Title: The Python's Lunch: geometric obstructions to decoding Hawking radiation

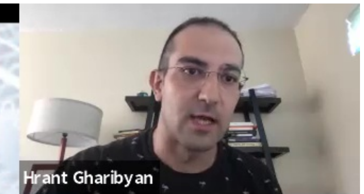
Speakers: Hrant Gharibyan

Series: Quantum Fields and Strings

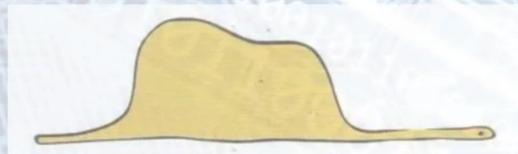
Date: May 21, 2020 - 3:00 PM

URL: http://pirsa.org/20050061

Abstract: Harlow and Hayden [arXiv:1301.4504] argued that distilling information out of Hawking radiation is computationally hard despite the fact that the quantum state of the black hole and its radiation is relatively un-complex. I will trace this computational difficulty to a geometric obstruction in the Einstein-Rosen bridge connecting the black hole and its radiation. Inspired by tensor network models, I will present a conjecture that relates the computational hardness of distilling information to geometric features of the wormhole - specifically to the exponential of the difference in generalized entropies between the two non-minimal quantum extremal surfaces that constitute the obstruction. Due to its shape, this obstruction was dubbed "Python's Lunch", in analogy to the reptile's postprandial bulge.



Python's Lunch:
geometric obstructions to decoding Hawking radiation



Adam Brown, **Hrant Gharibyan**, Geoff Penington, Leonard Susskind

arXiv: 1912.00228

Caltech

It from Qubit
Simons Collaboration on Quantum Fields, Gravity and Information

Hrant Gharibyan
IQIM Caltech
May 21, 2020 – IFQ Seminar

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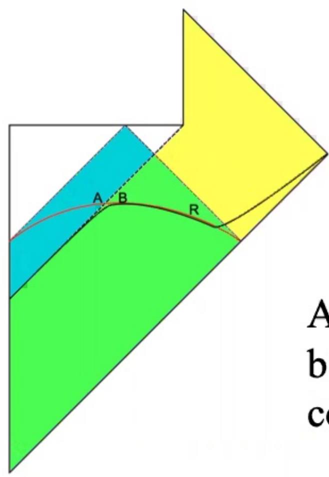
Harlow and Hayden (2013)

Daniel Harlow, Patrick Hayden,

“ Quantum Computation vs. Firewalls”; arXiv:1301.4504



Decoding the information that fell into black hole by only action on radiation is hard.

$$\text{Complexity} \sim e^{S}$$

Argument relies on the mixing assumption of black hole and radiation as well as complexity class reduction.

[Harlow-Hayden 2013; Aaronson 2014; Kim, Tang, Preskill et. al 2020]

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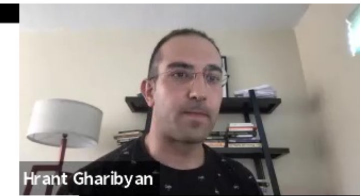


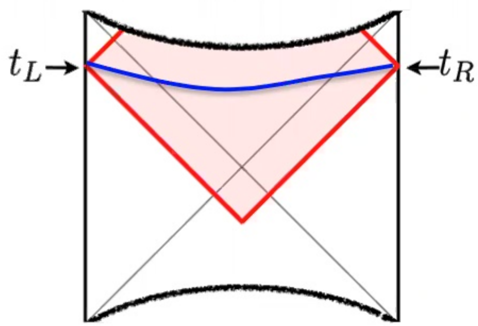
Susskind, Brown, and et. al (2014)

A. Brown, D. Roberts, L. Susskind, B. Swingle, Y. Zhao

"Complexity Equals Action"; arXiv: 1509.07876

D. Stanford, L. Susskind,

"Complexity and Shock Wave Geometries"; arXiv: 1406.2678



$$|\psi(t_L, t_R)\rangle$$

State Complexity = Volume of ER Bridge

State Complexity = Action of WDW Patch

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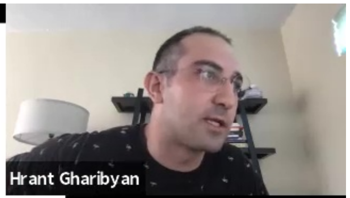
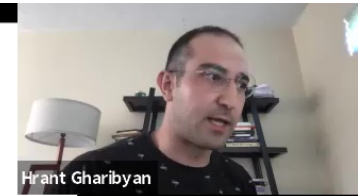




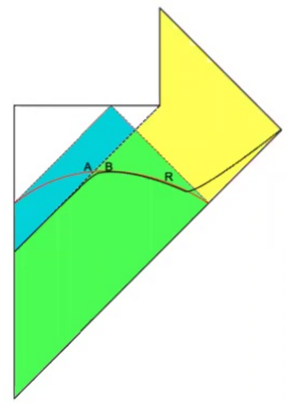
Apparent Contradiction

For evaporating black hole volume/action is polynomial and those complexity is polynomial.

ploy(S)

Harlow and Hayden suggested it is exponentially hard to decode the radiation.

exp(S)

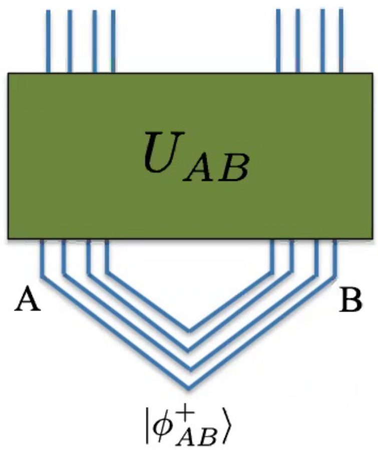


Where is this contradiction coming from?

Restricted vs Unrestricted Complexity

We are talking about two different notions of complexity

\- - - - - - unrestricted vs restricted - - - - - -



Unrestricted complexity

$$U_{AB} = g_1 g_2 \cdots g_C$$

Restricted complexity

$$V_A = g_1 g_2 \cdots g_{C_A}$$

$$V_A|\phi^+_{AB}\rangle \approx U_{AB}|\phi^+_{AB}\rangle$$

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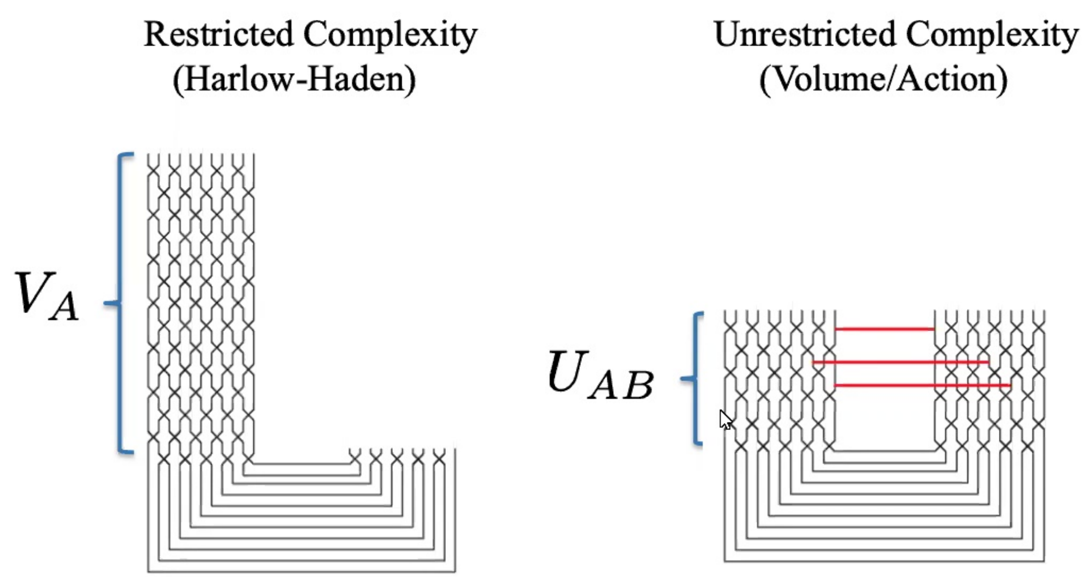
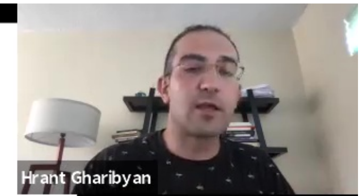
Restricted vs Unrestricted Complexity





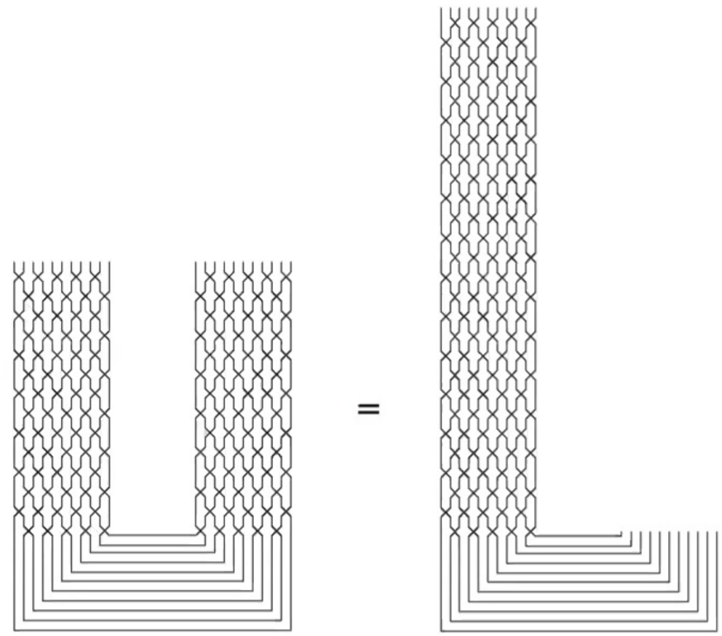
When are restricted and unrestricted complexities very different?

Time evolution of the thermofield double.

Restricted Complexity

=

Unrestricted Complexity



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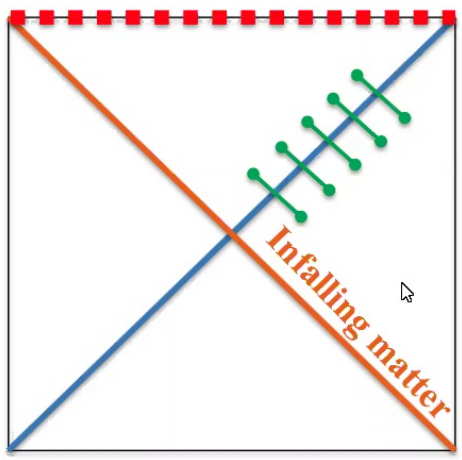
When are restricted and unrestricted complexities very different?

What about **evaporating black hole?**



Radiation and black hole modes are **coupled**!

A bit after **Page time**,
volume/action is polynomial in **S**.

Restricted complexity is **exponential**.

What the difference between semi-classical gravity thermofield double and evaporating black hole?

When are restricted and unrestricted complexities very different?

Python's lunch

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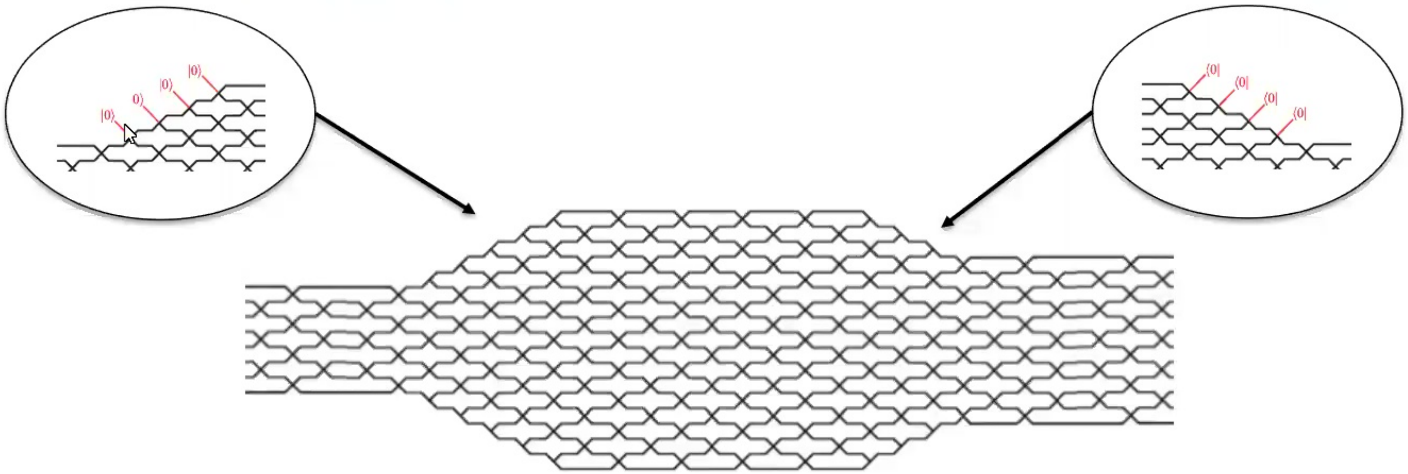
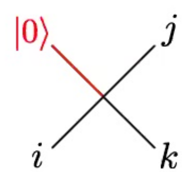
The Python's Lunch in Tensor Network



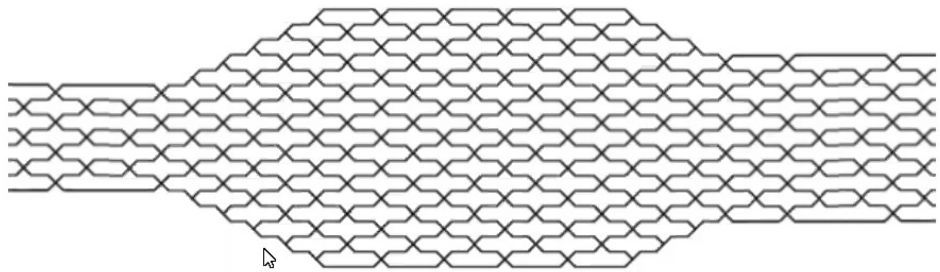
Tensor network is built off **unitary gates** and has **isometries** in endpoints.

Isometry:



The Python's Lunch in Tensor Network

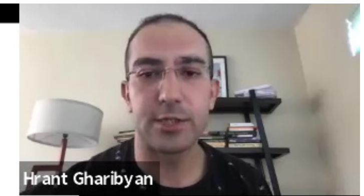
$|\psi_{in}\rangle$



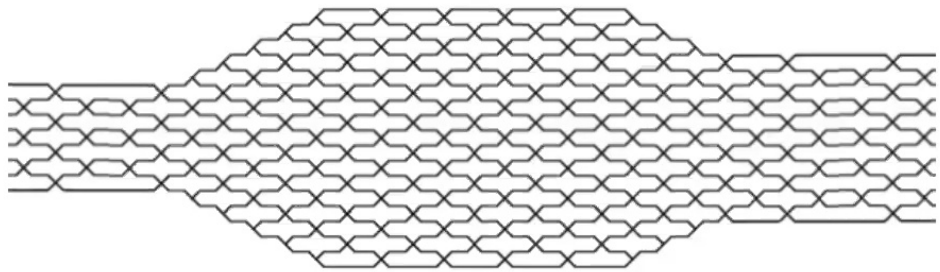
$|\psi_{out}\rangle$

$$|\psi_{out}\rangle \propto \langle 0|^{m_R}\, U_{TN}|\psi_{in}\rangle|0\rangle^{m_L}$$

Ancillary Input Qubits



The Python's Lunch in Tensor Network

$|\psi_{in}\rangle$  $|\psi_{out}\rangle$

$$|\psi_{out}\rangle \propto \langle 0|^{m_R}\, U_{TN} |\psi_{in}\rangle |0\rangle^{m_L}$$

Post-selection

Polynomial Complexity Unitary

Ancillary Input Qubits

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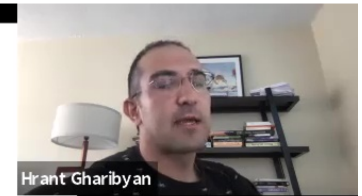
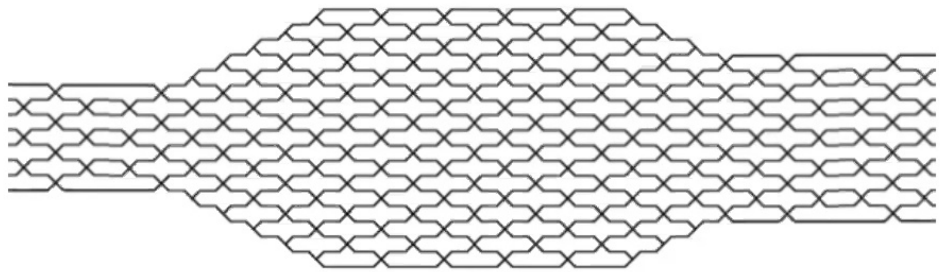


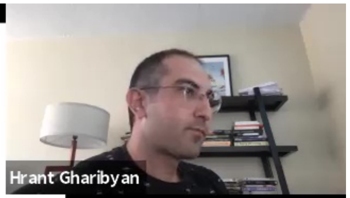
The Python's Lunch: NO POST-SELECTION

$|\psi_{in}\rangle$  $|\psi_{out}\rangle$

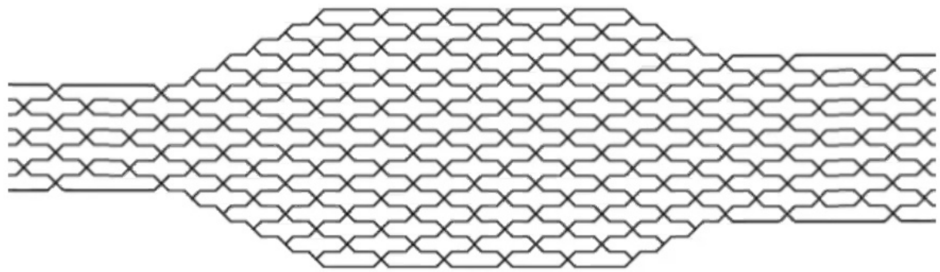
$$|\psi_{out}\rangle|0\rangle^{m_R} = U_{PL}|\psi_{in}\rangle|0\rangle^{m_L}$$

Unitary

Ancillary Input Qubits



The Python's Lunch: NO POST-SELECTION

$|\psi_{in}\rangle$  $|\psi_{out}\rangle$

$$|\psi_{out}\rangle|0\rangle^{m_R} = U_{PL}|\psi_{in}\rangle|0\rangle^{m_L}$$

How complex is U_{PL}?

Unitary

Ancillary Input Qubits

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What's the complexity of removing post-selection / final state projection?

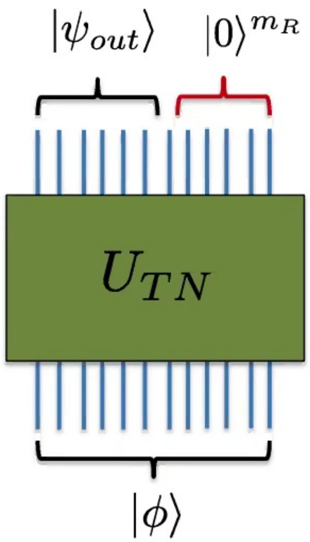
We can measure the qubits and hope to get the right answers. Probability of the right answer is $1/2^m$ and expected complexity

$$C \propto 2^{m_R} \cdot C_{TN}$$

It is not a fixed unitary and complexity can be improved.

We can do a generalized **Grover-like search.**

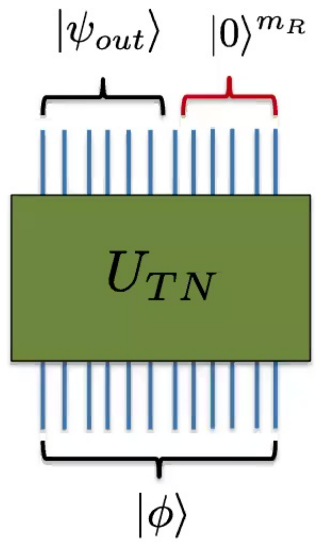
Grover-like search: state dependent



$$U_\phi = 2|\phi\rangle\langle\phi| - I$$

$$V = U_{TN}(2|0\rangle\langle 0|^{m_R} - I)U_{TN}^\dagger$$

Grover-like search: state dependent



$$U_\phi = 2|\phi\rangle\langle\phi| - I$$

$$V = U_{TN}(2|0\rangle\langle 0|^{m_R} - I)U_{TN}^\dagger$$

Repeat sequence: $U_\phi V$

$l = \sqrt{2^{m_R}}$ **~ times**

[Kitaev - Yoshida 2017]

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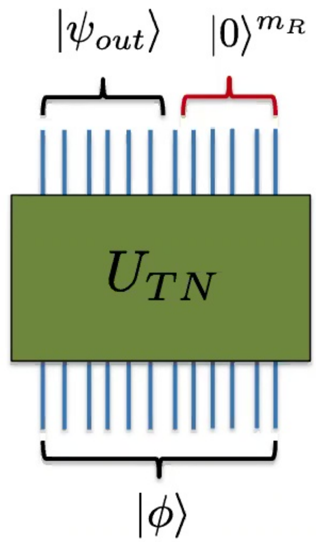
Grover-like search: state dependent



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$$V = U_{TN}(2|0\rangle\langle 0|^{m_R} - I)U_{TN}^\dagger$$

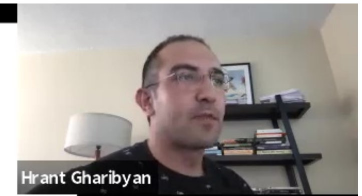
Repeat sequence: $U_\phi V$

$l = \sqrt{2^{m_R}}$ **~ times**

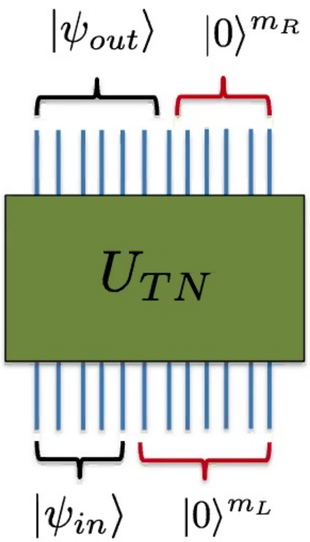
$$U_{PL} = U_{TN}\left[U_\phi V\right]^l$$

[Kitaev - Yoshida 2017]

This algorithms depends on the state $|\phi\rangle$



Grover-like search: state independent



$$U = 2|0\rangle\langle 0|^{m_L} - I$$

$$V = U_{TN}(2|0\rangle\langle 0|^{m_R} - I)U_{TN}^{\dagger}$$

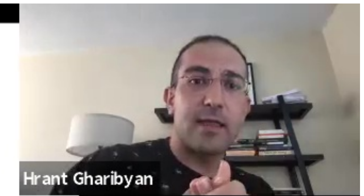
$$U_{PL} = U_{TN}\Big[UV\Big]^{l}$$

$$\mathcal{C}(U_{PL}) \propto \sqrt{2^{m_R}} \cdot C_{TN}$$

[Berry et. al 2015, Gilyen et. al 2017]

Is Grover search the optimal strategy?

In special cases, for a specific unitary there might be an algorithm that can do better than Grover-like search.

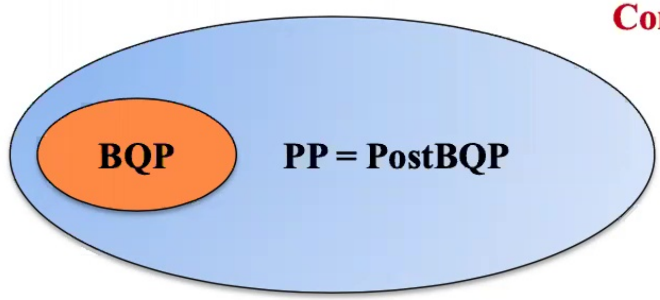


Is Grover search the optimal strategy?

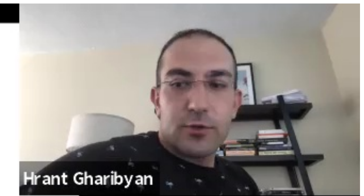
In special cases, for a specific unitary there might be an algorithm that can do better than Grover-like search.

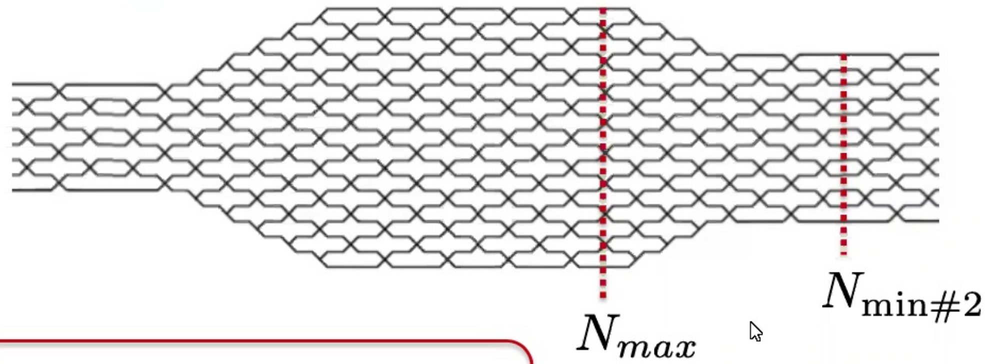
However, for generic, **scrambling tensor network** U_{TN} polynomial algorithm is extremely unlikely to exist.

Complexity class collapse!



Conjecture: restricted complexity in tensor network



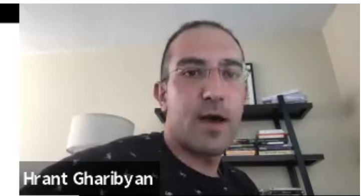


Minimax: minimal of all maximal cuts for all the possible slicing.

Largest minimal cut

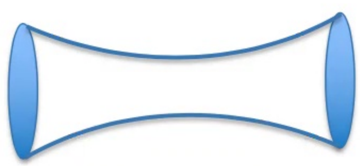
The restricted complexity of scrambling tensor network is

$$\mathcal{C}_R \propto C_{TN} \cdot \exp\left[\frac{1}{2}\left(N_{\max} - N_{\min\#2}\right)\right]$$



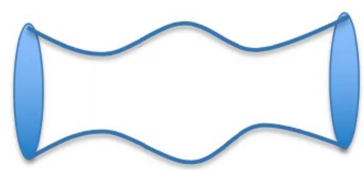
Conjecture: restricted complexity in gravity

Small (polynomial) restricted complexity



Time evolved thermo field double (TFD)

Large (exponential) restricted complexity

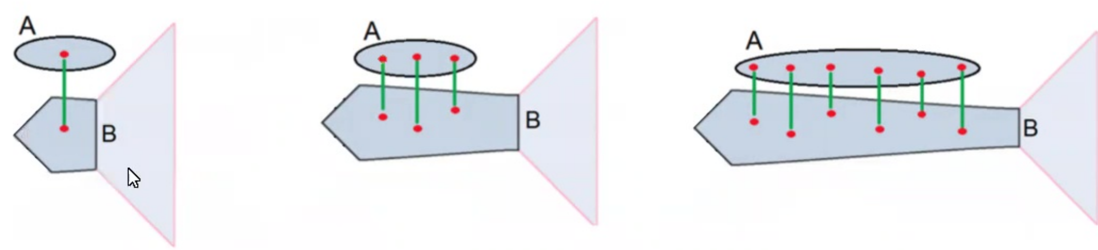


Python's lunch geometry

Evaporating Black Hole

Python's lunch in Evaporating Black Hole

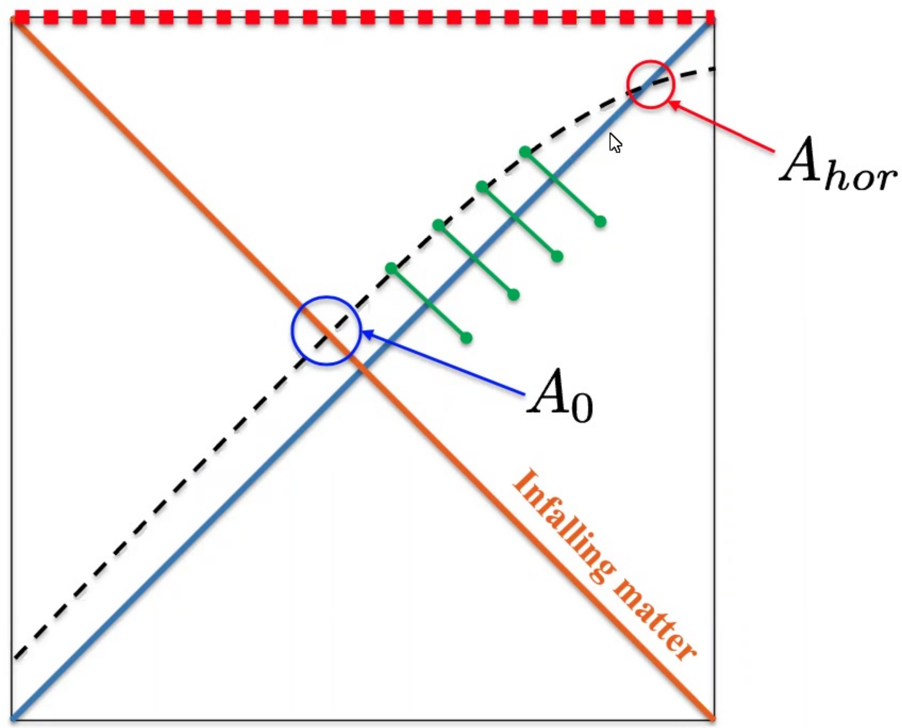
Naïve picture of evaporation process



A is the radiation modes. **B** is the remaining black hole, some nice Cauchy slice going into the horizon.

Hrant Gharibyan

Python's lunch in Evaporating Black Hole



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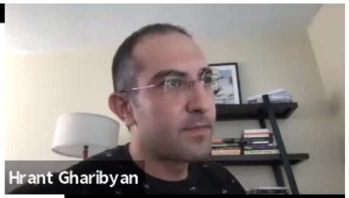
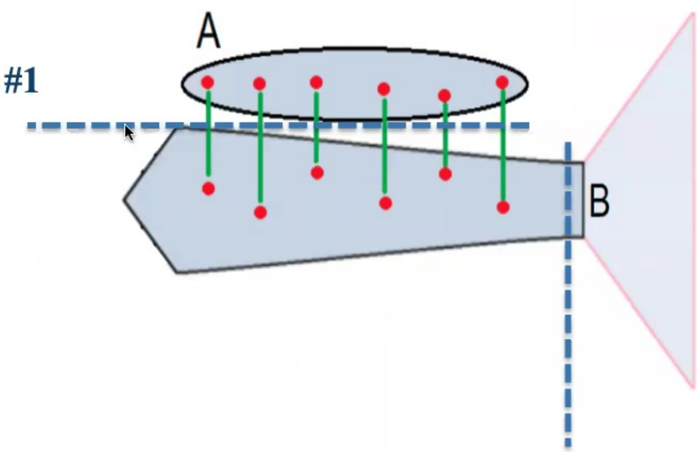


Python's Lunch in Evaporating Black Hole

Minimal Cut #1

$$S_{rad}$$



Page time

$$S_{rad} = \frac{A_{hor}}{4\hbar G}$$

Minimal Cut #2

$$\frac{A_{hor}}{4\hbar G}$$

What about the maximal cut?

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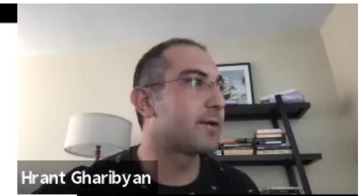
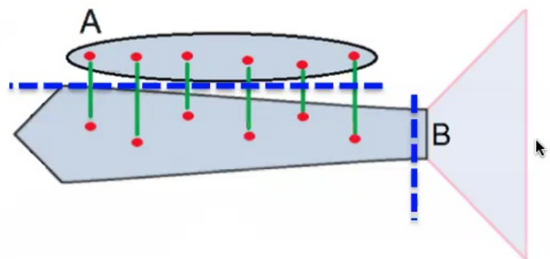


Covariant definition of python's lunch



Both **minimal cuts** **are quantum extremal surfaces (extremize the generalized entropy).**
[Engelhardt, Wall 2014]

Maximin prescription: **find the surfaces with smallest and second smallest generalized entropy then maximize over choice of Cauchy surfaces.**
[Wall 2012, AEPU 2019]

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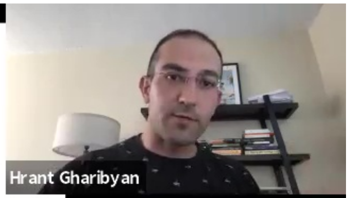
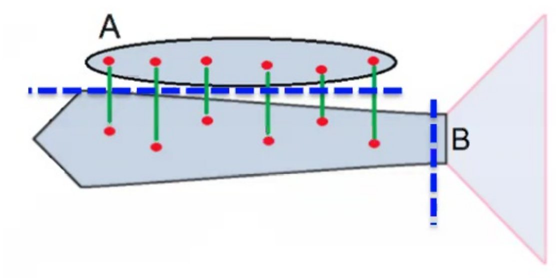


Covariant definition of python's lunch



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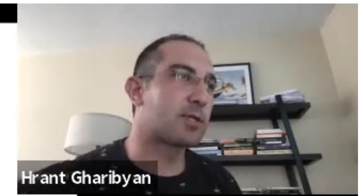
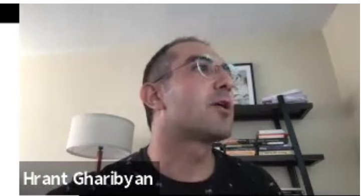
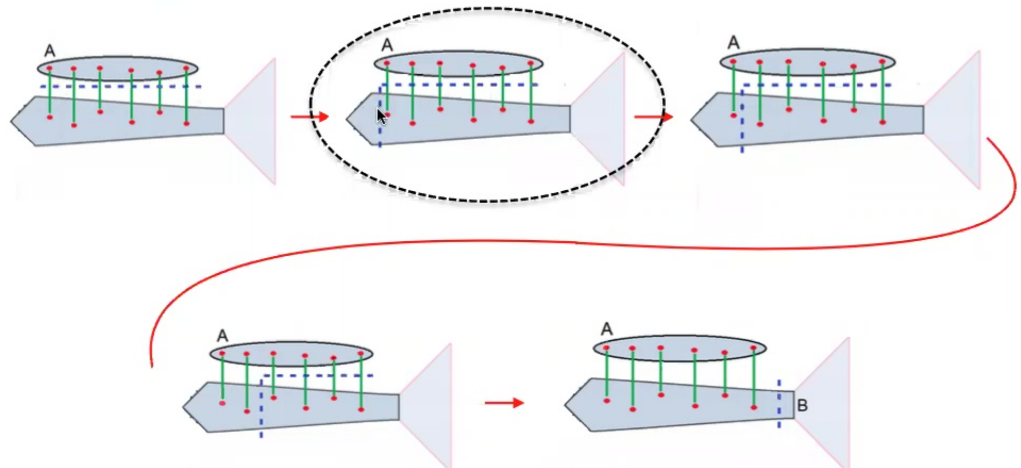




Python's lunch search for maximal cut

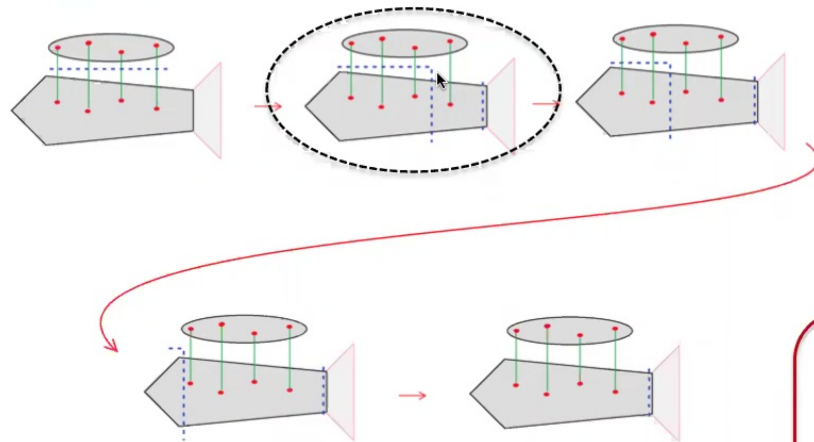


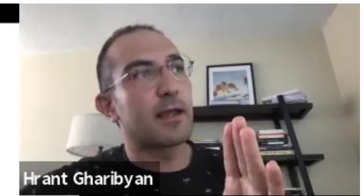
Before half time, we will be to forward slicing. The maximal generalized entropy.

$$S_{max}^{(gen)} = S_{rad} + \frac{A_0}{4\hbar G}$$

***Minimax*: *find* minimal of maximums of all slices.**

Python's lunch search for maximal cut





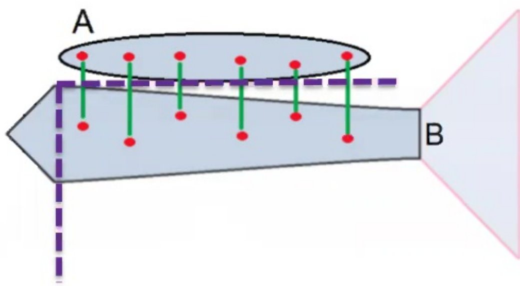
Half time

$$A_{hor} = \frac{A_0}{2}$$

After half time, we will perform reverse slicing.

$$S^{(gen)}_{max} = S_{rad} + 2\frac{A_{hor}}{4\hbar G}$$

Covariant definition of python's lunch



Minimax: **maximal cut for each Cauchy slice find the minimum of the all maximal cuts for all possible slicing methods. Analogous to the tensor network story.**

Maximinimax: **maximize again over all Cauchy surfaces, giving non-minimal quantum extremal surface.**

We find this cuts explicitly for **JT gravity plus non-interacting fermions** and verify the picture from the nice Cauchy slice.

Conjecture: restricted complexity for evaporating black hole

The restricted complexity of evaporating black hole is

$$C_R \propto C_{TN} \cdot \exp\left[\frac{1}{2}\left(S^{(gen)}_{max} - S^{(gen)}_{min}\right)\right]$$

- C_{TN}: **Volume/Action (unrestricted complexity)**
- $S^{(gen)}_{max}$: **Maximinimax prescription**
- $S^{(gen)}_{min}$: **Maximin prescription**

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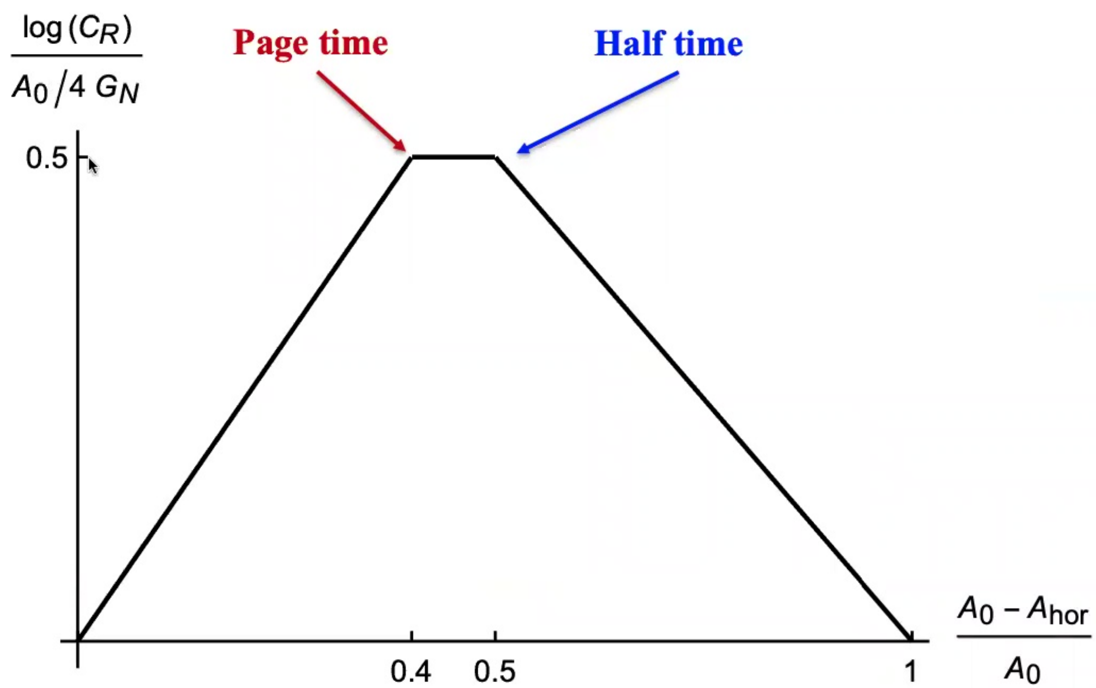
Restricted complexity for evaporating black hole



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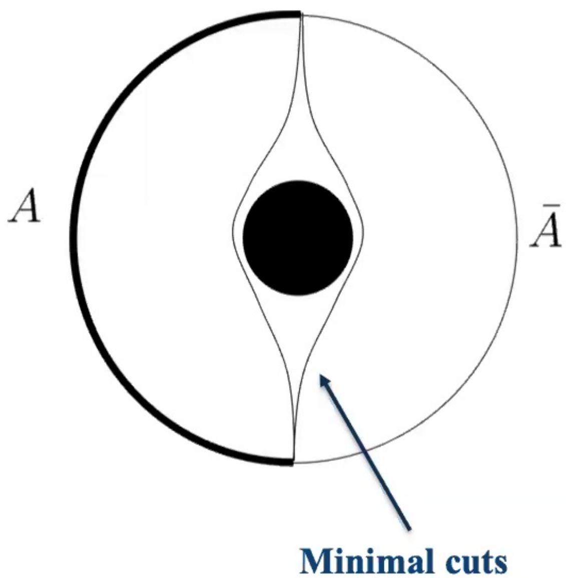
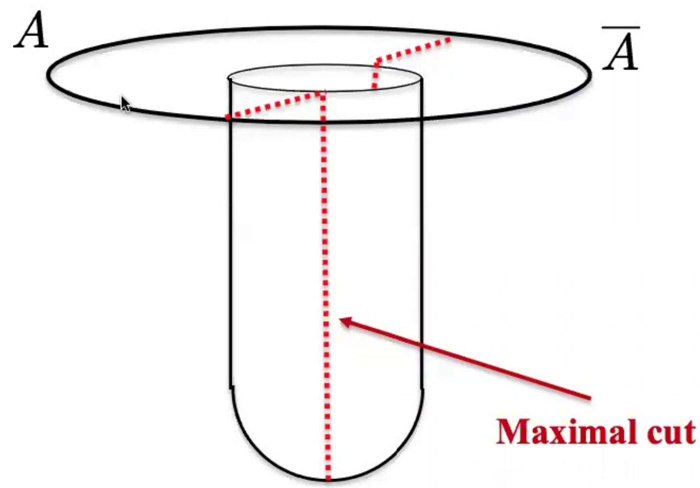
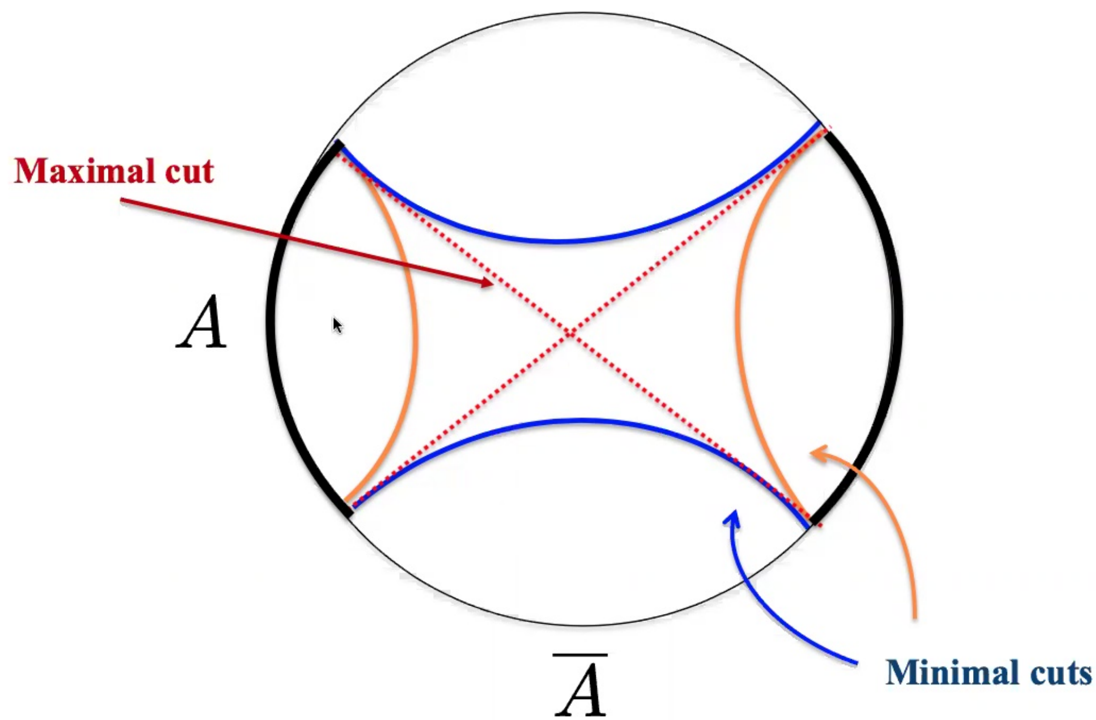
Python's lunch in pure state black hole





Python's lunch without black holes – empty AdS



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Final remarks

Summary: The restricted complexity of python's lunch is

$$C_R \propto C_{TN} \cdot \exp\left[\frac{1}{2}\left(S_{max}^{(gen)} - S_{min}^{(gen)}\right)\right]$$

Future directions

- Study other examples when python's lunches and non-minimal quantum extremal surfaces appear; pure state black holes and empty AdS.
  - What does this imply about reconstruction of operators in the python's lunch?
  - Relationship of python's lunch to complexity of holographic dictionary and quantum extended Church-Turing thesis and.
- Understand the connection between python's lunch, Petz map and Island formula.