Title: Sigma-VOA correspondence

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Abstract: In this talk I will discuss an interesting phenomenon, namely a correspondence between sigma models and vertex operator algebras, with the two related by their symmetry properties and by a reflection procedure, mapping the right-movers of the sigma model at a special point in the moduli space to left-movers. We will discuss the examples of N=(4,4) sigma models on T^4 and on $K3$. The talk will be based on joint work with Vassilis Anagiannis, John Duncan and Roberto Volpato.

Σ - VOA Correspondence

w. V. Anagiannis, J. Duncan, R. Volpato, to appear soon

I.1. mm

$H:\mathbb{H}\to\mathbb{C}$

$$F_2^{(2)}(\tau) := \sum_{\substack{r>s>0 \\ r\equiv s+1\,(2)}} (-1)^r \, q^{\frac{rs}{2}}$$

$$H(\tau) := -2E_2(\tau)+48F_2^{(2)}(\tau)$$

$\hat{H}$: transforms like a wt 1/2 mf

<u>Obs</u> (EOT, 2010): 45, 231, 770, ...

$= \dim$ irreps M_{24} R_{24}

$\cap$

S_{24}

Thm (Gannon) such R exists.

Q: How to construct R?

Umbral Moonshine

Recall, 24 rk 24 lattices even, self-dual, (+)-def.

$\supset \Lambda_{\text{Leech}}$

II.2 MM & K3

Def. $(EG,\ X;\ CY)$

$N=2$ $\Sigma(X;\mu)$

$\cancel{EG(\tau,z;X)}$

$$\mathrm{Tr}\ (-1)^{J_0+\bar J_0}\, y^{J_0}\, q^{L_0-\frac{c}{24}}\, \bar q^{\bar L_0-\frac{\bar c}{24}}$$

Note: M_{24} does not act on $\Sigma(MB)$

$EG = 20\, ch_{0,0} - 2\, ch_{0,\frac{1}{2}} + \cdots$

Thm: (Kondo

Thm (Kondo)

$\underline{S_{grp}(M_{23})} = \{$ grp of hyper-Käh preserving sym. of any K3 $\}$

II. $V^{s\natural}$ ~ ~~SVOA~~, $\mathrm{Aut}(V^{s\natural}) = Co_0$

Can define a trace on ~~it~~ ~~sth~~

$$\parallel$$

$$EG(K3)$$

(Duncan - Mack-Crane)

$$V^{s\natural} \cong \underline{\mathbb{Z}_2}\text{-orb}\,(V^{s}_{E_8}) \cong \underline{\mathbb{Z}_2}\text{-orb}\,(24 \text{ chiral fermions})$$

$(x^i \to -x^i)$ $\qquad$ $(\psi^i \to -\psi^i)$

$$\phi(\tau, z) = \mathrm{Tr}_{V^{SS}_{tw}}\ (-1)^{F+1}\, y^{J_0}\, q^{L_0 - \frac{c}{24}}$$

$NS \oplus R$

$$= \frac{1}{2}\left\{ q^{-\frac{1}{2}} \prod_n (1 + y q^{n-\frac{1}{2}})^2 (1 + y^{-1} q^{n-\frac{1}{2}})^2 (1 + q^{n-\frac{1}{2}})^{20} \right.$$

$$- q^{-\frac{1}{2}} \prod (\ \ )$$

$$- q\,(1+1)^{10}\,(y^{\frac{1}{2}} + y^{-\frac{1}{2}})^2 \prod (1 + y q^n)(1 + y^{-1} q^n)(1 + q^n)^{2}$$

Great!

But ① ~~full sol. to M_{24}~~

② Not all $EG_g(\tau, z; K3; \mu) = \phi_g$

eg. $\pi_g^{(K3)}$ ... 8 4 ...

[redacted]

Correspondence.

1. Sym : group

$$\mathrm{Aut}(\Sigma'(X;\mu)) \subset \mathrm{Aut}(\nu(X)) \quad \forall \mu \in M_X$$

defined to preserve some structure

e.g. SUSY

3. $\boxed{\text{Reflection}}$

$\exists\ \mu^* \in \mathcal{M}_X$, s.t.

$$\Sigma'(X;\mu^*) \xrightarrow{\text{reflect}} \tilde{\Sigma}'(X;\mu) \simeq V(X)$$

In particular ~~$\Sigma'(X;\mu^*)$~~

$$\Sigma'(X;\mu^*) \simeq V(X)$$

Examples:

X	$V(X)$
$\Sigma'(T^4)$	$\longleftrightarrow V^s_{E_8}$
	$\big\downarrow \mathbb{Z}_2$
$\Sigma'(K3)$	$\longleftrightarrow V^{s\natural}$

chiral alg: $U(1)^4$ $\subset$ translations

$SO(4)^L_1$: $:\psi^a \psi^b:$

$SO(4)^R_1$

σ-model

$$SO(4)^L = \left(SU(2)^L_J \times \underline{SU(2)^L_A} \right) / (-1)^{J_0 + A_0}$$

Thm 2.8

$$EG_g(\tau, z; T^4; \mu) = \phi_{g_D}(\tau, z) \quad \forall g \in G_0(4)$$
$$\forall \mu \in \mathbb{H}_4$$

Note: $EG(\tau, z; T^4) = 0$ but $EG_g(\tau, z; T^4)$ can be non-zero.

Thm 3 :

Thm 4: $g \in \mathrm{Aut}(\Sigma(T^4;\mu))$

$$EG\big(g\text{-orb}(\Sigma(T^4;\mu))\big)(\tau,z) = \begin{cases} 0 = EG(T^4) \\ EG(K3) \end{cases}$$

iff. $$PF\big(g_V\text{-orb}(V^s_{E_8})\big) = \begin{cases} PF(V^s_{E_8}) \\ PF(V^{s\natural}) \end{cases}$$

Lore: T is $N=(4,4)$ SCFT $c=(6,6)$, w. 4 gener. spectral