Title: LQG in diagonal gauge: A modern approach

Speakers: Gioele Botta

Series: Quantum Gravity

Date: May 13, 2020 - 4:00 PM

URL: http://pirsa.org/20050023

Abstract: The full theory of LQG presents enormous challenge to create physical computable models. In this talk we will present the new modern version of Quantum Reduced Loop Gravity. We will show that this framework provide an arena to study the full LQG in a certain limit, where the quantum computations are possible. We will analyze all the major step necessary to build this framework, how is connected with the full theory, its mathematical consistency and the physical intuition behind It.

LQG in diagonal gauge: a modern approach

Gioele Botta, university of warsaw

13/05/2020

Main investigator: Emanuele Alesci

Past collaborators: Francesco Cianfrani, Gabriele V. Stagno

Current collaborators: Prof. Jerzy Lewandodowski, Ilkka Mäkinen

QRLG framework: past and present

The QRLG program was initiated by Francesco and Emanuele in 2013 with the goal to build a bridge to connect the cosmological sector of the full theory and LQC

During the years this program showed that it was much more general with respect to the initial idea.

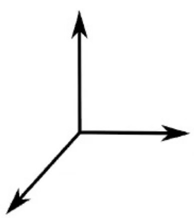
The first consistent realization of this program was performed in a framework suitable to study spherically symmetric spacetimes (Alesci,Pranzetti,Bahrami 2019). **Many surprises with respect to midisuperspace models!**

We are now reformulating the first idea of Emanuele and Francesco in a solid and consistent way. In this talk I will present these results which answer to many questions, like: what is the relation between QRLG and the full theory? Is QRLG consistent from a mathematical point of view? What is the physical intuition behind QRLG? (paper in preparation)

General Relativity

No background system fixed a priori → $G_{\mu\nu} = T_{\mu\nu}$ → ~~$g_{\mu\nu}$~~

Gauge fixing

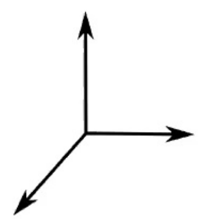


→ $\tilde{G}_{\mu\nu} = \tilde{T}_{\mu\nu}$ → $\tilde{g}_{\mu\nu}$?

Advantages?

Choose a coordinate system through a gauge fixing
(block diagonal metric. Vickers and Grant 2009)

Symmetry reduction



→ $g^{R}_{\mu\nu}$ ✓

Coordinate system adapted to the symmetries

Any 3 dimensional metric can be diagonalized. (Yang 1984, Vickers and Grant 2009)

LQG in diagonal gauge: the first step

Old formulation from Francesco and Emanuele (2013)

$$(A_a^i, E_i^a) \longrightarrow (A_a^i, E_i^a)_{diag}$$

Mix of symmetry reduction and gauge fixing

If $R = 0$ this phase space is preserved by the dynamics

Reduction compatible only with homogeneous spacetime

New formulation from Emanuele and Warsaw group (2017-2020) (paper in preparation)

$$\begin{matrix} G_i \\ H_a \end{matrix} \xrightarrow{\text{Partial gauge fixing}} \begin{matrix} \chi_i = \epsilon_{ij}^k E_k^a \delta_{ij} \\ E_i^a = 0 \quad i \neq a \end{matrix} \xrightarrow{\text{Gauge fixed space}} (A_a^i, E_i^a|_{diag})$$

Partial gauge fixing well defined ✓

Can this gauge fixing be preserved by the dynamics ?

How to deal with gauge fixing at the quantum level ?

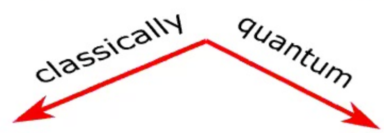
How to deal with a second class system

1) Dirac Bracket: $\{f,g\}_P \to \{f,g\}_D = \{f,g\}_P - \sum_{a,b} \{f,\phi_a\}_P M^{-1}_{ab} \{\phi_b, g\}_P$ $\xrightarrow{\text{Quantum}}$ **?**

2) Gauge unfixing: $\{f,g\}_P, H_{system} \to H_{system} + H_{GU}$ $\xrightarrow{\text{Quantum}}$ reduced quantization is problematic

3) Gubta bleuer: $(\phi_a)_{a=1,\dots,n} \to \hat{\phi}_a, \quad \langle \Psi_{phy} | \hat{\phi}_a | \Psi_{phy} \rangle = 0$ physical states?

QRLG strategy



Find $H_{system} + H_{GU}$ $\quad\quad$ $\langle \Psi_{kin} | \hat{\phi}_a | \Psi_{kin} \rangle \overset{0}{=} \Psi_{GF}$ use $\hat{H}_{system} + \hat{H}_{GU}$

Quantize

Valid procedure since we performed a PARTIAL GAUGE FIXING

Application of the strategy to QRLG

Classical step: find the Dirac matrix

$$\gamma\begin{pmatrix} 0 & 0 & -E_2^2 & -E_1^1\partial_1 & 0 & 0 \\ -E_3^3 & 0 & 0 & 0 & -E_2^2\partial_2 & 0 \\ 0 & E_3^3 & 0 & -E_1^1\partial_1 & 0 & 0 \\ E_2^2 & 0 & 0 & 0 & 0 & -E_3^3\partial_3 \\ 0 & -E_1^1 & 0 & 0 & 0 & -E_3^3\partial_3 \\ 0 & 0 & E_1^1 & 0 & -E_2^2\partial_2 & 0 \end{pmatrix}$$

Study the associated differential problem →

$$\begin{pmatrix} (E_2^2)^2\partial_2 & (E_1^1)^2\partial_1 & 0 \\ 0 & (E_3^3)^2\partial_3 & (E_2^2)^2\partial_2 \\ (E_3^3)^2\partial_3 & 0 & (E_1^1)^2\partial_1 \end{pmatrix}\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

Classificable like a generalized symmetric hyperbolic system: the solution exists, is unique and depend continuously on the initial datas

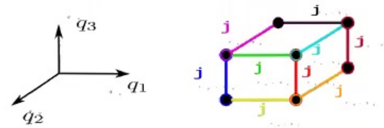
Quantum step:

Since the diagonal gauge is preservable, let us fix a "general" orthogonal coordinate system

Let us look for the solutions of these Gubta Bleuer conditions: $\langle\Psi_{GI}|\hat{\chi}_i|\Psi_{GI}\rangle = 0$ $\langle\Psi_{GI}|\hat{E}^a_k|\Psi_{GI}\rangle = 0$ $k \neq a$

Let us look for a space where any geometry connected to diagonal line element can be represented.

In the large j limit we find:

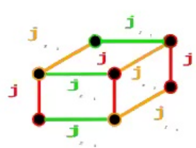


$$= \prod_{e\in\Gamma} D^{(j_e)}_{\sigma_e j_e\,\sigma_e j_e}(h_e)_{i_e}$$

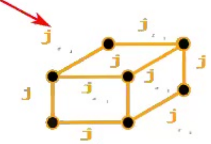
where $\sigma_e = +1$ or -1, and $i_e = q_1, q_2, q_3$

most general non homogeneous geometry

Only intersection at nodes should be cubical!



homogeneous anisotrpic geometry



homogeneous isotrpic geometry

Is the lost of the full gauge invariance a problem?

Two arguments to say no:

1) Copenhagen interpretation on QM: states are not physical, they contain informations about physical quantities ⟶ operators should be gauge invariant

2) Let us think at the knotted spinnetwork states, These are invariants with respect to spatial diffeomorphsims.

To be still in full GR it is necessary that this invariance should be respected from all possible backgrounds??

NO!, GR is a gauge theory, and like we have seen there are some possible gauge fixing who are accessible and preservable, this tells us that many background are just redundant, so to be still in the full GR theory we have just to select a subclass of coordinates systems which they still contain all the physical degrees of freedom.

In our framework we have seen that a subclass of these backgrounds are the orthogonal ones, so we were looking for state adapted to these backgrounds and all we need is a gauge invariance with respect to this class. No restriction with respect to the full GR arise since we have shown that these backgrounds are "gauge selected", so contain all the physical relevant informations of the full GR.

So since the states are not measurable objects but they contain measurable informations, if we have a class of coordinates systems where all the relevant degrees of freedoms are preserved, we can restrict to write the states in these coordinates systems and we are still in full GR.

And the operators?

Emanuele and Francesco old approach

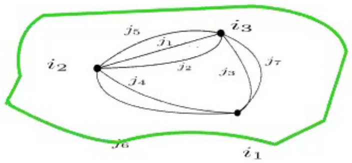
Full theory operators

$\hat{A}$ $\hat{D}^{(j_e)}_{mn}$

$\hat{V}_{AL}$ $\hat{E}_i(S^k)$

$\hat{H}_E$

Full theory Hilbert space



Gauge fixing →

Reduced operators

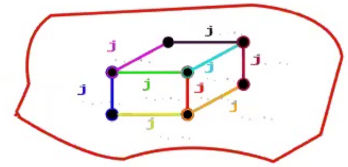
${}^{R}\hat{A}$ ${}^{R}\hat{D}^{(j_e)}_{mn}$

${}^{R}\hat{V}$ ${}^{R}\hat{E}_i(S^k)$

${}^{R}\hat{H}_E$

action →

Full theory gauge fixed hilbert space



$\hat{H}_E$ → ${}^{R}\hat{H}_E$: Projection

Volume is diagonal: ${}^{R}\hat{V}\,|j_1, ..., j_6, N_1\rangle \propto \sqrt{(j_1+j_4)(j_2+j_j)(j_3+j_6)}$

Not clear the relation with the full operators

Projection of non elementary operators is done component by component

Ilkka's new approach (2020)

Full theory operators

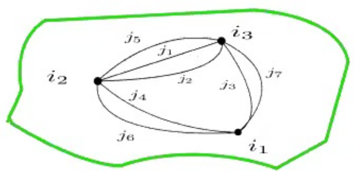
$\hat{D}^{(j_e)}_{mn}$

$\hat{A}$ $\hat{E}_i(S^k)$

$\hat{V}_{AL}$

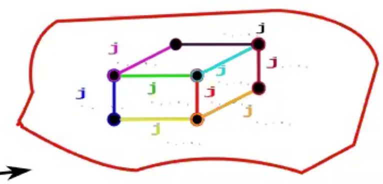
$\hat{H}_E$

Full theory Hilbert space



Gauge fixing →

Full theory gauge fixed hilbert space



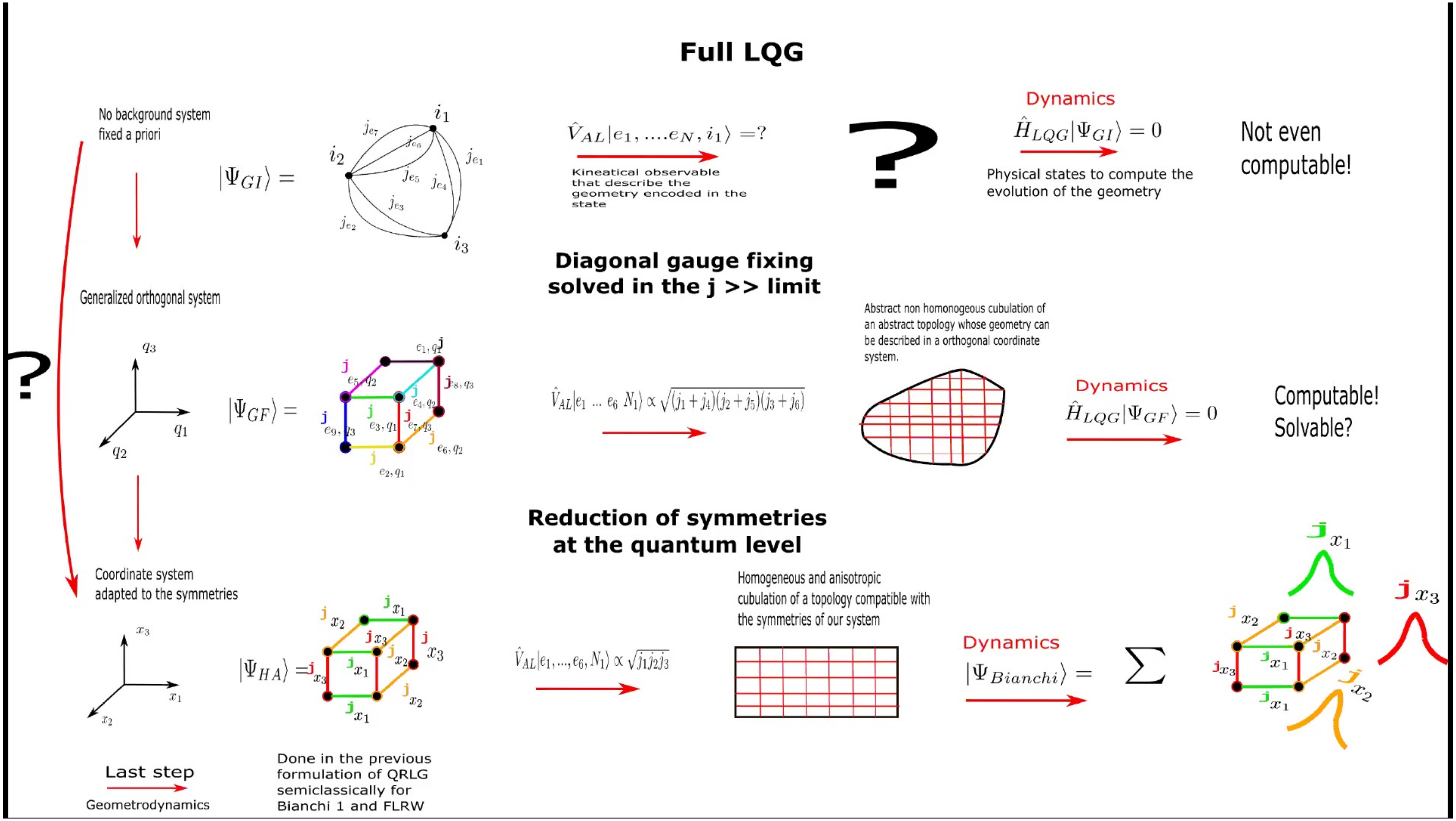
$\hat{H}_E$ → action

In the j>> limit the full theory operators reproduce the results of the reduced operators

They are the full theory operators

No need of projections

Computations are possible!!!


Full LQG
No background system fixed a priori
$|\Psi_{GI}\rangle =$
i_1 i_2 i_3 j_{e_1} j_{e_2} j_{e_3} j_{e_4} j_{e_5} j_{e_6} j_{e_7}
$\hat{V}_{AL}|e_1, ...e_N, i_1\rangle = ?$
Kineatical observable that describe the geometry encoded in the state
?
Dynamics
$\hat{H}_{LQG}|\Psi_{GI}\rangle = 0$
Physical states to compute the evolution of the geometry
Not even computable!
Diagonal gauge fixing solved in the j >> limit
Generalized orthogonal system
q_1 q_2 q_3
$|\Psi_{GF}\rangle =$
$\hat{V}_{AL}|e_1 \dots e_6\ N_1\rangle \propto \sqrt{(j_1+j_4)(j_2+j_5)(j_3+j_6)}$
Abstract non homonogeous cubulation of an abstract topology whose geometry can be described in a orthogonal coordinate system.
Dynamics
$\hat{H}_{LQG}|\Psi_{GF}\rangle = 0$
Computable! Solvable?
?
Reduction of symmetries at the quantum level
Coordinate system adapted to the symmetries
x_1 x_2 x_3
$|\Psi_{HA}\rangle =$
$\hat{V}_{AL}|e_1, ..., e_6, N_1\rangle \propto \sqrt{j_1 j_2 j_3}$
Homogeneous and anisotropic cubulation of a topology compatible with the symmetries of our system
Dynamics
$|\Psi_{Bianchi}\rangle = \sum$
Last step
Geometrodynamics
Done in the previous formulation of QRLG semiclassically for Bianchi 1 and FLRW

Conclusions

QRLG can be considered the leading order in j of the full theory, written in a specific coordinate system

This coordinate system is accessible through a gauge fixing, so we are considering all the physical degrees of freedom without imposing any kind of restriction.

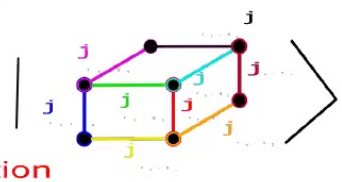
QRLG represents an ideal arena to study the full discrete version of general relativity, it should not be considered just a coherent states model. Coherent states dynamic is just one possible application of this framework.

QUANTUM COMPUTATIONS ARE POSSIBLE!!

Future goals

$$\hat{H}_{DLQG} = \hat{H}_E + \hat{H}_L + \hat{H}_{GU}$$

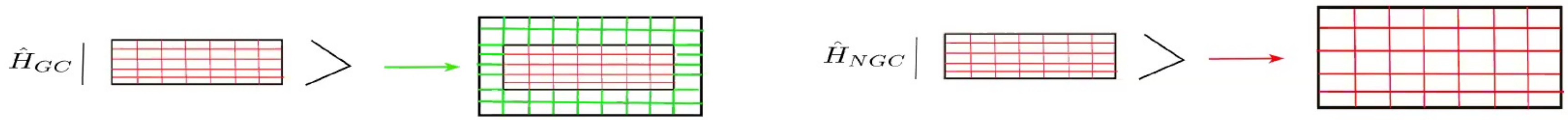
1) Find H_{GU} non local terms may arise ⟶ $\hat{H}_{GU}$

2) $H_L \propto R$ ⟶ $\hat{H}_L$ ⟶ $R(\hat{E}_i(S^a))$  regularizable

Thiemann's regularization   Warsaw regularization

Graph changing Vs non graph changing

Cosmologiacal Bianchi 1 example



✓ Physical evolution better compatible with the property of homogeneity of our classical spacetime

✗ To be done in QRLG

✗ Picture hardly compatible with the continuous classical spacetime

✓ Done in QRLG

The j>> limit

Reduced Hilbert space found neglecting $\mathcal{O}\left(\frac{1}{j}\right)$ → lowest order to neglect to have a consistent model

The order to build the Hilbert space is the important one to have access to this formalism

Ilkka's approach is able to estimate what is the actual order of validity of QRLG:

$$D^{(s)}_{mm}(h_e)D^{(j)}_{jj}(h_e) = D^{(j+m)}_{j+m\,j+m}(h_e) + \mathcal{O}\left(\frac{1}{j}\right)$$
$$D^{(s)}_{mn}(h_e)D^{(j)}_{jj}(h_e) = \mathcal{O}\left(\frac{1}{\sqrt{j}}\right) \qquad (m \neq n)$$

$$E_i(S_i)D^{(j)}_{jj}(h_e)_i = \pm(8\pi\beta G)\frac{1}{2}jD^{(j)}_{jj}(h_e)_i$$
$$E_k(S_i)D^{(j)}_{jj}(h_e)_i = \mathcal{O}(\sqrt{j}) \qquad (i \neq k)$$
$$E_k(S_i)D^{(j)}_{jj}(h_e)_l = 0 \qquad (i \neq l)$$

$$V_v|\Psi_0\rangle = \sqrt{\frac{1}{8}(j_1+j_4)(j_2+j_5)(j_3+j_6)}\,|\Psi_0\rangle + \mathcal{O}(\sqrt{j})$$

COMPUTABLE!! NO CLASSICAL ANALOGY.

Partial quantum gravity thermometer (homogeneous coherent sector)

All order in j	Maximum domain of QRLG	Intermediate domain of QRLG	Explored domain of QRLG	Validity of Emanuele and Francesco approach to the operators	Regularized LQC
$j = 1/2$	$\mathcal{O}(1/j)$	$\mathcal{O}(\sqrt{1/j})$		$\mathcal{O}(\sqrt{j})$	Coherent state sharply peaked during the evolution

Every domain tell us which is the lowest scale accessible to the model in terms of a fundamental cell (regularization schemes consists of fixing this cell choosing j)

QRLG + $\mathcal{O}(\sqrt{j})$ → Discretized classical GR, (non trivial discretization)
Quantum effect? Probably no
Effects which modify the dynamics comes from the discretization → LQC $\bar{\mu}$, $j = 1/2$?

Description by coherents states valid because we are not exploring a quantum regime?

minisuperspace models not only neglect the microscopic description of the gravitational field, but are they even able to explore only this regime of gravity?

QRLG + $\mathcal{O}(\sqrt{1/j})$ / $\mathcal{O}(1/j)$ → Explorable!
Computations are still easy? (fluxes and volume yes, holonomies?)
Finally some quantum effects?
Coherent states description still valid everywhere?

QRLG + All order in j → More general solutions of the Gubta Bleuer conditions
New Hilbert space