

Title: Summer Undergrad 2020 - Quantum Information - Lecture 1

Speakers: Alioscia Hama

Collection: Summer Undergrad 2020 - Quantum Information

Date: May 25, 2020 - 10:00 AM

URL: <http://pirsa.org/20050033>

Abstract: Foundations of Quantum Statistical Mechanics - Entanglement



Text Mode

Lasso Select

Insert Space



+



Quick Notes Edit

Untitled Page

No additional text



Untitled Page

No additional text



Monday, May 25, 2020 6:08 AM

Thermodynamics

Information

Quantum

Relativity  
QFT  
Strings

how can I make a better engine?

+ Page





Text Mode

Lasso Select

Insert Space



+



Quick Notes Edit

Untitled Page

No additional text



Untitled Page

No additional text



Monday, May 25, 2020 6:08 AM

Thermodynamics

Information

Quantum

What is possible?



+ Page



Text Mode

Lasso Select

Insert Space



Quick Notes Edit

Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Monday, May 25, 2020 10:12 AM

Schrödinger Cat

$$| \text{alive} \rangle + | \text{dead} \rangle$$


---


$$\frac{1}{\sqrt{2}} ( | \uparrow \rangle + | \downarrow \rangle )$$

decoherence



Aliscia Hamma

+ Page



Text Mode

Lasso Select

Insert Space



+



Quick Notes Edit

Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Monday, May 25, 2020 10:15 AM

Basic Factor of QM

MATH  $\mathcal{H} \cong \mathbb{C}^d$   $\dim \mathcal{H} = d$

$= \text{span} \{ |1\rangle, \dots, |d\rangle \}$

d-level system

$|\psi\rangle \in \mathcal{H} \quad \hat{A} : \mathcal{H} \rightarrow \mathcal{H}$

$ii) \quad \hat{A} = \sum_{ij} A_{ij} |i\rangle\langle j|$  linear

+ Page



Alioscia Hama



Monday, May 25, 2020 10:18 AM

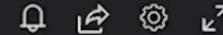
Positive  $\langle \psi | \hat{A} | \psi \rangle \geq 0$  for all  $|\psi\rangle$   
 Hermitian  $\hat{A}^\dagger = \hat{A}$   
 Spectral resolution  $|\hat{A}_i\rangle$   

$$\hat{A} = \sum_i a_i |\hat{A}_i\rangle\langle\hat{A}_i|$$

$$= \sum_i a_i P_i^A$$

ket-bra  
is a projector





Text Mode

Lasso Select

Insert Space



Quick Notes Edit

Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Monday, May 25, 2020 10:20 AM

Norm (Schatten  $p$ -norms)

$$\|A\|_p := \left( \text{tr}(A^\dagger A)^{p/2} \right)^{1/p}$$

$p=1$  trace norm  $\|A\|_1 = \text{tr} \sqrt{A^\dagger A}$

$p=2$  Frobenius  $\|A\|_2 = \sqrt{\text{tr} A^\dagger A}$

$p=\infty$  operator norm  $\|A\|_\infty = \sup_{|\psi\rangle} \frac{\|A|\psi\rangle\|}{\| |\psi\rangle \|}$

+ Page



Alicia Hama



A Text Mode

Lasso Select

Insert Space



Aliscia Hamma

Quick Notes Edit

Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



+ Page

Monday, May 25, 2020 10:20 AM

Norm (Schatten  $p$ -norms)

$$\|A\|_p := \left( \text{tr}(A^\dagger A)^{p/2} \right)^{1/p}$$

$p=1$  Trace norm  $\|A\|_1 = \text{tr} \sqrt{A^\dagger A}$

$p=2$  Frobenius  $\|A\|_2 = \sqrt{\text{tr} A^\dagger A}$

$p=\infty$  operator norm  $\|A\|_\infty = \sup_{\|\psi\rangle} \|A|\psi\rangle\|$

$\|\psi\rangle\| = \langle \psi | \psi \rangle$



A Text Mode

Lasso Select

Insert Space



+



Quick Notes Edit

Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Monday, May 25, 2020 10:23 AM

QM in one slide  
 states Hermitian positive  $\psi \geq 0$

$$\text{Tr } \psi = 1$$

$$\hat{A} = \sum_i a_i P_i^A ; P_i(a_i) = \text{Tr}[\psi P_i^A]$$

possible outcomes  $\equiv P_i$   
 of the measurement

$a_i, P_i$

$$E(A) = \sum_i a_i P_i$$

$$E(A^2) = \sum_i a_i^2 P_i$$

$$\Delta A^2 = E(A^2) - E(A)^2 \geq 0$$



Aliscia Hamma



Text Mode

Lasso Select

Insert Space



+



Quick Notes Edit

Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Monday, May 25, 2020 10:30 AM

$$\psi = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$\psi \geq 0$$

$$p_i \geq 0$$

vector

$$|\psi\rangle \in \mathcal{H}$$

$$p_e = \langle \psi_e | \psi | \psi_e \rangle \geq 0 \quad \text{steter } \psi \in \mathcal{B}(\mathcal{H})$$

$$\text{tr } \psi = 1 = \sum_e p_e \quad \text{prob. distribution}$$



Alioscia Hamma



Text Mode

Lasso Select

Insert Space



Quick Notes Edit

Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



+ Page

Monday, May 25, 2020 10:34 AM

$$\langle \psi | \psi \rangle = 1$$

$$P_i = |\langle \psi | A_i \rangle|^2$$

$$E(A) = \langle \psi | \hat{A} | \psi \rangle$$

$$\{ \underbrace{|\psi_e\rangle}, \psi_e \} *$$

$$E(A) = \sum_e \psi_e \langle \psi_e | A | \psi_e \rangle$$





Home Insert Draw View



Text Mode

Lasso Select

Insert Space



Quick Notes Edit

No additional text

Untitled Page

No additional text

+ Page

Monday, May 25, 2020 10:36 AM

$$\psi = \sum_e q_e |u_e\rangle \langle u_e|$$

Exercise  $E(A) = \text{Tr}[\psi A \psi]$

||

$$\sum_e q_e \langle u_e | \hat{A} | u_e \rangle$$

$$\{ \langle q_e, |u_e\rangle \} \quad q_e, |u_e\rangle$$



Aliscia Hamma



A Text Mode

Lasso Select

Insert Space



Quick Notes Edit

No additional text

Untitled Page  
No additional text

+ Page

Monday, May 25, 2020 10:38 AM

$$\psi = |\psi\rangle\langle\psi|$$



Alioscia Hamma



A Text Mode

Lasso Select

Insert Space



+



Quick Notes Edit

No additional text

Untitled Page

No additional text

+ Page

Monday, May 25, 2020 10:38 AM

$$|\psi_e\rangle \quad \underline{q_e} \quad \rightarrow \quad \psi = \sum_e q_e |\psi_e\rangle \otimes |q_e\rangle$$

$$q_e = (1, 0, \dots, 0)$$

$$\psi = |\psi_0\rangle \otimes |\psi_0\rangle \quad \text{pure state}$$

$$\mu = \text{span} \{ |0\rangle, |1\rangle \}$$

$$\psi = \frac{1}{\sqrt{2}} |0\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \otimes |1\rangle$$



Aliscia Hamma



Text Mode

Lasso Select

Insert Space



Quick Notes Edit

No additional text

Untitled Page

No additional text

+ Page

Monday, May 25, 2020 10:43 AM

$$\psi = \sum_i p_i |\psi_i\rangle \quad \alpha \geq 1$$

$$\sum_i p_i = 1 \quad \sum_i p_i^\alpha \leq 1$$

$$\alpha = 2 \quad \frac{1}{d} \leq \sum_i p_i^2 \leq 1$$

purity





Text Mode

Lasso Select

Insert Space



+



Quick Notes Edit

No additional text

Untitled Page

No additional text

+ Page

Monday, May 25, 2020 10:45 AM

Composite systems

Span  $\{|0\rangle, |1\rangle\} = \mathbb{C}^2$ ;  $\mathbb{C}^2$

$$\mathcal{H} = \underbrace{\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \dots \otimes \mathbb{C}^2}_N$$

$$\cong \mathbb{C}^{2 \otimes N}$$

$$= \text{span} \{ |i_1\rangle \otimes \dots \otimes |i_N\rangle \}$$


Aliscia Hamma



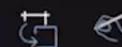
Text Mode

Lasso Select

Insert Space



+



Alioscia Hama

Quick Notes Edit

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



Untitled Page

No additional text



+ Page

Monday, May 25, 2020 10:46 AM  
What is Entropy? Von Neumann

N particles  $\mathbb{C}^{d_i} = \mathbb{H}_i$

$$\mathcal{H} = \bigotimes_{i=1}^N \mathcal{H}_i = \mathbb{C}^{2 \otimes N}$$

e.g.

$$|\psi_i\rangle = \sum_k w_k^i |e_k\rangle$$





Text Mode

Lasso Select

Insert Space



Quick Notes Edit

No additional text

Untitled Page

No additional text

+ Page

Monday, May 25, 2020 10:49 AM

$\mathbb{C}^2$   $|\psi_i\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$   $w_+, w_-$

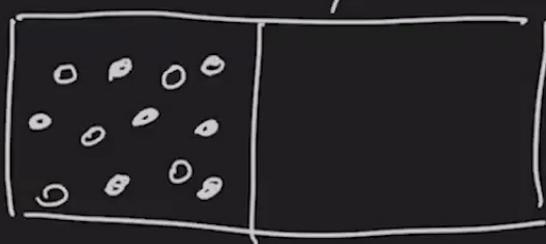
$|\psi\rangle = \sum_i |\psi_i\rangle$   $S(|\psi\rangle) = 0$

Quantum gas

$P_+ = |+\rangle\langle +|$ ;  $P_- = |-\rangle\langle -|$

$n_{\pm} = N |w_{\pm}|^2$

$\psi = |w_+|^2 P_+ + |w_-|^2 P_-$




Alioscia Hamma



Home Insert Draw View



Text Mode

Lasso Select

Insert Space



+



Quick Notes Edit

No additional text

Untitled Page

No additional text

+ Page

Monday, May 25, 2020 10:49 AM

$\mathbb{C}^2$   $|\psi_i\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$   $w_+, w_-$

$|\psi\rangle = \sum_i |\psi_i\rangle$   $S(|\psi\rangle) = 0$

Quantum gas

$P_+ = |+\rangle\langle +|$ ;  $P_- = |-\rangle\langle -|$   $S_2$

$n_{\pm} = N |w_{\pm}|^2$

$\psi = |w_+|^2 P_+ + |w_-|^2 P_-$

$\Delta S ?$





A Text Mode

Lasso Select

Insert Space



Quick Notes Edit

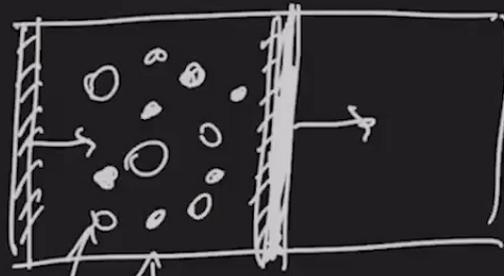
No additional text

Untitled Page

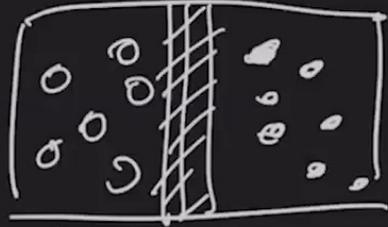
No additional text

+ Page

Monday, May 25, 2020 10:53 AM



$V = \text{const}$   
 $Q, W = 0$



# Isothermal Compression





Home Insert Draw View



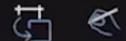
Text Mode

Lasso Select

Insert Space



+



Quick Notes Edit

No additional text

Untitled Page  $\psi =$

No additional text  $\psi > 0$

Untitled Page  $\langle \psi | \psi \rangle$

No additional text  $P_i =$

Untitled Page  $\psi =$

No additional text  $E_{\text{total}}$

Untitled Page  $|\psi\rangle$

No additional text  $a_0 =$

Untitled Page  $\psi = \sum_i$

No additional text

Untitled Page  $L_{\pm}$

No additional text  $S_1, S_2$

No additional text  $N$

Untitled Page  $\mathbb{C}^2$

No additional text

Untitled Page  $\psi =$

No additional text

+ Page

Monday, May 25, 2020 10:49 AM

$\mathbb{C}^2$   $|\psi_i\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |- \rangle)$   $w_+, w_-$

$|\psi\rangle = \sum_i |\psi_i\rangle$   $S(|\psi\rangle) = 0$

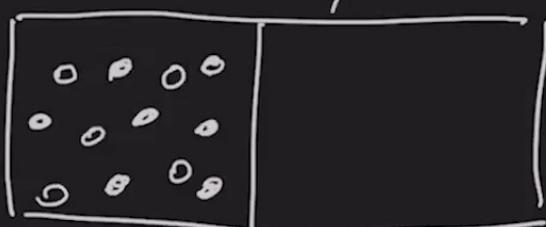
Quantum gas

$L_+ = |+\rangle\langle +|$ ;  $L_- = |- \rangle\langle -|$   $S_2$

$n_{\pm} = N |w_{\pm}|^2$

$\psi = |w_+|^2 P_+ + |w_-|^2 P_-$

$S_2 - S_1 = \Delta S ?$




Alioscia Hamma



Text Mode

Lasso Select

Insert Space



Quick Notes Edit

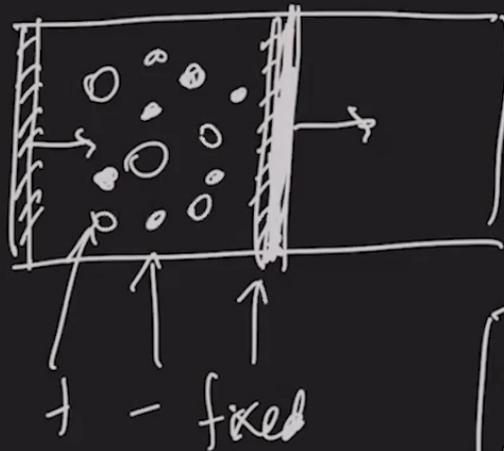
No additional text

Untitled Page

No additional text

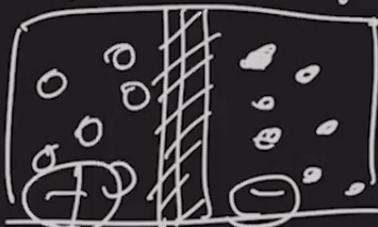
+ Page

Monday, May 25, 2020 10:53 AM



$$V = \text{const}$$

$$Q, W = 0$$



Isothermal Compression

$$\oplus V \rightarrow V_{\pm}' = |W_{\pm}|^2 V$$





Home Insert Draw View



A Text Mode

Lasso Select

Insert Space



Quick Notes Edit

No additional text

Untitled Page

No additional text

+ Page

Monday, May 25, 2020 11:01 AM

$$\Delta S_{\text{ew.}} = \Delta S_+^{\text{ew.}} + \Delta S_-^{\text{ew.}}$$

$$= -n_+ k \log \frac{V'_+}{V} - n_- k \log \frac{V'_-}{V}$$

$$n_{\pm} = |W_{\pm}|^2 N$$

$$V'_{\pm} = |W_{\pm}|^2 V$$

$$= -N k |W_+|^2 \log |W_+|^2$$

$$- N k |W_-|^2 \log |W_-|^2$$



Aliscia Hama



Text Mode

Lasso Select

Insert Space



Quick Notes Edit

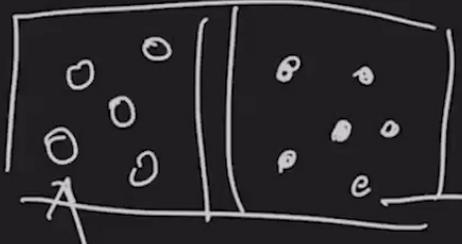
No additional text

Untitled Page

No additional text

+ Page

Monday, May 25, 2020 11:04 AM



$$S_2 = \Delta S^{ew}$$

$$= -Nk (W_+^2 \log W_+)$$

$$|+\rangle \langle +| \xrightarrow{U} |W_+ \rangle \langle +| + |W_- \rangle \langle -|$$

$$\Delta S^{ew} + \Delta S^{gas} = 0$$

$$\Delta S^{ew} = -\Delta S^{gas} = -(S_1 - S_2)$$





Text Mode

Lasso Select

Insert Space



Quick Notes Edit

No additional text

Untitled Page

No additional text

+ Page

Monday, May 25, 2020 11:07 AM

Von Neumann Entropy

$$S(\psi) = -N \sum_i |w_i|^2 \log |w_i|^2$$

$$S\left(\frac{1}{d}\right) = \log d$$

$$S_\alpha = \frac{1}{1-\alpha} \log \text{Tr } \psi^\alpha$$



Aliscia Hama