Title: Topological Modular Forms and Quantum Field Theory

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Collection: Elliptic Cohomology and Physics

Date: May 25, 2020 - 7:00 PM

URL: http://pirsa.org/20050051

Topological Modular Forms and Quantum Field Theory

Work in collaboration with T. Johnson-Freyd and E. Witten

arXiv:1904.05788 arXiv:1902.10249 arXiv:1811.00589

Homotopy and QFT

- Dynamics is hard. Deformation invariants help.
- What is the (homotopy of) the space of physical systems with property X?
    - Which topology? Deformations of couplings, RG flow, but also adding or removing ``trivial” degrees of freedom.
- Today: 2d physical systems with (0,1) SUSY
    - SUSY-preserving deformations, drop superselection sectors which spontaneously break SUSY.

Elliptic genus and beyond

- (Witten) The torus partition function with odd spin structure is a deformation invariant: elliptic genus $Z(\tau)$

- Depends homomorphically on complex structure, independent of area.

    - Rescale to get Modular form of weight $\frac{c_L - c_R}{2}$

- Evidence that the space [SQFT] of 2d physical systems with (0,1) SUSY has many connected components. What are they?

TMF and SQFT

- SQFT is a ``spectrum”.
- Conjecture (Stolz and Teichner): SQFT coincides with TMF, the spectrum for elliptic cohomology.
- For every SQFT T there should be a TMF class [T], deformation invariant
  - Sigma model in string manifold X => [X] understood, at least in principle
  - Other weakly coupled SQFTs?
  - Rational SCFTs?

Holomorphic CFTs

- Any holomorphic spin-VOA gives a (0,1) theory ``living on the 0 side"
  - Elliptic genus = vacuum character
  - Negative weight
- Any holomorphic N=1 sVOA gives a (0,1) theory ``living on the 1 side"
  - Elliptic genus = Integer nr. of Ramond vacua
  - Positive weight, power of eta function

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TMF and SQFT II

- What is the recipe to compute [T]?
  - Which data does it use?
  - Does it have a transparent physical meaning?
  - There are torsion TMF classes!
    - Direct sum of non-trivial theories may be trivial…
    - What physics calculation can produce a torsion class?

TMF and symmetries

- SQFTs can have symmetries, discrete or continuous.
  - Restrict to deformations which preserve the symmetry
  - Specify the anomaly as group supercohomology class, naively $H^4(G, U(1))$
- Equivariant TMF classes recently defined. Match?
- Rational CFTs often have large discrete symmetries…

Non-perturbative symmetries

- If M is string manifold with geometric G action, [M] should be a class in equivariant TMF
- Non-geometric actions exist, though.
- If G is finite and Abelian, Orbifold M/G sigma model has a dual G action.
    - Dual G action is not geometric. Acts on twist operators.
    - Does [M/G] admit lift to equivariant TMF?

Generalized symmetries

- When G is finite and non-Abelian, T/G has ``generalized symmetry"
  - Given by fusion category Rep_G
- General: 2d QFTs can have generalized symmetry F for any spherical (super)fusion category F.
- F-equivariant-TMF?

Boundary conditions of TFTs

- Further generalization: (0,1)-Susy boundary conditions for 3d TFT
- 3d TFT: (super) Modular Tensor Category C
- Elliptic genus lives in Hilbert space on torus, i.e. vector-valued modular form controlled by C
- Lift to ``vector-valued" TMF?
- Ideal to study RCFTs!

Action of topological interfaces

- Topological interfaces: modular functors C => C'
- Maps boundary SQFT for C to boundary SQFT for C'
- TMF[C] should be functorial

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Continuous symmetries

- Elliptic genus is promoted to a weak Jacobi form: function of complex structure and G-bundle z
- Some theories are ``sick” or ``non-compact”, but regularized by non-trivial G-bundle: partition function has poles as a function of z
  - Example: $\mathbb{R}^2$ with SO(2) action. $\frac{\eta^3(\tau)}{\theta(z;\tau)}$
  - Useful intermediate step, ingredients in non-sick gauge theories
- Lift to TMF?

Recent work: GJFW

- Verify that homotopy of manifolds is a deformation of sigma models
- Find TMF classes for some rational SCFTs: (0,1) $WZW[G]_k$
  - RG flow from G sigma model with H-flux k
  - Simple examples of torsion classes. $[S^3_k] = k\alpha$ $24\alpha = 0$

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Recent Work: GJF II

- Find physical proof that (0,1) WZW$[G]_k$ is non-trivial modulo 24
- Proof based on Mock-modular forms
    - Assume that M is a boundary of N, with cylindrical end.
    - Sigma model on N is sick! Elliptic genus defined, but not holomorphic
    - Holomorphic anomaly equation controlled by M sigma model
    - $\bar{\tau} \to \infty$: mock-modular form with integrality properties.