

Title: Equivariant elliptic cohomology with integral coefficients

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Collection: Elliptic Cohomology and Physics

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Abstract: Thirteen years ago, Lurie has sketched a way to obtain equivariant elliptic cohomology and equivariant topological modular forms without the need to restrict to rational or complex coefficients. Recently, David Gepner and I have found one way to flesh out the details and provide computations in the $U(1)$ -equivariant case. On this work I will report.

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Equivariant Elliptic Cohomology with integral coefficients

(jt work with D. Gepner, much is based on a sketch given by J. Lurie 13 years ago)

$$\cdot T = S^1$$

· In doubt, all π -spaces are finite π -CW-complexes, e.g. every cpt mtd with a π -action.

The situation over $\mathbb{C}$

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Rem: This has been used in geom rep theory
by Grojnowski, Ginzburg-Kapranov-Vasserot, ...

Over the integers

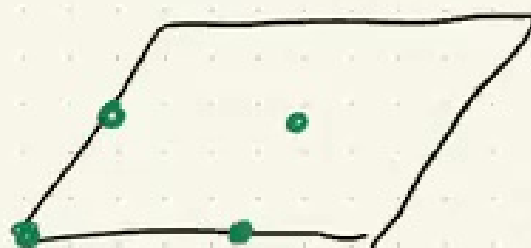
One can define the notion of an ell. curve
over any comm ring, scheme, stacks, ...

Analogue of perspective i): Replace ell curve / $\mathbb{C}$
with ell curve / $\mathbb{Z}$. Problems! Eg. there are

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E_X : $p \mid = T/\mathbb{T} \longrightarrow G_E$
 $T/C_n \longrightarrow \text{pushforward of } G_E[C_n] \text{ along } E[C_n] \longrightarrow E$



"skyscraper sheaf"

More precisely should work with
 $H_{\text{ét}}(\hat{Z}/n, E)$ instead of

$$H_{\text{ét}}(Z/n, E) = E[C_n]$$



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 $\mathcal{E}$ at the cent section

Rem: Say $\mathcal{E}$ is an ell curve over any base

$$S \text{ (e.g. } \mathbb{F}_p) \rightsquigarrow S \xrightarrow{f} \mathcal{M}_{\text{ell}}$$

$$\text{s.t. } \mathcal{E} = f^* \mathcal{E}.$$

$$(\Pi\text{-spaces})^{\text{op}} \longrightarrow \text{Coh}(\mathcal{E}) \xrightarrow{f^*} \text{Coh}(\mathcal{E})$$

this is in general no longer a cohomology theory

as f^* is not exact if f is not flat.

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Enrichments:

E.g. in Berwick-Evans - Tripathy one obtains $\mathcal{E}_{\pi}^i$
 in a 2-step process ($\mathcal{E}$ univ. ell. curve)

$$\pi\text{-sp}^{\text{op}} \xrightarrow{\mathcal{E}_{\pi}} \text{sheaves of chain complexes}_{/\mathbb{C}} \xrightarrow{H^i} \text{Coh}(\mathcal{E})$$

(cdga's)

Mell. 5

Need integral replacent of $\text{ch } X / \mathbb{C}$.

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$$\text{Ch } \mathcal{O}_X / \mathcal{O}_X$$

$$H^i$$

$$\text{Spectra}$$

$$\pi - i$$

Thm (Lurie; Gepner-M.) Let $\mathcal{E}$ be the universal curve over M_{ell} . Then the functor $\mathcal{E}_T^i$ factors as

π -spaces of $\xrightarrow{\mathcal{E}_\pi}$ sheaves of spectra on $\mathcal{E} \xrightarrow{\pi-i} \text{Coh}(\mathcal{E})$

Moreover, $\mathcal{E}_T(X \times Y) = \mathcal{E}_T(X) \otimes_{\mathcal{E}(pt)} \mathcal{E}_T(Y)$



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Rev: Non-equivariantly

Spaces^{op} $\xrightarrow{\mathcal{E}_e}$ Sheaves of spectra
on $\mathcal{M}ell$

pt $\xrightarrow{\quad}$ G^{top}

The global sections are $\Gamma(G^{top}) =: TMF$
Topological modular forms

Why modular forms?

"Classically": modular forms are sections of line

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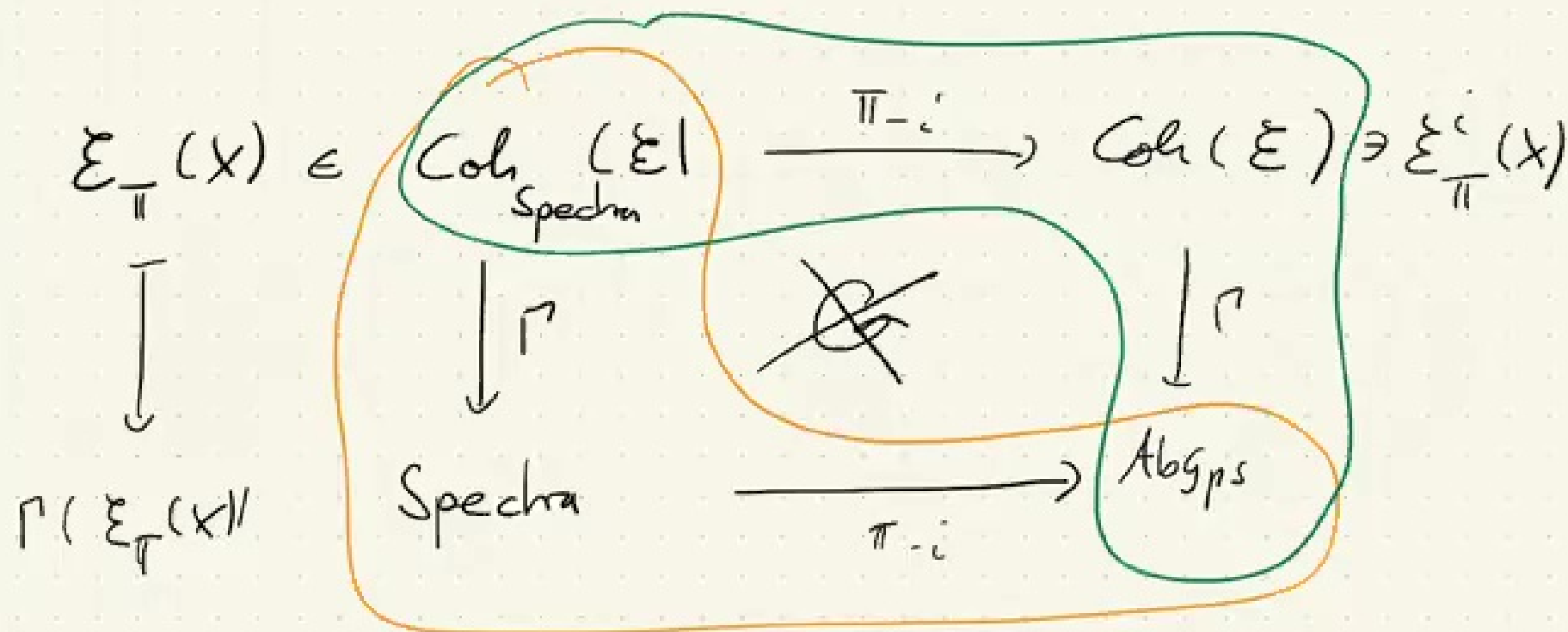


Integrally, modular forms are sections of line bundles $w \otimes k$ on M_{ell} / spectrum

$$\pi_{2k} G^{top} = w \otimes k \quad \text{Thus} \quad \Gamma(G^{top}) = \overline{TMF}$$

"contains" all global sections of the $w \otimes k$.

More precisely: $\pi_{2k} TMF \longrightarrow MF_k$ mod form of wt k
 (weakly hom.)
 becomes iso rationally.


$$m \ell \ell \rightarrow \ell$$


Green composition is not an equiv coh theory



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Green composition is not an equiv coh theory
as Γ not exact

In contrast, orange composition forms an equiv
cohomology theory with target abelian groups.

$$TMF_{\pi}^i(X)$$

implicitly "derived"

Q: What is $TMF_{\pi}^i(pt)$?

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Rem: E ell one over $\mathbb{C}$

$$\Gamma(G_\Sigma) = H^0(\Sigma; G_\Sigma) = \mathbb{C}$$

$$H^1(\Sigma; G_\Sigma) \cong \mathbb{C}.$$

Rem: There is a spectral seq relating "green and orange cap":

$$H^j(\Sigma; \Sigma_{\Pi}^i(X)) \Rightarrow TMF_{\Pi}^{i \pm j}(X).$$

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This is different in TMF

$$TMF^\pi := \Gamma(G_E^{\text{top}}) \rightsquigarrow \pi_* TMF^\pi = TMF_{\pi}^{-*}(\text{pt})$$

$$\begin{array}{ccc} \textcircled{TMF^\pi} = TMF \otimes \textcircled{\Sigma TMF} & \xrightarrow{\text{rationally}} & \textcircled{TMF} \otimes \textcircled{BT} \\ \text{finite over } TMF & & \text{in finite over } TMF \end{array}$$

conc. in even degrees if $\otimes \mathbb{Q}$

closed manifold M with π -actionmanifold with a map to BT

π -equiv
with gens
class in

$$\begin{array}{ccc} \pi_* TMF^\pi & \xrightarrow{\text{rationally}} & \pi_* TMF = BT \\ \uparrow \text{Hope} & & \uparrow \text{with gens} \end{array}$$

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Rem: Instead of taking usual global sections of $\mathcal{O}_E^{\text{top}}$
 I can also allow "poles" at the identity
 $\leadsto$ Topological Jacobi Forms

Can replace $\mathbb{T}$ easily by other abelian groups

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$$\mathbb{T}^n \longleftrightarrow \sum_{m \in \mathbb{Z}} x^m$$

$$\mathbb{P}^1 \longleftrightarrow \{ \infty \}$$

$$A \longleftrightarrow \text{Hom}(\hat{\Lambda}, \mathbb{Z})$$

Slogan:

$$U(n) \longleftrightarrow \text{degree } n \text{ divisors on } \mathbb{P}^1$$

$$\Gamma_{\text{divisor}}(\pi, \text{TUF}) = \text{TUF} \oplus \pi^{-1} \text{TUF}$$