Title: Summer Undergrad 2020 - Path Integrals (M) - Lecture 2

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Collection: Summer Undergrad 2020 - Path Integrals

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URL: http://pirsa.org/20050037

Abstract: Propagator in real and imaginary time

Lecture 2

$$K(q_f,t_f,q_i,t_i) = \int \prod_{j=1}^{N-1} dq_j \, \overbrace{K(q_{j+1},t_{j+1},q_j,t_j)}^{\text{rename } K_{q_{j+1},q_j}}$$

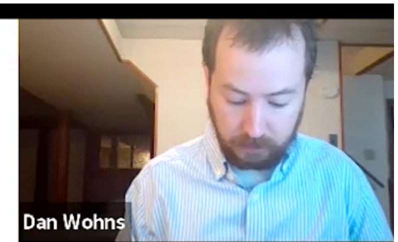
$$K_{q_{j+1},q_j} = \langle q_{j+1}| e^{-iH\Delta t/\hbar} |q_j\rangle$$

$$= \langle q_{j+1}| 1 - \frac{iH\Delta t}{\hbar} + O(\Delta t^2) |q_j\rangle$$

breakout rooms

$$\langle q_{j+1}|q_j\rangle = \delta(q_{j+1}-q_j)$$

$$= \int \frac{dp_j}{2\pi\hbar} e^{ip_j(q_{j+1}-q_j)/\hbar}$$



$$K_{q_{j+1},q_j} = \int \frac{dp_j}{2\pi\hbar}\, e^{i p_j (q_{j+1}-q_j)/\hbar} \underbrace{\left(1 - \frac{i\Delta t H}{\hbar} + O(\Delta t^2)\right)}_{e^{-\frac{i\Delta t}{\hbar}H}}$$

$H = \frac{p_j^2}{2m} + V(q_j)$

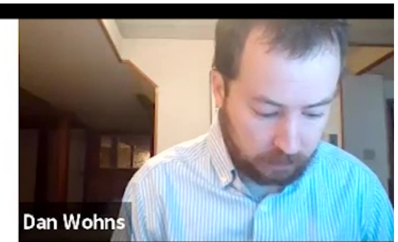
$$= \int \frac{dp_j}{2\pi\hbar}\, e^{\frac{i\Delta t}{\hbar}\left(p_j \dot q_j - \frac{p_j^2}{2m} - V(q)\right)}$$

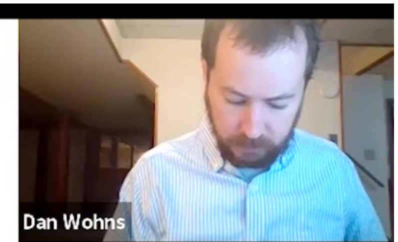
Gaussian in p

$\int e^{ax^2+bx+c}\,dx$

$$= \sqrt{\frac{m}{2\pi i\hbar\Delta t}}\; e^{\frac{i\Delta t}{\hbar}\left(\frac{m}{2}\dot q_j^2 - V(q_j)\right)}$$

$$K = \left(\frac{m}{2\pi i\hbar}\right)^{N/2} \int \left(\prod_{j=1}^{N-1} dq_j\right) \exp\left[\frac{i\Delta t}{\hbar}\sum_{j=0}^{N-1}\left(\frac{m\dot q_j^2}{2} - V(q_j)\right)\right]$$





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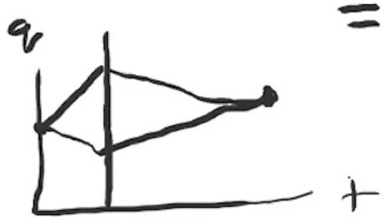

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$$S = \int dt\left(\frac{m\dot{q}^2}{2} - V\right)$$

$$\boxed{K = \int \mathcal{D}q(t)\, e^{iS[q(t)]/\hbar}}$$

$\nwarrow$ i leads to oscillations

$$T = -i T_E$$

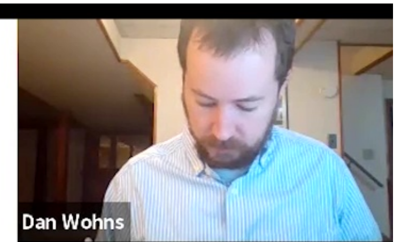
$$K_E = \langle q_f | e^{-T_E H/\hbar} | q_i \rangle$$

$t = -i\tau$ everywhere

$$iS[q(t)] = i \int_0^T dt \left(\frac{m}{2} \left(\frac{dq}{dt} \right)^2 - V(q) \right)$$

$$\rightarrow -\int_0^{T_E} d\tau \left(\frac{m}{2} \left(\frac{dq}{d\tau} \right)^2 + V(q) \right)$$

$$= -S_E[q(\tau)]/\hbar$$



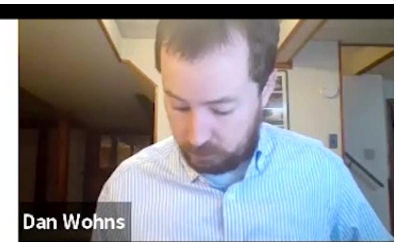
$$Z[\beta] = \sum_j e^{-\beta E_j}$$

$$K_E = \int \mathcal{D}q(\tau)\, e^{-S_E[q(\tau)]/\hbar}$$

$$Z = \int dq\, K_E(q, \beta\hbar, q, 0)$$ ← breakout rooms

$\tau_E = \beta\hbar$

$$\boxed{E_0 = -\lim_{\beta \to \infty} \frac{1}{\beta} \log Z}$$



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ground state energy

Free path integrals

If S is at most quadratic in q

→ N coupled Gaussian integrals

→ exact results

$$K_E = \sqrt{\frac{m}{2\pi\beta}}\, e^{-S[q_c]/\hbar}$$

q_c = classical trajectory

breakout rooms

$$Z[\beta] = \int dq \, \overbrace{\langle q | e^{-HT_E/\hbar}}^{K_E(q, T_E; q, 0)} | q \rangle$$

$$1 = \sum_j |j\rangle\langle j|$$

$N \to \infty$ limit $\to$ algebra

$N = 1$ can understand much of physics

next: perturbation theory

$$y = \sqrt{\frac{m\,\Delta t\,\hbar}{2}}\,q$$