Title: Summer Undergrad 2020 - Path Integrals (M) - Lecture 5

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Collection: Summer Undergrad 2020 - Path Integrals

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Abstract: Topology and path integrals

Lecture 5 - Topology + Particle Statistics

$$A = \sum_{paths} e^{iS[q_1(t),\, q_2(t)]}$$

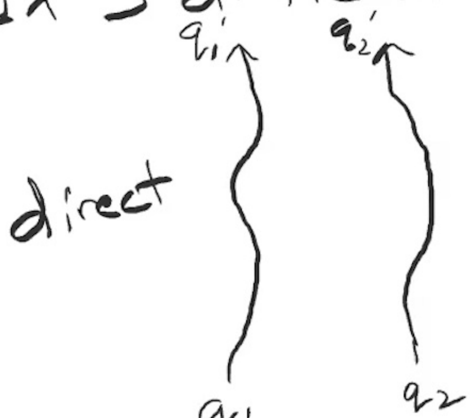
↑

amplitude for two identical particles to go from

q_1, q_2 at $t=0$

q_1', q_2' at $t=T$

In 3 dimensions of space

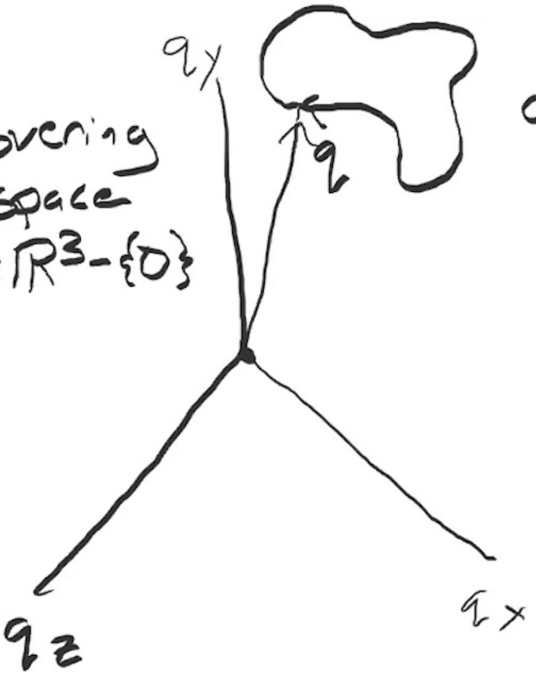


$q(t) = q_2(t) - q_1(t)$

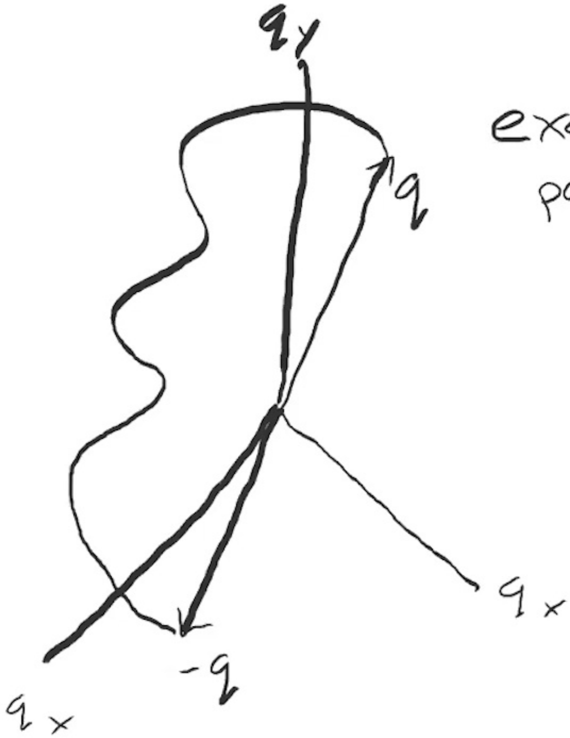
↑ relative position

Assume $q(T) = q(0)$

covering space $= \mathbb{R}^3 - \{0\}$



direct path



exchange path

configuration space $= (\mathbb{R}^3 - \{0\}) / \mathbb{Z}_2$

↖ identifying $q \leftrightarrow -q$

$$A(q,T,q,0) = \sum_{\text{direct}} e^{iS[q]} + \sum_{\text{exchange}} e^{iS[q]}$$

↑ in configuration space

$$= \bar{A}(q,T,q,0) + \bar{A}(-q,T,q,0)$$

↑ in covering space

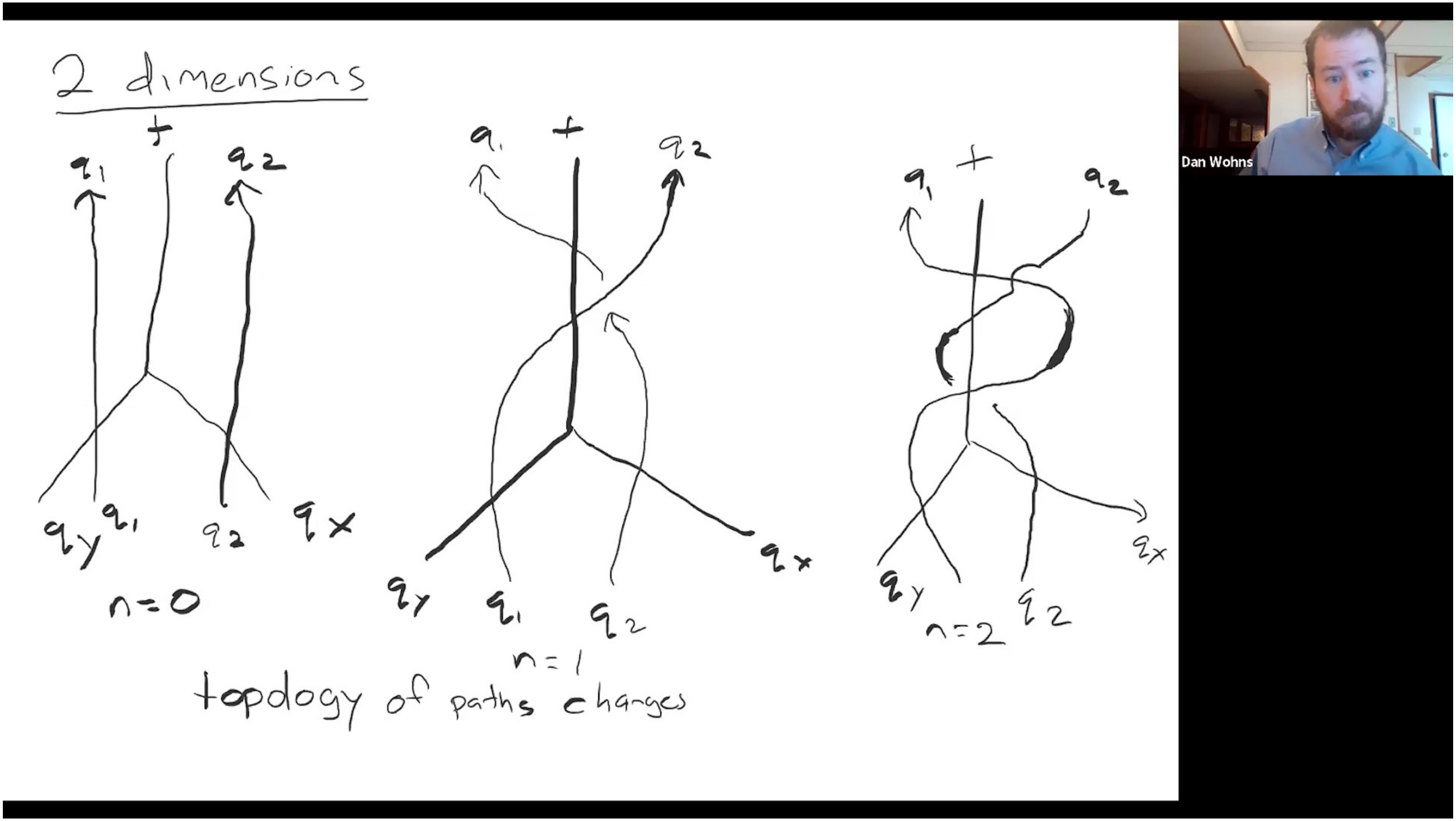
$$A^{\phi}(q',T,q,0) = \bar{A}(q',T,q,0) + e^{i\phi}\bar{A}(-q',T,q,0)$$

$$A^{\phi}(q',T,q,0) = e^{i\alpha}A^{\phi}(-q',T,q,0)$$

$$\bar{A}(q',T,q,0) + e^{i\phi}\bar{A}(-q',T,q,0) = e^{i\alpha}\left(\bar{A}(-q',T,q,0) + e^{i\phi}\bar{A}(+q',T,q,0)\right)$$

$$e^{i\phi} = e^{i\alpha} \rightarrow \phi = \alpha \ (+2\pi n)$$

$$1 = e^{i(\alpha+\phi)} = e^{2i\phi} \rightarrow \phi = 0 \text{ or } \pi \ (+2\pi n$$


2 dimensions
t
n=0
n=1
n=2
topology of paths changes
Dan Wohns

$$A(\vec{q}, T, \vec{q}, 0) = \sum_{n=-\infty}^{\infty} c_n \bar{A}_n(\vec{q}, T, \vec{q}, 0)$$

↑ paths in covering space in class n

$$\rightarrow A(q', \theta' + \gamma_1, T, q, \theta, 0) = e^{-i\phi} A(q', \theta', T, q, \theta, 0)$$

$$\bar{A}_n(q', \theta' + \gamma_1, T, q, \theta, 0) = \bar{A}_{n+1}(q', \theta', T, q, \theta, 0)$$

$$\sum_{n=-\infty}^{\infty} c_n \bar{A}_{n+1} = e^{-i\phi} \sum_{n=-\infty}^{\infty} c_n \bar{A}_n$$

$$\sum_{n=-\infty}^{\infty} c_{n-1} \bar{A}_n = e^{-i\phi} \sum_{n=-\infty}^{\infty} c_n \bar{A}_n$$

$$\boxed{c_{n-1} e^{i\phi} = c_n}$$

$\phi \in \mathbb{R}$

if $\phi \neq 0$ or π $\rightarrow$ anyons

Paths not taken

- application
- fermions
- fields