Title: Bulk observables in JT gravity

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Series: Quantum Gravity

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Abstract: Using a definition of bulk diff-invariant observables, we go into the bulk of 2d Jackiw-Teitelboim gravity. By mapping the computation to a Schwarzian path integral, we study exact bulk correlation functions and discuss their physical implications. We describe how the black hole thermal atmosphere gets modified by quantum gravitational corrections. Finally, we will discuss how higher topological effects further modify the spectral density and detector response in the Unruh heat bath.

Bulk observables in JT gravity

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Based on	arXiv:1902.11194	with A. Blommaert and H. Verschelde
	arXiv:1903.10485	
	arXiv:2005.?????	with A. Blommaert and H. Verschelde

Outline

Jackiw-Teitelboim gravity

Jackiw-Teitelboim (JT) 2d dilaton gravity

$S = \frac{1}{16\pi G} \int d^2x \sqrt{-g}\Phi(R+2) + \frac{1}{8\pi G} \int d\tau \sqrt{-\gamma}\Phi_{bdy} K$

Teitelboim '83, Jackiw '85

Motivation:

- Dimensional reduction (s-wave) of 3d pure $\Lambda < 0$ gravity
- Appears as near-horizon theory of near-extremal higher-dimensional black holes
- Describes low-energy sector of all (known) SYK-like models
- Solvable including coupling to bulk matter fields

Here: Discuss bulk QG physics

Path integrate over $\Phi \Rightarrow R = -2$:
Geometry fixed as AdS_2: $ds^2 = \frac{-dF^2 + dZ^2}{Z^2}, \quad Z \geq 0$
Poincaré patch (frame) of AdS_2, boundary at $Z = 0$

Important frames in AdS$_2$ (1)

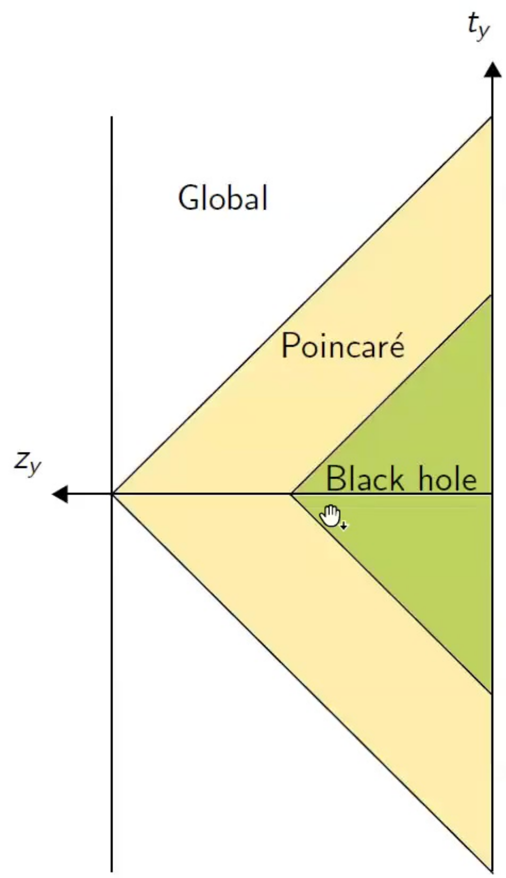
Lightcone coordinates $U = F + Z$ and $V = F - Z$

Major classical frames:

- Poincaré patch:
  $ds^2 = -\frac{4dUdV}{(U-V)^2}$
  Found in near-horizon regime of extremal black hole
  Isometries: $SL(2,\mathbb{R})$: $U \to \frac{AU+B}{CU+D}$, $V \to \frac{AV+B}{CV+D}$
- BH frame: $U(u) = \tanh\left(\frac{\pi}{\beta}u\right)$, $V(v) = \tanh\left(\frac{\pi}{\beta}v\right)$
  $ds^2 = -\frac{\pi^2}{\beta^2}\frac{4}{\sinh^2(\frac{\pi}{\beta}(u-v))}dudv$
  Found in near-horizon regime of near-extremal black hole with $T \equiv 1/\beta \sim \sqrt{M}$

Important frames in AdS_2 (2)

Penrose diagram



Jackiw-Teitelboim gravity and the Schwarzian

Path-integrate out Φ:
$\Rightarrow$ Only boundary term survives: $S = \frac{1}{8\pi G}\int d\tau \sqrt{-\gamma}\,\Phi_{bdy} K$
Consider boundary curve $(F(\tau), Z(\tau))$ as UV cutoff, carving out a shape from AdS_2
Conditions:

- asymptotic Poincaré: $Z(\tau) = \epsilon \dot{F}(\tau)$
- boundary along constant large value of dilaton $\Phi_{bdy} = a/(2\epsilon)$

Using $\sqrt{-\gamma} = 1/\epsilon, \quad K = 1 + \epsilon^2 \{F, \tau\} + \dots$

$$\boxed{\Rightarrow S = -C \int d\tau \, \{F, \tau\}}, \quad C = \frac{a}{16\pi G}, \ \{F, \tau\} = \frac{F'''}{F'} - \frac{3}{2}\left(\frac{F''}{F'}\right)^2$$

Almheiri-Polchinski '15, Jensen '16, Maldacena-Stanford-Yang '16, Engelsöy-TM-Verlinde '16

$F(\tau) =$ time reparametrization

Compare to CS / WZW topological duality
Semi-classical regime: $C \to \infty \equiv G, \hbar \to 0$
Note: C has dimension length $\to$ quantum effects important in IR

JT disk path integral

JT gravity reduces to an integral over boundary frames $F(\tau)$
Boundary correlators of the thermal JT theory are of the form:

$$\langle \mathcal{O}_{\ell_1} \mathcal{O}_{\ell_2} \dots \rangle_\beta = \frac{1}{Z} \int_{\mathcal{M}} [\mathcal{D}F] \mathcal{O}_{\ell_1} \mathcal{O}_{\ell_2} \dots \, e^{C \int_0^\beta d\tau \{F,\tau\}}$$

with $F \equiv \tan\left(\frac{\pi f(\tau)}{\beta}\right)$, $\quad \{F,\tau\} = \{f,\tau\} + \frac{2\pi^2}{\beta^2} f'^2$

$\mathcal{M} = \mathrm{Diff}(S^1)/SL(2,\mathbb{R})$, $\quad f(\tau+\beta) = f(\tau) + \beta, \; f' \geq 0$

$SL(2,\mathbb{R})$: $\quad F \to \frac{aF+b}{cF+d}$ comes from isometry group of AdS_2

Q: What are the natural operators to consider?

Boundary two-point function

Take massive scalar field in bulk, asymptotic expansion (AdS_2/CFT_1):

$$\phi(Z,F) \quad \to Z^{1-\Delta}\tilde{\phi}_b(F) = \epsilon^{1-\Delta}F'^{1-\Delta}\tilde{\phi}_b(F(\tau)) = \epsilon^{1-\Delta}\phi_b(\tau)$$

Generating functional:

$$I \sim \int dF_1 \int dF_2 \frac{1}{(F_1-F_2)^{2\Delta}}\tilde{\phi}_b(F_1)\tilde{\phi}_b(F_2)$$

$$= \int d\tau_1 \int d\tau_2 \frac{F'(\tau_1)^{\Delta}F'(\tau_2)^{\Delta}}{(F(\tau_1)-F(\tau_2))^{2\Delta}}\phi_b(\tau_1)\phi_b(\tau_2)$$

Bilocal operator:

$$\mathcal{O}_\ell(\tau_1,\tau_2) \equiv \left(\frac{F'(\tau_1)F'(\tau_2)}{(F(\tau_1)-F(\tau_2))^2}\right)^{\ell} \equiv \left(\frac{f'(\tau_1)f'(\tau_2)}{\frac{\beta}{\pi}\sin^2\frac{\pi}{\beta}[f(\tau_1)-f(\tau_2)]}\right)^{\ell}$$

Other origin of this operator:

- Boundary-anchored Wilson line Blommaert-TM-Verschelde '18, Iliesiu-Pufu-Verlinde-Wang '19

Approaches to JT correlators: an overview

Several approaches to obtain JT boundary correlators exist:

- 1d Liouville $f' = e^{\phi}$ Bagrets-Altland-Kamenev '16, '17
- 2d Liouville CFT between ZZ-branes TM-Turiaci-Verlinde '17, TM '18
- 2d BF bulk Blommaert-TM-Verschelde '18, Iliesiu-Pufu-Verlinde-Wang '19
- Particle in infinite B-field in AdS_2 Yang '18, Kitaev-Suh '18
- Minimal string TM-Turiaci '19, WIP

Result for $\langle \mathcal{O}_\ell(\tau_1, \tau_2) \rangle_\beta$:

$$\frac{1}{Z} \int dE_2 e^{-\beta E_2} \rho_0(E_2) \int dE_1 \rho_0(E_1) e^{-\tau_{12}(E_1 - E_2)} \frac{\Gamma(\ell \pm i\sqrt{E_1} \pm i\sqrt{E_2})}{\Gamma(2\ell)}$$

$Z=$ Schwarzian disk partition function, $\rho_0(E) = \frac{1}{2\pi^2} \sinh 2\pi\sqrt{E}$
Fixed energy E_2 (microcanonical) answer by stripping off the Laplace E_2-integral

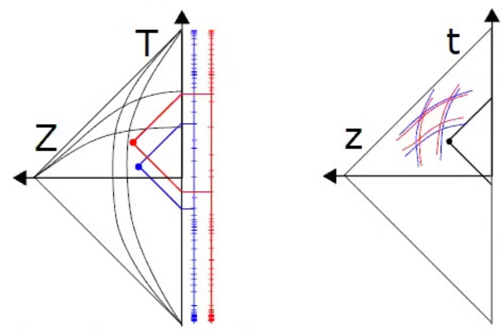
Bulk observables: defining the bulk frame

Goal of this talk: compute bulk observables

In QG defining bulk observable (= diff-invariant operator) requires care. E.g. scalar field $\phi(x)$, but what is x here ?

Need to specify bulk location in a geometrically invariant way
Holography $\rightarrow$ preferably boundary-intrinsic way

Choice: given 2 times u and v, define bulk point (U, V) from the boundary reparametrization F using
Radar definition of bulk point:
$U = F(u)$, $V = F(v)$ Blommaert-TM-Verschelde '19



Observables $\mathcal{O}(F(u), F(v)) \rightarrow$ Contribution in correlator from implicit dependence on geometry F through this construction
Visible in e.g. commutator computations Donnelly-Giddings '15

Application: bulk matter two-point function (1)

Couple JT gravity to a bulk matter action, take massless scalar for simplicity:

$$\frac{1}{2}\int d^2x\sqrt{-g}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + S_{\mathrm{JT}}[g,\Phi]$$

Matter two-point function in a fixed frame F:

$$G_{bb}(x,x') = \langle \phi_1\phi_2\rangle_{\mathrm{CFT}} = \ln\left|\frac{(F(u)-F(u'))(F(v)-F(v'))}{(F(v)-F(u'))(F(u)-F(v'))}\right|$$

= CFT two-point function on UHP with image charge to implement Dirichlet boundary condition

Integrate over frames: $\langle G_{bb}(x,x')\rangle = \int [\mathcal{D}F]\, G_{bb}(x,x')e^{-S[F]}$

Two-step process:

1. Integrate over matter to get a gravitational operator
2. Integrate over gravity with this operator insertion

Application: bulk matter two-point function (2)

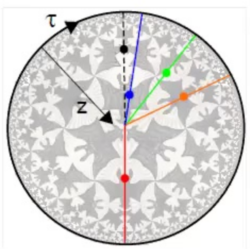
Trick:

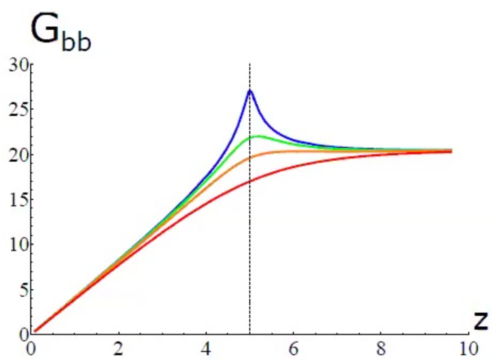
$$\ln \left| \frac{(F(u)-F(u'))(F(v)-F(v'))}{(F(v)-F(u'))(F(u)-F(v'))} \right| = \int_v^u dt \int_{v'}^{u'} dt' \frac{F'(t)F'(t')}{(F(t)-F(t'))^2}$$

Coincides with HKLL prescription to map bulk operators into boundary observables Hamilton-Kabat-Lifschytz-Lowe '05

Doing the double integral:

$$\langle G_{bb}(t,z,z')\rangle_\beta = \int_0^\infty dE_2 \rho_0(E_2) e^{-\beta E_2} \int_0^\infty dE_1 \rho_0(E_1) e^{it(E_1-E_2)}$$
$$\times \frac{\sin z(E_2-E_1)}{E_2-E_1} \frac{\sin z'(E_2-E_1)}{E_2-E_1} \Gamma(1 \pm i\sqrt{E_1} \pm i\sqrt{E_2})$$





- Singularities only at lightlike separated points
- Log-divergences close to the lightcones

Generalizations: CFT primaries and massive fields

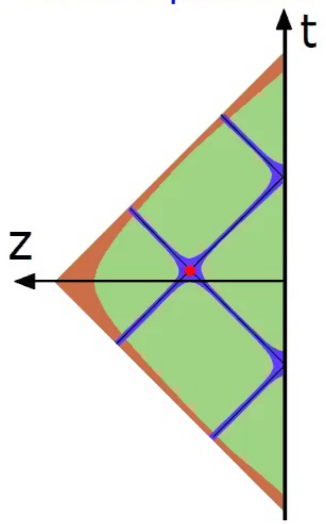
Generalization to matter CFT primaries:

$$G_{h,\bar{h}}(u,u',v,v') = \left(\frac{F'(u)F'(u')}{(F(u)-F(u'))^2}\right)^h \left(\frac{F'(v)F'(v')}{(F(v)-F(v'))^2}\right)^{\bar{h}} - (u' \leftrightarrow v')$$

Generalization to massive bulk fields :

$G(x,x') \sim \frac{1}{\sigma^\Delta} {}_2F_1\left(\frac{\Delta}{2}, \frac{\Delta+1}{2}; \frac{2\Delta+1}{2}; \frac{1}{\sigma^2}\right)$ with invariant distance function $\sigma = 1 - 2\frac{(F(u)-F(u'))(F(v)-F(v'))}{(F(u)-F(v))(F(u')-F(v'))}$ and $m^2 = \Delta(\Delta-1)$

Generic picture:



blue: UV singularities
red: IR region where strong QG fluctuations appear

Bulk locality

Local operators: should commute for spacelike separation

$$[\phi(t_1, z_1), \phi(t_2, z_2)] = 0, \qquad (t_1, z_1) \text{ and } (t_2, z_2) \text{ spacelike}$$

Commutator $\equiv$ Difference of two orderings for bulk two-point function

$\Rightarrow$ Satisfied here: these observables are (mutually) local operators in the full QG in the bulk

See also Lin-Maldacena-Zhao '19 for other construction of diff-invariant operators that turn out to be local

$\Rightarrow$ JT gravity is more local than generically expected in QG (as in e.g. Donnelly-Giddings '15)

Breakdown of Rindler geometry

Near-horizon region is IR region, similar to late time regime $\Rightarrow$ strong quantum gravitational effects, deviations from Rindler correlators

Further intuition: Operational definition of infinitesimal distance[2]:

$$ds^2 = \ln \left| 1 - \frac{(F(u)-F(u+du))}{(F(u)-F(v))} \frac{(F(v)-F(v+dv))}{(F(u)-F(v))} \right| = \frac{\dot{F}(u)\dot{F}(v)}{(F(u)-F(v))^2} du\, dv$$

Indeed strong deviations close to horizon in this quantum geometry

Implications for information paradox:

Defining bulk operators in a diff-invariant way can lead to the near-horizon region being very different from Rindler

$\rightarrow$ breakdown effective field theory

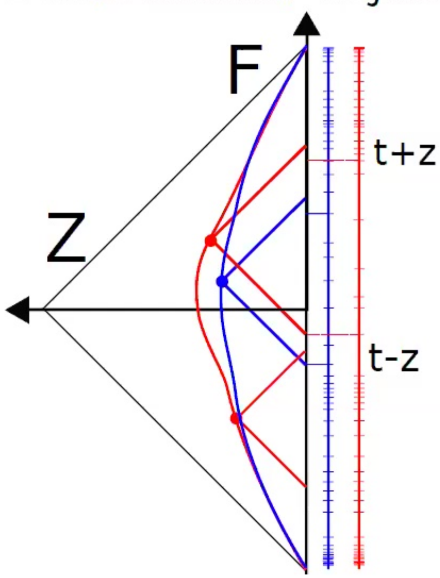
Reservations:

- for this specific model $\leftrightarrow$ universality JT
- for these specific bulk operators $\rightarrow$ (less natural) bulk operators (presumably) exist that do not have this property

Unruh heat bath: bulk detector (1)

Now: spectral content of the bulk two-point function
$\rightarrow$ probes black hole thermal atmosphere (Unruh bath)
Two quantities: detector measurement, and bath spectral energy density

Define detector trajectory of Unruh-DeWitt detector operationally:



- Use radar definition to define entire trajectory $(Z(t), F(t))$ of detector worldline
- Along worldline, introduce interaction $H_{\text{int}} = \mu(t)\phi(\mathbf{x}(t))$ coupling the bulk quantum field ϕ to a detector QM system μ Unruh '79, DeWitt '80

Unruh heat bath: bulk detector (2)

Transition probability for detector to go from ground state $|0_{\text{det}}\rangle$ to $|\omega_{\text{det}}\rangle$, without any information on the excitation of the QFT matter state:

$$P(\omega) = \sum_{\phi_{\text{QFT}}} \left| \langle \omega_{\text{det}}, \phi_{\text{QFT}}| - i \int_{-\infty}^{+\infty} dt H_{\text{int}}(t) \, |0_{\text{det}}, 0_F\rangle \right|^2$$

Transition rate:

$$R(\omega) \equiv \lim_{T\to+\infty} P(\omega)/T =$$

$$|\langle\omega| \, \mu(0) \, |0\rangle_{\text{det}}|^2 \lim_{T\to+\infty} \frac{1}{T} \int_{-T}^{+T} dt dt' e^{-i\omega(t-t')} \, \langle \phi(\mathbf{x}(t))\phi(\mathbf{x}(t')) \rangle_{\text{CFT}}$$

in terms of CFT bulk matter two-point function

Strategy: insert in Schwarzian path integral and Fourier transform

For simplicity, consider the microcanonical ensemble for a fixed energy M black hole state

Unruh heat bath: bulk detector (3)

Answer:
$R(\omega) = 2\left(\frac{\sin z\omega}{\omega}\right)^2 \frac{\sinh 2\pi\sqrt{M-\omega}}{2\pi^2}\Gamma(1 \pm i\sqrt{M} \pm i\sqrt{M-\omega})\,\Theta(M-\omega)$

Interpretation:

- $2\left(\frac{\sin z\omega}{\omega}\right)^2$ is interference factor from the image charges across the AdS_2 boundary
- $\frac{\sinh 2\pi\sqrt{M-\omega}}{2\pi^2}\Gamma(1 \pm i\sqrt{M} \pm i\sqrt{M-\omega})\,\Theta(M-\omega)$ is the matter emission probability
- $\Theta(M-\omega)$ signals backreaction effects: no energy greater than the original black hole mass can be emitted to the detector probe

In semi-classical regime $M \gg 1$, $M \gg \omega$, we approximate:
$R(\omega) \approx 2\left(\frac{\sin z\omega}{\omega}\right)^2 \frac{\omega}{e^{\beta\omega}-1}$ in terms of the Bose-Einstein (Planckian) black body spectrum

Higher topology (1)

Up to now, we only considered the genus 0 topology (disk)
Including higher topology in gravity is important:
Typically strongly suppressed by $\sim e^{-(2g-1)S_0}$, coming from the EH term $S_0 \frac{1}{4\pi} \int d^2x \sqrt{g} R$ with S_0 the extremal entropy
But can be important in dynamical regimes where this suppression is compensated by an enhancement (late time, small energy separation). Recent examples:

- Replica wormholes to find the Page curve Almheiri et al. '19, Penington et al '19
- Ramp regime that governs late-time averaged correlations of the boundary theory Saad-Shenker-Stanford '19

Higher genus expansion is asymptotic, requires non-perturbative completion
$\Rightarrow$ For JT gravity, a double-scaled random matrix integral completes the genus expansion Saad-Shenker-Stanford '19

Higher topology (2)

Studied for multi-boundary amplitudes in Saad-Shenker-Stanford '19

Studied for boundary correlators Blommaert-TM-Verschelde '19, Saad '19

Computations show that these contributions only correct the n-density factor in the correlator, e.g. in the two-point function:

$$\rho_0(E_1)\rho_0(E_2) \to \rho_{JT}(E_1, E_2), \qquad \rho_0(E) = \frac{e^{S_0}}{2\pi^2} \sinh 2\pi\sqrt{E}$$

where $\rho(E_1, E_2)$ is the JT random matrix pair density correlator

$\rho_{JT}(E_1, E_2)$ very well approximated by the GUE random matrix structure of the pair density correlator:

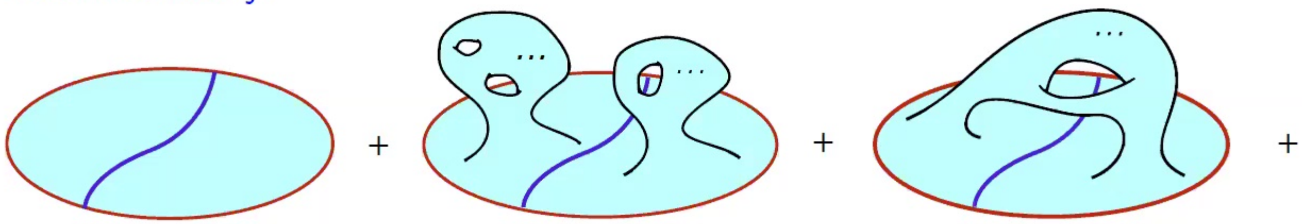
$$\rho(E_1, E_2) = \rho(E_1)\rho(E_2) - \frac{\sin^2 \pi\rho(\bar{E})(E_1 - E_2)}{\pi^2(E_1 - E_2)^2} + \rho(E_2)\delta(E_1 - E_2)$$

Important features:

- level repulsion: $\rho(E_1, E_2) \overset{E_1 \approx E_2}{\approx} (E_1 - E_2)^2 + \dots$
- high-frequency wiggles: spacing $\sim e^{-S_0}$

Higher topology (3)

Geometrically:



Interpretation:

First two diagrams: disk topology + disconnected higher topology on each side of the line: $\rho(E_1)\rho(E_2)$

Last diagram: connected higher topology across the line: $\rho_{\text{conn}}(E_1, E_2)$

Since bulk correlator was written through HKLL and the radar construction in terms of a boundary correlator, we insert the higher genus effects directly in the boundary correlator

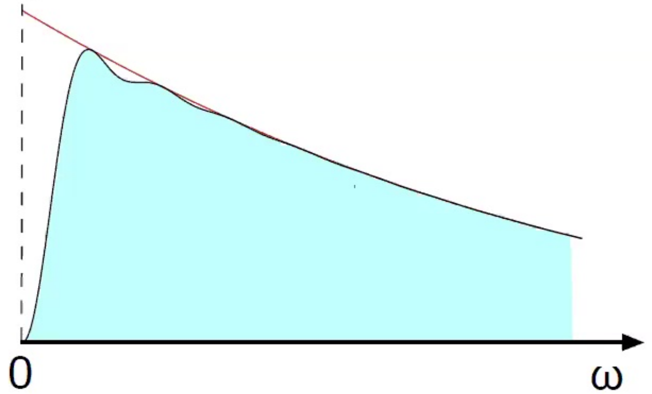
We obtain for the detector response rate:

$$R(\omega) = 2\left(\frac{\sin z\omega}{\omega}\right)^2 \frac{\rho(M, M-\omega)}{\rho(M)} \Gamma(1 \pm i\sqrt{M} \pm i\sqrt{M-\omega})$$

Higher topology (4)

$$R(\omega) = 2\left(\frac{\sin z\omega}{\omega}\right)^2 \frac{\rho(M, M-\omega)}{\rho(M)} \Gamma(1 \pm i\sqrt{M} \pm i\sqrt{M-\omega})$$

$$R(\omega) = 2\left(\frac{\sin z\omega}{\omega}\right)^2 \times$$



Interpretation as product of probabilities:

- Probability of black hole system containing two levels spaced by ω, $\sim \frac{\rho(M, M-\omega)}{\rho(M)\rho(M-\omega)}$
- Probability of matter emission from such a system $\sim \rho(M-\omega)\Gamma(1 \pm i\sqrt{M} \pm i\sqrt{M-\omega})$
- AdS_2 interference factor

Unruh bath energy

Energy flux densities:

As coincident limit of two-point function

$\langle : T_{uu}(u) : \rangle_{\mathrm{CFT}} = \lim_{u' \to u} \langle : \partial_u \phi(u) \partial_u \phi(u') : \rangle_{\mathrm{CFT}}$

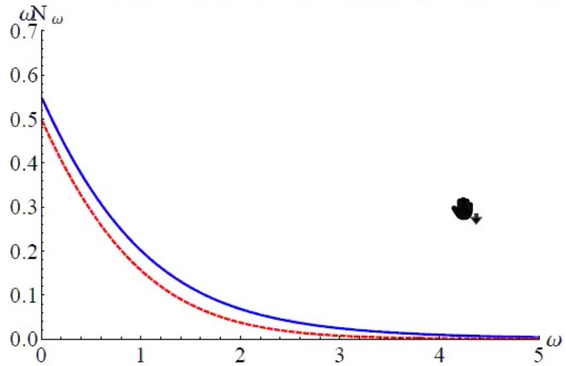
Unruh spectral energy density (1)

From two-point function, we can extract the energy occupation number $\omega N_\omega[f] \equiv \langle 0_F | \omega a_\omega^\dagger a_\omega | 0_F \rangle$ by Fourier transforming from the two-point function to the oscillators:

$$-\frac{1}{8\pi^2}\int du_1 \int du_2 e^{-i\omega(u_1-u_2)}\left[\frac{F'(u_1)F'(u_2)}{(F_1-F_2+i\epsilon)^2}-\left(\frac{1}{u_{12}+i\epsilon}\right)^2\right]+(\epsilon\to-\epsilon)$$

Unruh spectral energy density (2)

Insert this expression in Schwarzian path integral (only disk)
$\Rightarrow$ Canonical ensemble result:



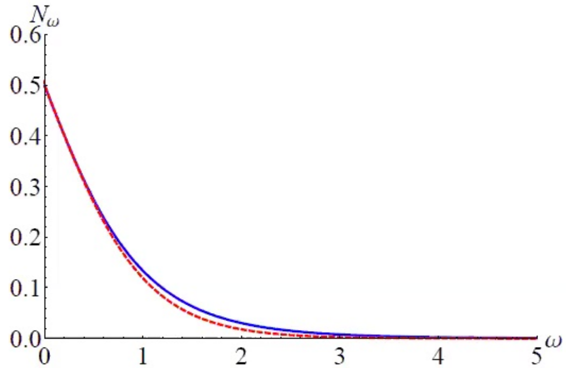
red: Planckian black-body spectrum in 1+1d with $\beta = 4C$
blue: JT disk result

$\rightarrow$ Slightly higher population

Check: $\int_0^{+\infty} d\omega\, \omega \left\langle N_\omega \right\rangle_\beta = \int_0^{+\infty} du \left\langle : T_{uu} : \right\rangle_\beta + \left\langle : T_{vv} : \right\rangle_\beta$

Unruh spectral energy density: fermions

Generalization to bulk massless Majorana fermion field TM '19:



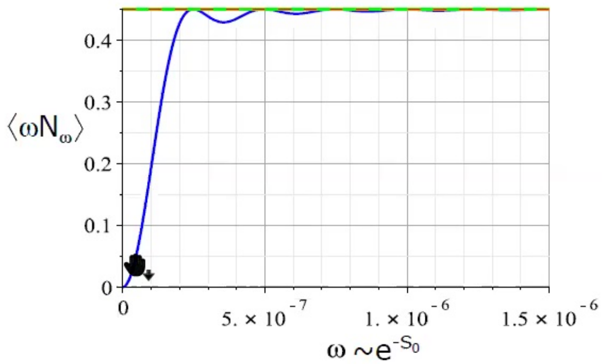
red: Fermi-Dirac spectrum in 1+1d with $\beta = 4C$
blue: Exact result

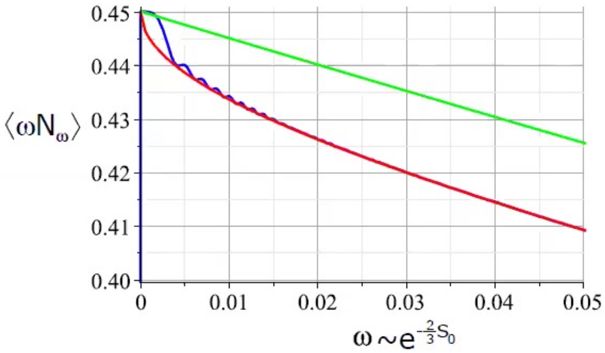
Interpretation: low-energy spectrum has competition between gravity and Pauli-exclusion preventing any major modifications to these highly occupied levels

Further generalizations to charged fields, SUSY possible WIP

Going beyond the disk, we choose to work microcanonically and refer our energy density w.r.t. the $M = 0$ energy density

Results in level repulsion and high-frequency wiggles:





Green: semi-classical result, Red: Schwarzian result, Blue: full result
$M = 2\,(= 1/C)$, $S_0 = 10$

Conclusion

Jackiw-Teitelboim gravity is toy model of quantum gravity, striking the ideal balance between relevance and solvability

- Relevance: low-energy sector of all SYK-type models
  Most basic non-trivial holographic 2d gravity model
  Universal in near-extremal near-horizon regimes
- Solvability: gravitational dofs reduce to boundary time reparametrizations F, with explicit analytic solution for correlators, non-perturbatively in G_N. Explicit understanding of higher topology and resulting random matrix effects

Computed bulk two-point functions (strongly dependent on the definition of our bulk observables) that exhibit:

- Bulk microcausality
- Gravitational corrections to the Unruh heat bath and detector response, with level repulsion at ultra-low emission energies

JT gravity is ideal test case to study conceptual questions about quantum gravity