Title: Emergent criticality in non-unitary random dynamics

Speakers: Xiao Chen

Series: Quantum Matter

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Abstract: In this talk, I will discuss emergent criticality in non-unitary random quantum dynamics. More specifically, I will focus on a class of free fermion random circuit models in one spatial dimension. I will show that after sufficient time evolution, the steady states have logarithmic violations of the entanglement area law and power law

correlation functions. Moreover, starting with a short-range entangled many-body state, the dynamical evolution of entanglement and correlations quantitatively agrees with the predictions of two-dimensional conformal field theory with a space-like time direction. I will argue that this behavior is generic in non-unitary free quantum dynamics with time-dependent randomness, and show that the emergent conformal dynamics of two-point functions arises out of a simple" nonlinear master equation".

Emergent criticality in non-unitary random dynamics

Xiao Chen
Boston College

(Virtual) Perimeter Institute, 06/16/2020

Collaborators:

Yaodong Li

Matthew Fisher

Andy Lucas

Andreas Ludwig

Li-Chen-Fisher, Phys. Rev. B 98, 205136 (2018):

Quantum Zeno Effect and the Many-body Entanglement Transition

Li-Chen-Fisher, Phys. Rev. B 100, 134306 (2019):

Measurement-driven entanglement transition in hybrid quantum circuits

Li-Chen-Ludwig-Fisher, arXiv: 2003.12721

Conformal invariance and quantum non-locality in non-unitary dynamics

Chen-Li-Fisher-Lucas, arXiv: 2004.09577, Emergent conformal symmetry in non-unitary random dynamics of free fermions

Outline

- Introduction on quantum dynamics
- Measurement induced entanglement phase transition
- Emergent criticality in non-unitary random free fermion dynamics
- Conclusion and outlook

Quantum thermalization in pure state

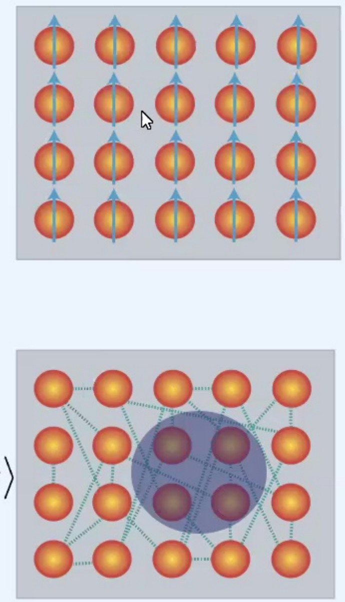
In a generic chaotic quantum system, the wave function can thermalize under its own dynamics

Global unitary evolution

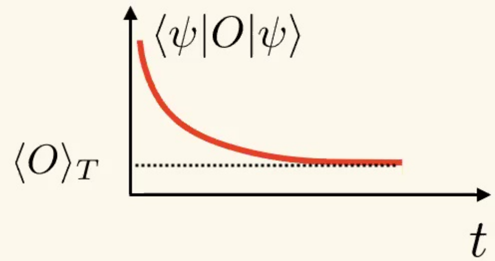
$|\psi_0\rangle$

e^{-iHt}

$|\psi_t\rangle = e^{-iHt}|\psi\rangle$



Local thermalization



$\langle\psi_0|H|\psi_0\rangle \sim T$

Reduced density matrix

$$\rho_A(t) = \mathrm{Tr}_B\left[|\psi_t\rangle\langle\psi_t|\right] \longrightarrow \rho_T \sim e^{-H/T}$$

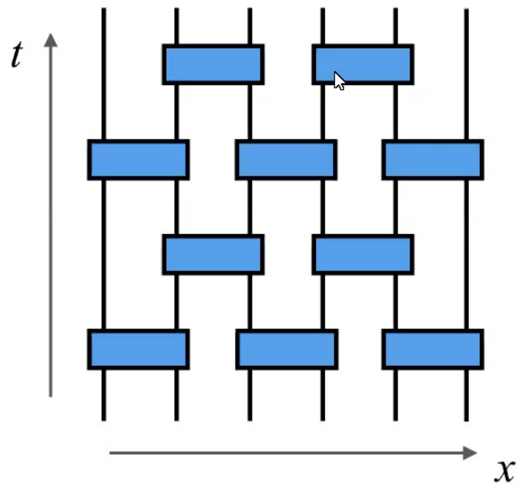
Srednicki, 1994
Deutsch, 1991

The steady state has volume law entanglement entropy

Random unitary circuit

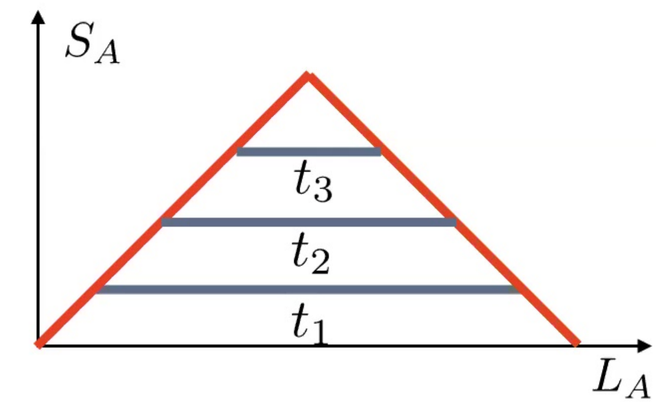
Minimal model to study quantum thermalization, entanglement dynamics, information scrambling….

Random Haar circuit



$$|\psi\rangle \rightarrow U(x,t)|\psi\rangle$$

Nahum, Ruhman, Vijay and Haah, 2016
Nahum, Vijay and Haah, 2017
Keyserlingk, Rakovszky, Pollmann, Sondhi, 2017



Unitary evolution will **increase** the entanglement

Question:
What will happen in non-unitary dynamics?

Projective measurement tends to reduce entanglement

Apply the projection operator P_α on the wave function $\sum_\alpha P_\alpha = 1$

The outcome $|\psi\rangle \to \frac{P_\alpha|\psi\rangle}{||P_\alpha|\psi\rangle||}$ with probability $p_\alpha = \langle\psi|P_\alpha|\psi\rangle$

One example:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$P_+ = |\uparrow\rangle\langle\uparrow|$

$\langle P_+\rangle = \frac{1}{2}$

$$|\psi\rangle_+ = |\uparrow\downarrow\rangle$$

$P_- = |\downarrow\rangle\langle\downarrow|$

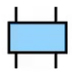
$\langle P_-\rangle = \frac{1}{2}$

$$|\psi\rangle_- = |\downarrow\uparrow\rangle$$

$S = \log 2$

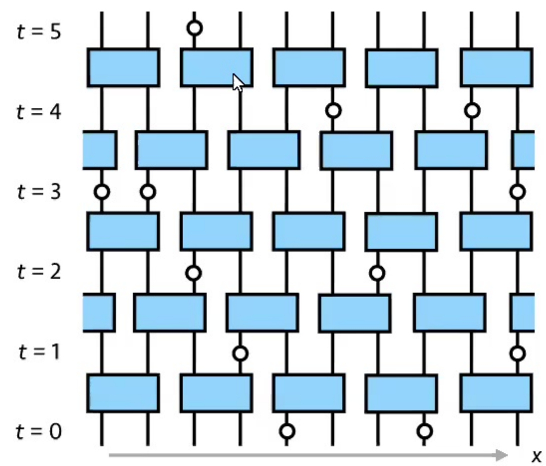
$S = 0$

Hybrid quantum circuit


Two-qubit unitary gate

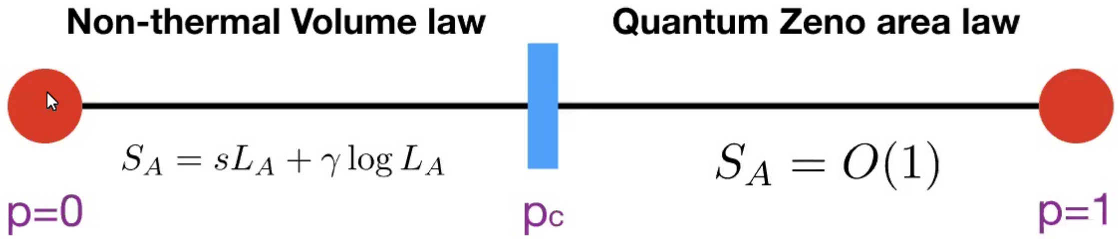
Local projective measurement

- Build up a quantum circuit with both local unitary gate and projective measurement
- Quantum trajectory of the pure wave function
- Tune the measurement rate p
- Two limits: p=0 and p=1

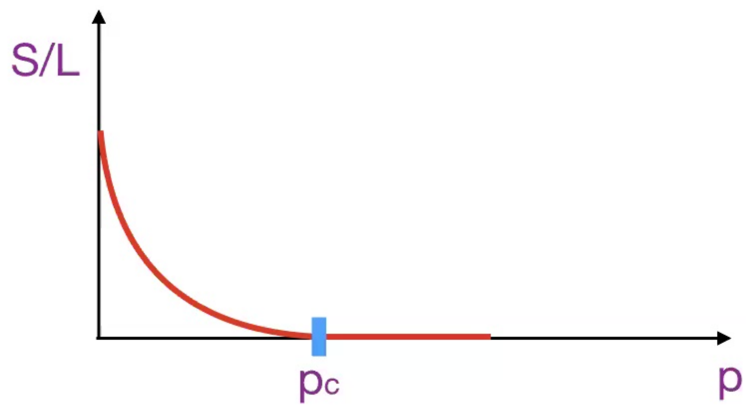


Skinner-Ruhman-Nahum, Phys. Rev. X 9, 031009 (2019)
Chan-Nandkishore-Pretko-Smith, Phys. Rev. B 99, 224307 (2019)
Li-Chen-Fisher, Phys. Rev. B 98, 205136 (2018)

Entanglement transition



$$S_A \sim \log L_A$$

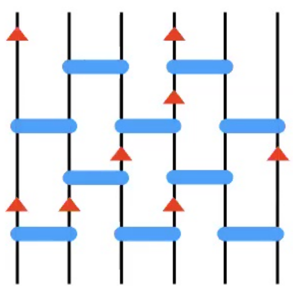


Skinner-Ruhman-Nahum, Phys. Rev. X 9, 031009 (2019)
Chan-Nandkishore-Pretko-Smith, Phys. Rev. B.99. 224307 (2019)
Li-Chen-Fisher, Phys. Rev. B 98, 205136 (2018)

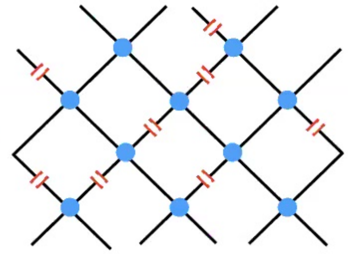
Emergent conformal symmetry at the critical point

Minimal put picture: Skinner-Ruhman-Nahum, Phys. Rev. X 9, 031009 (2019)

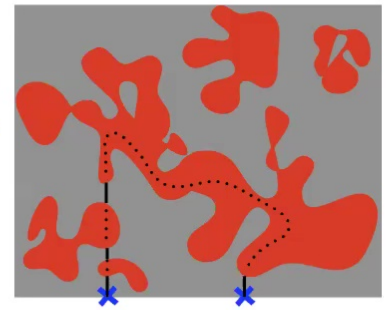
Haar random circuit



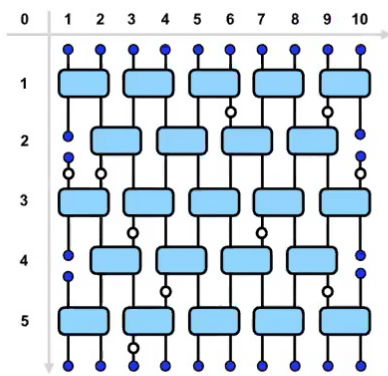
Bond percolation problem

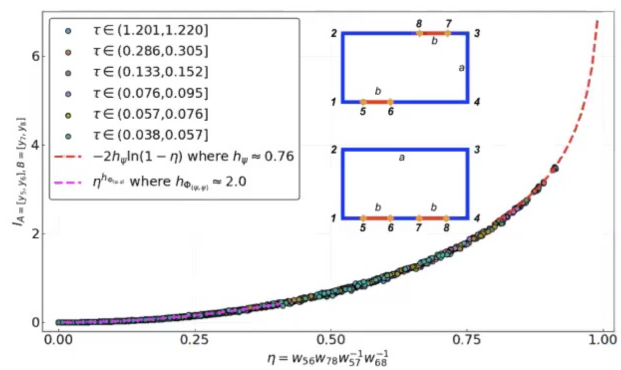


S_0: The minimal path separating the lattice into two pieces



Data collapse on Clifford circuit: Li-Chen-Ludwig-Fisher, arXiv: 2003.12721





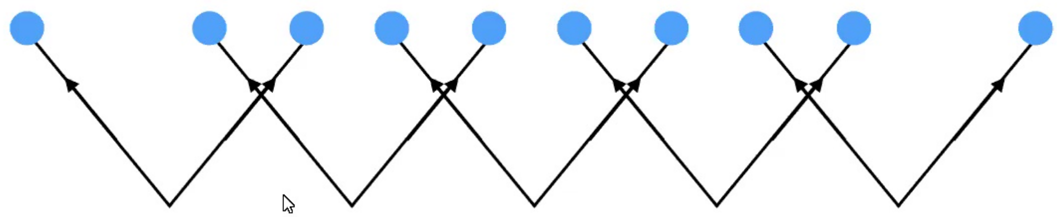
Two dimensional conformal symmetry

Free fermion system

Unitary dynamics: **Spreading of quasiparticle pairs** Calabrese-Cardy: J.Phys.A 42:504005,2009

(1) Early time linear growth

(2) Late time volume law entanglement

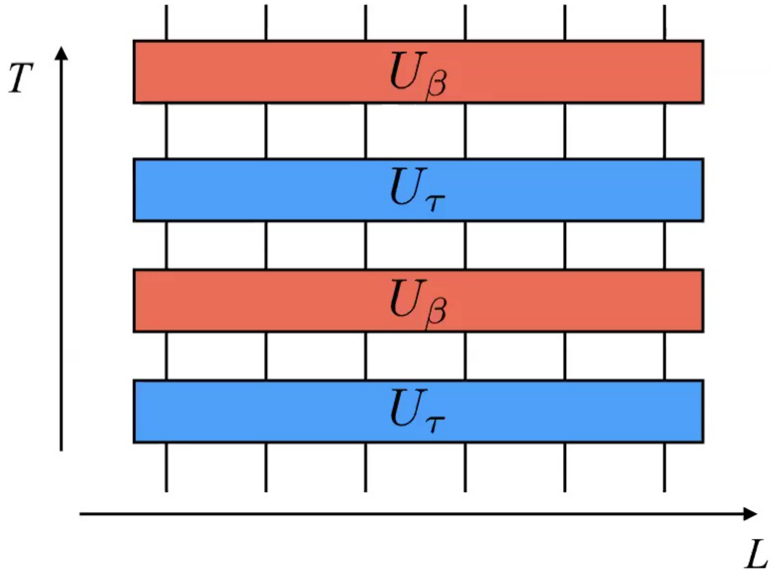


The nonlocal quasiparticle pair is broken under the local projective measurement

The volume law phase is unstable

Chan-Nandkishore-Pretko-Smith, Phys. Rev. B 99, 224307 (2019)

Non-unitary free fermion dynamics



$$U_\tau(t) = \exp(-2i\tau H_1(t))$$

$$U_\beta(t) = \exp(-2\beta H_2(t))$$

$$H_1(t) = \sum_x \kappa_{x,t} c_x^\dagger c_{x+1} + \text{H.c.}$$

$$H_2(t) = \sum_x \lambda_{x,t} c_x^\dagger c_x.$$

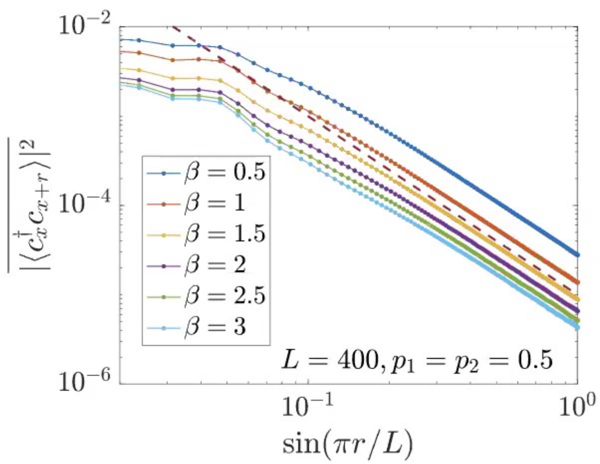
$$P_\kappa(\kappa_{x,t}) = p_1 \delta(\kappa_{x,t} - 1) + (1 - p_1)\delta(\kappa_{x,t} + 1)$$
$$P_\lambda(\lambda_{x,t}) = p_2 \delta(\lambda_{x,t} - 1) + (1 - p_2)\delta(\lambda_{x,t})$$

Chen-Li-Fisher-Lucas, arXiv: 2004.09577

Steady state

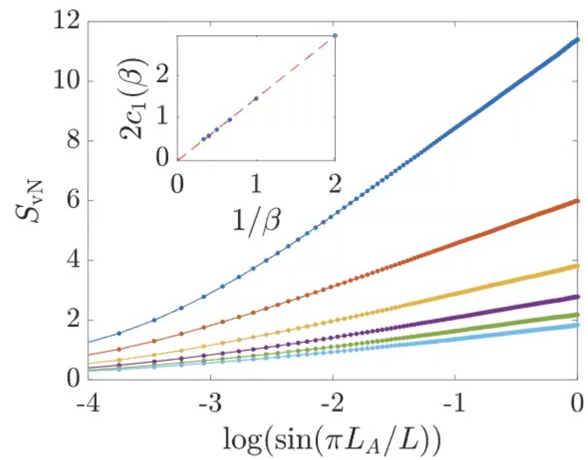
Squared correlation function

$$\overline{|\langle c_x^\dagger c_{x+r}\rangle|^2} \sim \frac{1}{r^2}$$



Entanglement entropy

$$S_n = c_1\left(1+\frac{1}{n}\right)\log L_A$$

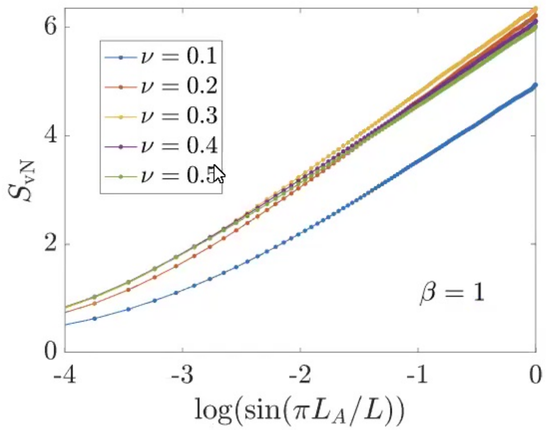


$$c_1(\beta) \propto \frac{1}{\beta}$$

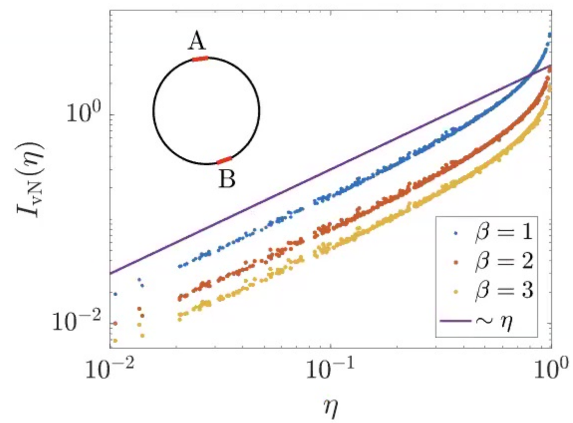
This model remains critical for finite β

Steady state

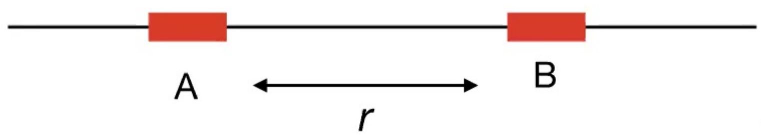
Insensitive to the parameter:
Different filling factor



Mutual information



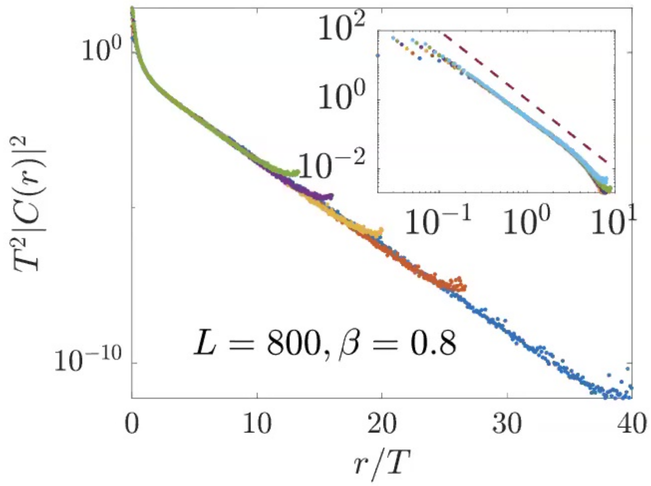
$$I(\eta) \sim \eta, \quad \text{when } \eta \to 0$$



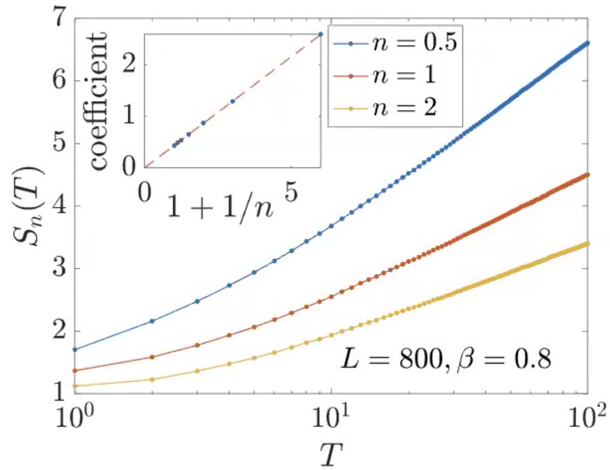
$$I(r) \sim 1/r^2$$

Consistent with the correlation function result

Dynamics



$$\overline{|\langle c_x^\dagger c_{x+r}\rangle|^2} \sim \frac{\mathrm{e}^{-ar/T}}{T^2}$$



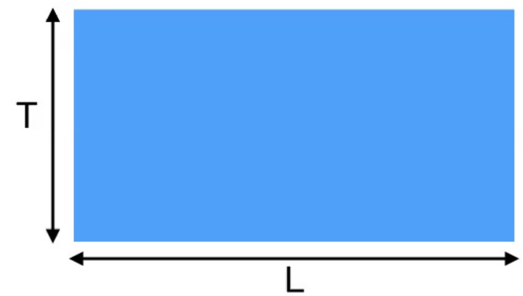
$$S_n = \frac{c_1}{2}\left(1+\frac{1}{n}\right)\log T$$

The coefficient is the same as the steady state

The emergent conformal symmetry

$$|\psi(T)\rangle \approx \frac{\mathrm{e}^{-TH_{\mathrm{CFT}}}|\psi_0\rangle}{\|\mathrm{e}^{-TH_{\mathrm{CFT}}}|\psi_0\rangle\|}$$

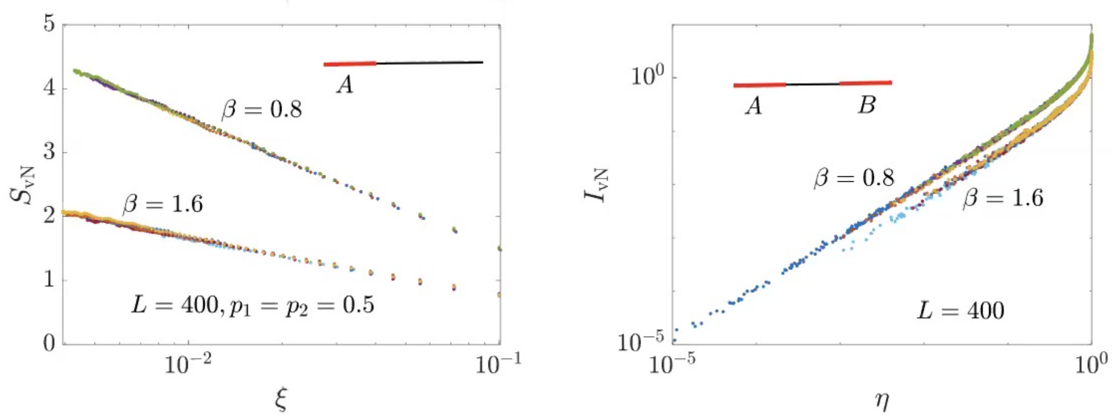
CFT defined on rectangle



Early time: T <<L, T is the correlation length: $\overline{|\langle c_x^\dagger c_{x+r}\rangle|^2} \sim \frac{\mathrm{e}^{-ar/T}}{T^2}$

Finite when the ratio r/T is finite!

Data collapse for entanglement and mutual information



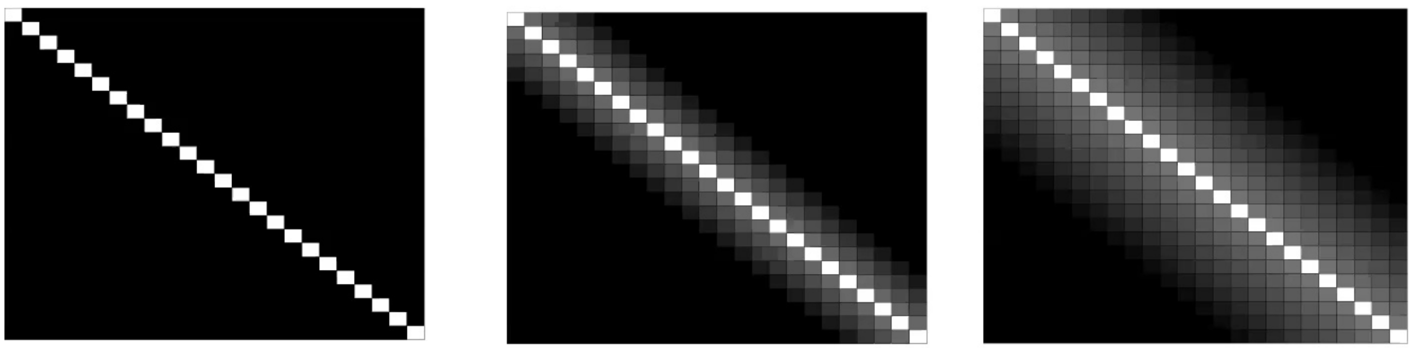
Spreading of correlation function

This dynamics is (1) Markovian and (2) has a conservation law $\mathrm{Tr}C^2 = N$

In this free fermion system with U(1) symmetry, C is a projector

$$C_{xy}(T) \equiv \langle \psi(T)| c_x^\dagger c_y |\psi(T)\rangle \qquad \mathrm{Tr}C = \mathrm{Tr}C^2 = N$$

The dynamics of C matrix

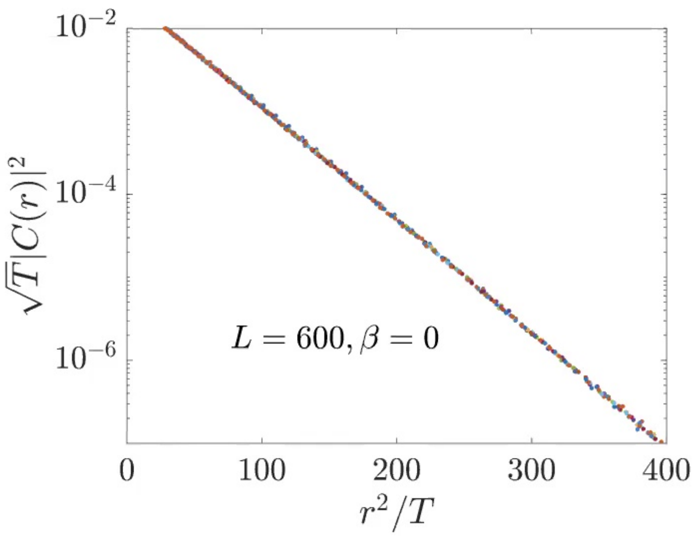


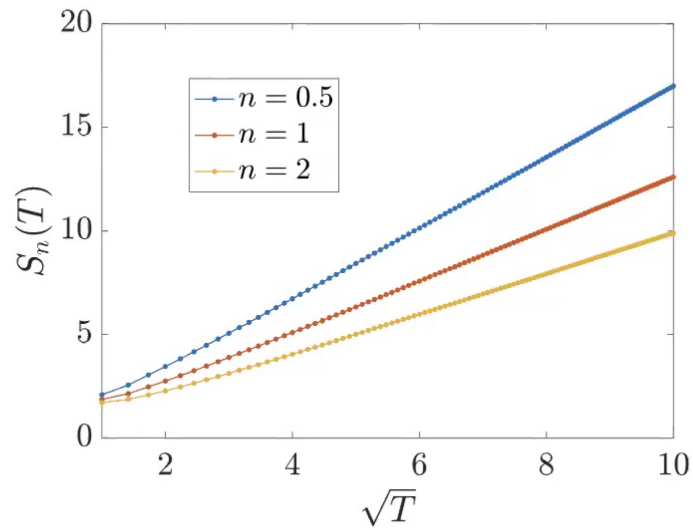
Dynamics of C matrix

Pure unitary dynamics: Diffusive spreading

$$|C(r,T)|^2 \sim \frac{\exp(-\frac{r^2}{T})}{\sqrt{T}}$$

$$S(T) \sim \sqrt{T}$$





Brownian circuit and master equation

$$\mathrm{d}H(t) = \sum_j \Big(c_{j+1}^\dagger c_j \mathrm{d}W_j(t) + c_j^\dagger c_{j+1} \mathrm{d}\overline{W}_j(t) - \mathrm{i} c_j^\dagger c_j \mathrm{d}W_j'(t) \Big) ,$$

with $\mathrm{d}W_j$, $\mathrm{d}\overline{W}_j$ and $\mathrm{d}W_j'$ representing three different Brownian motions.

$$f_n \equiv \begin{cases} \sum_a \frac{|C_{a,a}|^2}{N}, & \text{when } n=0 \\ \sum_a \frac{|C_{a,a+n}|^2 + |C_{a,a-n}|^2}{N}, & \text{when } n>0 \end{cases}$$

Pure unitary case

$$\partial_t f_n = f_{n+1} + f_{n-1} - 2 f_n$$

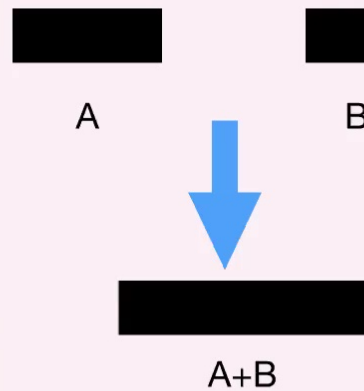
Non-unitary dynamics

Nonlinear master equation:

$$\partial_t f_1 = \mu + \theta(f_2 - 2f_1) - 2f_1 \sum_{m=1}^{\infty} f_m + \sum_{m=1}^{\infty} f_m f_{m+1},$$

$$\partial_t f_n = \theta(f_{n+1} + f_{n-1} - 2f_n) - 2f_n \sum_{m=1}^{\infty} f_m + \sum_{m=1}^{\infty} f_m f_{m+n} + \frac{1}{2}\sum_{m=1}^{n-1} f_m f_{n-m}, \quad (n > 1)$$

Nonlocal domain wall aggregation dynamics



P. Krapivsky, Physica A198, 157 (1993).

$\theta = 0$ Steady state

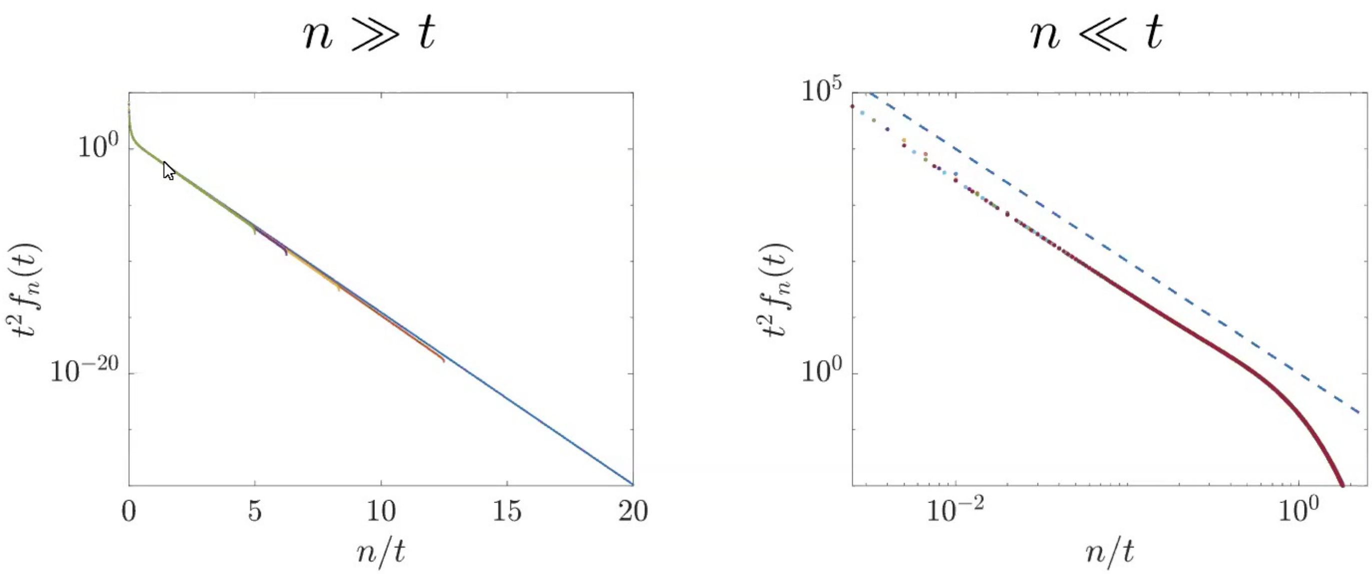
$$-f(x) + A\int_{-\infty}^{\infty} dy f(y) f(x-y) = 0$$

$$f(x) \sim \frac{1}{x^2}$$

Nonlinear master equation

$$f_n(t) \sim \begin{cases} n^{-2} & n \ll t \\ t^{-2}\exp(-n/t) & n \gg t \end{cases}$$

Consistent with numerical results for the discrete non-unitary random dynamics



Numerical simulation of the master equation

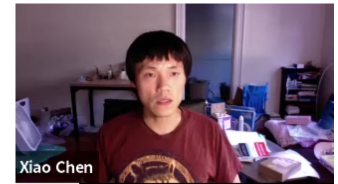
Conclusion

- Emergent criticality in non-unitary random free fermion dynamics

- This critical point is very robust and insensitive to the parameters of the model (self-organized criticality)

- A master equation approach to interpret the spreading of correlation function

Two classes of non-unitary CFT



Free fermion	Measurement driven transition

Full conformal symmetry in two dimensions

(1) Causality is broken

(2) Steady state log L_A scaling and early time log t growth

(3) Mutual information as a function of cross ratio

Difference: critical exponent

Free system $\frac{1}{r^2}$

Interacting system $\frac{1}{r^4}$

Haar random circuit & Clifford random circuit

Skinner-Ruhman-Nahum, Phys. Rev. X 9, 031009 (2019)

Li-Chen-Fisher, Phys. Rev. B 100, 134306 (2019):

Open questions

- Importance of randomness -> non-unitary CFT

  Gurarie, Nucl. Phys. B 410, 535 (1993)

  Cardy, J. Phys. A, 46, 494001 (2003)   Gurarie-Ludwig, J. Phys. A, 35, L377 (2002)

- Other non-unitary dynamics

- Experimental relevance