Title: Summer Undergrad 2020 - Quantum Information - Lecture 5

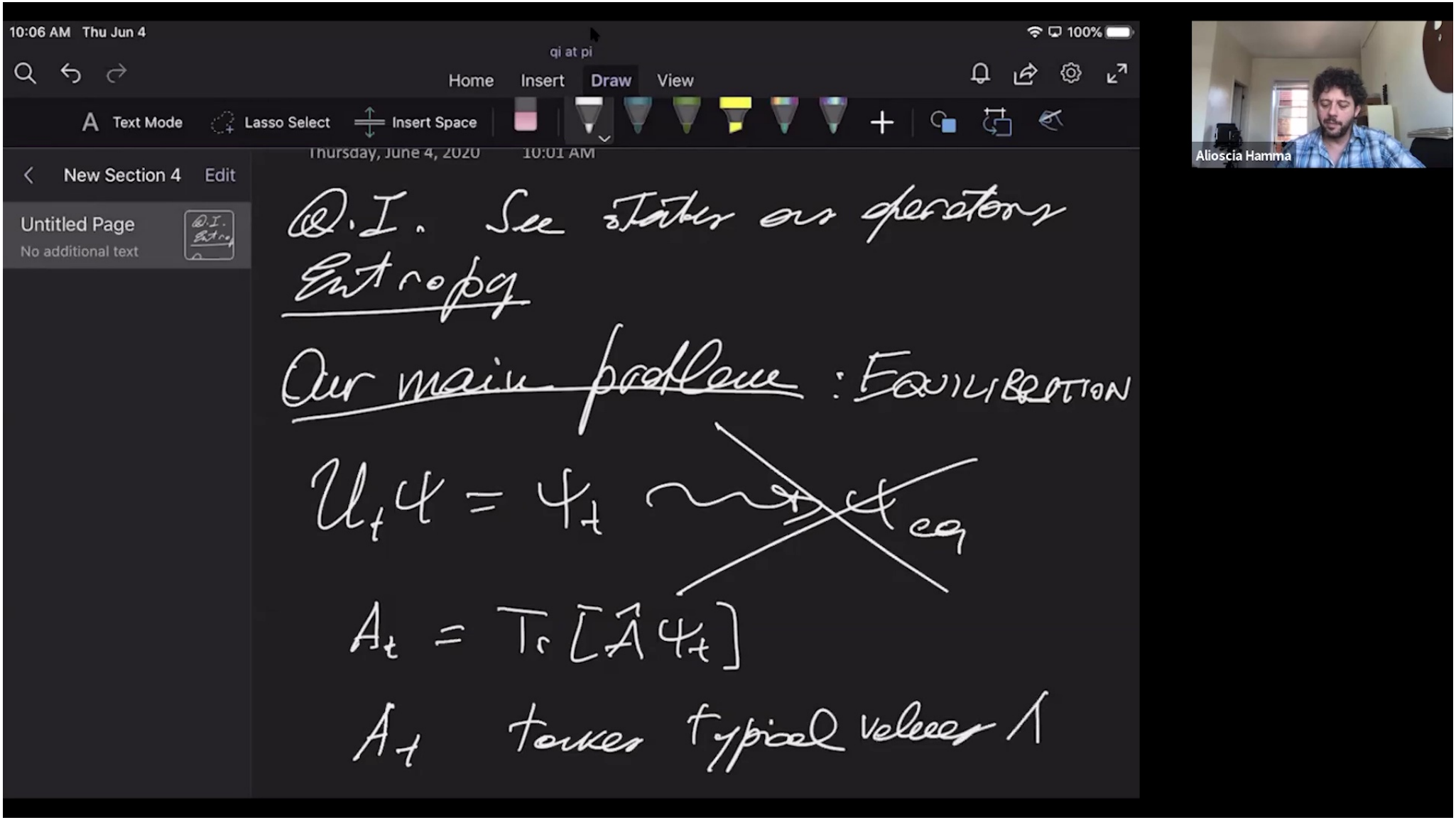
Speakers: Alioscia Hamma

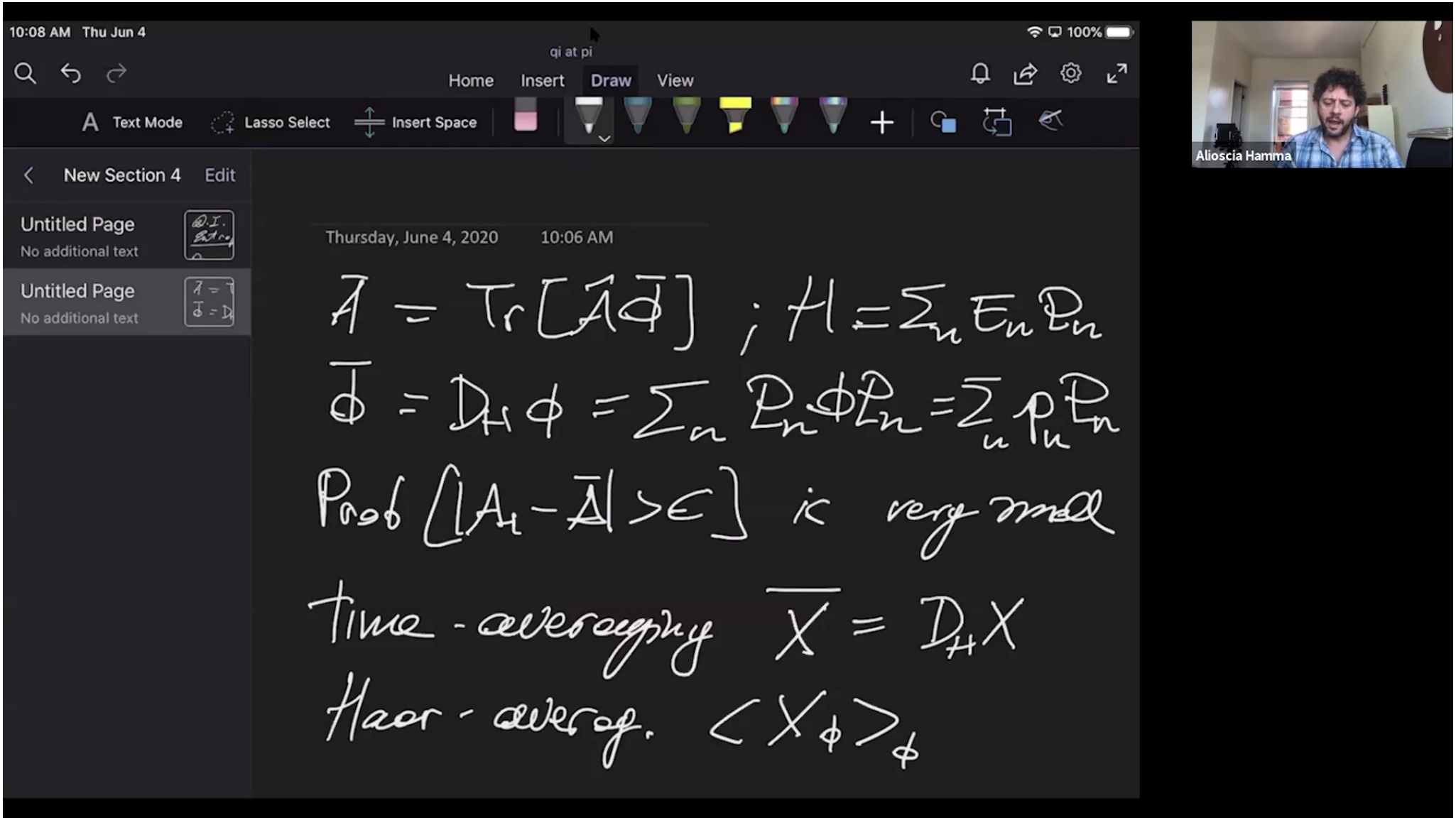
Collection: Summer Undergrad 2020 - Quantum Information

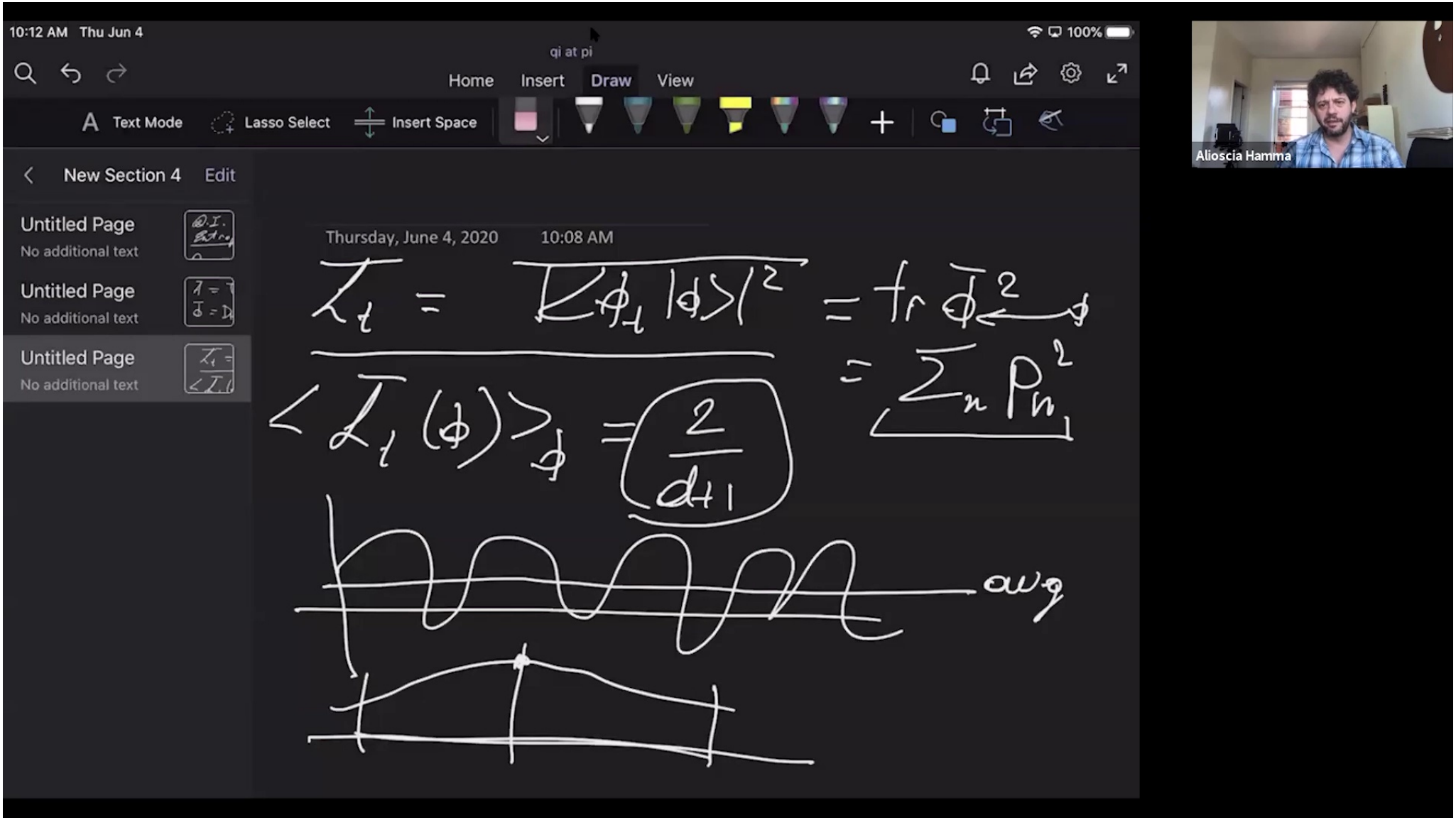
Date: June 03, 2020 - 10:00 AM

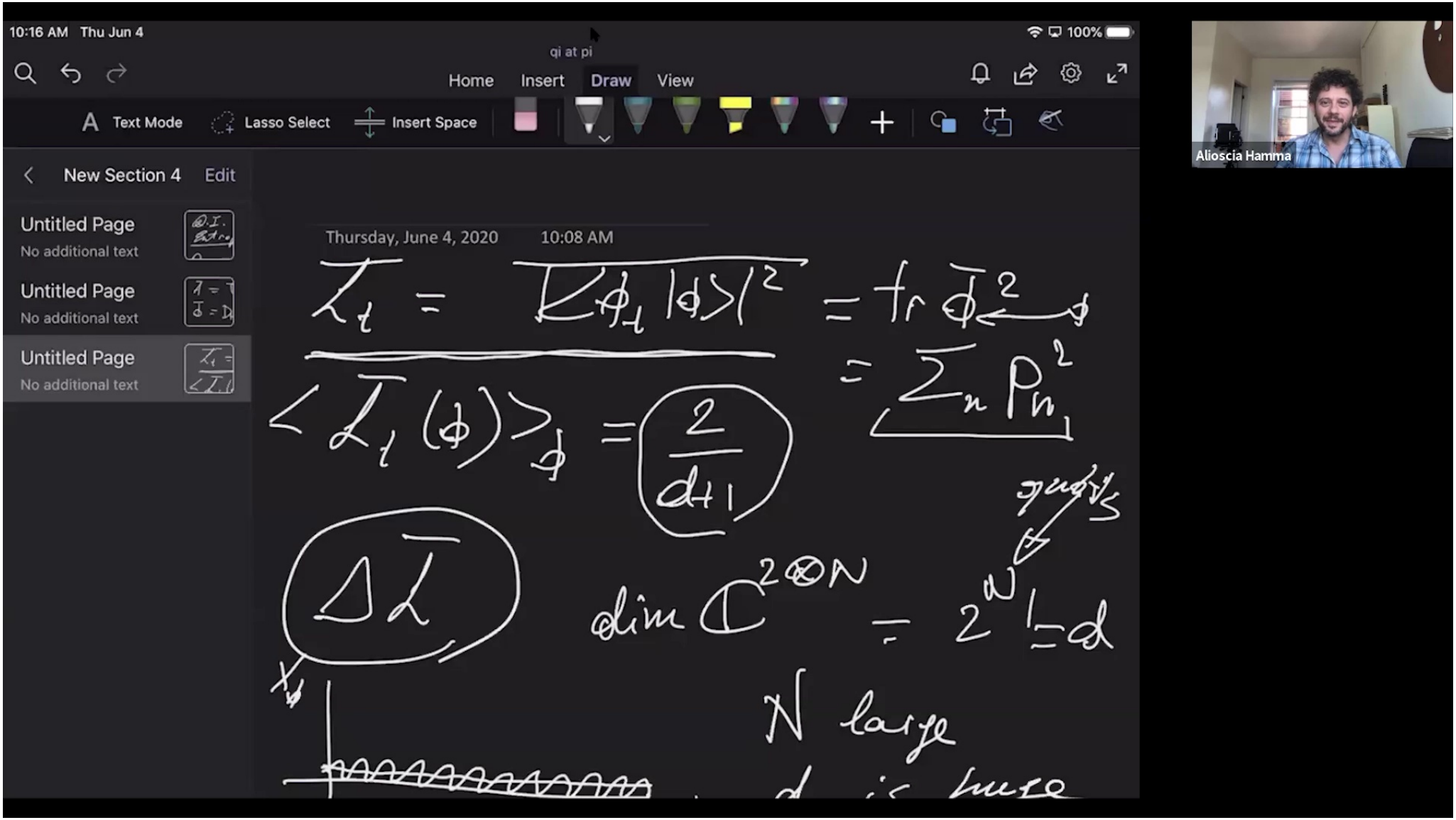
URL: http://pirsa.org/20060004

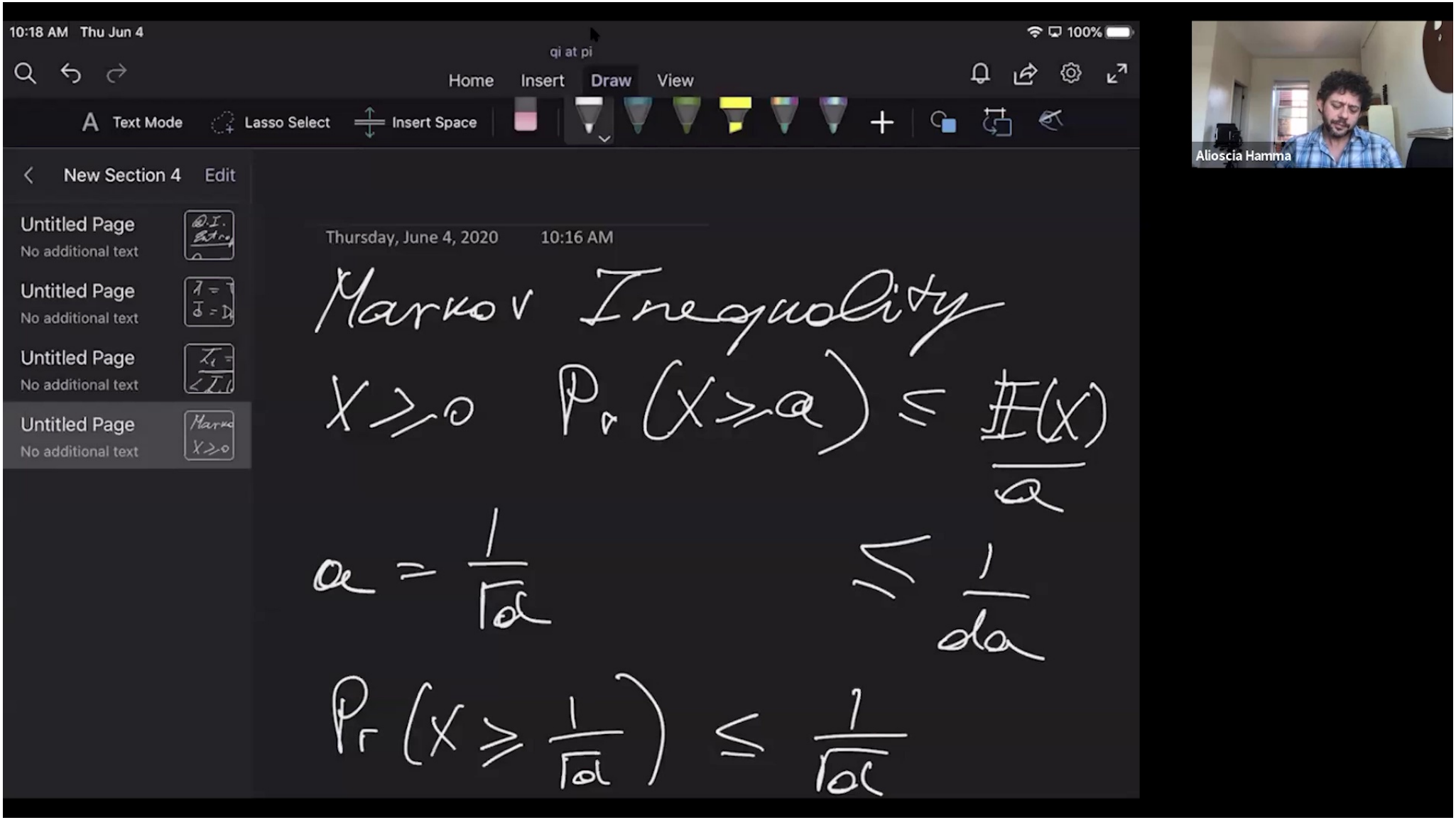
Abstract: Quantum Thermal Machines

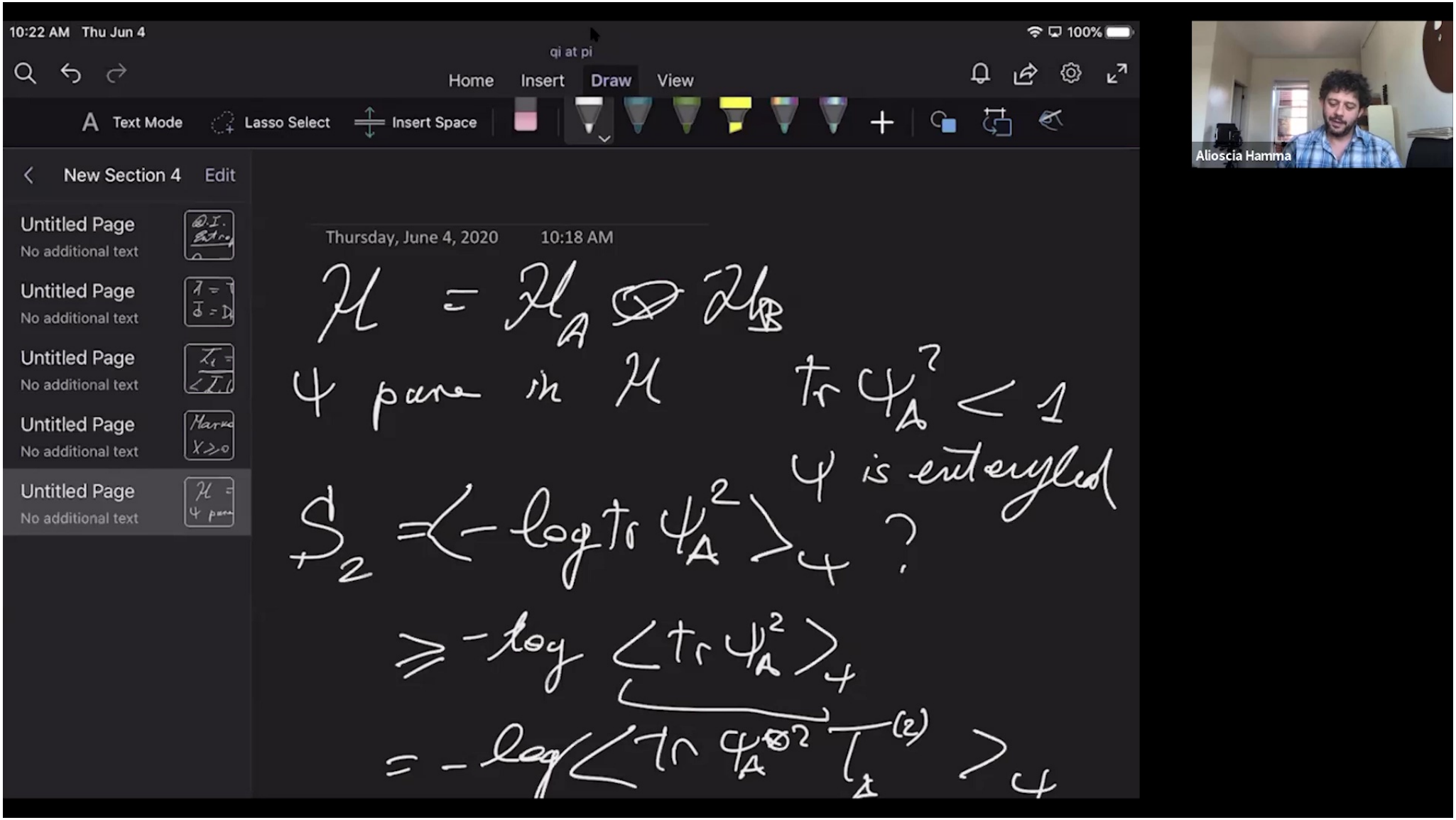

10:06 AM Thu Jun 4
100%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:01 AM
Q.I. See other operators
Entropy
Our main problem : EQUILIBRATION
$U_t \Psi = \Psi_t$
$\Psi_t \to \Psi_{eq}$
$A_t = Tr[\hat{A} \Psi_t]$
A_t tomes typical values
Alioscia Hamma


10:08 AM Thu Jun 4
100%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Untitled Page
No additional text
Thursday, June 4, 2020 10:06 AM
$\bar{A} = Tr[\hat{A}\hat{\phi}]$; $H = \sum_n E_n P_n$
$\bar{\phi} = D_H \phi = \sum_n P_n \phi P_n = \sum_n p_n P_n$
$Prob[|A_t - \bar{A}| > \epsilon]$ is very small
time-averaging $\overline{X} = D_H X$
Haar-averag. $\langle X_\phi \rangle_\phi$
Alioscia Hamma

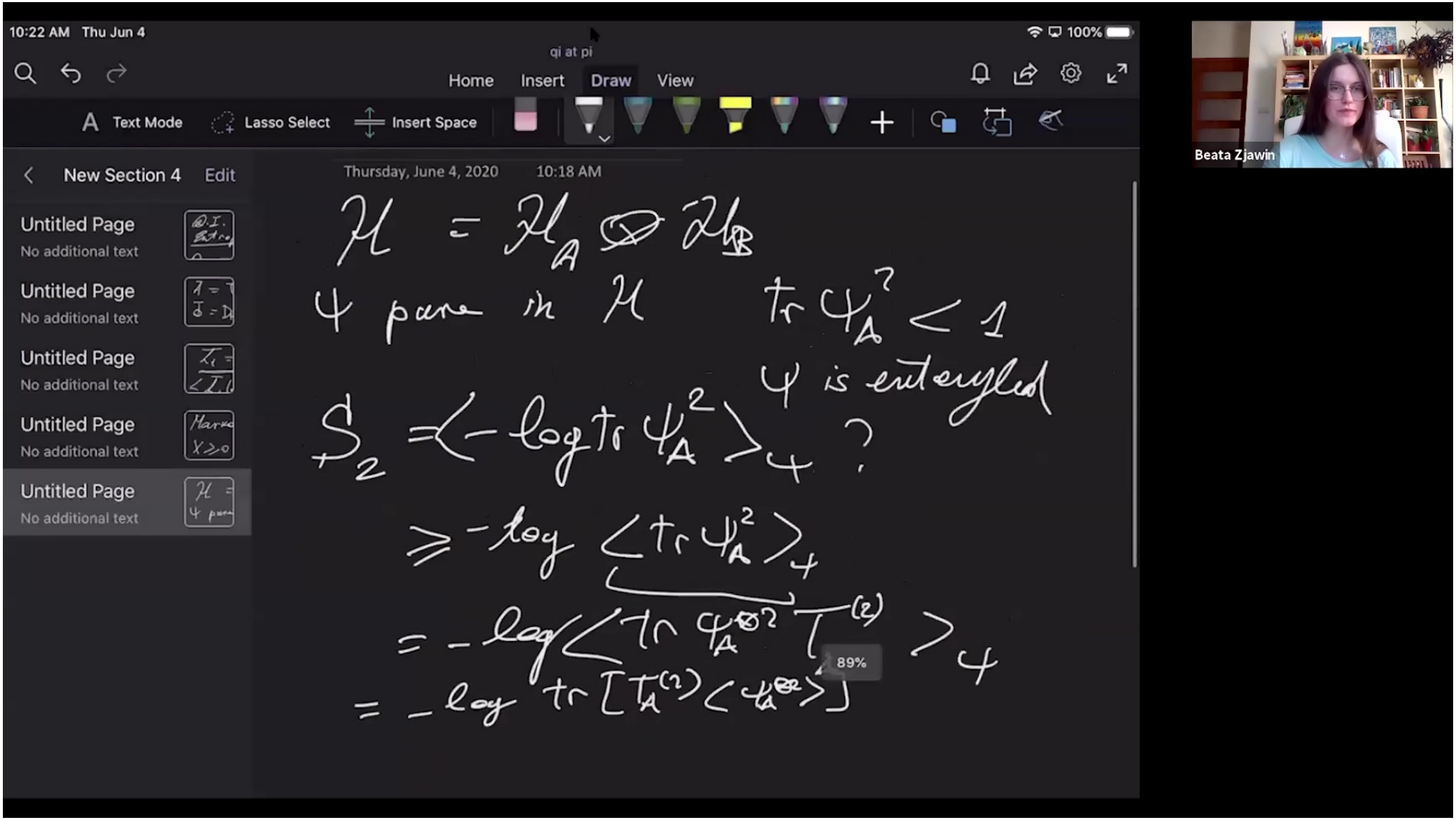

10:12 AM Thu Jun 4
100%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Thursday, June 4, 2020
10:08 AM
Alioscia Hamma

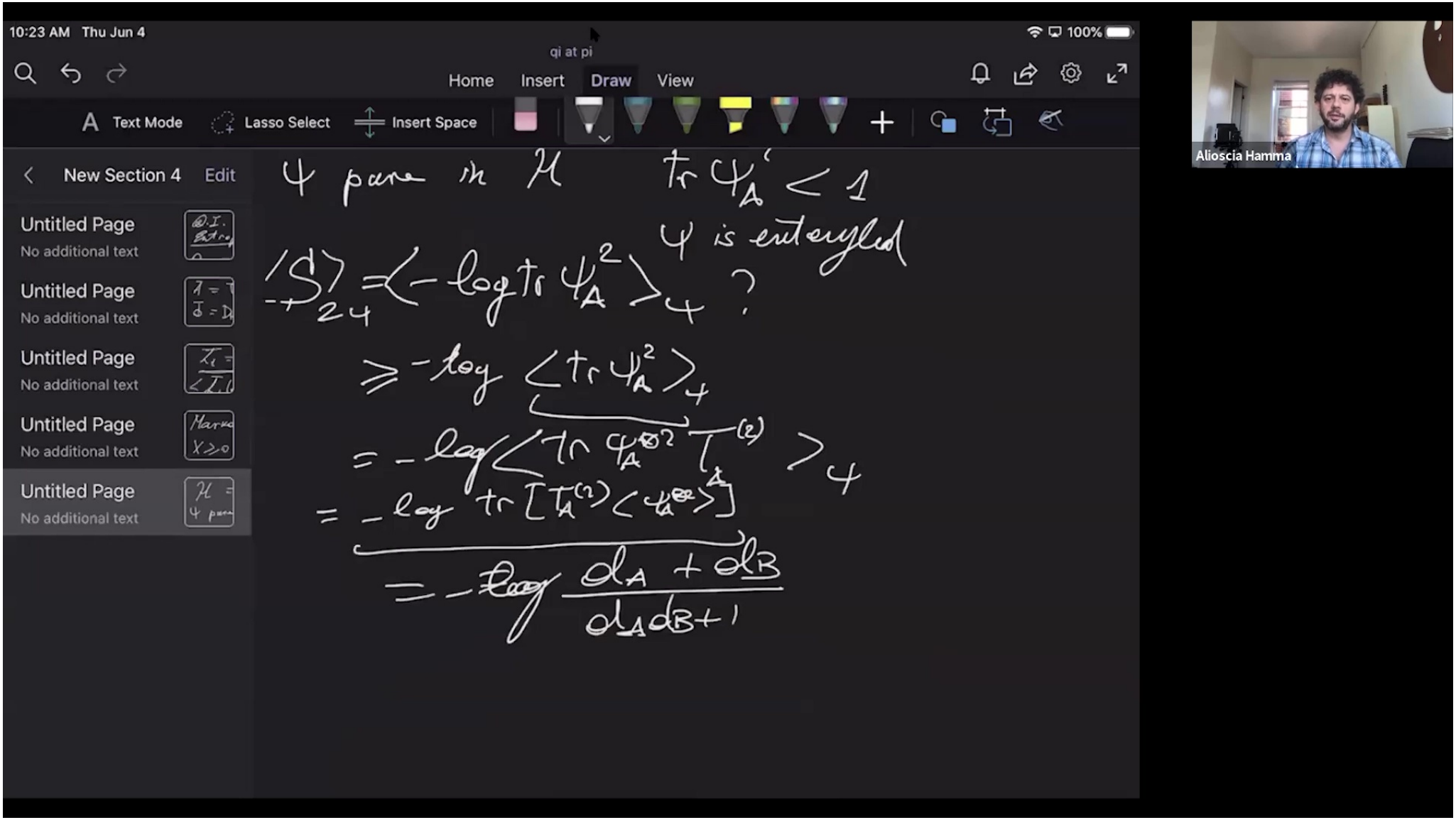

10:16 AM Thu Jun 4
100%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Thursday, June 4, 2020
10:08 AM
Alioscia Hamma

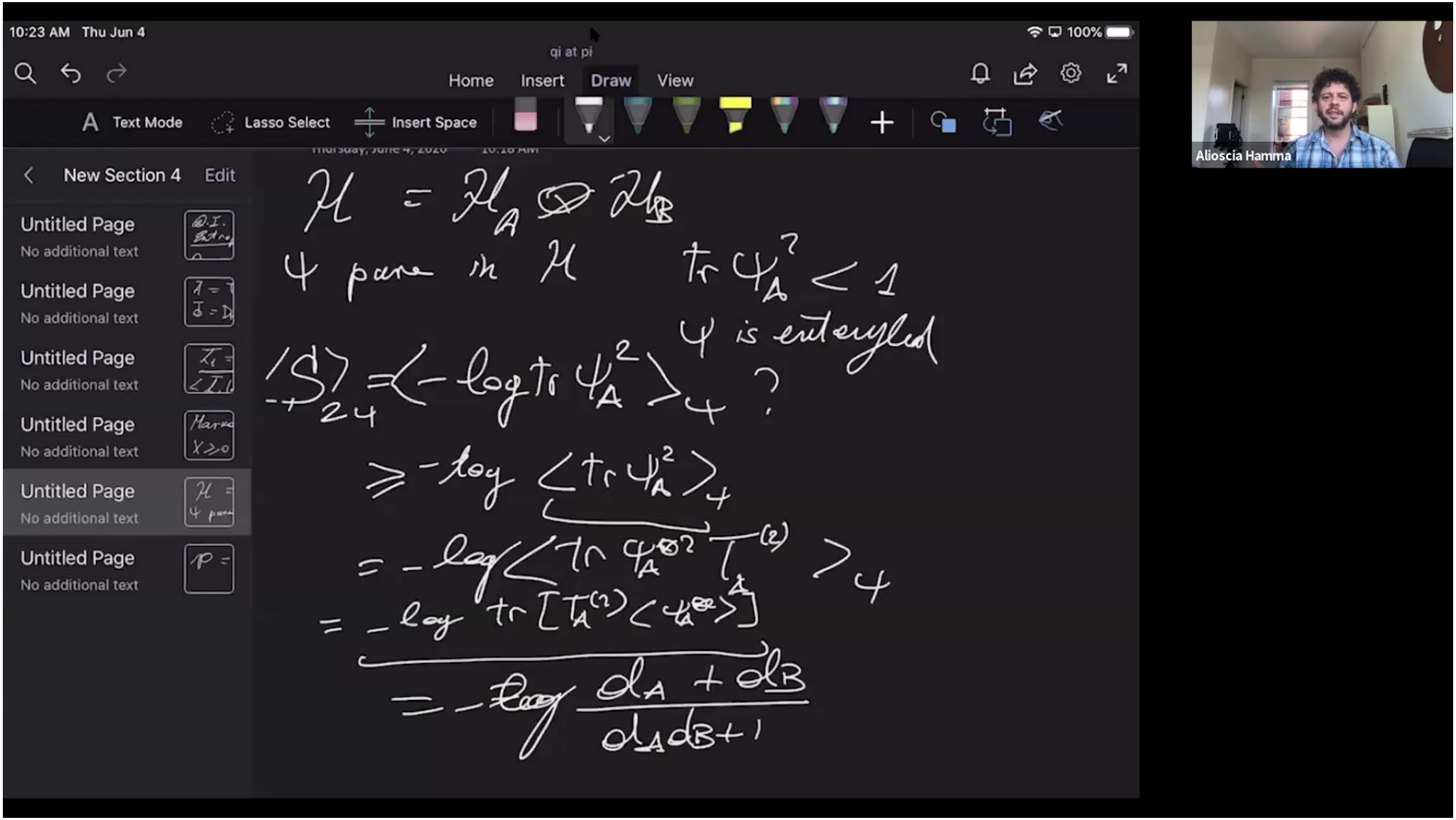

10:18 AM Thu Jun 4
100%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Thursday, June 4, 2020 10:16 AM
Markov Inequality
$X \geq 0$ $P_r(X \geq a) \leq \frac{E(X)}{a}$
$a = \frac{1}{10}$ $\leq \frac{1}{da}$
$P_r(X \geq \frac{1}{10}) \leq \frac{1}{10}$
Alioscia Hamma

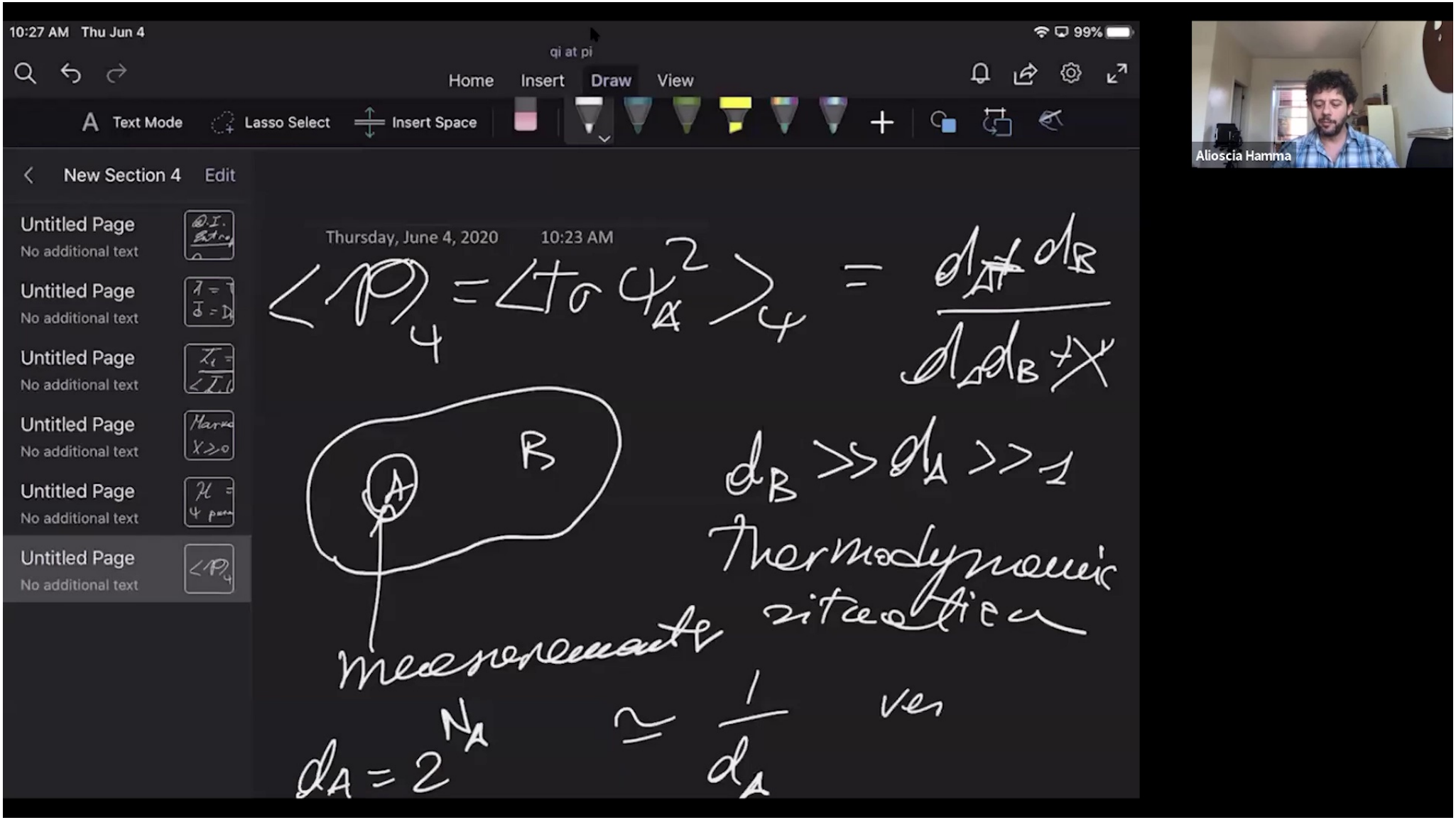

10:22 AM Thu Jun 4
100%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Thursday, June 4, 2020 10:18 AM
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
ψ pure in $\mathcal{H}$
$\mathrm{tr}\,\psi_A^2 < 1$
ψ is entangled
$S_2 = \langle -\log \mathrm{tr}\,\psi_A^2 \rangle_\psi$?
$\geq -\log \langle \mathrm{tr}\,\psi_A^2 \rangle_\psi$
$= -\log \langle \mathrm{tr}\,\psi_A^{\otimes 2} T_A^{(2)} \rangle_\psi$

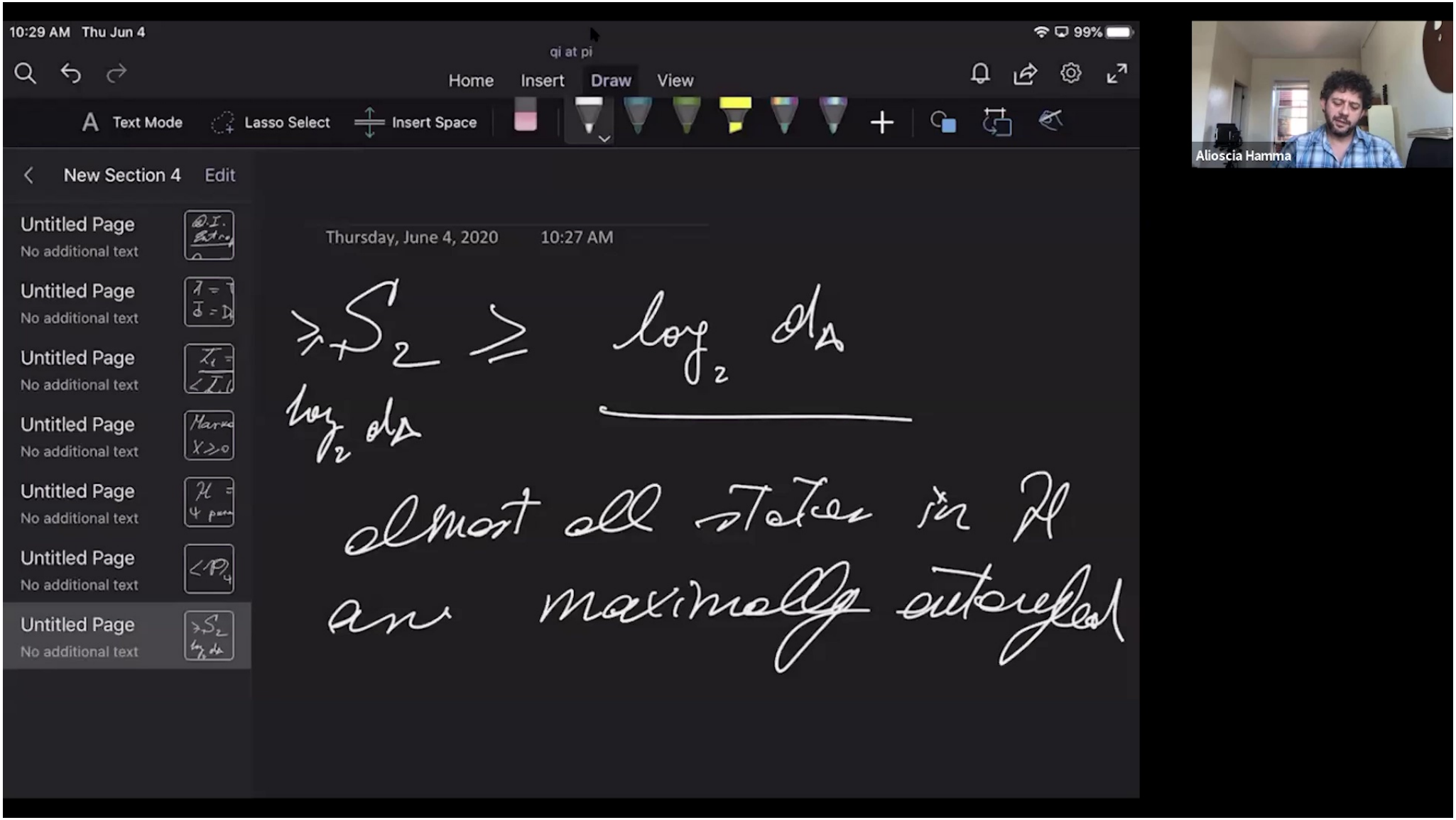
Alioscia Hamma

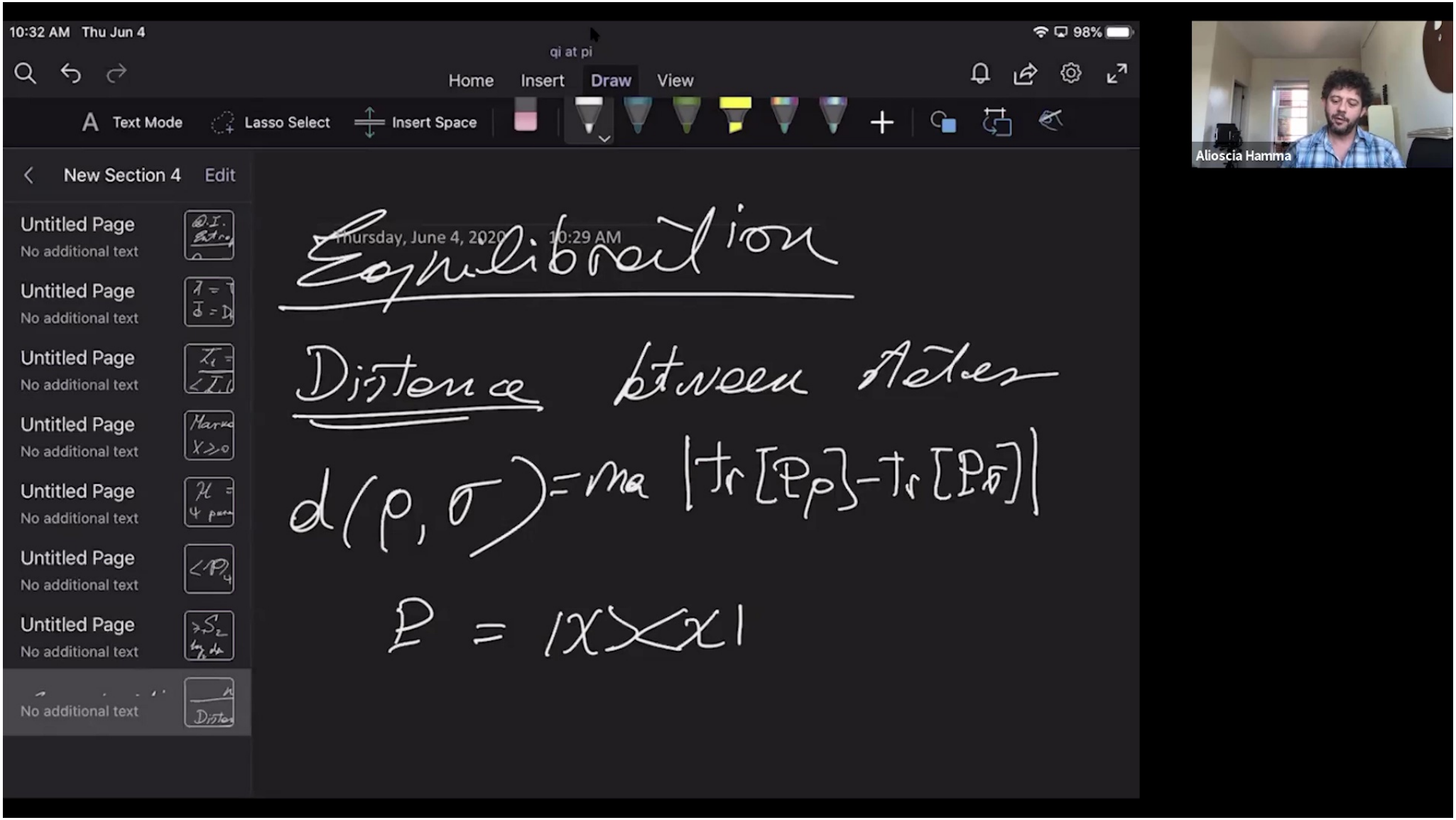

10:22 AM Thu Jun 4
100%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020 10:18 AM
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
ψ pure in $\mathcal{H}$
$\mathrm{tr}\,\psi_A^2 < 1$
ψ is entangled
$S_2 = \langle -\log \mathrm{tr}\,\psi_A^2 \rangle_\psi$?
$\geq -\log \langle \mathrm{tr}\,\psi_A^2 \rangle_\psi$
$= -\log \langle \mathrm{tr}\,\psi_A^{\otimes 2} T^{(2)} \rangle_\psi$
$= -\log \mathrm{tr}\,[T_A^{(2)} \langle \psi_A^{\otimes 2} \rangle]$
89%
Beata Zjawin

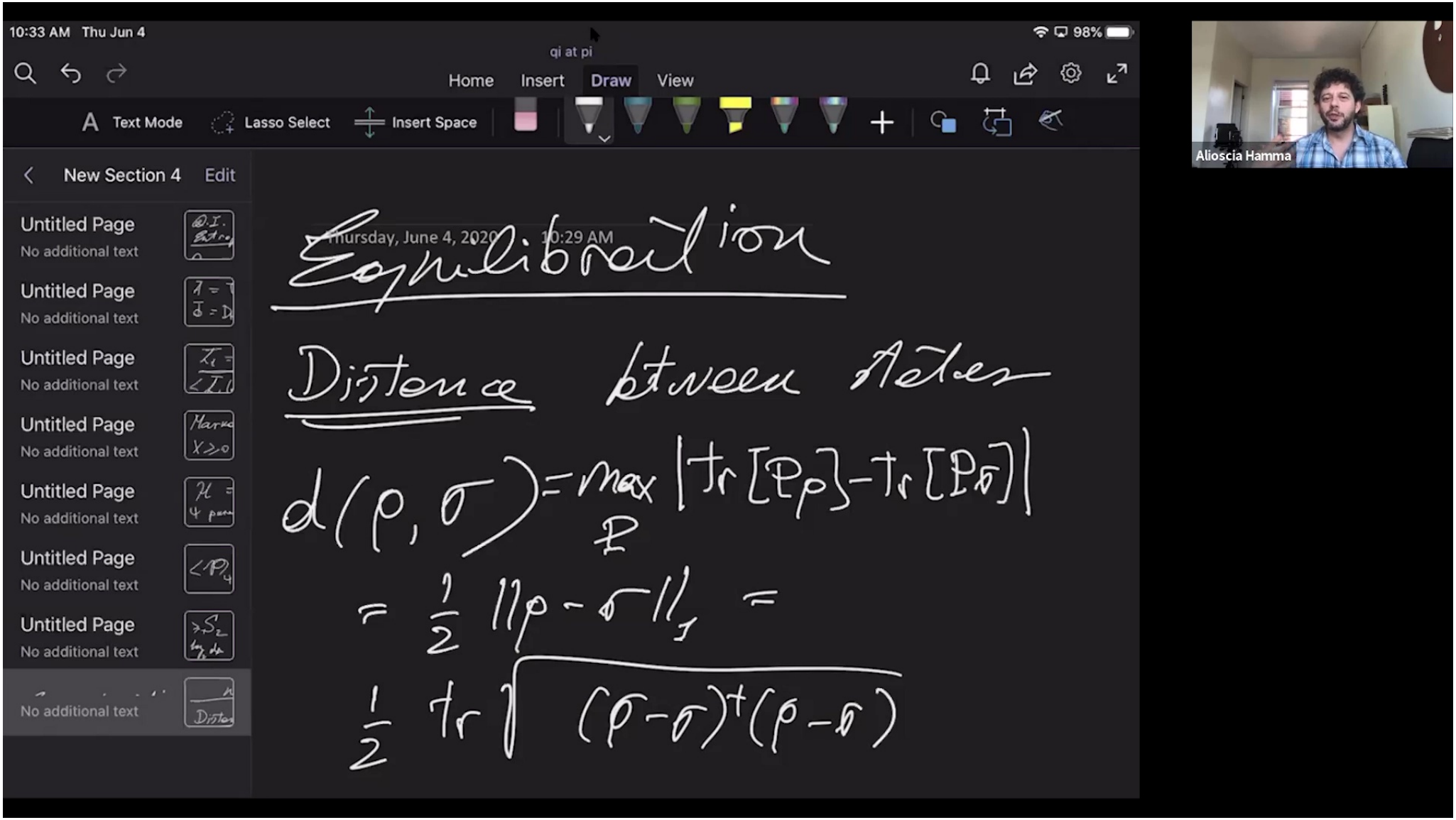

10:23 AM Thu Jun 4
100%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
ψ pure in H
tr ψ_A^2 < 1
ψ is entangled
S_2 = ⟨ − log tr ψ_A^2 ⟩_ψ ?
≥ − log ⟨ tr ψ_A^2 ⟩_ψ
= − log ⟨ tr ψ_A^⊗2 T_A^(2) ⟩_ψ
= − log tr [T_A^(2) ⟨ ψ_A^⊗2 ⟩]
= − log (d_A + d_B) / (d_A d_B + 1)
Alioscia Hamma

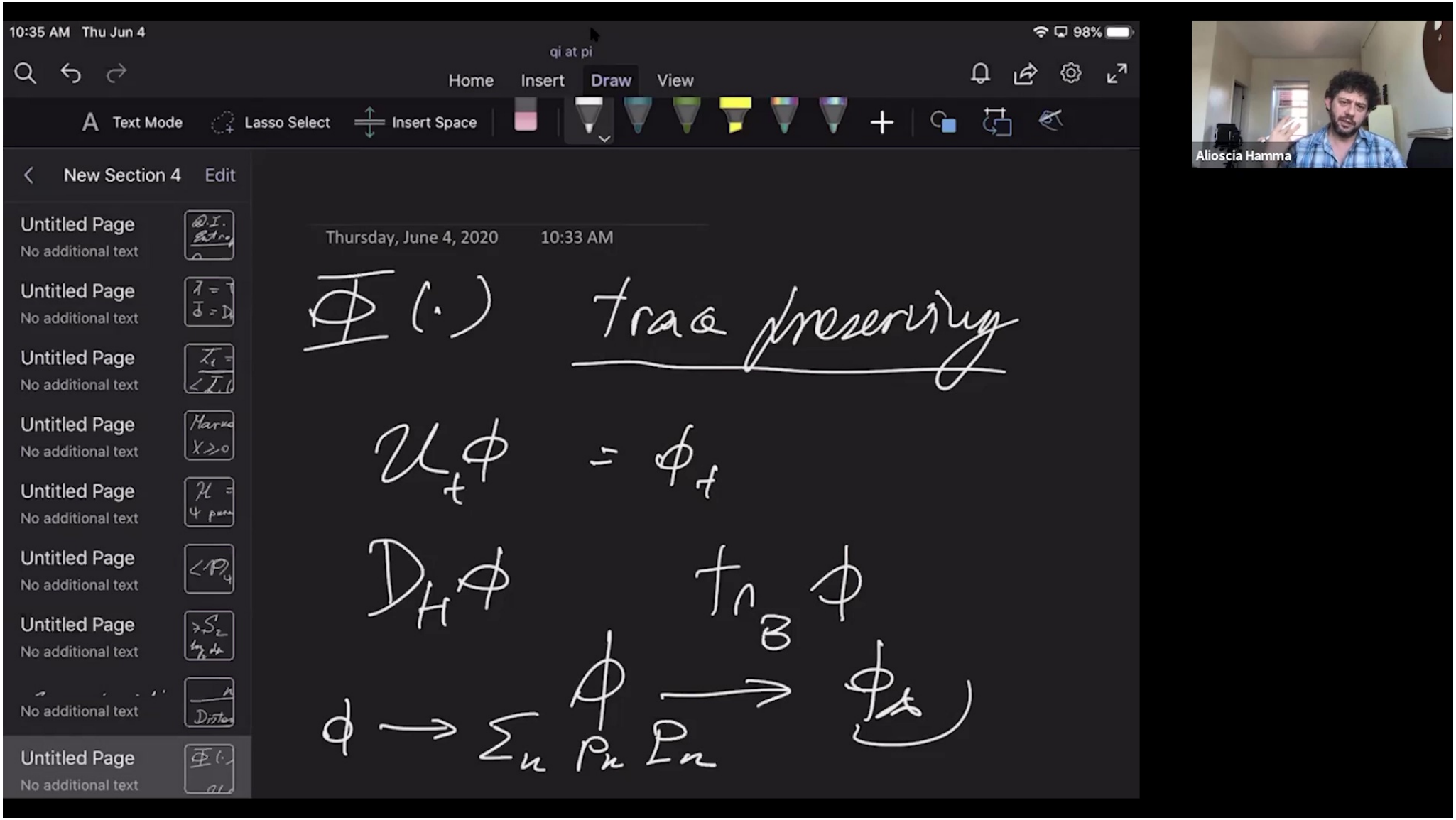

10:23 AM Thu Jun 4
100%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Untitled Page
No additional text
Alioscia Hamma

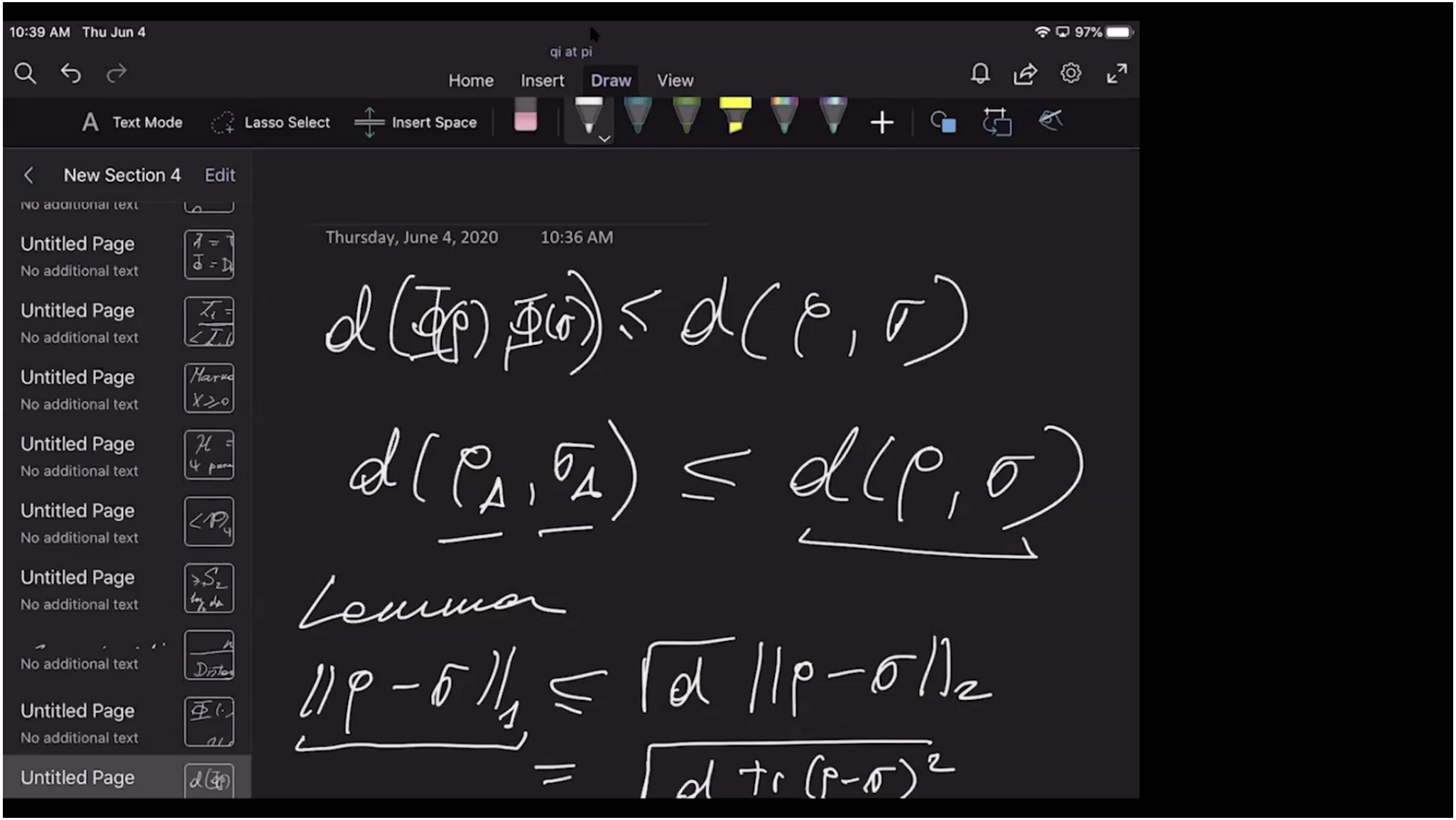

10:27 AM Thu Jun 4
99%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:23 AM
A
B
Thermodynamic
Alioscia Hamma

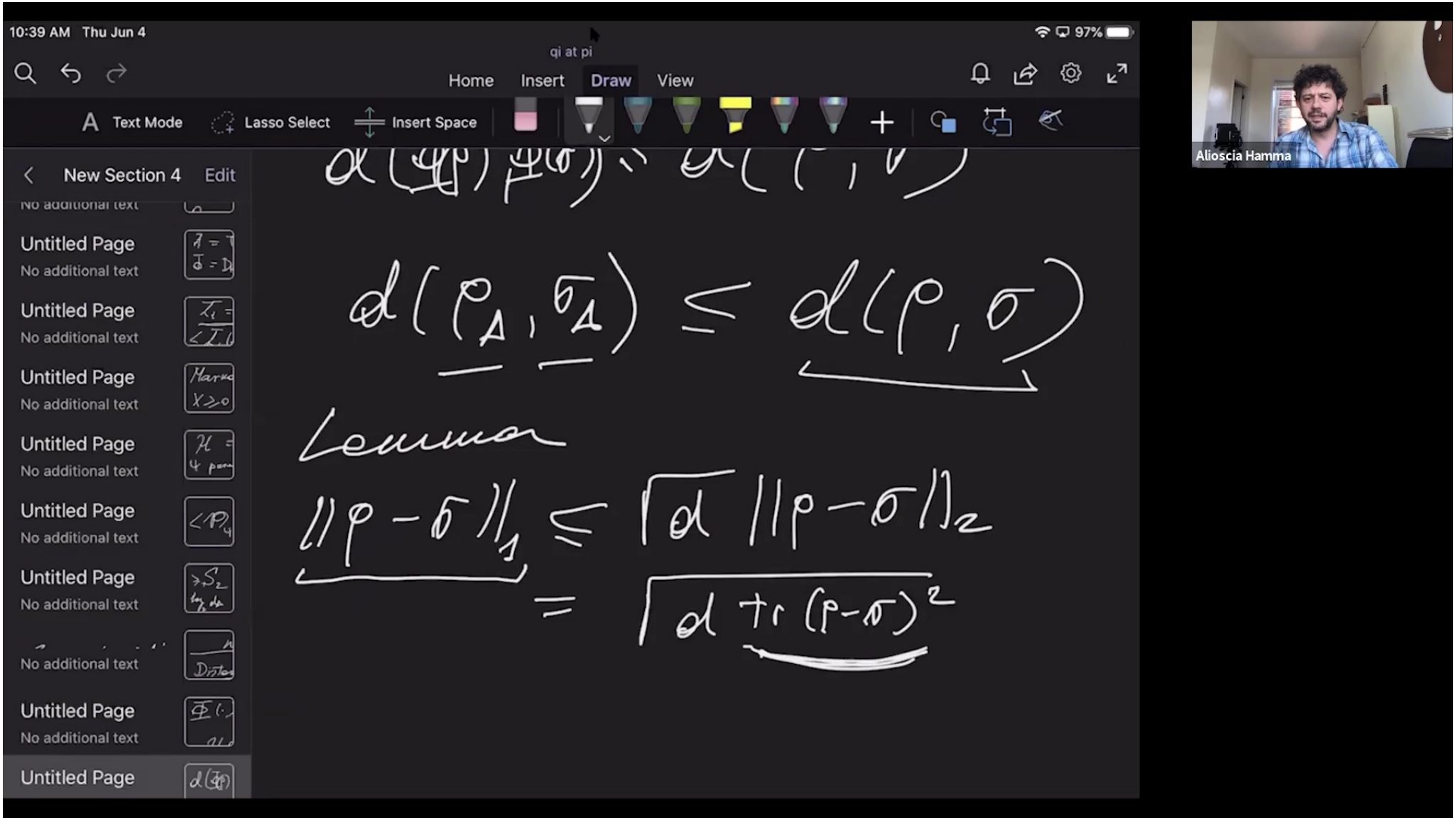

10:29 AM Thu Jun 4
99%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:27 AM
almost all states in H
are maximally entangled
Alioscia Hamma

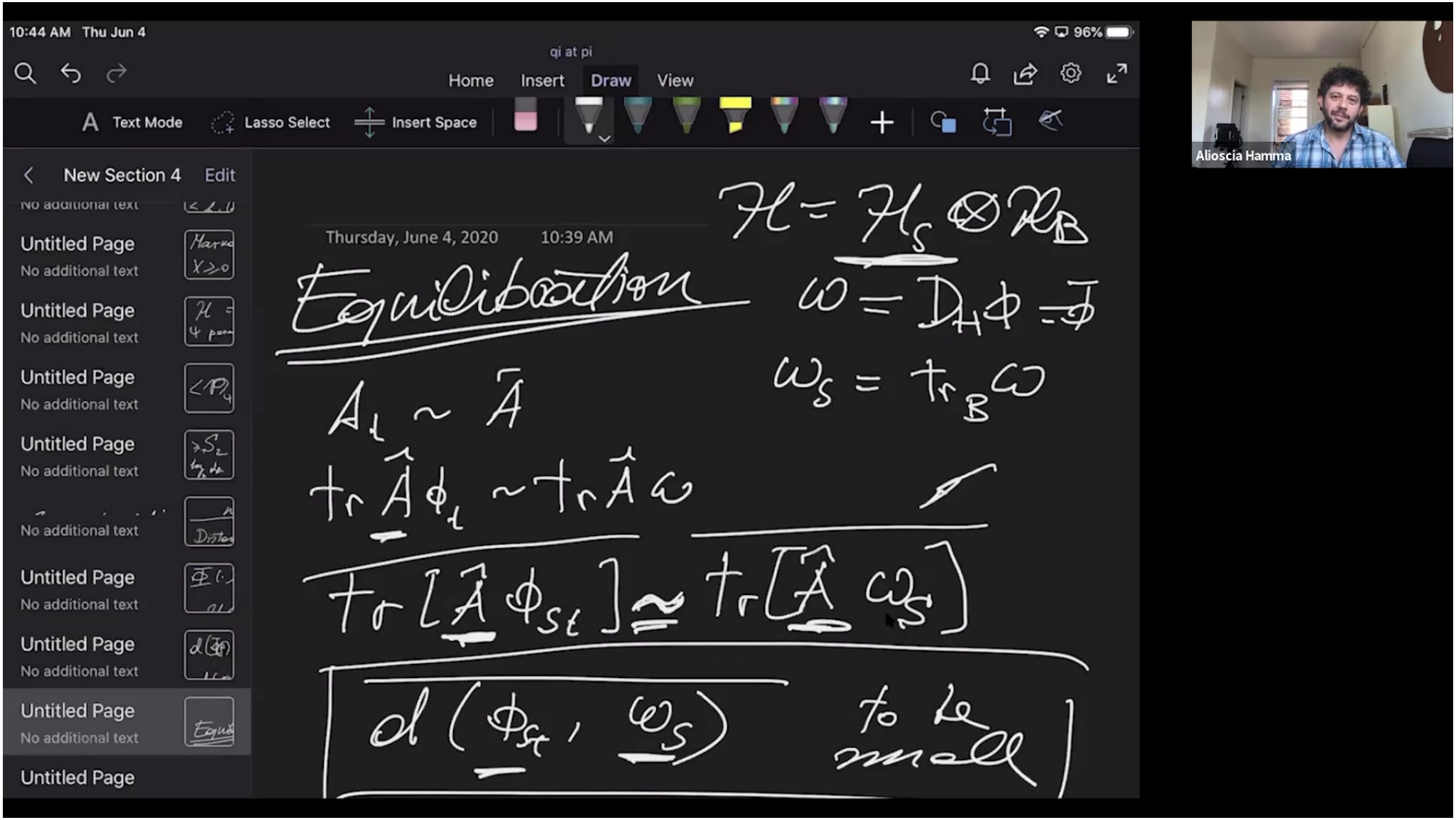

10:32 AM Thu Jun 4
98%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020 10:29 AM
Equilibration
Distance between states
$d(\rho,\sigma) = \max |\mathrm{tr}[P\rho] - \mathrm{tr}[P\sigma]|$
$P = |X\rangle\langle X|$
Alioscia Hamma

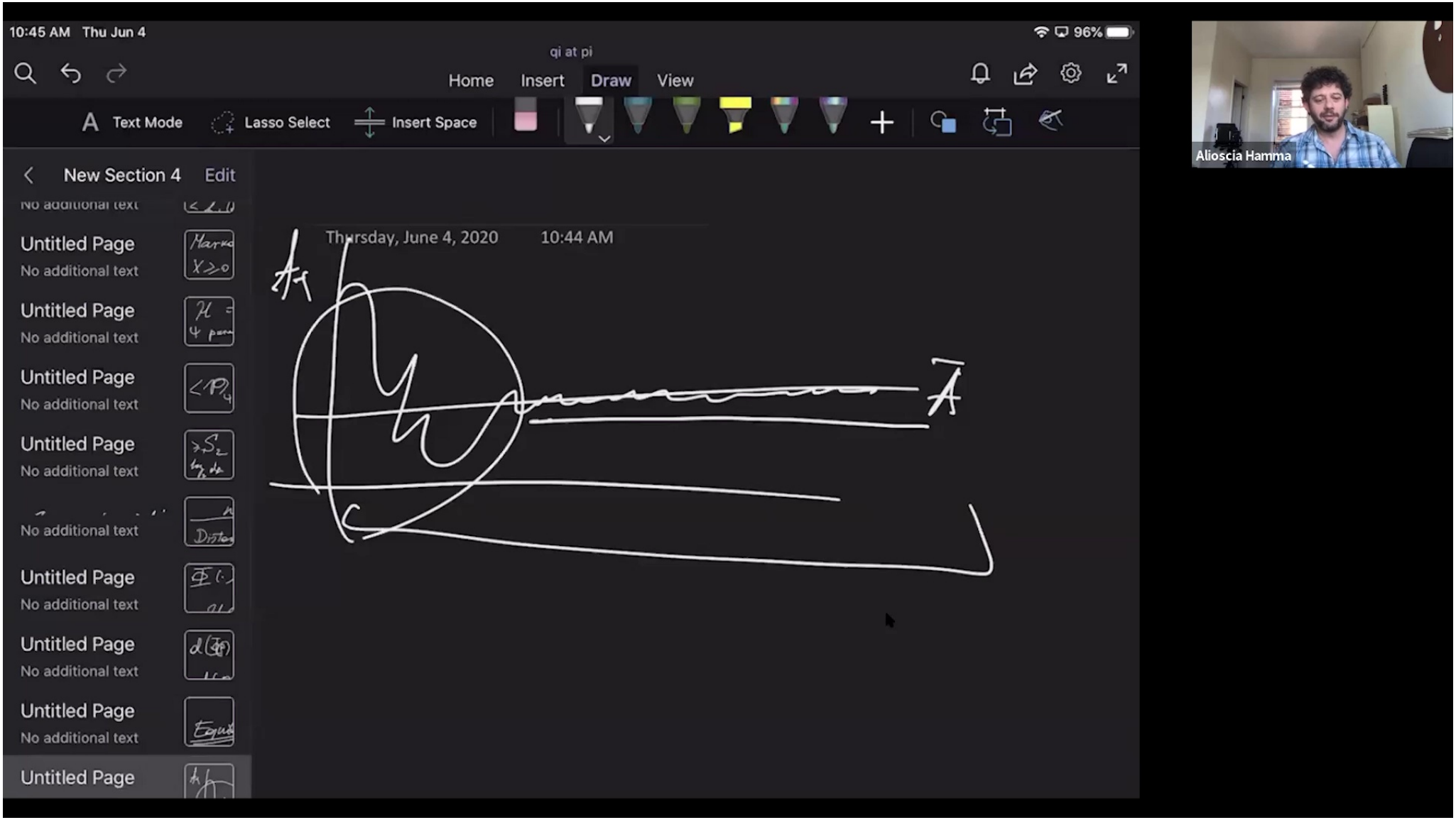

10:33 AM Thu Jun 4
98%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020 10:29 AM
Equilibration
Distance between States
$d(\rho,\sigma) = \max_{\Pi} |\mathrm{tr}[\Pi\rho] - \mathrm{tr}[\Pi\sigma]|$
$= \frac{1}{2}\|\rho-\sigma\|_1 =$
$\frac{1}{2}\mathrm{tr}\sqrt{(\rho-\sigma)^\dagger(\rho-\sigma)}$
Alioscia Hamma

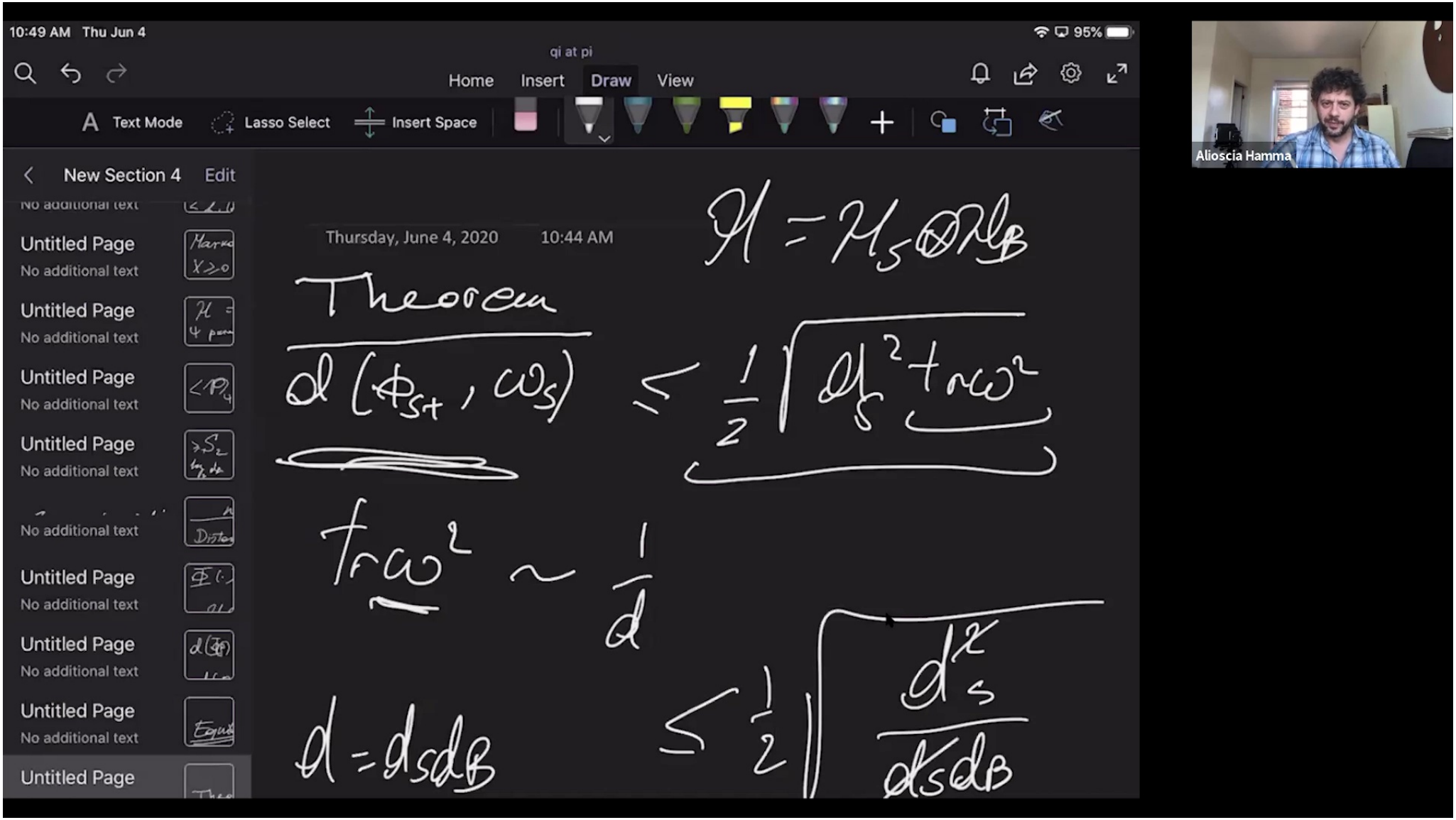

10:35 AM Thu Jun 4
98%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:33 AM
Trace preserving
Alioscia Hamma


10:39 AM Thu Jun 4
97%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020 10:36 AM
Lemma

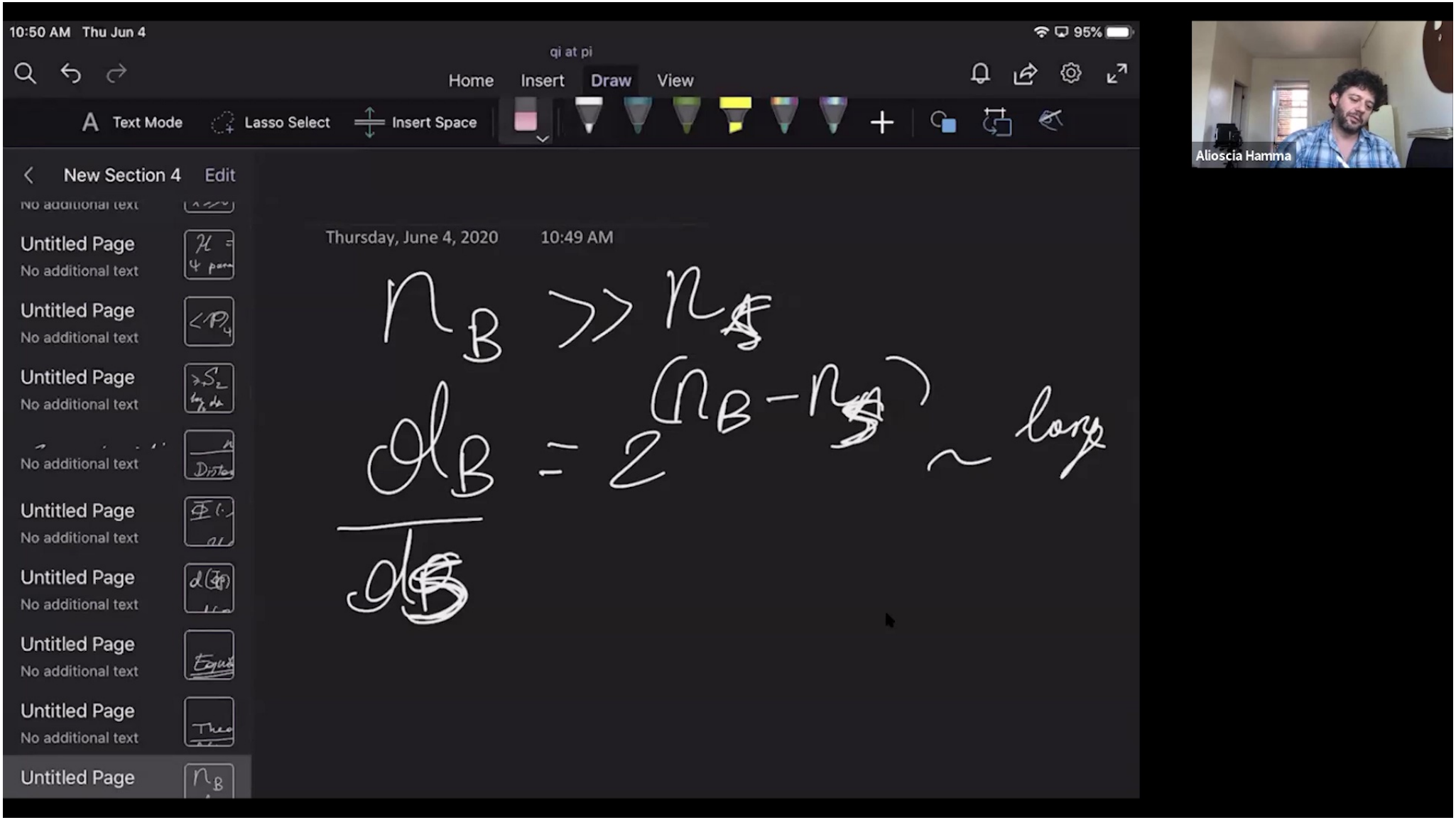

10:39 AM Thu Jun 4
97%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Lemma
Alioscia Hamma

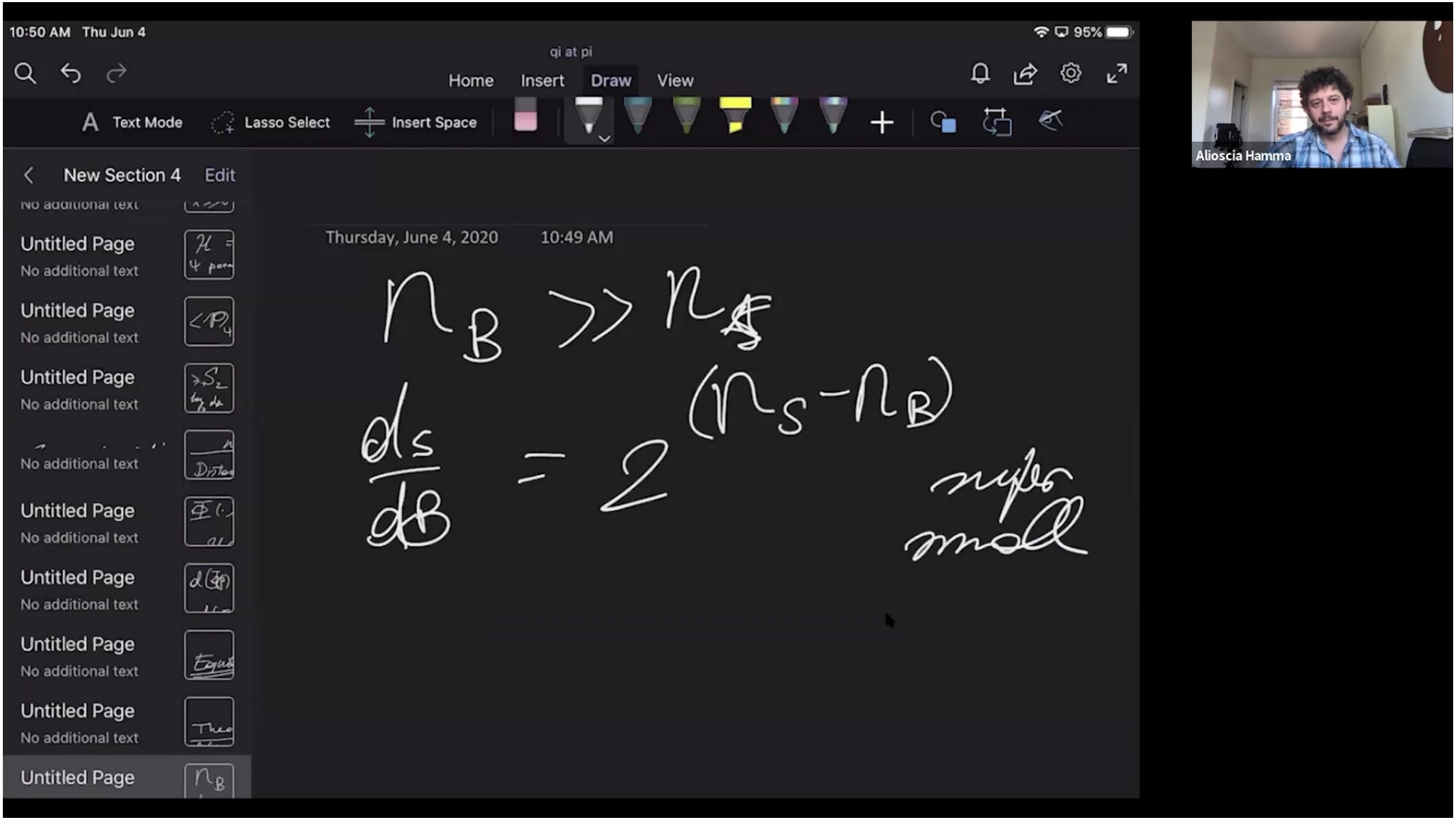

10:44 AM Thu Jun 4
96%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020 10:39 AM
Equilibration
$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B$
$\omega = D_H \Phi = \mathcal{J}$
$\omega_S = tr_B \omega$
$A_L \sim \bar{A}$
$tr \hat{A} \phi_L \sim tr \hat{A} \omega$
$tr[\hat{A} \phi_{St}] \simeq tr[\hat{A} \omega_S]$
$d(\phi_{St}, \omega_S)$ to be small
Alioscia Hamma

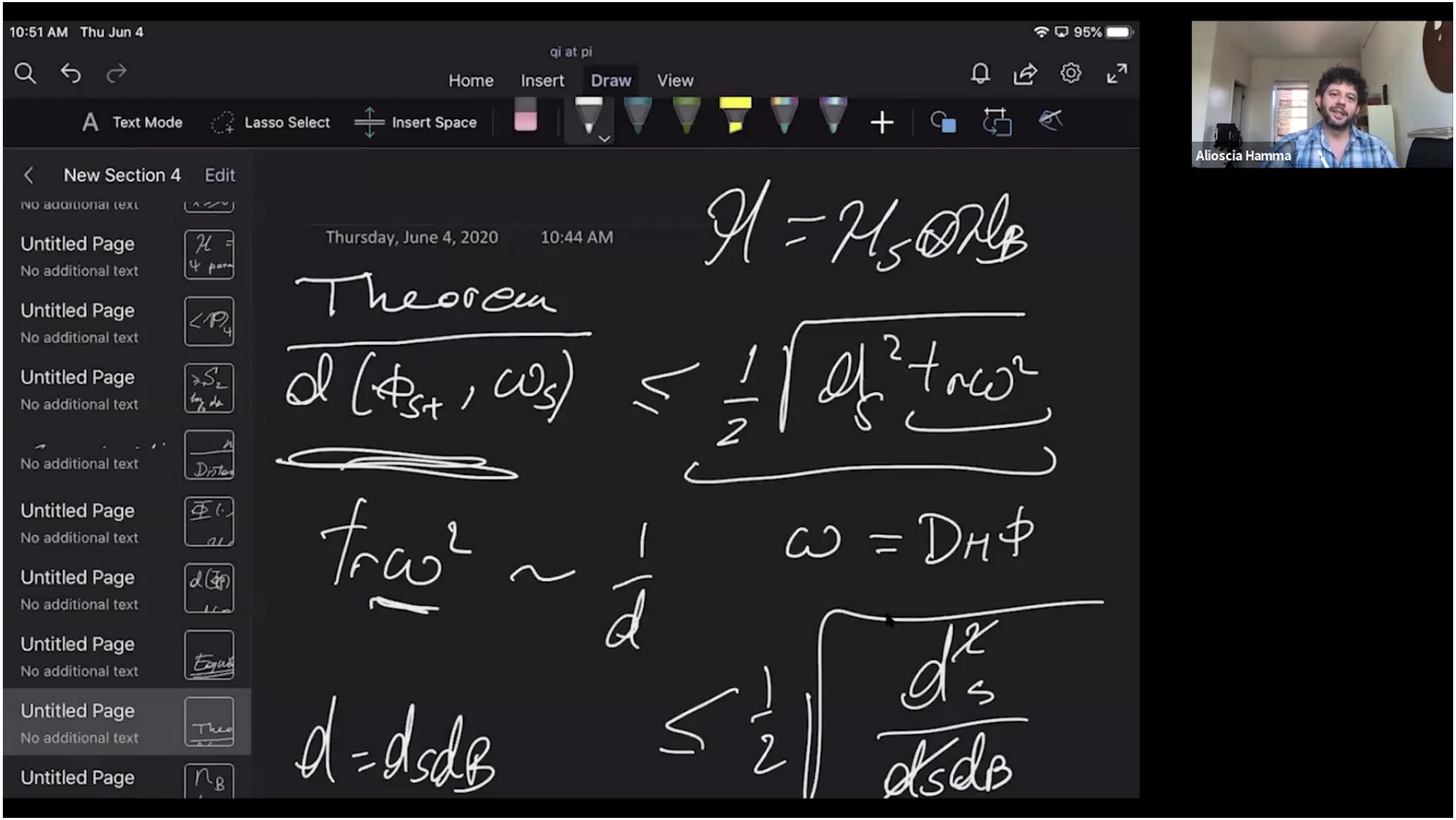

10:45 AM Thu Jun 4
96%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:44 AM
Alioscia Hamma


10:49 AM Thu Jun 4
95%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:44 AM
Theorem
$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B$
$d(\Phi_{st}, \omega_s) \leq \frac{1}{2}\sqrt{d_S^2 + r\omega^2}$
$r\omega^2 \sim \frac{1}{d}$
$d = d_S d_B$
$\leq \frac{1}{2}\sqrt{\frac{d_S^2}{d_S d_B}}$

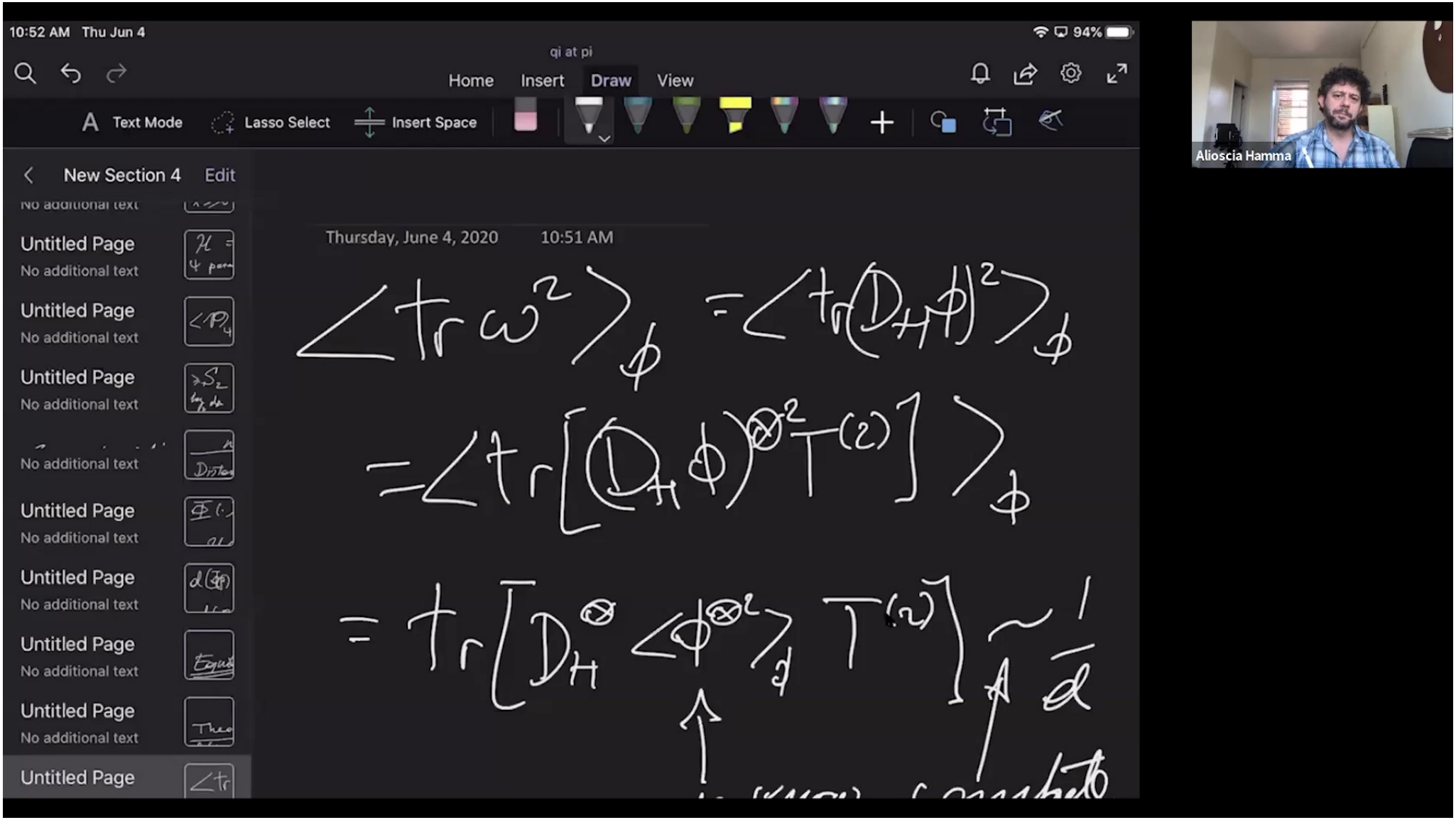
Alioscia Hamma

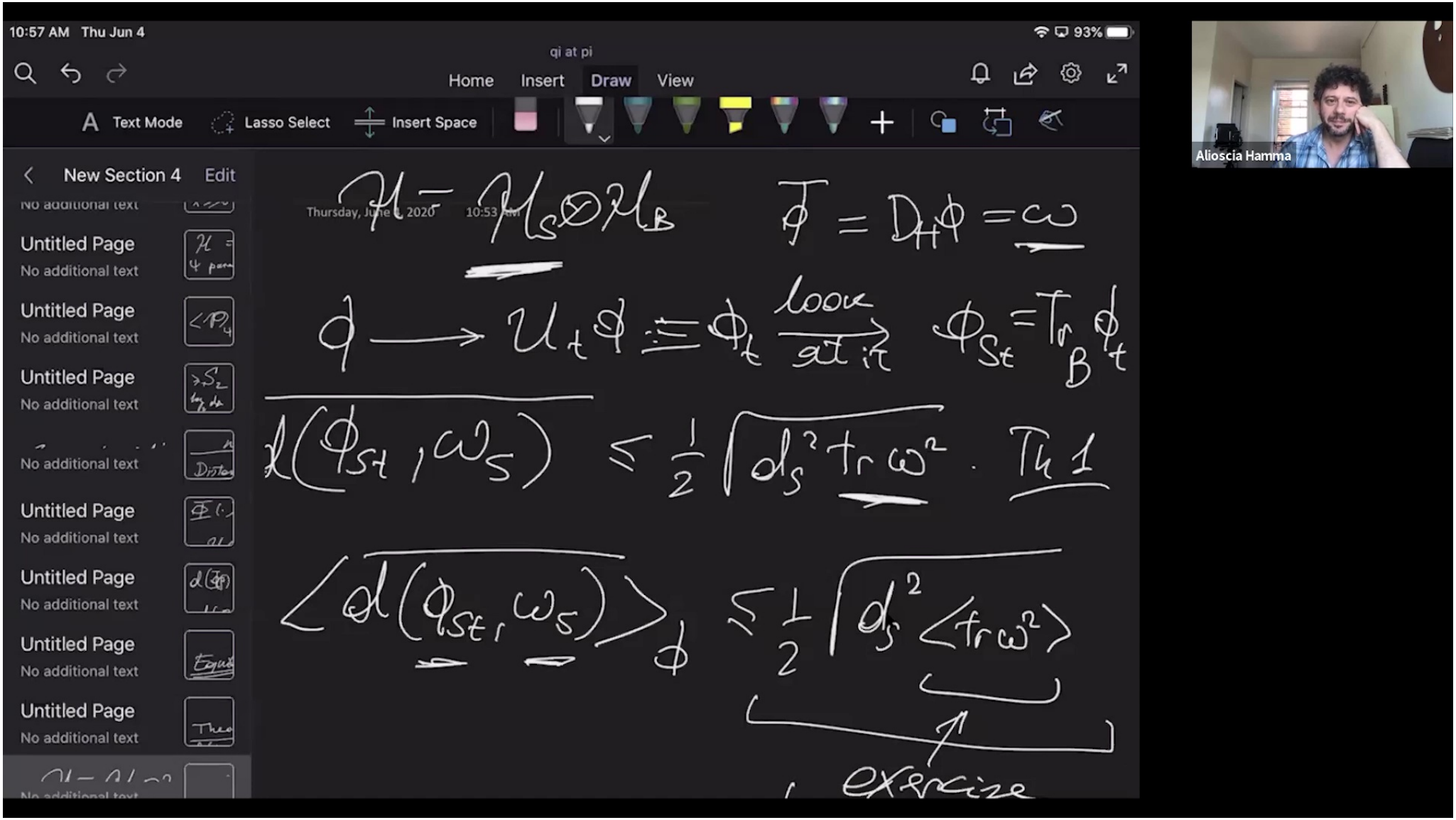

10:50 AM Thu Jun 4
95%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:49 AM
$n_B >> n_F$
$d_B = 2^{(n_B - n_F)} \sim$ large
$\frac{d_B}{d_F}$
Alioscia Hamma


10:50 AM Thu Jun 4
95%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:49 AM
$n_B >> n_S$
$\frac{ds}{dB} = 2^{(n_S - n_B)}$
super
small
Alioscia Hamma

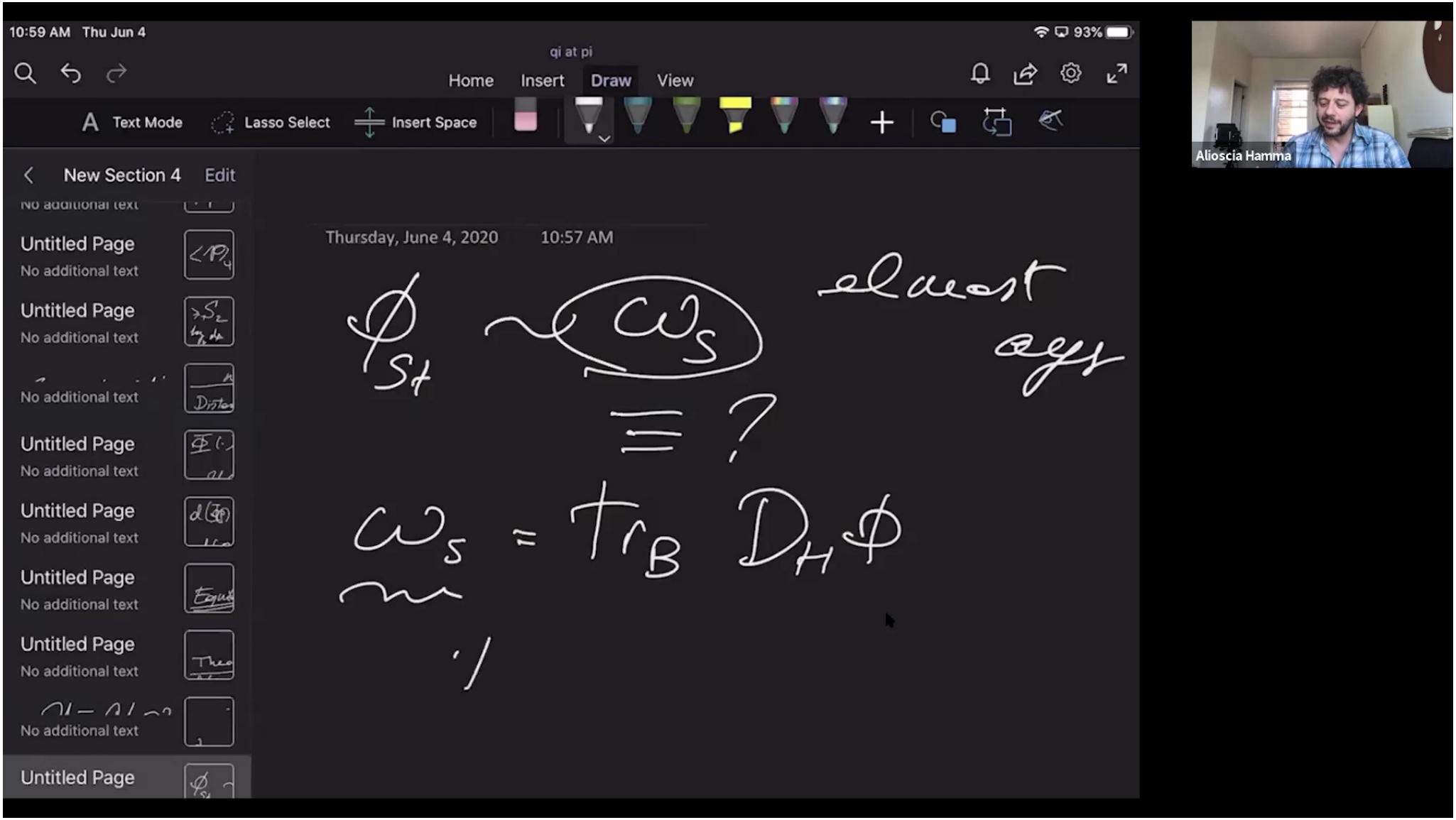

10:51 AM Thu Jun 4
95%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:44 AM
Theorem
$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B$
$d(\Phi_{St}, \omega_S) \leq \frac{1}{2}\sqrt{d_S^2 + r\omega^2}$
$r_F\omega^2 \sim \frac{1}{d}$
$\omega = D_H \Phi$
$d = d_S d_B$
$\leq \frac{1}{2}\sqrt{\frac{d_S^2}{d_S d_B}}$

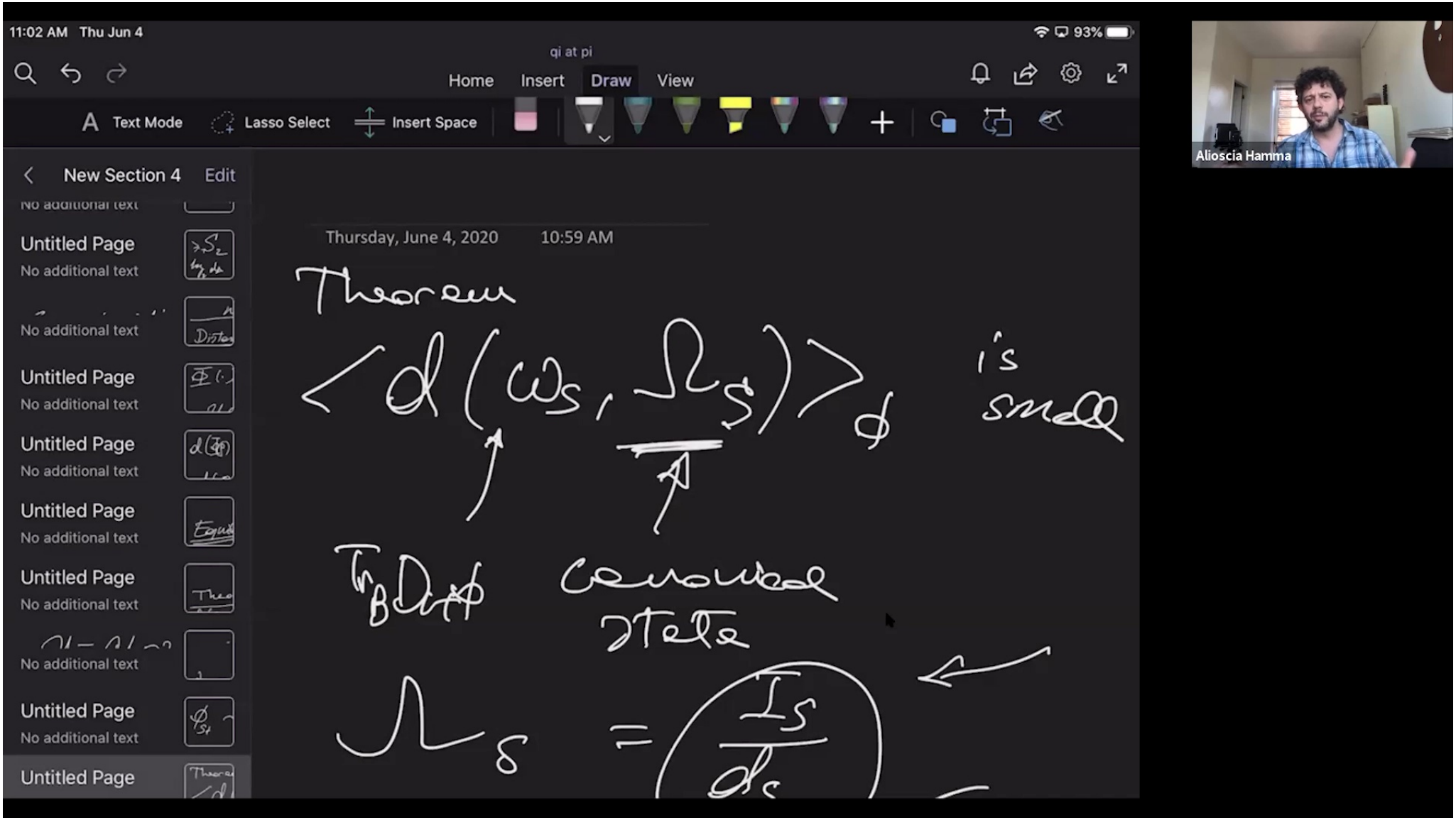
Alioscia Hamma

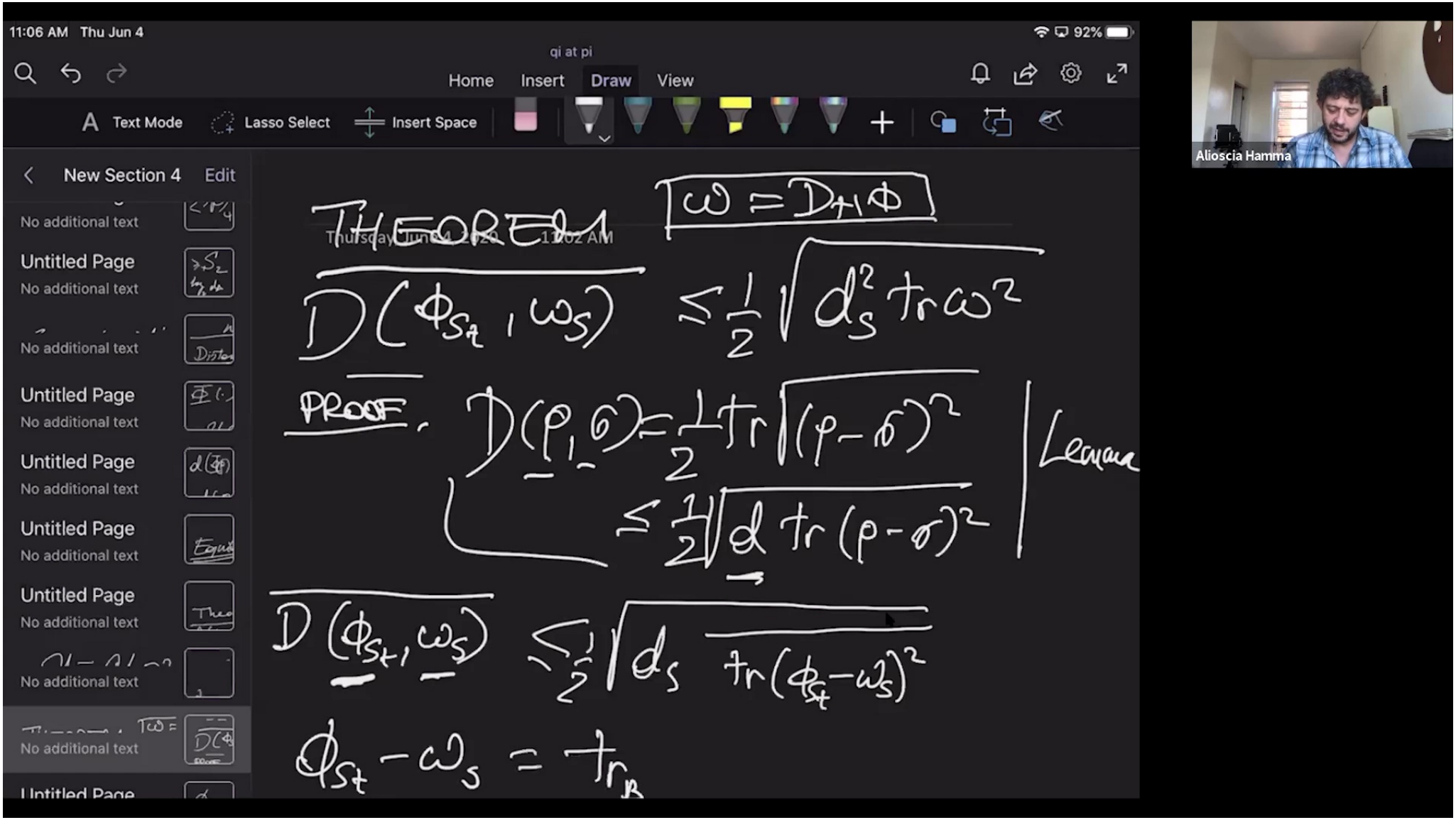

10:52 AM Thu Jun 4
94%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020 10:51 AM
Alioscia Hamma

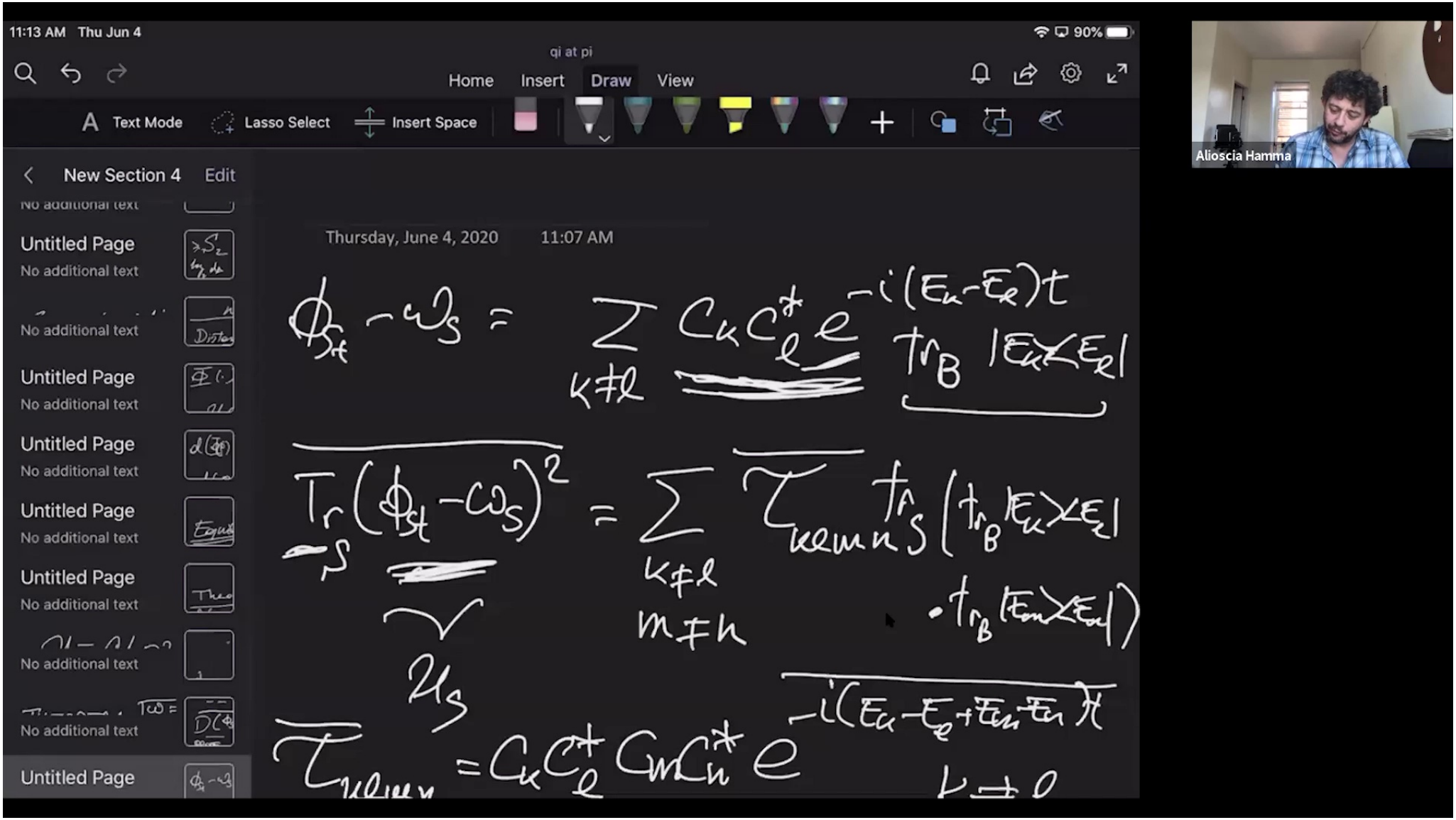

10:57 AM Thu Jun 4
93%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
exercise

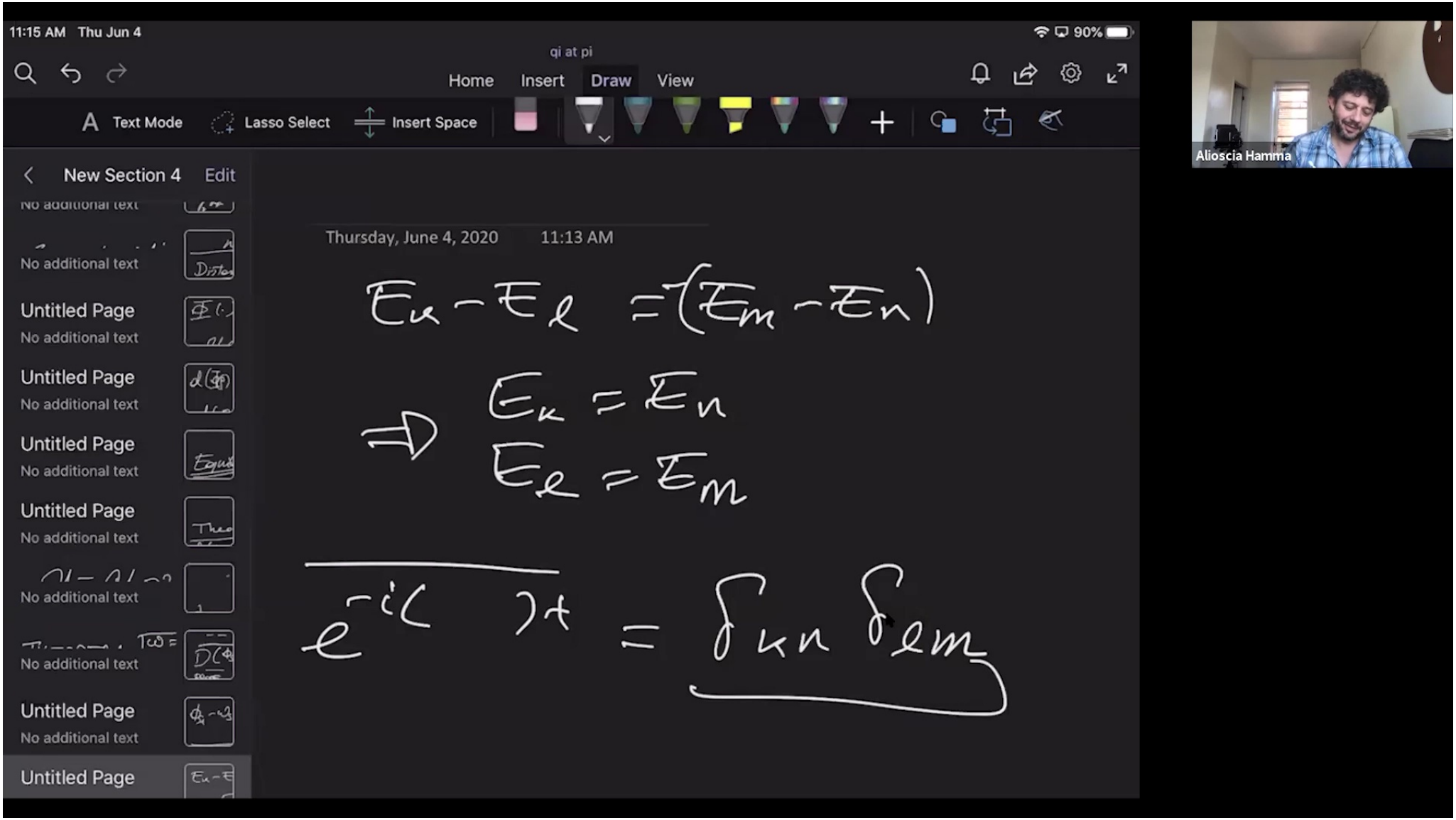
Alioscia Hamma

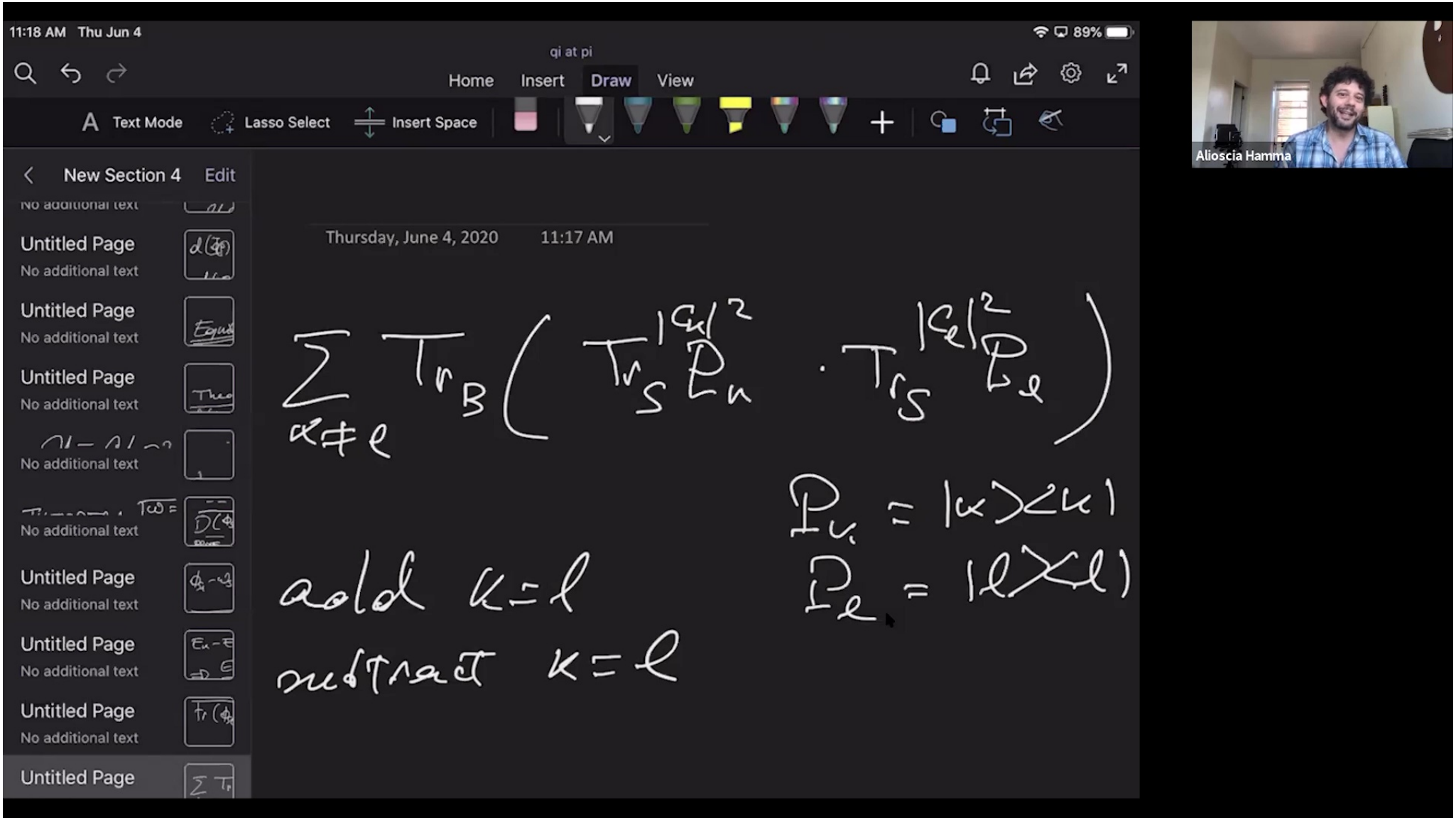

10:59 AM Thu Jun 4
93%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:57 AM
almost
ays
ω_s
$\equiv ?$
$\omega_s = Tr_B D_H \phi$
Alioscia Hamma


11:02 AM Thu Jun 4
93%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
10:59 AM
Theorem
is
small
Alioscia Hamma

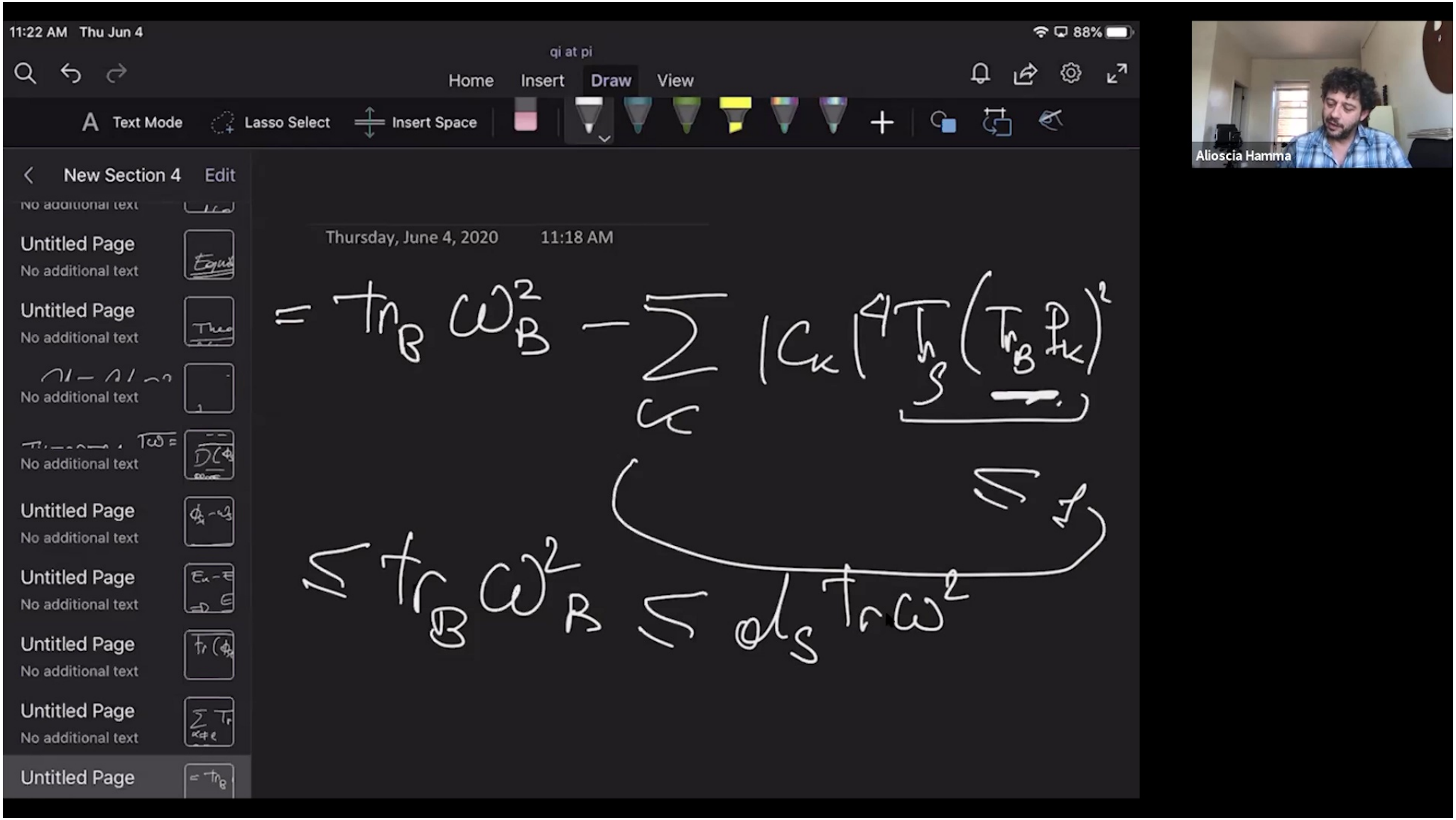

11:06 AM Thu Jun 4
92%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
No additional text
Untitled Page
No additional text
THEOREM
$\omega = D_{t+1}\Phi$
$D(\Phi_{S_t}, \omega_S) \leq \frac{1}{2}\sqrt{d_S^2 \operatorname{tr}\omega^2}$
PROOF.
$D(\rho,\sigma) = \frac{1}{2}\operatorname{tr}\sqrt{(\rho-\sigma)^2}$
$\leq \frac{1}{2}\sqrt{d \operatorname{tr}(\rho-\sigma)^2}$
Lemma
$D(\Phi_{S_{t+1}}, \omega_S) \leq \frac{1}{2}\sqrt{d_S \operatorname{tr}(\Phi_{S_t} - \omega_S)^2}$
$\Phi_{S_t} - \omega_S = \operatorname{tr}_B$
Alioscia Hamma

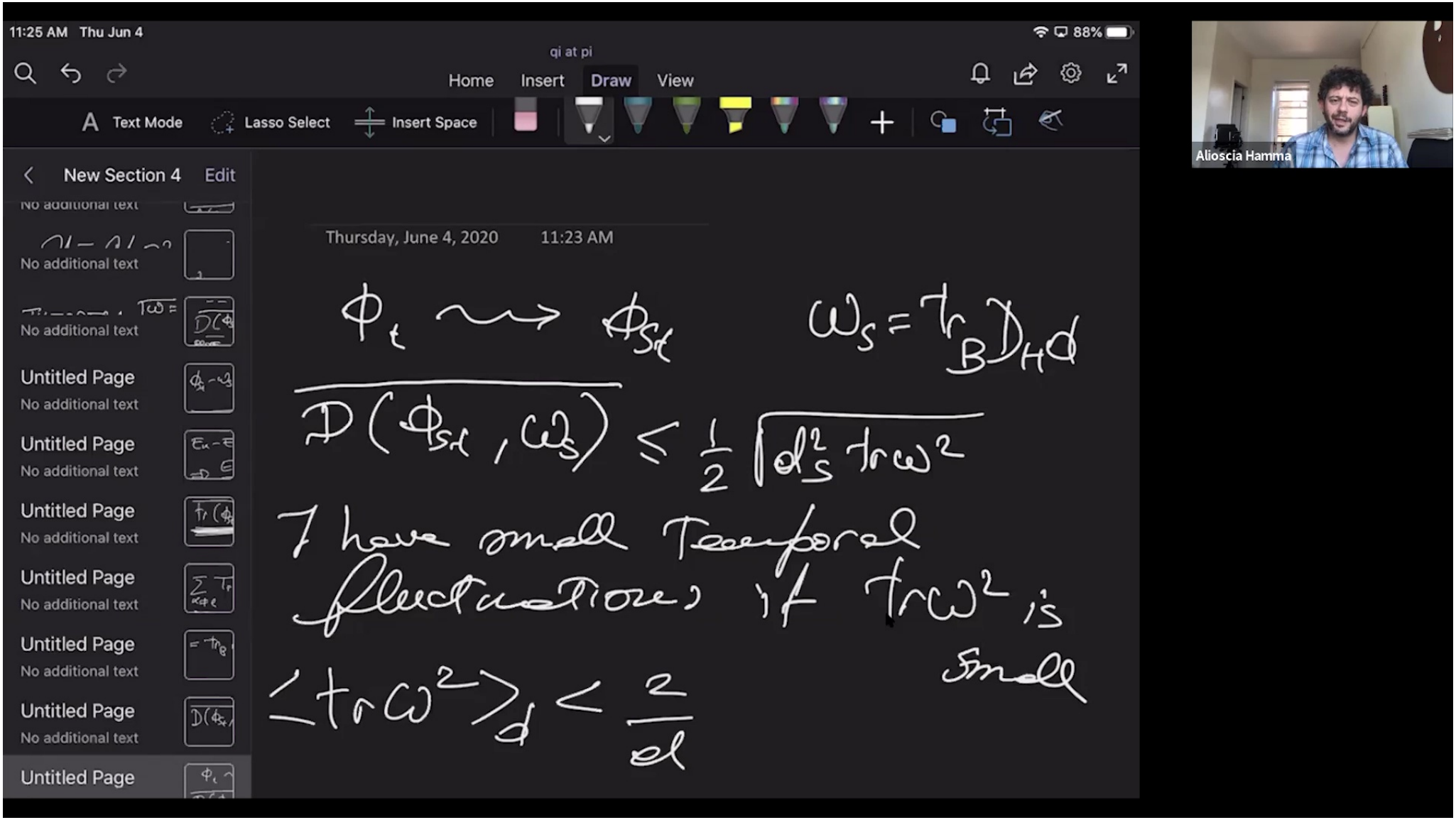

11:13 AM Thu Jun 4
90%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
11:07 AM
Alioscia Hamma


11:15 AM Thu Jun 4
90%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
No additional text
Untitled Page
No additional text
Thursday, June 4, 2020
11:13 AM
$E_k - E_l = (E_m - E_n)$
$\Rightarrow E_k = E_n$
$E_l = E_m$
$e^{-i(\quad)t} = \delta_{kn}\delta_{lm}$
Alioscia Hamma

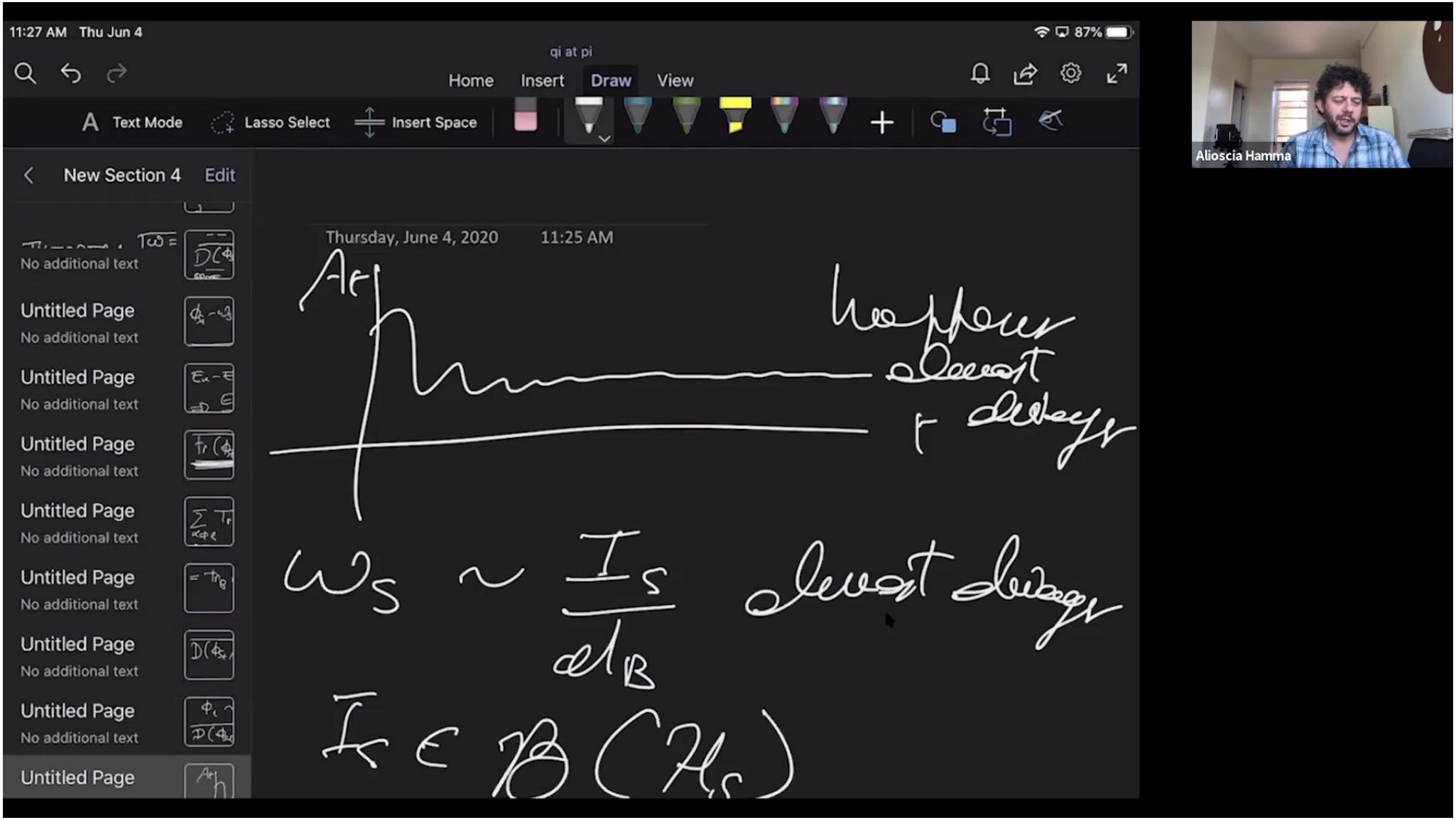

11:18 AM Thu Jun 4
89%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020 11:17 AM
$\sum_{k \neq l} Tr_B \left(Tr_S^{|\langle k|\psi\rangle|^2} P_k \cdot Tr_S^{|\langle l|\psi\rangle|^2} P_l \right)$
$P_k = |k\rangle\langle k|$
$P_l = |l\rangle\langle l|$
add $k = l$
subtract $k = l$

Alioscia Hamma


11:22 AM Thu Jun 4
88%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
Untitled Page
No additional text
Thursday, June 4, 2020
11:18 AM
Alioscia Hamma


11:25 AM Thu Jun 4
88%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
No additional text
Untitled Page
No additional text
Thursday, June 4, 2020 11:23 AM
$\Phi_t \rightsquigarrow \Phi_{St}$
$\omega_S = \mathrm{Tr}_B \rho_{HA}$
$\overline{D(\Phi_{St}, \omega_S)} \leq \frac{1}{2}\sqrt{d_S^2 \mathrm{Tr}\omega^2}$
I have small Temporal fluctuations if $\mathrm{Tr}\omega^2$ is small
$\overline{\langle \mathrm{Tr}\omega^2 \rangle_d} < \frac{2}{d}$

Alioscia Hamma


11:27 AM Thu Jun 4
87%
qi at pi
Home
Insert
Draw
View
Text Mode
Lasso Select
Insert Space
New Section 4
Edit
No additional text
Untitled Page
No additional text
Thursday, June 4, 2020
11:25 AM
Alioscia Hamma