Title: Summer Undergrad 2020 - Numerical Methods (A) - Lecture 2

Speakers: Aaron Szasz

Collection: Summer Undergrad 2020 - Numerical Methods

Date: May 28, 2020 - 3:45 PM

URL: http://pirsa.org/20050041

Abstract: Tensor product spaces & many-body quantum states and operators; Limiting cases of two-site Ising model

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Note May 28, 2020 (2)
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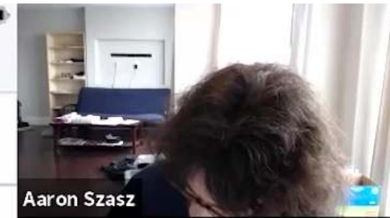

Tensor product spaces

A B 2 spin-$\frac{1}{2}$

$\{|\uparrow\rangle_A, |\downarrow\rangle_A\}$ $\{|\uparrow\rangle_B, |\downarrow\rangle_B\}$

Question: What are the allowed states of the 2-spin system?

Answer: Any state in the "ten·

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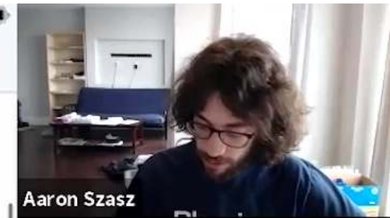

<u>Question</u>: What are the allowed states of the 2-spin system?

<u>Answer</u>: Any state in the "tensor product space":

vector space with basis

$$\{ \underbrace{|\uparrow\rangle_A \otimes |\uparrow\rangle_B}, \; |\uparrow\rangle_A \otimes |\downarrow\rangle_B, \; |\downarrow\rangle_A \otimes |\uparrow\rangle_B, \; |\downarrow\rangle_A \otimes |\downarrow\rangle_B \}$$

basis vector is an ordered pair:

(basis vector for A, b.v. for B)

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$$\{ \underbrace{|\uparrow\rangle_A \otimes |\uparrow\rangle_B}, \; |\uparrow\rangle_A \otimes |\downarrow\rangle_B, \; |\downarrow\rangle_A \otimes |\uparrow\rangle_B, \; |\downarrow\rangle_A \otimes |\downarrow\rangle_B \}$$

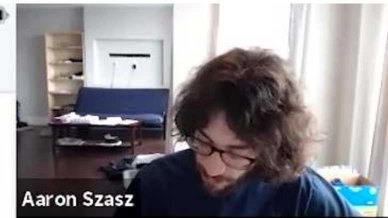
basis vector is an ordered pair:

(basis vector for A, b.v. for B)

$$(|\uparrow\rangle_A, |\uparrow\rangle_B)$$

General state is a superposition:

$$|\psi\rangle = a|\uparrow\rangle\otimes|\uparrow\rangle + b|\uparrow\rangle\otimes|\downarrow\rangle + c|\downarrow\rangle\otimes|\uparrow\rangle + d|\downarrow\rangle\otimes|\downarrow\rangle$$



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- General state is a superposition:

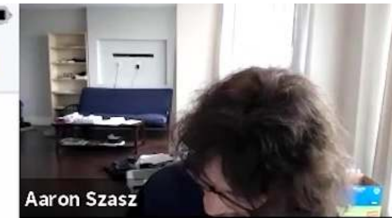
$$|\psi\rangle = a|\uparrow\rangle\otimes|\uparrow\rangle + b|\uparrow\rangle\otimes|\downarrow\rangle + c|\downarrow\rangle\otimes|\uparrow\rangle + d|\downarrow\rangle\otimes|\downarrow\rangle$$

- Inner product:

$$|\psi_A\rangle\otimes|\psi_B\rangle$$

$$|\phi_A\rangle\otimes|\phi_B\rangle$$

$$\left(\langle\psi_A|\otimes\langle\psi_B|\right)\left(|\phi_A\rangle\otimes|\phi_B\rangle\right)$$



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$$|\phi_A\rangle \otimes |\phi_B\rangle$$

$$\left(\langle\psi_A| \otimes \langle\psi_B|\right)\left(|\phi_A\rangle \otimes |\phi_B\rangle\right)$$

$$= \langle\psi_A|\phi_A\rangle \cdot \langle\psi_B|\phi_B\rangle$$

is the state of A the same?

is the state of B the same?

A B

$|\psi_A\rangle \otimes |\psi_B\rangle$

$|\phi_A\rangle \otimes |\phi_B\rangle$

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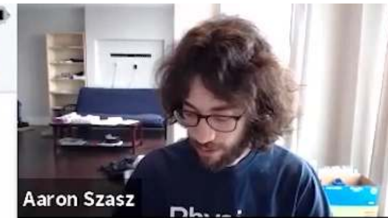

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is the state of A the same?

is the state of B the same?

One important consequence:

If A part is $\perp$ or B part is $\perp$, the whole state is

Ex: $|\uparrow\rangle_A \otimes |\uparrow\rangle_B \;,\; \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A \otimes |\uparrow\rangle_B + |\uparrow\rangle_A \otimes |\downarrow\rangle_B\right)$

$\langle\psi|\left(a|\phi\rangle + b|\tilde{\phi}\rangle\right)$

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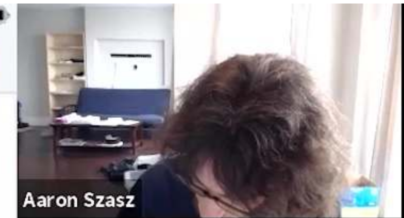

the whole state is

$$\underline{\text{Ex}}: \quad \overset{|\psi\rangle}{|\uparrow\rangle_A \otimes |\uparrow\rangle_B}, \quad \overset{|\phi\rangle}{\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A \otimes |\uparrow\rangle_B + |\uparrow\rangle_A \otimes |\downarrow\rangle_B\right)}$$

$$\left[\langle\varphi|\left(a|\phi\rangle + b|\tilde{\phi}\rangle\right) = a\langle\varphi|\phi\rangle + b\langle\varphi|\tilde{\phi}\rangle\right]$$

$$\langle\varphi|\phi\rangle = \frac{1}{\sqrt{2}}\left(\langle\uparrow|\otimes\langle\uparrow|\right)\left(|\uparrow\rangle\otimes|\uparrow\rangle\right)$$

$$+ \frac{1}{\sqrt{2}}\left(\langle\uparrow|\otimes\langle\uparrow|\right)\left(|\uparrow\rangle\otimes|\downarrow\rangle\right)$$

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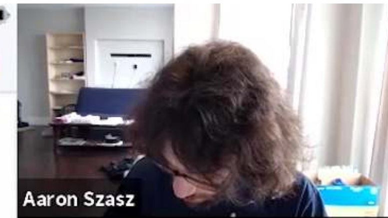

the whole state is

Ex: $\overset{|\psi\rangle}{|\uparrow\rangle_A \otimes |\uparrow\rangle_B}$, $\overset{|\phi\rangle}{\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A \otimes |\uparrow\rangle_B + |\uparrow\rangle_A \otimes |\downarrow\rangle_B\right)}$

$$\left[\langle\psi|\left(a|\phi\rangle + b|\tilde{\phi}\rangle\right) = a\langle\psi|\phi\rangle + b\langle\psi|\tilde{\phi}\rangle\right]$$

$$\langle\psi|\phi\rangle = \frac{1}{\sqrt{2}}\left(\langle\uparrow|\otimes\langle\uparrow|\right)\left(|\uparrow\rangle\otimes|\uparrow\rangle\right)$$

$$+\frac{1}{\sqrt{2}}\left(\langle\uparrow|\otimes\langle\uparrow|\right)\left(|\uparrow\rangle\otimes|\downarrow\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\Big(\underbrace{\langle\uparrow|\uparrow\rangle}_{A}\cdot\underbrace{\langle\uparrow|\uparrow\rangle}_{B} + \underbrace{\langle\uparrow|\uparrow\rangle}_{A}\cdot\underbrace{\langle\uparrow|\downarrow\rangle}_{B}\Big)$$

$$= \frac{1}{\sqrt{2}}(1+0) = \frac{1}{\sqrt{\ }}$$

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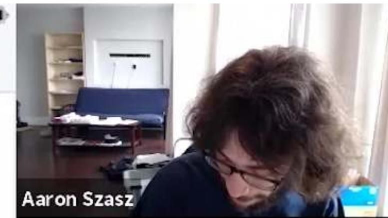

$$+\frac{1}{\sqrt{2}}\left(\langle\uparrow|\otimes\langle\uparrow|\right)\left(|\uparrow\rangle\otimes|\downarrow\rangle\right)$$

$$=\frac{1}{\sqrt{2}}\Big(\underbrace{\langle\uparrow|\uparrow\rangle}_{A}\cdot\underbrace{\langle\uparrow|\uparrow\rangle}_{B}+\underbrace{\langle\uparrow|\uparrow\rangle}_{A}\cdot\underbrace{\langle\uparrow|\downarrow\rangle}_{B}\Big)$$

$$=\frac{1}{\sqrt{2}}(1+0)=\frac{1}{\sqrt{2}}$$

- Apply operat

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- <u>Apply operators</u>

An operator looks like

$$\sum_i \mathcal{O}_i^A \otimes \mathcal{O}_i^B$$

$\mathcal{O}^A \otimes \mathcal{O}^B$ acts on a state like this:

$$(\mathcal{O}^A \otimes \mathcal{O}^B)(|\psi_A\rangle \otimes |\varphi_B\rangle) = ($$

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An operator looks like

$$\sum_i \mathcal{O}_i^A \otimes \mathcal{O}_i^B$$

$\mathcal{O}^A \otimes \mathcal{O}^B$ acts on a state like this:

$$(\mathcal{O}^A \otimes \mathcal{O}^B)(|\varphi_A\rangle \otimes |\varphi_B\rangle) = (\mathcal{O}_A |\varphi_A\rangle) \otimes (\mathcal{O}_B |\varphi_B\rangle)$$

$$\begin{array}{ccc} |\varphi_A\rangle & \otimes & |\varphi_B\rangle \\ \uparrow & & \uparrow \\ \mathcal{O}_A & & \mathcal{O}_B \end{array}$$

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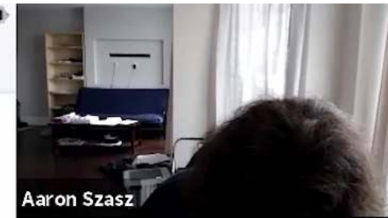

$$|\varphi_A\rangle \otimes |\varphi_B\rangle$$
$$\uparrow \qquad \uparrow$$
$$\mathcal{O}_A \qquad \mathcal{O}_B$$

Corollary:

$$(\mathcal{O}_A \otimes \mathcal{O}_B)\left[(\tilde{\mathcal{O}}_A \otimes \tilde{\mathcal{O}}_B)\left(|\varphi_A\rangle \otimes |\varphi_B\rangle\right)\right]$$
$$= (\mathcal{O}_A \otimes \mathcal{O}_B)\left(\tilde{\mathcal{O}}_A |\varphi_A\rangle \otimes \tilde{\mathcal{O}}_B |\varphi_B\rangle\right)$$
$$= \mathcal{O}_A \tilde{\mathcal{O}}_A |\varphi_A\rangle \otimes \mathcal{O}_B \tilde{\mathcal{O}}_B |\varphi_B\rangle$$
$$= \left(\mathcal{O}_A \tilde{\mathcal{O}}_A \otimes \mathcal{O}_B \tilde{\mathcal{O}}_B\right)\left(|\varphi_A\rangle \otimes |\varphi_B\rangle\right)$$

$$(\mathcal{O}_A \otimes \mathcal{O}_B)$$

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$$= (O_A \tilde{O}_A \otimes O_B \tilde{O}_B)(|\varphi_A\rangle \otimes |\varphi_B\rangle)$$

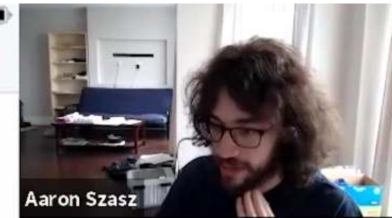
$$(O_A \otimes O_B)(\tilde{O}_A \otimes \tilde{O}_B) = O_A \tilde{O}_A \otimes O_B \tilde{O}_B$$

(b) $O_A \otimes Id_B$, $Id_A \otimes O_B$ ← always commute

$$[\ \ ,\ \ ] = (O_A \otimes Id)(Id \otimes O_B) - (Id \otimes O_B)(O_A \otimes Id)$$

$$= O_A \otimes O_B - O_A \otimes O_B = 0$$

"Operators on different sites commute"



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"Operators on different sites commute"

A   B

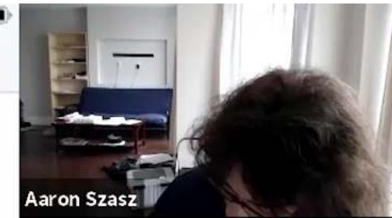
σ^z σ^x

<u>Either</u> 1st apply σ^z to A, then σ^x to B

<u>or</u> opposite order

<u>or</u> apply at the same time

$$(Id \otimes \sigma^x)(\sigma^z \otimes Id),$$



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or opposite order

or apply at the same time

$$(Id\otimes\sigma^x)(\sigma^z\otimes Id),\ (\sigma^z\otimes Id)(Id\otimes\sigma^x),\ (\sigma^z\otimes\sigma^x)$$

Note σ^z_A is not a valid operator

must use $\sigma^z_A\otimes Id_B$

- Born rule for probabilities

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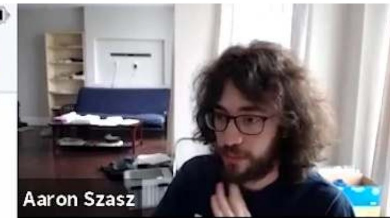


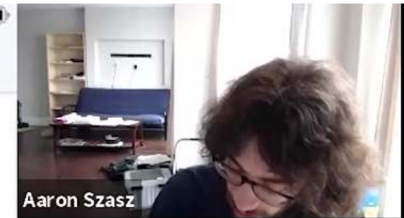
- Born rule for probabilities

<u>1 spin</u> $|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$

$$P(\uparrow) = |a|^2$$
$$P(\downarrow) = |b|^2$$

<u>2 spin</u> $|\psi\rangle = a|\uparrow\rangle\otimes|\uparrow\rangle + b|\uparrow\rangle\otimes|\downarrow\rangle$
$+ c|\downarrow\rangle\otimes|\uparrow\rangle + d|\downarrow\rangle\otimes|\downarrow\rangle$

$$P(\uparrow_A, \uparrow_B)$$



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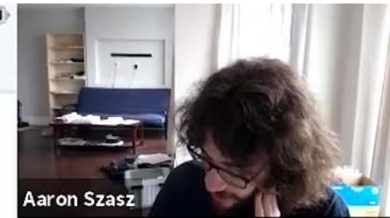

<u>2 spin</u> $|\psi\rangle = a|\uparrow\rangle\otimes|\uparrow\rangle + b|\uparrow\rangle\otimes|\downarrow\rangle$

$+ c|\downarrow\rangle\otimes|\uparrow\rangle + d|\downarrow\rangle\otimes|\downarrow\rangle$

$$\left.\begin{array}{l} P(\uparrow_A, \uparrow_B) = |a|^2 \\ P(\uparrow_A, \downarrow_B) = |b|^2 \end{array}\right\} P(\uparrow_A) = |a|^2 + |b|^2$$

$$= \sum_{S_B} P(\uparrow_a, S_B)$$

$$P(\downarrow_A) = |c|^2 + |d|^2$$

$\langle \sigma^z_A \rangle \leftarrow$ really means $\langle \sigma^z_A \otimes Id$

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$$P(\downarrow_A) = |c|^2 + |d|^2$$

$\langle \sigma^z_A \rangle \leftarrow$ really means $\langle \sigma^z_A \otimes Id_B \rangle$

$$= 1 \cdot P(\uparrow_A) + (-1)\, P(\downarrow_A)$$

$$= |a|^2 + |b|^2 - \left(|c|^2 + |d|^2\right)$$

Application.

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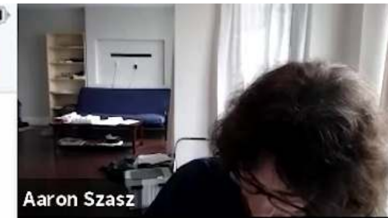

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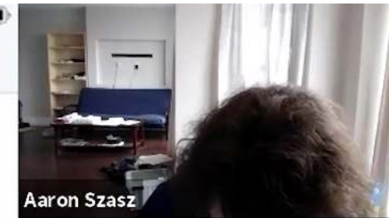

Application: In $|\psi\rangle$, calculate $\langle \sigma^z_A \otimes Id_B \rangle$ directly:

$$\left(a\langle\uparrow|\otimes\langle\uparrow| + \dots\right)\underbrace{\left(\sigma^z_A \otimes Id_B\right)\left(a|\uparrow\rangle\otimes|\uparrow\rangle + \dots\right)}$$

$$\left(\sigma^z_A \otimes Id_B\right)|\uparrow\rangle\otimes|\uparrow\rangle = \sigma^z|\uparrow\rangle \otimes Id|\uparrow\rangle$$

$$= |\uparrow\rangle\otimes|\uparrow\rangle$$

σ^z

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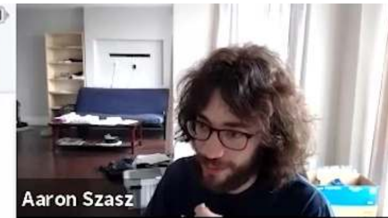

$$(a\langle\uparrow|\otimes\langle\uparrow| + \dots)\underbrace{(\sigma^z_A\otimes Id_B)(a|\uparrow\rangle\otimes|\uparrow\rangle + \dots)}$$

$$(\sigma^z_A \otimes Id_B)\,|\uparrow\rangle\otimes|\uparrow\rangle = \sigma^z|\uparrow\rangle\otimes Id\,|\uparrow\rangle$$
$$= |\uparrow\rangle\otimes|\uparrow\rangle$$

$$(\sigma^z_A\otimes Id_B)(|\downarrow\rangle\otimes|\uparrow\rangle) = -|\downarrow\rangle\otimes|\uparrow\rangle$$

$$\langle\psi| \quad (a|\uparrow\rangle\otimes|\uparrow\rangle + b|\uparrow\rangle\otimes|\downarrow\rangle - c|\downarrow\rangle\otimes|\uparrow\rangle - d|\downarrow\rangle\otimes|\downarrow\rangle)$$

= 16 terms: 12 are 0

$$+|a|^2+|b|^2-|c|^2-|d|^2$$

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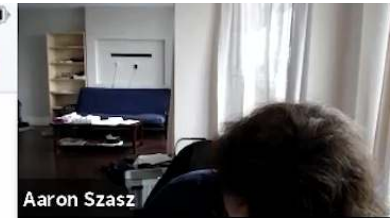
$= 16$ terms ; 12 are 0

$+ |a|^2 + |b|^2 - |c|^2 - |d|^2$

Expectation value (3rd method):

use matrix representation.

- assign basis vectors
- find matrix for $\sigma_A^z \otimes Id_B$
- find matrix/vector representation of $|\psi\rangle$
- multiply



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- assign basis vectors
- find matrix for $\sigma^z_A \otimes Id_B$
- find matrix/vector representation of $|\psi\rangle$
- multiply out

$$|\uparrow\rangle_A \otimes |\uparrow\rangle_B \xrightarrow{\text{shorthand notation}} |\uparrow\uparrow\rangle$$

$$|\uparrow\uparrow\rangle \longrightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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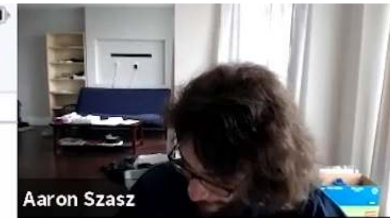
- multiply out

- $|\uparrow\rangle_A \otimes |\uparrow\rangle_B \xrightarrow{\text{shorthand notation}} |\uparrow\uparrow\rangle$

$|\uparrow\uparrow\rangle \longrightarrow \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $|\downarrow\uparrow\rangle \rightarrow \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$

$|\uparrow\downarrow\rangle \longrightarrow \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$ $|\downarrow\downarrow\rangle \rightarrow \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

- Find matrix for $\sigma^z_A \otimes Id_B$



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$$|\uparrow\downarrow\rangle \longrightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad |\downarrow\downarrow\rangle \longrightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

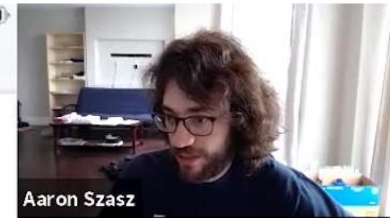
- Find matrix for $\sigma^z_A \otimes Id_B$

1^{st} col is $(\sigma^z \otimes Id)$ on $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$(\sigma^z \otimes Id)\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = (\sigma^z \otimes Id)|\uparrow\uparrow\rangle$$

$$= \sigma^z|\uparrow\rangle \otimes Id|\uparrow\rangle$$

$$= |\uparrow\rangle \otimes |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



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$$= |\uparrow\rangle \otimes |\uparrow\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

<u>2nd col</u>: act on $\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = |\uparrow\rangle \otimes |\downarrow\rangle$

$$\big\downarrow \sigma^z \otimes Id$$

$$|\uparrow\rangle \otimes |\downarrow\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$\vdots$$

$$\begin{pmatrix}1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}$$

- Find a vector for $|\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle$

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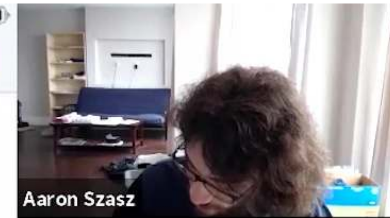
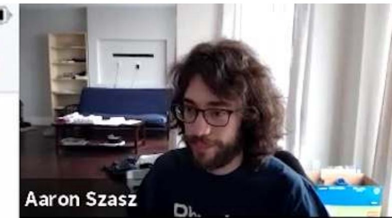


$+c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle$

$$= \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$



- $\langle\psi| \sigma^z \otimes Id |\psi\rangle =$

$$(a^* \; b^* \; c^* \; d^*) \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$= |a|^2 + |b|^2 - |c|^2 - |d|^2$$

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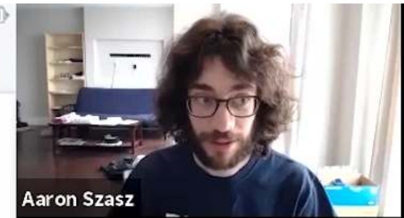

Exercise Find 4x4 matrices in this basis for:

① $\sigma^y_A \otimes \sigma^z_B$, ② $\sigma^z_A \otimes \sigma^y_B$

Answers

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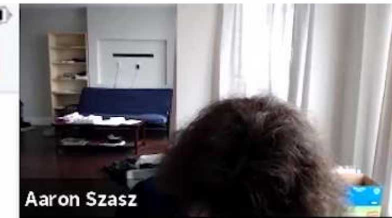

① $\sigma^Y_A \otimes \sigma^Z_B$, ② $\sigma^Z_A \otimes \sigma^Y_B$

$\boxed{\text{Answers}}$ ① $\begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$

$0\cdot\sigma^Z$, $-i\cdot\sigma^Z$, $i\cdot\sigma^Z$, $0\cdot\sigma^Z$

② $\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}$

$1\cdot\sigma^Y$, $0\cdot\sigma^Y$, $0\cdot\sigma^Y$, $-1\cdot\sigma^Y$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \to \sigma^Y \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \to \sigma^Z$$

$\sigma^Y \otimes$

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$i \cdot \sigma^z \qquad 0 \cdot \sigma^z \qquad 0 \cdot \sigma^y \qquad -1 \cdot \sigma^y$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \to \sigma^y \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \to \sigma^z$$

$$\sigma^y \otimes \sigma^z : \begin{pmatrix} 0 \cdot \sigma^z & -i \cdot \sigma^z \\ i \cdot \sigma^z & 0 \cdot \sigma^z \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$A \otimes B$$

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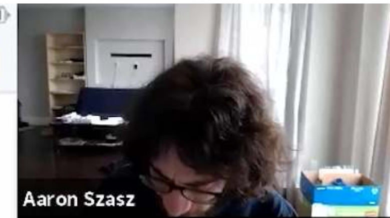
$$\begin{pmatrix} 0\cdot\sigma^z & -i\cdot\sigma^z \\ i\cdot\sigma^z & 0\cdot\sigma^z \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$A \otimes B = \begin{pmatrix} a_{11} B & a_{12} B \\ a_{21} B & a_{22} B \end{pmatrix}$$



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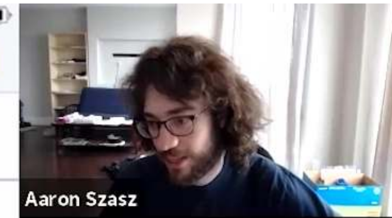

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$A \otimes B = \begin{pmatrix} a_{11} B & a_{12} B \\ a_{21} B & a_{22} B \end{pmatrix}$$

Always true in correct choice of basis

$$\begin{cases} |\uparrow\rangle \otimes |\uparrow\rangle \\ |\uparrow\rangle \otimes |\downarrow\rangle \end{cases} \qquad \begin{cases} |\downarrow\rangle \otimes |\uparrow\rangle \\ |\downarrow\rangle \otimes |\downarrow\rangle \end{cases}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underset{|\uparrow\rangle}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} \otimes \underset{|\uparrow\rangle}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} 1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 0 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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$A = \begin{pmatrix} \cdots \\ a_{21} & a_{22} \end{pmatrix}$

$$A \otimes B = \begin{pmatrix} a_{11} B & a_{12} B \\ a_{21} B & a_{22} B \end{pmatrix}$$

"Kronecker product"

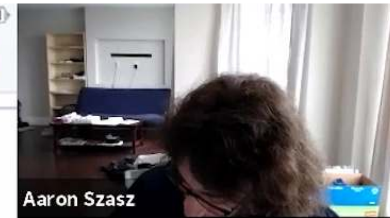
Always true in correct choice of basis

$$\left\{ \begin{array}{l} |\uparrow\rangle \otimes |\uparrow\rangle \\ |\uparrow\rangle \otimes |\downarrow\rangle \\ |\downarrow\rangle \otimes |\uparrow\rangle \\ |\downarrow\rangle \otimes |\downarrow\rangle \end{array} \right.$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underset{|\uparrow\rangle}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} \otimes \underset{|\uparrow\rangle}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} 1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 0 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Matrix for $O_A \otimes O_B$ is Kronecker produc



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$$H = -J\sum_i S_i^z S_{i+1}^z - h\sum_i S_i^x$$

$$S_i^z = \frac{\hbar}{2}\cdot\sigma_i^z$$

get rid of this

$$H = -\underbrace{J\cdot\left(\frac{\hbar}{2}\right)^2}_{\text{"}J\text{"}}\sum_i \sigma_i^z\sigma_{i+1}^z - h\left(\frac{\hbar}{2}\right)\sum_i\sigma_i^x$$

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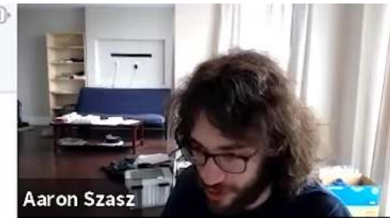

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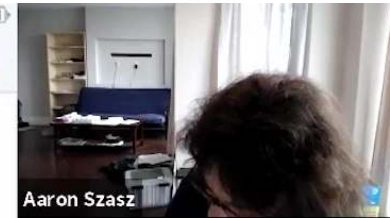

$$H = -\underbrace{J\left(\frac{\hbar}{2}\right)^2}_{\text{"}J\text{"}} \sum_i \sigma_i^z \sigma_{i+1}^z - \underbrace{h\left(\frac{\hbar}{2}\right)}_{\text{"}h\text{"}} \sum_i \sigma_i^x$$

$\downarrow$ rename constants

$$H = -J \sum_i \sigma_i^z \sigma_{i+1}^z - h \sum_i \sigma_i^x$$

$\downarrow$ make this precise

$$H = -J \sigma_0^z \sigma_1^z - h\left(\sigma_0^x + \sigma_1^x\right)$$

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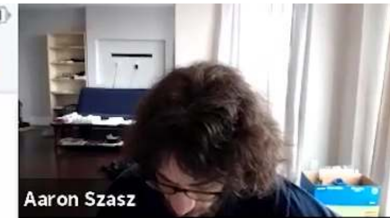
$\downarrow$ make this precise

$$H = -J\, \sigma_0^z \sigma_1^z - h\,(\sigma_0^x + \sigma_1^x)$$

really means:

$$H = -J\,(\sigma_0^z \otimes \sigma_1^z) - h\,(\sigma_0^x \otimes Id_1 + Id_0 \otimes \sigma_1^x)$$

We know how to write down a 4x4 matrix for this.



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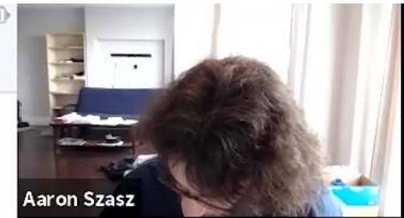

$$H = -J\,\sigma_0^z \sigma_1^z - h\,(\sigma_0^x + \sigma_1^x)$$

really means:

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We know how to write down a 4x4 matrix for this.

Exercises ① Find 4x4 matr

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Exercises

① • Find 4x4 matrix for

$$\sigma_0^z \otimes \sigma_1^z$$

• Find eigenstates & eigenvalues of H when $\lambda = 0$

• Show: in each of these eigenstates, $\langle \sigma_0^x \otimes Id_1 + Id_0 \otimes \sigma_1^x \rangle = 0$

② • Find 4x4 matrix for

$$\sigma_0^x \otimes Id_1 + Id_0 \otimes \sigma_1^x$$

• Find eig

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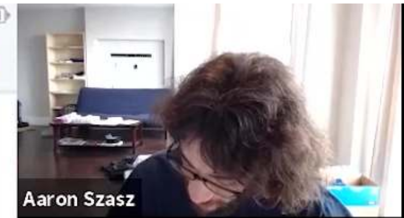

We know how to write down a 4×4 matrix for this.

Exercises ① • Find 4×4 matrix for $\sigma_0^z \otimes \sigma_1^z$

• Find eigenstates & eigenvalues of H when $\lambda=0$

• Show: in each of these eigenstates, $\langle \sigma_0^x \otimes Id_1 + Id_0 \otimes \sigma_1^x \rangle = 0$

② • Find 4×4 matrix for $\sigma_0^x \otimes Id_1 + Id_0 \otimes \sigma_1^x$

• Find eigenstates & eigenvalues of H when $J=0$

• Show: in each of these states, $\langle \sigma_0^z \otimes \sigma_1^z \rangle = 0$

③ Find the 4×4 for H