Title: Summer Undergrad 2020 - Numerical Methods (A) - Lecture 5

Speakers: Aaron Szasz

Collection: Summer Undergrad 2020 - Numerical Methods

Date: June 05, 2020 - 3:45 PM

URL: http://pirsa.org/20060013

Abstract: Exact diagonalization part 3: Expectation values and correlation functions in the Ising model GS, using these and energy gaps to identify a quantum phase transition; Demonstration of using matrix product states for the Ising model

Note Jun 5, 2020 (2)
Jun 5, 2020 at 1:55 PM

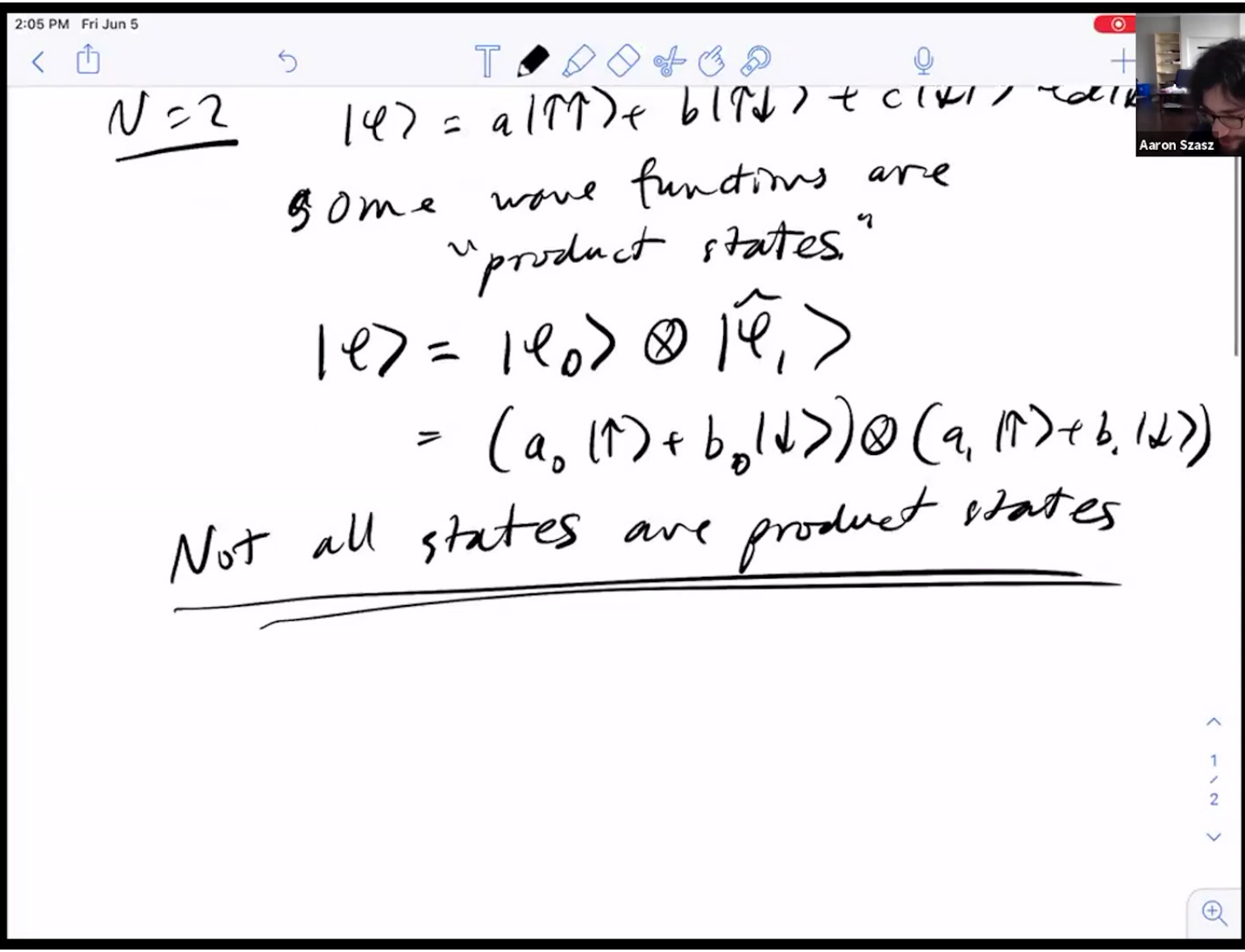
Matrix Product States

$N=2$ $\quad |\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle$

Some wave functions are "product states."

$$|\psi\rangle = |\psi_0\rangle \otimes |\psi_1\rangle$$

$$= (a_0|\uparrow\rangle + b_0|\downarrow\rangle) \otimes (\cdot$$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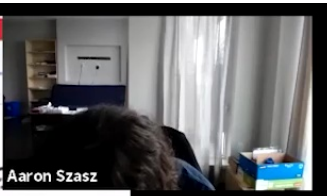
N = 2
$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c(\ldots)$
Some wave functions are "product states."
$|\psi\rangle = |\psi_0\rangle \otimes |\psi_1\rangle$
$= (a_0|\uparrow\rangle + b_0|\downarrow\rangle) \otimes (a_1|\uparrow\rangle + b_1|\downarrow\rangle)$
Not all states are product states
1
2



eg $|\psi\rangle = \frac{|\uparrow\rangle|\downarrow\rangle}{\sqrt{2}}$ not a product state

"entangled"

Product states are useful

N sites, $|\psi\rangle = |\psi_0\rangle \otimes |\psi_1\rangle \otimes \cdots \otimes |\psi_{N-1}\rangle$

$\langle \sigma_i^z \rangle = \langle Id \otimes \cdots \otimes \sigma_i^z \otimes \cdots \otimes Id \rangle$

$= \langle \psi_0 | Id | \psi_0 \rangle \cdot \langle \psi_1 | Id | \psi_1 \rangle \cdot \cdots$

$\cdot \langle \psi_i | \sigma^z | \psi_i \rangle \cdot \cdots \langle \psi_{N-1} | Id | \psi_N$

$$\cdot \langle \psi_i | \sigma^z | \psi_i \rangle \cdot \ldots \langle \psi_{N-1} | \mathrm{Id} | \psi_N \rangle$$

$$= 1 \cdot 1 \cdot \ldots \cdot \langle \psi_i | \sigma^z | \psi_i \rangle \cdot 1 \cdot \ldots 1$$

$$= \langle \psi_i | \sigma^z | \psi_i \rangle$$

only need to know $|\psi_i\rangle$

reduced from $2^N \to 2$ coeffs

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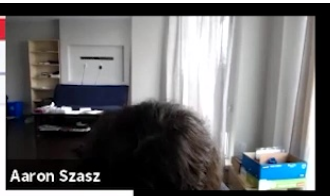
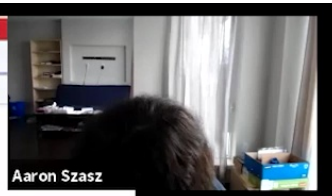


Matrix product states:

Try to write non-product states in a way that lets us do these same efficient calculations

eg $N=2$ $(a_0|\uparrow\rangle + b_0|\downarrow\rangle) +$



eg $N=2$

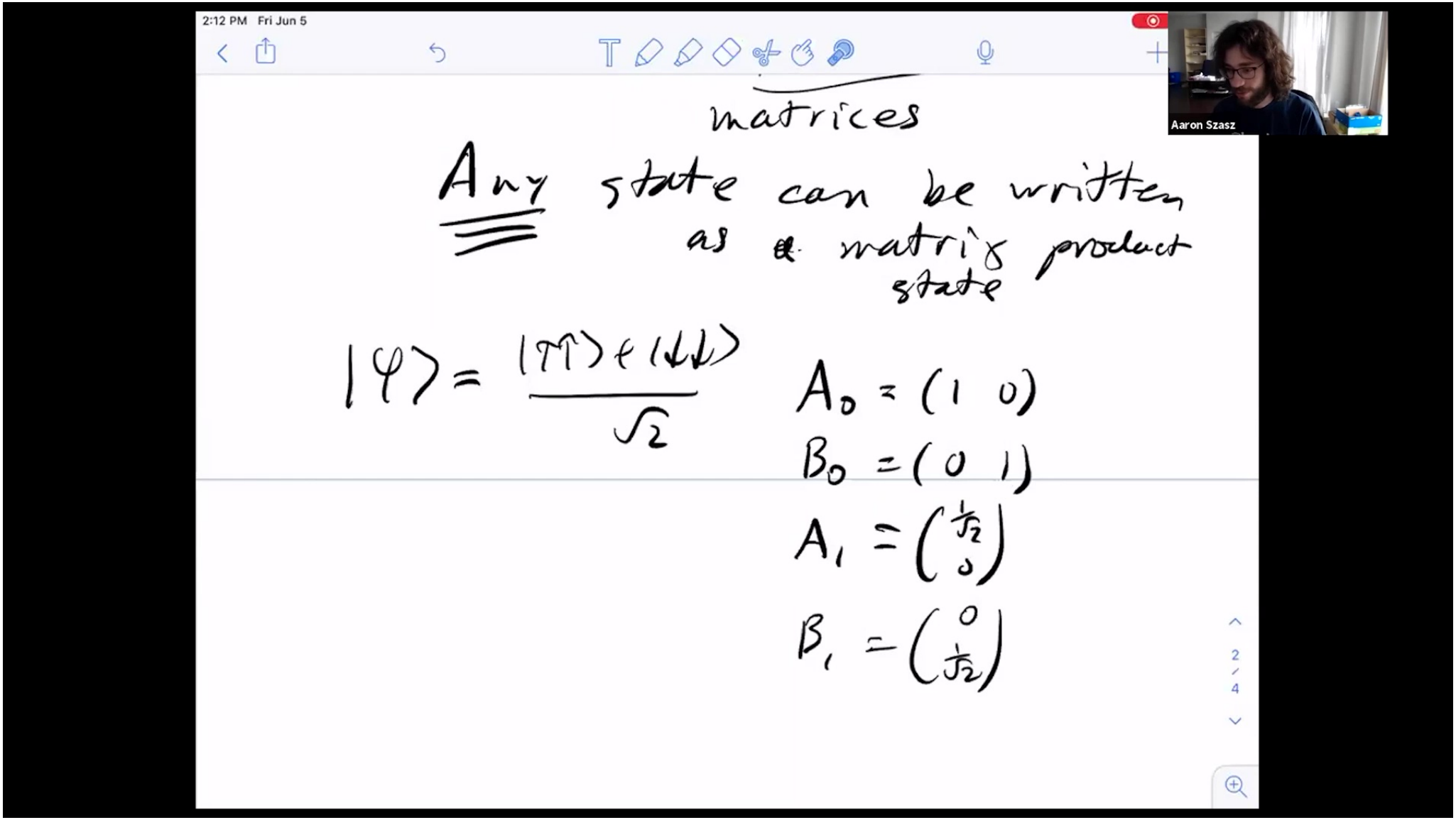
$$(a_0|\uparrow\rangle + b_0|\downarrow\rangle) \otimes (a_1|\uparrow\rangle + b_1|\downarrow\rangle)$$

$$\Downarrow$$

$$(A_0|\uparrow\rangle + B_0|\downarrow\rangle) \otimes (A_1|\uparrow\rangle + B_1|\downarrow\rangle)$$

matrices

Any state can be written as a matrix product state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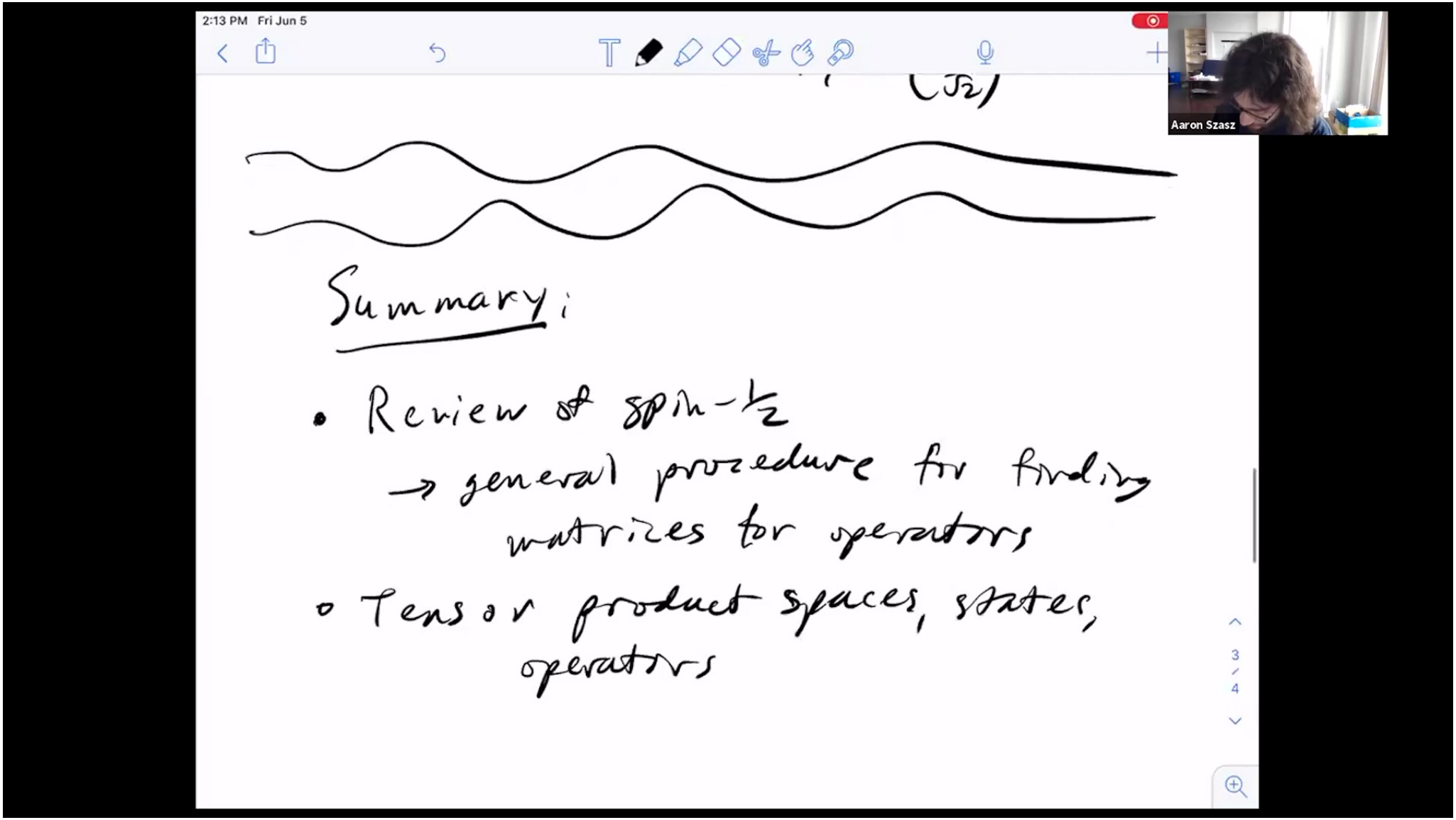
matrices
Any state can be written as a matrix product state
$|\psi\rangle = \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}$
$A_0 = (1 \;\; 0)$
$B_0 = (0 \;\; 1)$
$A_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$
$B_1 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
2 / 4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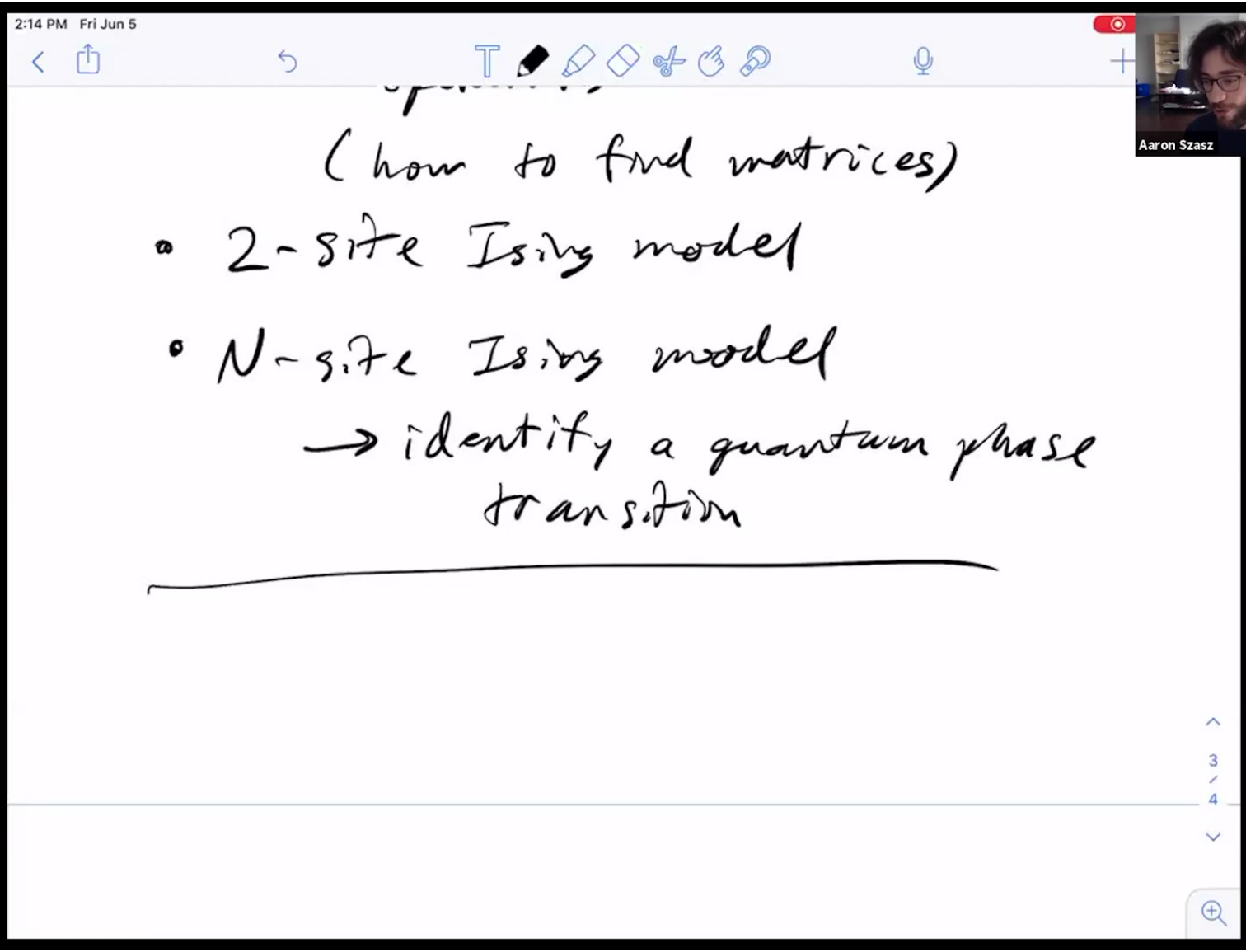
Summary:
• Review of spin-½
→ general procedure for finding matrices for operators
• Tensor product spaces, states, operators
3
4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(how to find matrices)
• 2-site Ising model
• N-site Ising model
→ identify a quantum phase transition
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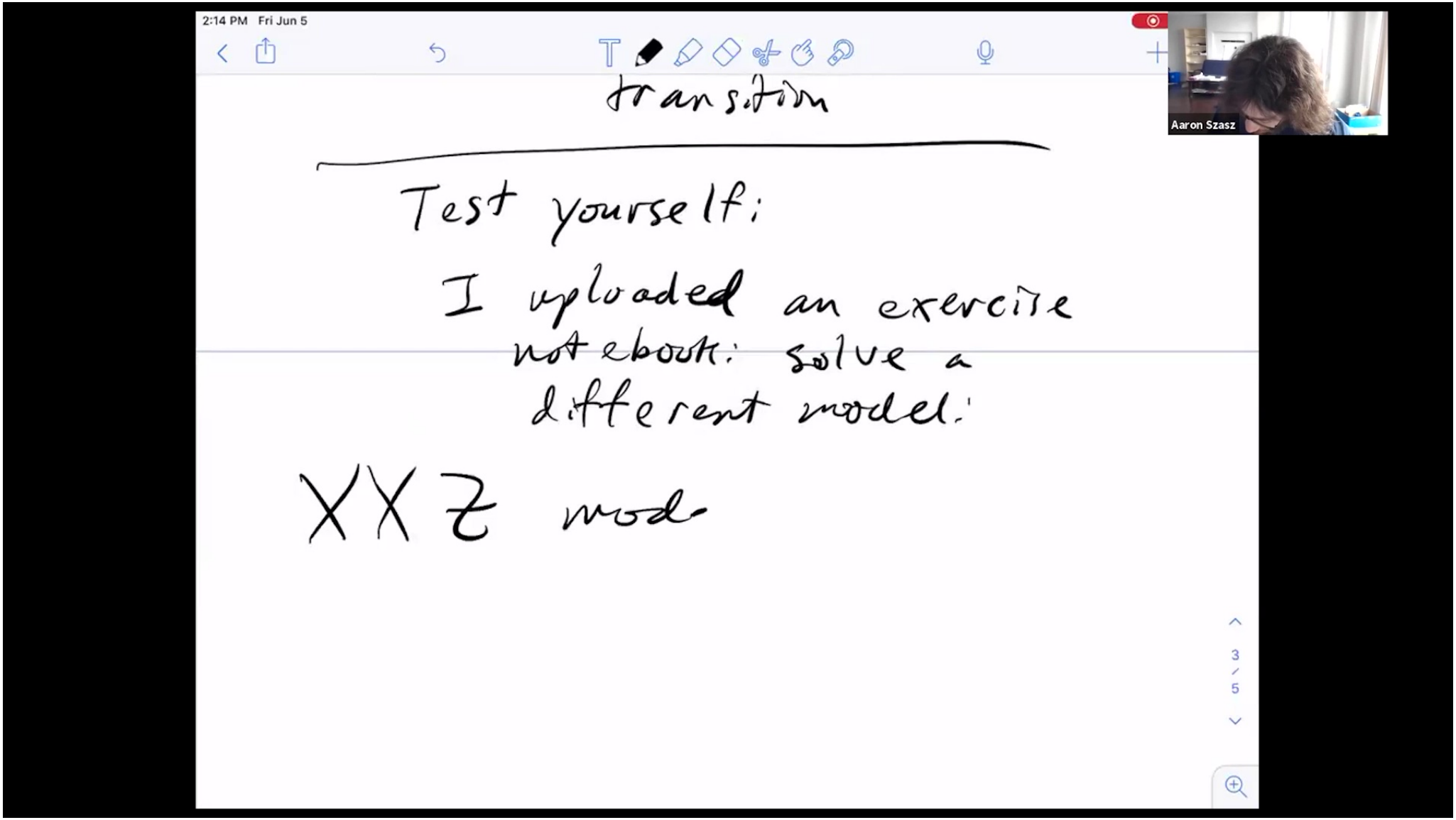
3
4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transition
Test yourself:
I uploaded an exercise notebook: solve a different model:
XYZ mod-
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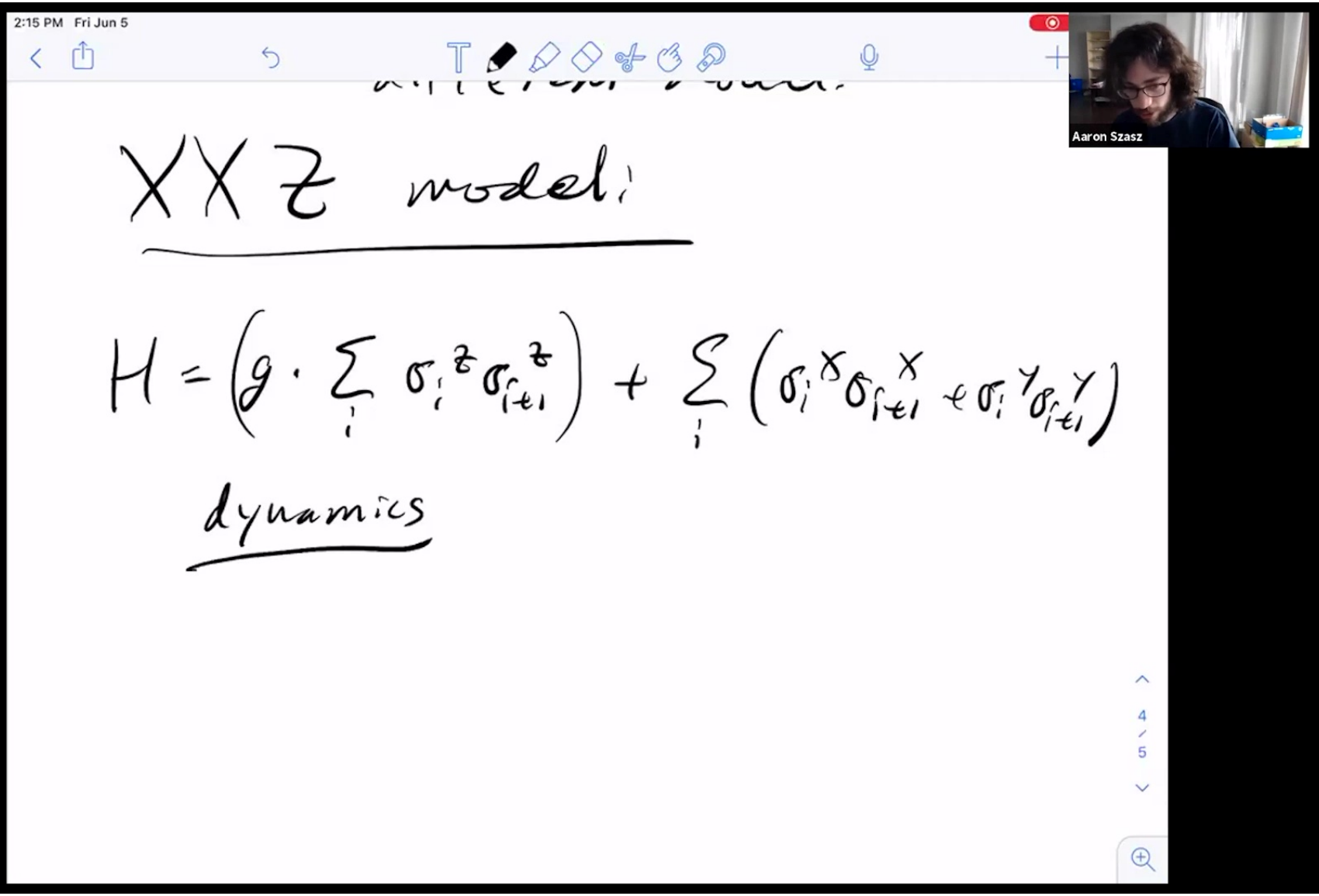
3
5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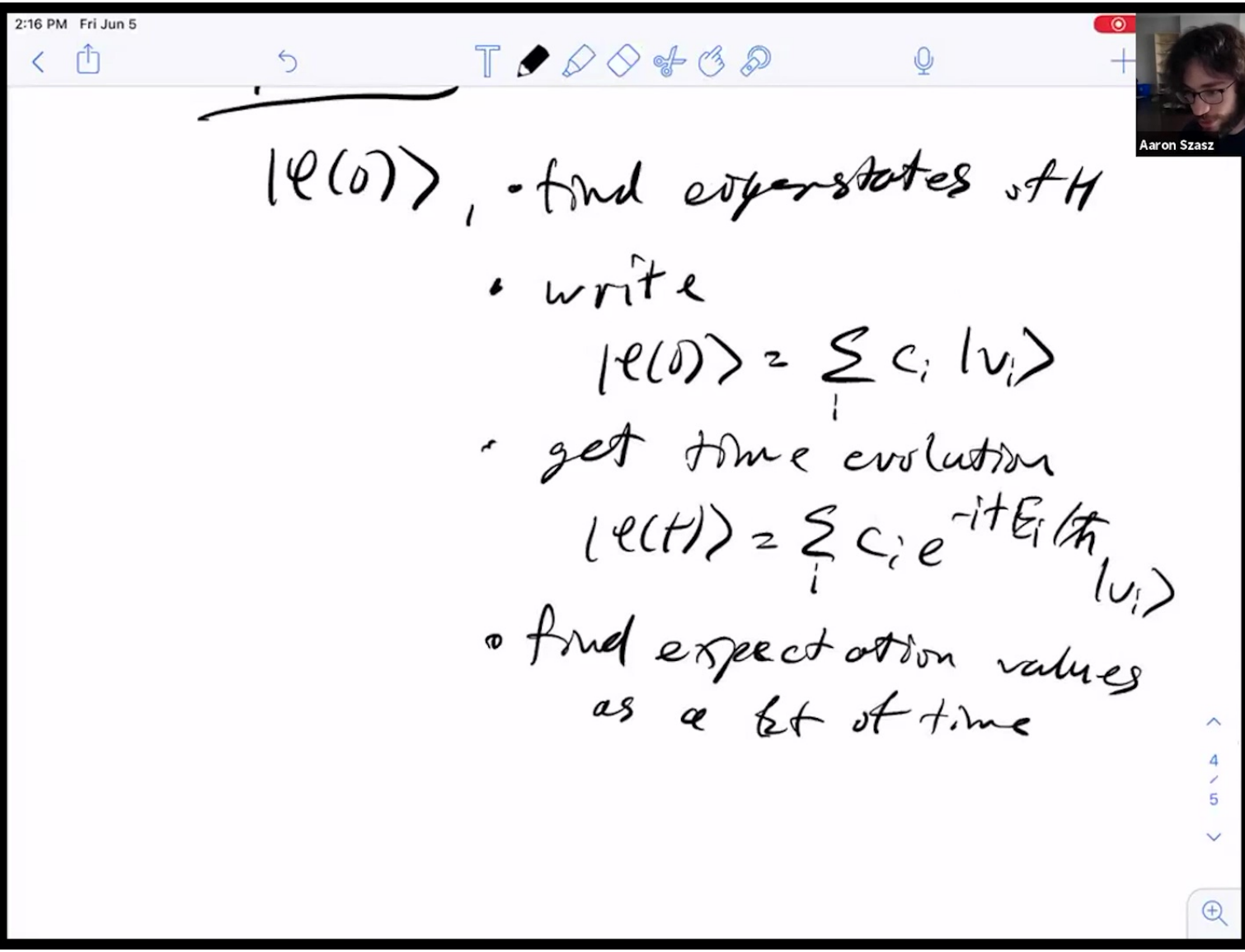
XXZ model:
$H = \left(g \cdot \sum_i \sigma_i^z \sigma_{i+1}^z\right) + \sum_i \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right)$
dynamics
4
5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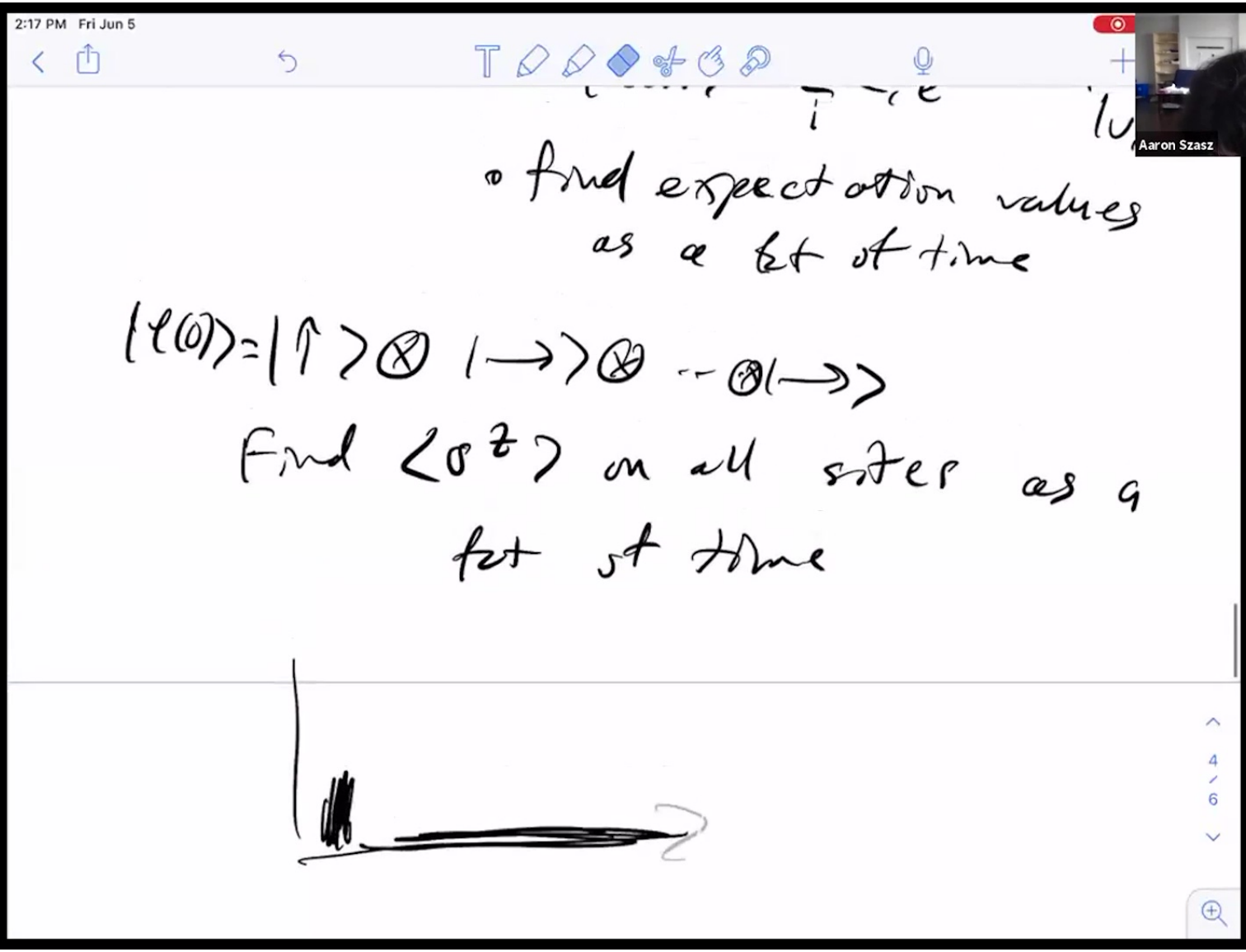
$|\psi(0)\rangle$, - find eigenstates of H
• write
$|\psi(0)\rangle = \sum_i c_i |v_i\rangle$
• get time evolution
$|\psi(t)\rangle = \sum_i c_i e^{-itE_i/\hbar} |v_i\rangle$
• find expectation values as a fct of time
4
5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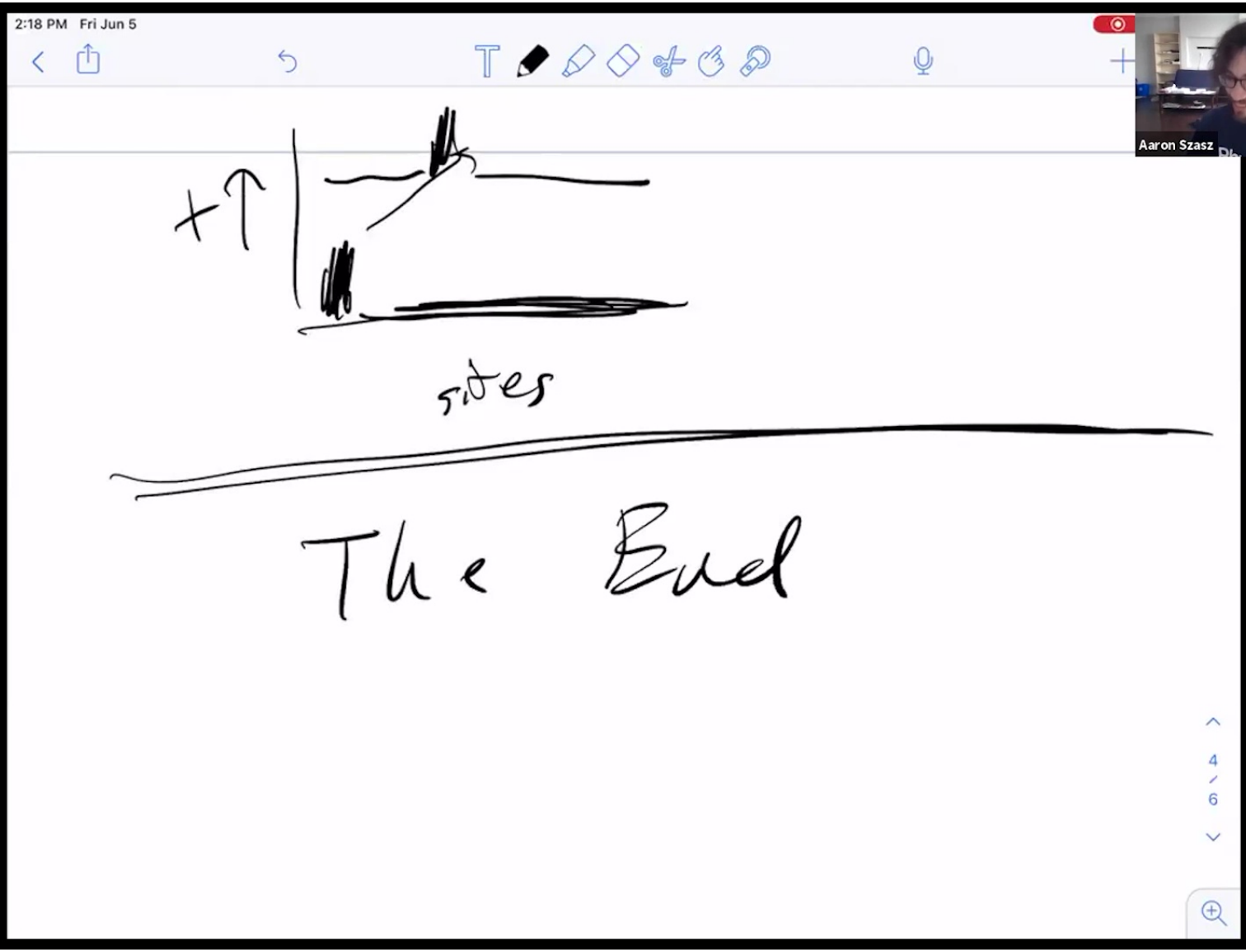
• find expectation values as a fct of time
$|\psi(0)\rangle = |\uparrow\rangle \otimes |\rightarrow\rangle \otimes \cdots \otimes |\rightarrow\rangle$
Find $\langle \sigma^z \rangle$ on all sites as a fct of time
4
6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