathcal{B} = \mathrm{Rep}\,\Pi_Q$ $\quad \Pi_Q$ = preprojective algebra $\quad \mathcal{B}$: CY2, gldim $\mathcal{B} = 2$

Thm (Davison-Meinhardt, SV) 1) $\mathrm{COHA}_{\Pi_Q} \simeq U^+_{(h)}(\mathfrak{g}^{BPS}_Q[u])$ "Yangian of $\mathfrak{g}^{BPS}_Q$"

2) $\mathrm{ch}(\mathfrak{g}^{BPS}_Q) = \mathrm{ch}(\tilde{\mathfrak{g}}_Q)$. (i.e. given by Kac pol of $A = \mathrm{Rep}\,Q$)

Conj: $\mathfrak{g}^{BPS}_Q \simeq \tilde{\mathfrak{g}}_Q$ ($\simeq$ Maulik-Okounkov algebra)

Conj $\mathrm{KHA}_{\Pi_Q} \simeq U^+_{(q)}(\tilde{\mathfrak{g}}_Q[u,u^{-1}])$

Rem:
- $\exists$ shuffle algebra presentations for CoHA, KHA, ...
- Conjectures are known for Q finite or affine.
- Cohomological Hall algebras act on cohomology of Nakajima quiver varieties

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"CY" example: $\quad Q = \cdot\circlearrowleft$

$\mathrm{Rep}\, Q = \mathrm{Coh}_0(\mathbb{A}^1)$ $\qquad H_{Q/\mathbb{F}_q} \simeq U_q^+(\widehat{\mathfrak{gl}(1)})$ (quantum Heisenberg)

$\mathrm{Rep}\, \Pi_Q = \mathrm{Coh}_0(\mathbb{A}^2)$

$$\begin{cases} COHA_{\Pi_Q} \simeq U^+_{\varepsilon_1,\varepsilon_2}(\widehat{\mathfrak{gl}(1)}[u]) & \text{(affine Yangian of } \mathfrak{gl}(1)) \\ KHA_{\Pi_Q} \simeq U^+_{q,t}(\widehat{\widehat{\mathfrak{gl}(1)}}) & (\simeq \text{Elliptic Hall algebra, EHA}) \end{cases}$$

$\circlearrowleft$
$SL(2,\mathbb{Z})$

These $COHA_{\Pi_Q}$ / KHA_{Π_Q} feature in various places as Hecke algebras of (pointwise) modifications on so

(SV, Rapcak-Soibelman-Yang-Zhao, Chuang-Creozig-Diaconescu-Soibelman, Negut (n big), ...)

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$\lfloor$ – cohomological Hall algebras act on cohomology of Nakajima quiver varieties

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$\mathrm{Rep}\, \Pi_Q = \mathrm{Coh}_0(\mathbb{A}^2)$ $\qquad \begin{cases} \mathrm{COHA}_{\Pi_Q} \simeq U^+_{\epsilon_1,\epsilon_2}(\hat{\mathfrak{gl}}(1)[u]) & \text{(affine Yangian of } \mathfrak{gl}(1)) \\ \mathrm{KHA}_{\Pi_Q} \simeq U^+_{q,t}(\hat{\hat{\mathfrak{gl}}}(1)) & (\Leftarrow \text{Elliptic Hall algebra, EHA}) \end{cases}$

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These COHA_{Π_Q} / KHA_{Π_Q} feature in various places as Hecke algebras of (pointwise) modifications on surfaces
([SV], Rapcak-Soibelman-Yang-Zhao, Chuang-Creozig-Diaconescu-Soibelman, Negut^ (a big), ...)

n to $\mathcal{A} = \mathrm{Coh}(X)$ $\quad X$ smooth projective curve of genus g $\qquad d = 1$

$\mathcal{B} = \mathrm{Higgs}(X)$

$\mathbb{P}^1$ $\qquad H^{sph}_{X/\mathbb{F}_q} = U_q^+(\hat{\mathfrak{sl}}_2)$ (Kapranov, Baumann-Kassel)

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Now, we turn to $\mathcal{A} = \mathrm{Coh}(X)$ X smooth projective curve of genus g $d=1$

$\mathcal{B} = \mathrm{Higgs}(X)$

Ex: • $X = \mathbb{P}^1$

$$H^{sph}_{X/\mathbb{F}_q} = U_q^+(\hat{sl}_2) \quad \text{(Kapranov, Baumann – Kassel)}$$

$$\cap \qquad\qquad \cap$$

$$H_{X/\mathbb{F}_q} = U_q^+(\hat{gl}_2)$$

Rem: Here "+" denotes Drinfeld's positive half.

• X = elliptic curve (cy)

$$H^{sph}_{X/\mathbb{F}_q} = U^+_{\sigma_1,\sigma_2}(\hat{\hat{gl}}(1)) \quad \text{Elliptic Hall algebra} \quad \text{(Burban-S.)} \quad \circlearrowleft sl_2(\mathbb{Z})$$

$$\cap \qquad\qquad \cap$$

$$H_{X/\mathbb{F}_q} = U^+_{\sigma_1,\sigma_2}(\hat{\hat{gl}}(1))^{\otimes H^*(X)} \quad \text{(Fratila)}$$

where $\{\sigma_1, \sigma_2\}$ are the Weil numbers of X

• $g>1$ "Conj": $H_{X/\mathbb{F}_q} = U^+_{\sigma_1,\dots,\sigma_{2g}}(\tilde{\mathfrak{g}}_g)$ For some Lie algebra $\tilde{\mathfrak{g}}_g$ (in the category of GSp

such that $ch(\tilde{\mathfrak{g}}_g) = \sum_{r,d} A_{g,r}(\sigma_1,\dots,\sigma_{2g})\, e^{(r,d)}$

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curve

(cy)

$$H_{X/\mathbb{F}_q} = U^{+}_{\sigma_1,\sigma_2}(\mathfrak{g}\hat{l}(1))^{\otimes H^*(X)}$$ (Fratila)

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Kac polynomial of genus g curves counting abs. indec coherent sheaves of rk r & deg d.

⇝ By analogy with quiver setup, it's natural to expect that

"Conj":

$$\mathrm{CoHA}_{\mathrm{Higgs}_X} \simeq U^{+}_{(\hbar)}(\tilde{\mathfrak{g}}_g[u])$$

$$\mathrm{KHA}_{\mathrm{Higgs}_X} \simeq U^{+}_{(q)}(\tilde{\mathfrak{g}}_g[u,u^{-1}])$$

Rem: 1) Here the def. parameters $(\underline{\hbar}), (\underline{q})$ are $g+1$-dim: in addition to the "dilat" along fibers "$\mathbb{C}^*$", they also incorporate the action of the mapping class group (or its quotient $Sp(2g,\mathbb{C})$).

2) One expects COHAs of Higgs_X to act on cohomology of moduli spaces of (semi)stable coh. sheaves on any surface S containing X and such that $N_X \simeq \Omega_X$ by Hecke modifications along X.

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II - The case of $\mathbb{P}^1$

$X = \mathbb{P}^1$ $\begin{cases} A = \mathrm{Coh}(\mathbb{P}^1) \\ B = \mathrm{Higgs}(\mathbb{P}^1) \end{cases}$ $\mathrm{OBM} = H_*^T$ $T = \mathbb{C}^\times \times \mathbb{C}^\times$ (dilat° along fibers + loop rotation)

$\mathrm{COHA}_B := \bigoplus_{r,d} H_*^T(\mathrm{Higgs}_{r,d}(\mathbb{P}^1)) =: \mathbb{Y}^+$ Equiv. param: $H_T^*(pt) = \mathbb{C}[\varepsilon_1, \varepsilon_2]$.

$H_{X/\mathbb{F}_q} \simeq U_q^+(\widehat{\mathfrak{gl}}_2)$, so we expect $\mathbb{Y}^+$ to be related to $Y^+_{\varepsilon_1\varepsilon_2}(\widehat{\mathfrak{sl}}_2)$.

- Affine Yangian of $\widehat{\mathfrak{sl}}_2$

$Y := Y_{\varepsilon_1,\varepsilon_2}(\widehat{\mathfrak{sl}}_2) = \langle e_{i,\ell}, h_{i,\ell}\ f_{i,\ell} \mid i=0,1,\ \ell \geq 0 \rangle$

$\cup$

$Y^+ := \langle e_{i,\ell} \mid i=0,1,\ \ell \geq 0 \rangle$ "Kac-Moody positive half"

$\hookrightarrow Y_0 := \langle e_{i,0}, h_{i,0} \mid i=0,1 \rangle \simeq U(\widehat{\mathfrak{sl}}_2) \subset Y$

$\hookrightarrow \mathbb{H} := \langle \mathrm{Ad}(h_{i,\ell}) \mid i=0,1,\ \ell \geq 0 \rangle \subset \mathrm{Aut}(Y)$

$\Big\}$ $\mathbb{H} \cdot Y_0$ generates Y

$H_{X/\mathbb{F}_q} = U_q(\hat{sl}_2)$, so we expect Y to be related to $Y_{\varepsilon_1\varepsilon_2}(\hat{sl}_2)$.

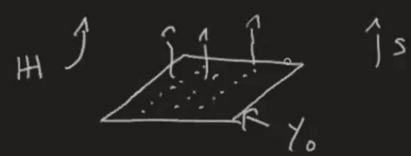
- <u>Affine Yangian of $\hat{sl}_2$</u>

$$Y := Y_{\varepsilon_1,\varepsilon_2}(\hat{sl}_2) = \langle e_{i,\ell}, h_{i,\ell}, f_{i,\ell} \mid i=0,1,\ \ell \geq 0 \rangle \qquad e_{i,\ell} \rightsquigarrow \begin{cases} e s^\ell & \text{if } i=1 \\ ft\, s^{\ell+1} & \text{if } i=0 \end{cases} \in \underbrace{\hat{sl}_2[s]}_{sl_2[t,t^{-1}]}$$

$$\cup$$

$$Y^+ := \langle e_{i,\ell} \mid i=0,1,\ \ell \geq 0 \rangle \qquad \text{"Kac-Moody positive half"}$$

$$\left.\begin{array}{l} \hookrightarrow Y_0^+ := \langle e_{i,0}, h_{i,0} \mid i=0,1 \rangle \simeq U^+(\hat{sl}_2) \subset Y^+ \\ \hookrightarrow \mathbb{H} := \langle \mathrm{Ad}(h_{i,\ell}) \mid i=0,1,\ \ell \geq 0 \rangle \subset \mathrm{Aut}(Y) \end{array}\right\} \quad \begin{array}{l} \mathbb{H}\cdot Y_0 \text{ generates } Y \\ \mathbb{H}\cdot Y_0^+ \text{ —— } Y^+ \end{array}$$



<u>Thm</u> (S-Vasserot, in progress)

1) Y^+ is isomorphic to the subalgebra of Y generated by $\mathbb{H}\cdot Y_0^{+,D}$ where $Y_0^{+,D} = U(t\mathfrak{h}[t] \oplus \mathfrak{n}[t,t^{-1}]) \subset U(\hat{sl}_2)$ is the Drinfeld positive half.

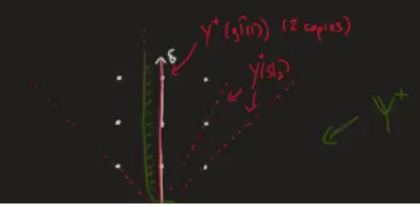
2) Y^+ contains two copies of $Y^+_{\varepsilon_1,\varepsilon_2}(\hat{\mathfrak{gl}(1)})$ ("Cartan double loops")

3) $\forall k \in \mathbb{Z}$, Y^+ contains a copy of $Y_\gamma^+(sl_2)$ (lifting semistable COHA of slope k)

<u>Picture</u>

(projection on blackboard)

$Y(\hat{sl}_2)$:



2) Y^+ contains two copies of $Y^+_{\delta_1,\delta_2}(\widehat{gl(1)})$ ("Cartan double loops")

3) $\forall k \in \mathbb{Z}$, Y^+ contains a copy of $Y^+_\gamma(sl_2)$ (lifting semistable COHA of slope k)

Picture

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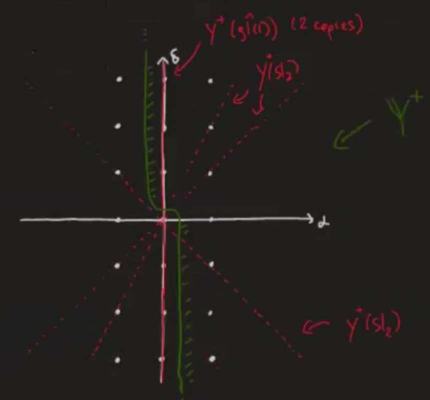
$Y(\hat{sl}_2)$:



Rem: - One expects a similar picture for KHA (replace $Y(\widehat{gl(1)})$ by EHA) $\to$ cf. Mounzer-Zegers

- One could as well call $Y(\hat{sl}_2)$ the affine Yangian of gl_2 (if in gl_2 we put one cartan gen. in degree 2)

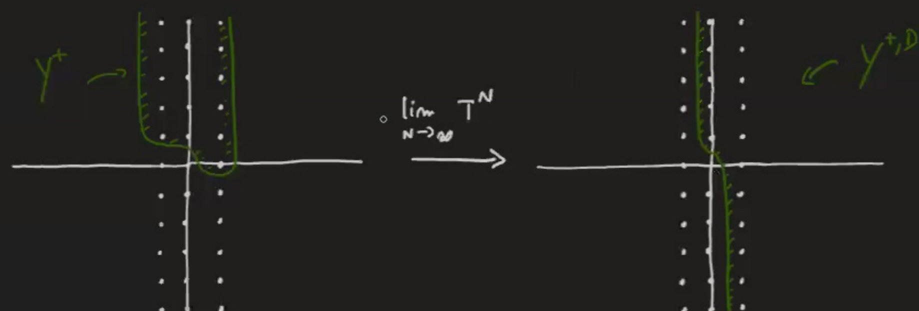
$\rightsquigarrow$ Heuristics: $Y(\hat{sl}_2)$ acts on cohomology of moduli spaces of sheaves on any $S \supset \mathbb{P}^1$ s.t. $\mathbb{P}^1 \cdot \mathbb{P}^1 = -2$.

Idea of proof: Use $\begin{cases} \text{derived equivalence} \quad D^b(Coh(\mathbb{P}^1)) \simeq D^b(Rep\, Q), \text{ with } Q = \bullet \rightrightarrows \bullet \\ \text{Thm (SV, Yang-Zhao)}: \ Y^+_Q := COHA_{Rep\, \Pi_Q} \simeq Y^+_{\varepsilon_1,\varepsilon_2}(\hat{sl}_2) \end{cases}$

Idea of proof: Use $\begin{cases} \text{derived equivalence} \quad D(Coh(\mathbb{P})) \simeq D(Rep\, Q), \text{ with } Q = \underset{1}{\bullet} \rightrightarrows \underset{0}{\bullet} \\ \text{Thm (SV, Yang-Zhao)}: \; Y^+_Q := COHA_{Rep\, \Pi_Q} \simeq Y^+_{\varepsilon_1, \varepsilon_2}(\hat{sl}_2) \end{cases}$

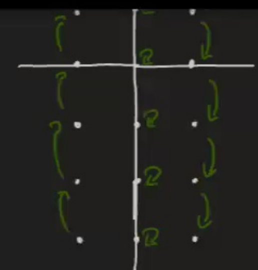
* Algebraically, passing from Y^+ to $Y^{+,D}$ amounts a (limit of) twist by braid group element

(still projection on blackboard plane)



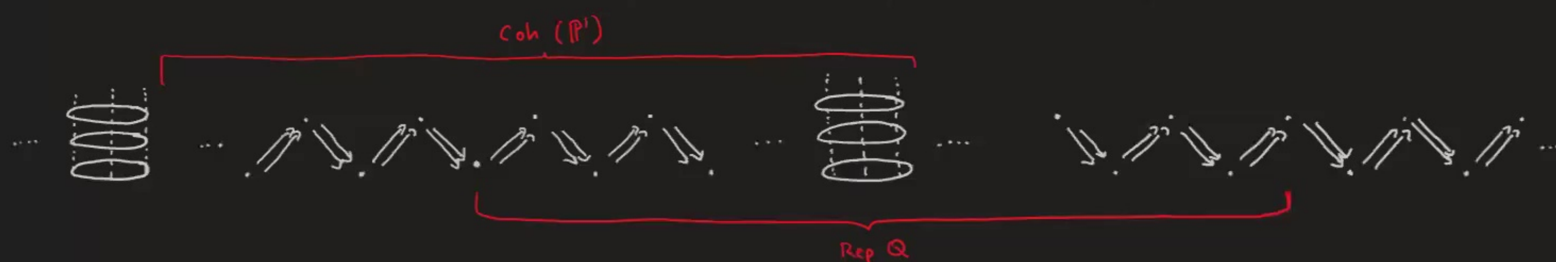
Here, T is a generator of the lattice part of the braid group B of $\hat{\mathfrak{S}}_2$.
(Thm (Kodera) B acts on $Y(\hat{sl}_2)$).

action of T



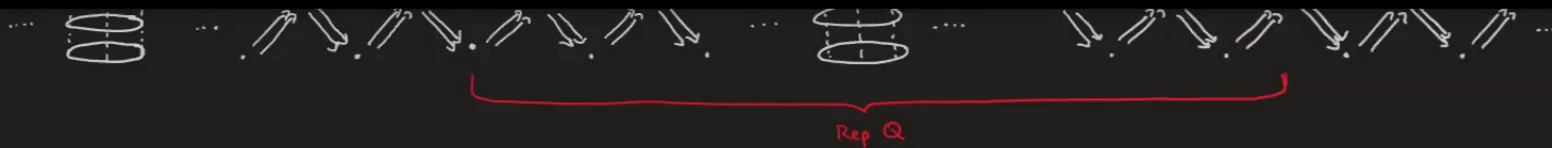
* Geometrically, B acts on $D^b(\mathrm{Rep}\,Q) \simeq D^b(\mathrm{Coh}(\mathbb{P}^1))$ (BGP reflexion functors)

In terms of the Auslander-Reiten quiver (indecomposables in $D^b(\mathrm{Rep}\,Q) = D^b(\mathrm{Coh}(\mathbb{P}^1))$) :



Action of T on objects:

In terms of $\mathrm{Coh}(\mathbb{P}^1)$: $T(\mathcal{F}) = \mathcal{F} \otimes \mathcal{O}(-1)$.



Action of T on objects:

In terms of $\mathrm{Coh}(\mathbb{P}^1)$: $T(\mathcal{F}) = \mathcal{F} \otimes \mathcal{O}(-1)$.

↪ We have to show that the "action" of BGP functors on the CoHAs match the algebraic one.

More precisely:

Conj Let $Q = (I, \Omega)$ be any quiver, $\begin{cases} Y^+ \subset Y & \text{the associated Yangian} \\ \mathcal{Y}^+ := \bigoplus_d H_*^{(T)}(\mathrm{Rep}_d \Pi_Q) & \text{CoHA.} \end{cases}$

Fix $i \in I$; put $\alpha_i =$ simple root , $S_i =$ simple Q-mod assoc. to i.

Then the following diagram is commutative:

$$Y^+ \cap T_i^{-1}(Y^+) \xrightarrow{T_i} T_i(Y^+) \cap Y^+$$

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Fix $i \in I$; put $\alpha_i =$ simple root, $S_i =$ simple Q-mod assoc. to i.

Then the following diagram is commutative:

$$\begin{tikzcd}
Y^+ \cap T_i^{-1}(Y^+) \arrow[r, "T_i", "\sim"'] \arrow[d, phantom, "\wr"] & T_i(Y^+) \cap Y^+ \arrow[d, phantom, "\wr"] \\
Y^+ / Y^+ Y^+_{\alpha_i} \arrow[d, "\wr"', "\text{Conj}" near start] \arrow[r, phantom, "\circlearrowleft"] & Y^+ / Y^+_{\alpha_i} Y^+ \arrow[d, "\wr"'] \\
Y^+ / Y^+ Y^+_{\alpha_i} \arrow[d, phantom, "\wr"] & Y^+ / Y^+_{\alpha_i} Y^+ \arrow[d, phantom, "\wr"] \\
H_*^{(T)}((\mathrm{Rep}\,\Pi_Q)_i) \arrow[r, "\sim", "\Phi_i"'] & H_*^{(T)}({}_i(\mathrm{Rep}\,\Pi_Q))
\end{tikzcd}$$

where $(\mathrm{Rep}\,\Pi_Q)_i = \{ (M, \varphi) \in \mathrm{Rep}\,\Pi_Q \mid \mathrm{Hom}(S_i, M) = \{0\} \}$

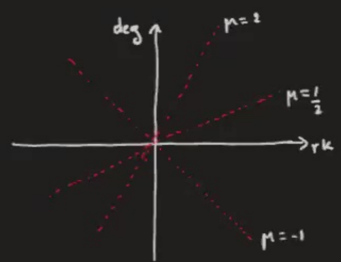
${}_i(\mathrm{Rep}\,\Pi_Q) = \{ (M, \varphi) \in \mathrm{Rep}\,\Pi_Q \mid \mathrm{Hom}(M, S_i) = \{0\} \}$

Rem: This is an analog of a classical result of Lusztig in [illegible] (uses canonical bases)

Rem:
- This is an analog of a classical result of Lusztig in $D^b_{const}(\mathrm{Rep}\,Q)$ (uses canonical bases)
- For semi-canonical bases & $\mathrm{Fun}(\Lambda_Q)$, this is a theorem of Baumann.
- OK for Q of finite type. Weak form ok for Q of affine type.

Next:
- Affine quivers (ALE spaces, $\widetilde{\mathbb{C}^2/\Gamma}$)
- Elliptic curves

$\hookrightarrow$



$\forall \mu \in \mathbb{Q} \cup \{\infty\}$ contains $Y(\widehat{\mathfrak{gl}(1)})$ (or EHA for KHA)

$\rightsquigarrow$ all together, expect $U_{(n)}(\hat{\hat{\mathfrak{gl}}}(1))$ "pagoda algebra"

Mironov - Morozov - Zenkevich

Note: For KHA, $\begin{cases} \forall \mu, \ SL(2,\mathbb{Z}) \text{ acts on each EHA (fixed } \mu) \\ SL(2,\mathbb{Z}) \text{ acts, permuting EHAs for various } \mu\text{s} \end{cases}$

$\rightsquigarrow \quad SL(3,\mathbb{Z}) \curvearrowright U_{(n)}(\hat{\hat{\mathfrak{gl}}}(1))$

* Algebraically, passing from Y^+ to $Y^{+,D}$ amounts a (limit of) twist by braid group element

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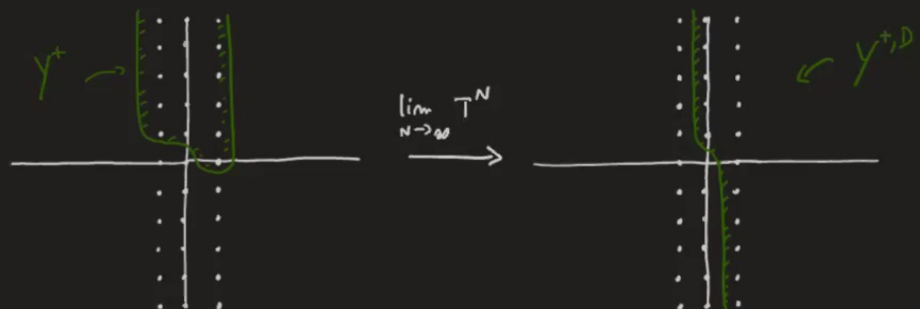
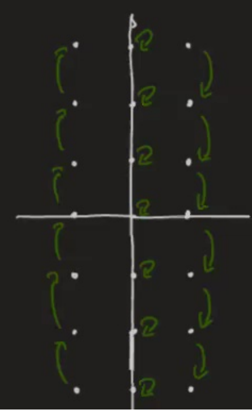


$$H^{sph}_{Q} = \langle 1_{[S_i]} \mid i \in I \rangle$$

simple rep of Q

$=$

Here, T is a generator of the lattice part of the braid group B of $\widehat{\mathfrak{S}}_2$.
(Thm (Kodera) B acts on $Y(\widehat{sl}_2)$).



action of T

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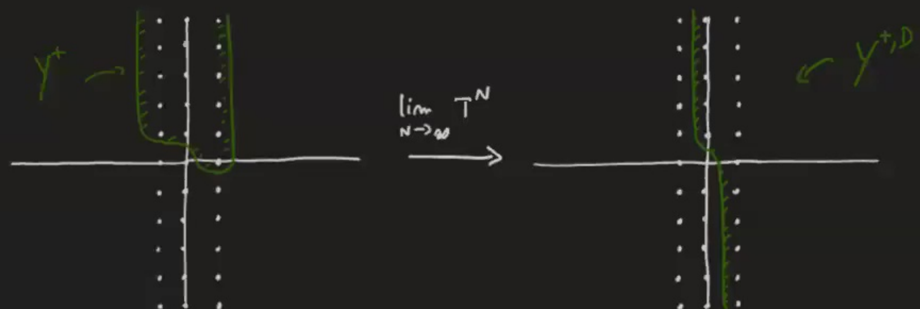

Here, T is a generator of the lattice part of the braid group B of $\hat{G}_2$.
(Thm (Kodera) B acts on $Y(\hat{sl}_2)$).

$$H^{sph}_{Q} = \langle 1_{[S_i]} \mid i \in I \rangle$$

simple rep of Q

$$= \langle 1_{M_d} \mid d \in \mathbb{N}^I \rangle$$

(if Q acyclic)

$$H^{sph}_{X} = \langle 1_{M_{r,d}} \mid r \geq 0, \; d \in \mathbb{Z} \rangle$$