Title: Twisted superconformal algebras and representations of higher Virasoro algebras

Speakers: Brian Williams

Collection: Elliptic Cohomology and Physics

Date: May 27, 2020 - 11:00 AM

URL: http://pirsa.org/20050056

Abstract: (Super)conformal algebras on two-dimensional spacetimes play a ubiquitous role in representation theory and conformal field theory. In most cases, however, superconformal algebras are finite dimensional. In this talk, we introduce refinements of certain deformations of superconformal algebras which share many facets with the ordinary (super) Virasoro algebras. Representations of these refinements include the higher dimensional Kacâ€"Moody algebras, and many more motivated by physics. Finally, we will show how these algebras enhance to higher dimensional chiral/factorization algebras which upon ``localization" deform to well-studied chiral algebras on Riemann surfaces.

9:41 AM Tue Jan 9 100%

Pi - elliptic
May 25, 2020 at 10:10 AM

Brian Williams
University of Edinburgh

Higher dimensional Virasoro algebras and superconformal symmetry

①

arXiv

1910.04120 } w/ Ingmar Saberi
1906.04221 } [SW]

1810.06534 } w/ Owen Gwilliam [GW]

Superconformal algebras describe symmetries of physical systems in d-dimensional spacetime.

$d = 2$ Superconformal algebras

$N = 1, 2, 4$

- Super enhancements of Virasoro Lie algebra.

Symmetry of states of the superstring.

- $N=2$ Superconformal index = elliptic genus.
  (From BV quantization w/ Matt Szczesny and James Ladouce (BU))
- Symmetry alg of familiar vertex algebras (CDR on CY, Chiral polyvector fields, ...)

$\underline{d>2}$ superconformal algebras

- Superconformal index still a good invariant.
  (Witten index = $S^3 \times S^1$ partition f^n
  Higher dim^l versions as well.)
- All <u>finite</u> <u>dimensional</u>.

☆ Much of the discussion today makes sense in arbitrary (complex)

9:41 AM Tue Jan 9

- All finite dimensional.

☆ Much of the discussion today makes sense in arbitrary (complex) dimensions but we focus on $\frac{1}{2}\dim_{\mathbb{R}} = \dim_{\mathbb{C}} = 2$.

②

After a holomorphic twist, 4d superconformal algebras admit ∞-dim^l refinements. (Higher dim^l Virasoro algebras)

- **Holomorphic twist**: Let $\mathfrak{g}$ = super Lie algebra

Data: An odd element $Q \in \mathfrak{g}$ s.t.

Virasoro algebras)

- Holomorphic twist : Let $\mathfrak{g}$ = super Lie algebra

Data : An odd element $Q \in \mathfrak{g}$ s.t.

$$[Q, Q] = 0 \in \mathfrak{g}.$$

Given a (dg) rep^n $M \overset{\rho}{\circlearrowleft} \mathfrak{g}$, can twist

$$(M, d) \rightsquigarrow (M, d + \rho(Q))$$

twist

SUSY theory $\rightsquigarrow$ module for the $N=k$ super Lie algebra

$$\mathfrak{g}_{N=k} = \mathbb{C}^4 \oplus \Pi \Sigma_{N=k}$$

complexified translations

$$\{\partial_{x_i}\}_{i=1,\dots,4}.$$

Spin(4)-rep.

$$\Sigma_{N=k} = (S_+ \oplus S_-) \otimes \mathbb{C}^k.$$

- Mathematical formulation of (holomorphic) twisting is due to Costello, w/ Elliott-Safronov have characterized the hol. twist of SYM in dimensions $2 \leq d \leq 10$.

Fact: In 4d SUSY $(N=1,2,4)$ there exists supercharges

9:41 AM Tue Jan 9 100%

dimensions $2 \leq d \leq 10$.

(3)

Fact: In 4d SUSY $(N = 1, 2, 4)$ there exists supercharges

$Q \in \Pi\Sigma$, such that:

$[Q, Q] = 0$

$$\mathrm{Im}([Q, -]) \cong \mathbb{C}^2 \subseteq \mathbb{C}^4 = \text{space-time translations.}$$

This is called a holomorphic supercharge. Choose complex coord's $(z_1, z_2) \in \mathbb{C}^2$, then for such a Q:

$$\frac{\partial}{\partial \bar{z}_i} = [Q, \eta_i], \text{ for some } \eta_i.$$

<u>Theorem</u> [SW] : Let $\mathfrak{E}(N=k)$ be the 4d $N=k$ superconformal Lie algebra.

Lie algebra. Then $\exists$ embeddings :

$$H^{\bullet}_{Q}\big(\mathfrak{E}(N=k)\big) \hookrightarrow \mathrm{Vect}^{hol}\big(\underbrace{\mathbb{C}^{2|k-1}}\big).$$

holomorphic supercharge

graded manifold w/ odd variables

$$\mathcal{O}(\mathbb{C}^{2|k-1}) = \mathcal{O}(\mathbb{C}^2)[\underbrace{\varepsilon_1, \ldots, \varepsilon_{k-1}}]$$

<u>Ex</u> : $\big(\mathfrak{E}(N=2), Q\big) \simeq \mathfrak{sl}(3|1)$, acts geometrically

as :

Euler v.f.

$$\mathfrak{sl}(3|1) \cong \mathrm{span}\Big\{ \partial \ldots,\ z_i \partial \ldots,\ z_i\big(E + \varepsilon \tfrac{\partial}{\ldots}\big),\ \tfrac{\partial}{\ldots},\ \varepsilon\tfrac{\partial}{\ldots} \Big\}$$

3 / 15

as :

$$\mathfrak{sl}(3|1) \cong \operatorname{span}\Big\{ \overbrace{\frac{\partial}{\partial z_i}, \; z_i \frac{\partial}{\partial z_j}, \; z_i\big(\overset{\text{Euler v.f.}}{E} + \varepsilon \frac{\partial}{\partial \varepsilon}\big)}^{\text{even}}, \; \overbrace{\frac{\partial}{\partial \varepsilon}, \; \varepsilon \frac{\partial}{\partial z_i}}^{\text{odd}} \Big\}.$$

$N = k$

Suggests following generalizations of the ordinary Virasoro / super Virasoro algebras: derivations of graded algebra: (4)

W:

$$\mathcal{O}(\mathbb{C}^2 \setminus 0)[\varepsilon_1, \ldots, \varepsilon_{k-1}].$$

holomorphic fns.

Problem: $\mathcal{O}(\mathbb{C}^2 \setminus 0) \cong \mathcal{O}(\mathbb{C}^2)$ "Hartog's Lemma"

Problem : $\mathcal{O}(\mathbb{C}^2 \setminus 0) \cong \mathcal{O}(\mathbb{C}^2)$ "Hartog's Lemma"

$H^i(\Lambda$

Better (see also: Kapranov-Faonte-Hennion) Use a dg model

$$\mathbb{R}\Gamma(\mathbb{C}^2 \setminus 0, \mathcal{O}) =: \underline{\underline{(A^\bullet_2, \bar{\partial})}} \quad \text{cdga}$$

Then, the dg Lie alg of graded vector fields is

of the form : $\mathrm{Der}(A^\bullet_2[\varepsilon_1, \dots \varepsilon_{k-1}]) =$

$$A^\bullet_2[\varepsilon_1 \dots \varepsilon_{k-1}]\left\{\frac{\partial}{\partial z_1}, \frac{\partial}{\partial z_2}\right\} \oplus A^\bullet_2[\varepsilon_1 \dots \varepsilon_{k-1}]\left\{\frac{\partial}{\partial \varepsilon_1} \cdots \frac{\partial}{\partial \varepsilon_{k-1}}\right\}.$$

$4/15$

- $\left[\frac{\partial}{\partial \varepsilon_i}, \varepsilon_j\right] = \delta_{ij}$ and more ...

9:41 AM Tue Jan 9

Then, the dg Lie alg of graded vector fields is of the form: $\mathrm{Der}\left(A^{\bullet}_{2}[\varepsilon_1, \ldots \varepsilon_{k-1}]\right) =$

$$A^{\bullet}_{2}[\varepsilon_1 \ldots \varepsilon_{k-1}]\left\{\frac{\partial}{\partial z_1}, \frac{\partial}{\partial z_2}\right\} \oplus A^{\bullet}_{2}[\varepsilon_1 \ldots \varepsilon_{k-1}]\left\{\frac{\partial}{\partial \varepsilon_1} \cdots \frac{\partial}{\partial \varepsilon_{k-1}}\right\}.$$

- $\left[\frac{\partial}{\partial \varepsilon_i}, \varepsilon_j\right] = \delta_{ij}$ and more ...
- $\left[\frac{\partial}{\partial \varepsilon_i}, A^{\bullet}_{2}\frac{\partial}{\partial z_j}\right] = 0$
- I'll refer to this as $\mathrm{Witt}^{N=k}$, except when $k=1$ just use Witt.

9:41 AM Tue Jan 9

5

- ## Central extensions

- $k=1$, we just have

$$\mathrm{Witt}^{\bullet} = \mathrm{Der}(A_2^{\bullet}).$$

A, C- cycles in physics

Central ext. are parametrized by

$$H^5_{GF}(W_2; \mathbb{C}) \cong \mathbb{C}\{\varphi_A, \varphi_C\}.$$

v.f.'s on formal disk

Define the following "2-cocycles":

$$\varphi_A(\xi_1,\xi_2,\xi_3) = \oint_{S^3} \mathrm{Tr}(J\xi_1) \wedge \partial\mathrm{Tr}(J\xi_2) \wedge \partial\mathrm{Tr}(J\xi_3) + \cdots$$

S^3

- $k > 2$. Have additional cocycle:

$$\varphi_{vir} \in H^2(\mathrm{Witt}^{N=k}; \mathbb{C})$$

of form:

$$i = 1, 2, \quad \varphi^i_{vir}(\overbrace{\xi}^{\text{even}}, \overbrace{\varepsilon \xi'}^{\text{odd}}) = \oint_{S^3} \mathrm{Tr}(J\xi) \wedge \partial \mathrm{Tr}(J\xi') \wedge \overbrace{dz_i}^{i=1,2}$$

Residue

Jacobian

hol. de Rham

$\xi, \xi' \in A^\bullet_2 \left\{ \frac{\partial}{\partial z_i} \right\}$ (so, $\varepsilon \xi'$ is an odd v.f.)

Def: 1) The higher Virasoro algebra of type $(a, c) \in \mathbb{C} \times \mathbb{C}$

9:41 AM Tue Jan 9 100%

6

Def : 1) The higher Virasoro algebra of type $\underbrace{(a,c)}_{} \in \mathbb{C} \times \mathbb{C}$ is the central extension :

$$\mathbb{C} \longrightarrow Vir_{a,c} \longrightarrow Witt$$

degree 2-cocycle $\underbrace{a \varphi_A + c \varphi_C}_{}$.

2) The higher (twisted) $N=2$ Virasoro algebra of type (a, c, c_i) is :

$$\mathbb{C} \longrightarrow Vir^{N=2}_{(a,c,c_i)} \longrightarrow Witt^{N=2}$$

degree 2-cocycle $\underbrace{a\,\varphi_A + c\,\varphi_c + c_i\,\varphi^i_{vir}}$.

In the second part of the talk, we will see how the higher $N=2$ virasoro localizes to the ordinary Virasoro Lie algebra.

For now, we indicate some natural representations of the $(N=1)$ higher Virasoro algebra.

7

Ex : Let V be a $\mathbb{C}$-vector space. Let $\mathcal{H}_V$ be the following dg Lie alg :

abelian dg Lie alg

$$\mathbb{C} \cdot \hbar \longrightarrow \mathcal{H}_V \longrightarrow \overbrace{A_2^\bullet \otimes \left(V \oplus V^*[1]\right)}$$

β γ

2-cocycle :

$$(\gamma \otimes v \,,\, \beta \otimes v^*) \longmapsto \langle v, v^* \rangle \oint_{S^3} \gamma \wedge \beta \wedge d^2 z$$

Have (abelian) sub dg Lie alg :

$$\mathcal{H}_{V,+} = A^\bullet_{2,+} \otimes \left(V \oplus V^*[1]\right) \oplus \mathbb{C}$$

quotient

where

9:41 AM Tue Jan 9

quotient

when

$$A^{\bullet}_{2,+} = \ker\left(A^{\bullet}_2 \longrightarrow H^1(A^{\bullet}_2) \right)$$

" positive modes of Laurent polynomials "

central term acts by 1.

Set :

$$\mathcal{V}_V = \mathrm{Ind}^{\mathcal{H}_V}_{\mathcal{H}_{V,+}} (\mathbb{C}_{t_h=1}) = U(\mathcal{H}_V) \otimes_{U(\mathcal{H}_{V,+})} \mathbb{C}_{t_h=1} .$$

☆ This example is motivated by the holomorphic twist of the free $N=1$ chiral multiplet

This example is motivated by the holomorphic twist

of the free $\mathcal{N}=1$ chiral multiplet

$\mathbb{C}^2 \setminus 0$ vs $\underline{\underline{\mathbb{C}^\times \times \mathbb{C}^\times}}$ $\qquad$ $\mathcal{H}_V$ ext. of $T^*[-1]\mathrm{Map}(\mathbb{C}^2\setminus 0, V)$

(8)

<u>Virasoro action</u> : Filtration on $\mathcal{U}(\mathcal{H}_V) \rightsquigarrow \mathrm{gr}\,\mathcal{U}(\mathcal{H}_V)$

is Poisson algebra.

By def^n $\mathrm{Witt}^\bullet \circlearrowright A_2^\bullet$, define

$$J : \mathrm{Witt}^\bullet \longrightarrow \mathrm{gr}\ \mathcal{U}(\mathcal{H}_V)$$

$$\xi \longmapsto \oint_{S^3} \langle \beta, \xi\cdot\gamma \rangle_V$$

... of Lie algebras. Want to lift

Virasoro action : Filtration on $\mathcal{U}(\mathcal{H}_v) \rightsquigarrow \mathrm{gr}\,\mathcal{U}(\mathcal{H}_v)$

is Poisson algebra.

By def^n $\mathrm{Witt}^\bullet \circlearrowright A_2^\bullet$, define

$$J : \mathrm{Witt}^\bullet \longrightarrow \mathrm{gr}\;\mathcal{U}(\mathcal{H}_v)$$

$$\xi \longmapsto \oint_{S^3} \langle p, \xi \cdot \gamma \rangle_v$$

Hamilt

Check this is map of Lie algebras. Want to lift

this to "quantization" $\mathcal{U}(\mathcal{H}_v)$ of $\mathrm{gr}\;\mathcal{U}(\mathcal{H}_v)$

$\tilde{J}$

$[\,,\,]$

$$\begin{tikzcd} \mathcal{U}(\mathrm{Vir}_{(a,c)}) \arrow[r, dashed] \arrow[d] & \mathcal{U}(\mathcal{H}_V) \circlearrowright \mathcal{V}_V \arrow[d] \\ \mathcal{U}(\mathrm{Witt}^\bullet) \arrow[r, "J", "\text{Poisson map}"'] & \mathrm{gr}\, \mathcal{U}(\mathcal{H}_V) \circlearrowright \mathrm{gr}\, \mathcal{V}_V \end{tikzcd}$$

For some combination (a, c) :

$$a\, \varphi_A + c\, \varphi_c = \dim(V) \times \left(\begin{array}{c} \text{“universal} \\ \text{Todd class”} \end{array} \right) \in H^5_{GF}(W_2) .$$

The module $\mathcal{V}_V$ is a "Free field realization" of $\mathrm{Vir}_{(a,c)}$.

9:41 AM Tue Jan 9

9

- **Character** : Cartan of $\mathrm{Vir}_{(a,c)}$ gen by $\left\{ z_1 \frac{\partial}{\partial z_1} , z_2 \frac{\partial}{\partial z_2} \right\}$.

"L_0"

$U(1)_t$- Charge of $V = +1$.

$$\mathrm{ch}(\mathcal{V}_V) = \underbrace{\prod_{n_1, n_2 \geq 0} \frac{1 - t^{-1} q_1^{n_1+1} q_2^{n_2+1}}{1 - t\, q_1^{n_1} q_2^{n_2}}}_{\text{elliptic } \Gamma\text{-function}}$$

- **Conjectural example** : for $\mathfrak{g}$ any Lie algebra obtain

dg Lie alg : $A_2^\bullet \otimes \mathfrak{g}$.

For $\Theta \in \mathrm{Sym}^3(\mathfrak{g})^{\mathfrak{g}}$ have a central extension

$$\mathbb{C} \longrightarrow \hat{\mathfrak{g}}_\Theta \longrightarrow A_2^\bullet \otimes \mathfrak{g}$$

- Superconformal localization

Local enhancement : the Lie algebra of holomorph. vf's on cplx surface X has natural local enhancement

$$\mathrm{Vect}^{hol}(X) \xhookrightarrow{\simeq} \Omega^{0,\bullet}(X, T_X^{1,0}) .$$

Dolbeault resolution

Obtain a factorization algebra on X :

$$\mathcal{V}ir_{(a,c)} := C_\bullet^{\varphi}\left(\Omega_c^{0,\bullet}(X, T^{1,0})\right)$$

Twisted Lie alg. homology

on cplx surface X

$$\varphi = a\,\varphi_A + c\,\varphi_c .$$

This is analog of "chiral enveloping algebra" of Beilinson-Drinfeld.

(Holomorphic fact alg's on Riem surfaces agree w/ the notions of chiral alg's / vertex alg's. In this case, the construction agrees w/ BD)

Can do same thing for $N=k$ version of Virasoro.

We are most interested in the factorization algebra

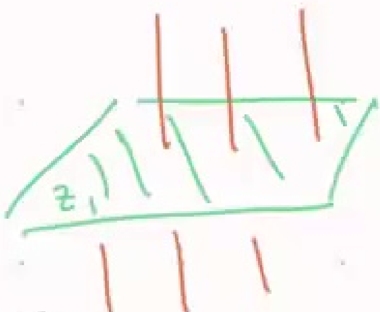
$\mathcal{V}ir^{N=2}$.

We are most interested in the factorization algebra

$\underline{\underline{V_{ir}^{N=2}}}$.

Fact. alg version of work [Beem et. al.]

★ **Localization**: Focus on $X = \mathbb{C}^2$. Consider plane:



$$\mathbb{C}_{z_1} \xhookrightarrow{i} \mathbb{C}^2_{z_1, z_2}.$$

There is a vector field: $z_2 \dfrac{\partial}{\partial \varepsilon} \in \mathrm{Vect}^{hol}(\mathbb{C}^{2|1})$.

Notice this is a MC element $\rightsquigarrow$ can deform any $V_{ir}^{N=2}$-module by such an element.

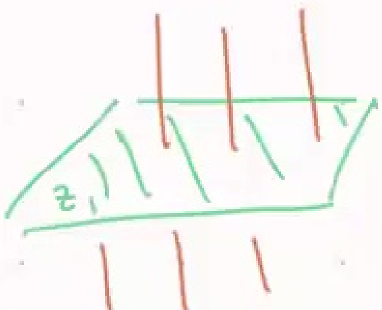
We are most interested in the factorization algebra

$\mathcal{V}_{ir}^{N=2}$.

Fact. alg version of work [Beem et. al.]

11

* **Localization**: Focus on $X = \mathbb{C}^2$. Consider plane:



$$\mathbb{C}_{z_1} \xhookrightarrow{i} \mathbb{C}^2_{z_1, z_2}.$$

This is a vector field: $\boxed{z_2 \frac{\partial}{\partial \varepsilon}} \in \mathrm{Vect}^{hol}(\mathbb{C}^{2|1})$.

Notice this is a MC element $\rightsquigarrow$ can deform any

$\mathcal{V}_{ir}^{N=2}$-module by such an element.

9:41 AM Tue Jan 9

Ex: Higher dim'l KM also has a factorization algebra enhancement:

$\varphi \in \mathrm{Sym}^3(\mathfrak{g})^{\mathfrak{g}} \oplus \mathrm{Sym}^2(\mathfrak{g})^{\mathfrak{g}}$

$$\mathcal{V}_{ir}^{N=2} \circlearrowright \mathcal{F}^{N=2}_{\mathfrak{g},\varphi} = C_\bullet^{\varphi}(\Omega^{0,*}_c(X, \underbrace{\mathfrak{g}}_{\text{Lie alg}})[\varepsilon])$$

Koszul resolution $\mathbb{C}_{z_1} \subseteq \mathbb{C}^2$

- Have deformation:

$$(\mathcal{F}^{N=2}_{\mathfrak{g},\varphi}, d_{\mathcal{F}}) \rightsquigarrow (\mathcal{F}^{N=2}_{\mathfrak{g},\varphi}, d_{\mathcal{F}} + z_2 \frac{\partial}{\partial \varepsilon})$$

$$\cong$$

$$i_* \mathrm{KM}^{-\kappa/2}_{\mathbb{C}_{z_1}}$$

$$\varphi = \underset{S^3(\mathfrak{g})}{\underset{\uparrow}{\Theta}} + \underset{S^2(\mathfrak{g})}{\underset{\uparrow}{\kappa}}$$

level

$$\mathrm{Vir}^{N=2} \circlearrowright \mathcal{Y}^{N=2}_{g,\varphi} = C^{\varphi}_{\bullet}\big(\mathfrak{n}^{\circ;\cdot}_{c}(x,\underbrace{g}_{\text{Lie alg}})[\varepsilon]\big)$$

Koszul resolution

- Have deformation:

$\mathbb{C}_{z_1} \subseteq \mathbb{C}^2$

$$\left(\mathcal{Y}^{N=2}_{g,\varphi}, d_{\mathcal{Y}}\right) \rightsquigarrow \left(\mathcal{Y}^{N=2}_{g,\varphi}, d_{\mathcal{Y}} + z_2 \frac{\partial}{\partial \varepsilon}\right)$$

$$\cong$$

$$\varphi = \underset{\substack{\uparrow \\ S^3(g)}}{\Theta} + \underset{\substack{\uparrow \\ S^2(g)}}{\kappa} \qquad \qquad i_* \mathrm{KM}^{-\kappa/2}_{\mathbb{C}_{z_1}}.$$

where $\mathrm{KM}_{\mathbb{C}_{z_1}}$ is the ordinary KM factorization algebra supported on $\mathbb{C}_{z_1}$.

level 4d

$$\mathrm{Vir}^{N=2} \circlearrowright \mathcal{F}^{N=2}_{\mathfrak{g},\varphi} = C^{\varphi}_{\bullet}\big(\Omega^{0,\bullet}_{c}(X, \underbrace{\mathfrak{g}}_{\text{Lie alg}})[\varepsilon]\big)$$

Koszul resolution

- Have deformation:

$\mathbb{C}_{z_1} \subseteq \mathbb{C}^2$

$$\big(\mathcal{F}^{N=2}_{\mathfrak{g},\varphi}, d_{\mathcal{F}}\big) \rightsquigarrow \big(\mathcal{F}^{N=2}_{\mathfrak{g},\varphi}, d_{\mathcal{F}} + z_2 \tfrac{\partial}{\partial \varepsilon}\big)$$

$$\cong$$

$$\varphi = \underset{S^3(\mathfrak{g})}{\Theta} + \underset{S^2(\mathfrak{g})}{\kappa} \qquad i_* \mathrm{KM}^{-\kappa/2}_{\mathbb{C}_{z_1}}$$

where $\mathrm{KM}_{\mathbb{C}_{z_1}}$ is the ordinary KM factorization algebra supported on $\mathbb{C}_{z_1}$.

9:41 AM Tue Jan 9

(12)

Thm : [SW] Let $\mathcal{V}ir^{c}_{z_1}$ be the ordinary Virasoro vertex alg thought of as a fact. alg on $\mathbb{C}_{z_1} \overset{i}{\subseteq} \mathbb{C}^2$.

Then, there is an equiv. of fact alg's :

$$\left(\mathcal{V}ir^{N=2}_{\varphi = c\varphi_1^{vir}} \;,\; \bar\partial + \left[z_2 \frac{\partial}{\partial \varepsilon} , - \right] \right)$$

$$\wr\!\wr$$

$$i_* \mathcal{V}ir^{-12c}_{z_1} .$$

A similar argument also works at the level

9:41 AM Tue Jan 9 100%

$$\left(\mathrm{Vir} \quad , \quad \left[z_2 \partial_z , \quad \right] \right)$$

$$\varphi = c\,\varphi_1^{\mathrm{Vir}}$$

4d central charge

$$\underline{\underline{12}}$$

2d new central charge

$$\underline{i_* \mathrm{Vir}^{-12c}_{z_1}} .$$

· A similar argument also works at the level of dg Lie algebras.

$$\mathrm{Vir}^{N=2} \rightsquigarrow \left(\mathrm{Vir}^{N=2} , \left[z_2 \frac{\partial}{\partial z} , - \right] \right)$$

$$12$$

$$\mathrm{Vir}$$

Usual Virasoro Lie algebra.

$\mathbb{C}_{z_1} \subseteq \mathbb{C}$.

higher A/C cocycles

Then, there is an equiv. of fact alg's :

$$\left(\mathcal{V}ir^{N=2}_{\varphi = c\varphi_1^{vir}} , \bar\partial + \left[z_2 \frac{\partial}{\partial \varepsilon} , - \right] \right)$$

4d central charge

$\wr 2$ ← new 2d central charge .

$$i_* \mathcal{V}ir^{-12c}_{z_1} .$$

· A similar argument also works at the level of dg Lie algebras .

$$Vir^{N=2} \rightsquigarrow \left(Vir^{N=2} , \left[z_2 \frac{\partial}{\partial \varepsilon} , - \right] \right)$$

$\wr 2$

$$\underline{i_* \mathcal{V}ir^{-12c}_{z_1}} \qquad \text{12}$$

4d central charge

2d new central charge.

- A similar argument also works at the level of dg Lie algebras.

$$\mathrm{Vir}^{N=2} \rightsquigarrow \left(\mathrm{Vir}^{N=2}, \left[z_2 \frac{\partial}{\partial z_1}, - \right] \right)$$

dg Lie

$$\mathrm{Vir}^{12}$$

Usual Virasoro Lie algebra.

9:41 AM Tue Jan 9 100%

13

Remarks : • Can perform same type of localization for $N=4$ version of higher Virasoro. Get super versions of 2d virasoro.

• Thm [SW] : Suppose an $N=k$ theory $\mathcal{T}^{N=k}$ has a global gauge symmetry by $\mathfrak{g}$, commuting w/ SUSY. Then, if $\mathcal{T}^{N=k}$ is superconformal

Symmetry enhancement = "Noether's Theorem for CFT"

Holomorphic twist

$$\mathrm{Vir}^{N=k} \ltimes \mathcal{J}_{\mathfrak{g}}^{N=k} \circlearrowright \left(\mathcal{T}^{N=2}, Q \right)^{\text{Holomorphic twist}}$$

holomorphic

Higher Virasoro

higher KM algebra

holomorphic supercharge

- At the level of holomorphic twist, the superconformal index is a character for $\mathrm{Vir}^{N=k}$.

Eg: 4d $N=1$ SUSY $\longrightarrow$ characters are elliptic Γ-functions

$\parallel$ [SW]

Partition f^n on Hopf mflds $(\mathbb{C}^2 \setminus 0)/\text{lattice}$.

9:41 AM Tue Jan 9

(14)

conformal blocks = global sections over cplx surfaces.

- Higher vertex structure on $\mathcal{V}_{ir}^{N=k}$, $\mathcal{F}_{g}^{N=k}$.

These factorization algebras define algebras over the (colored) operad of holomorphic 2-disks. $\rightsquigarrow$ higher OPE.

Coefficients of holomorphic OPE $\leftsquigarrow$? Higher Virasoro algebras

$\rightsquigarrow$ holomorphic gravitational anomalies $\rightsquigarrow$?

- Holomorphic twist of 6d $N = (2,0)$ theory:

localize