Title: The small mass-ratio expansion of the relativistic two body problem

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Series: Strong Gravity

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Abstract: To predict the gravitational waves emitted by a black hole binary, one needs to understand the dynamics of the binary in general relativity. No closed form solutions of this problem exist. Instead one must introduce some form of approximation. One such approximation, can be made if one of the components is much heavier than the other, suggesting a perturbative expansion in the mass-ratio. I will review this small mass-ratio (SMR) expansion of the dynamics, and the progress that has been made over the last two decades. In particular, I will discuss some recent results on the convergence of this series, suggesting that at relatively low orders this SMR expansion can be used to model even equal mass binaries.

The small mass-ratio expansion of the relativistic two body problem

Maarten van de Meent
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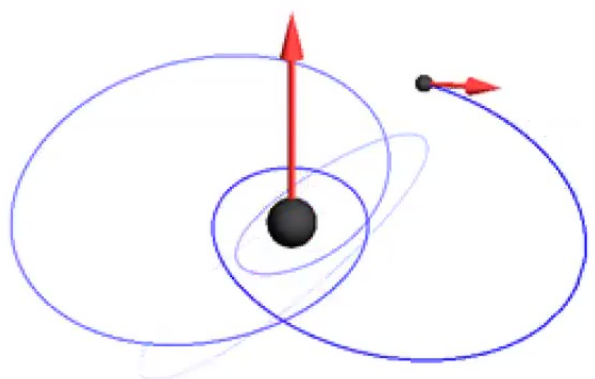
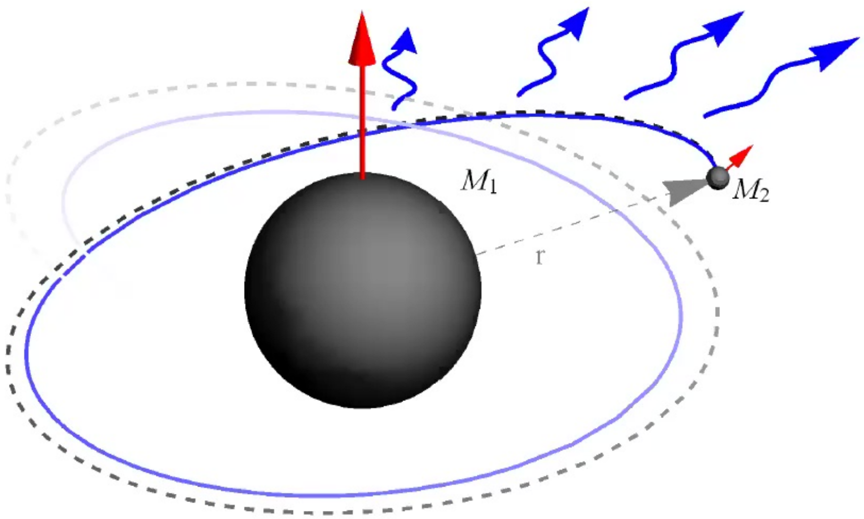


14 May 2020
PI Strong gravity seminar

The 2-body problem in General Relativity



General Setup

Two bodies (black holes) with masses $M_1 \geq M_2$ and spins $\vec{a}_1$ and $\vec{a}_2$ in mutual gravitational orbit.

Problem

No closed form exact solutions exist. => We need some form of approximation.

Numerical Relativity (NR)

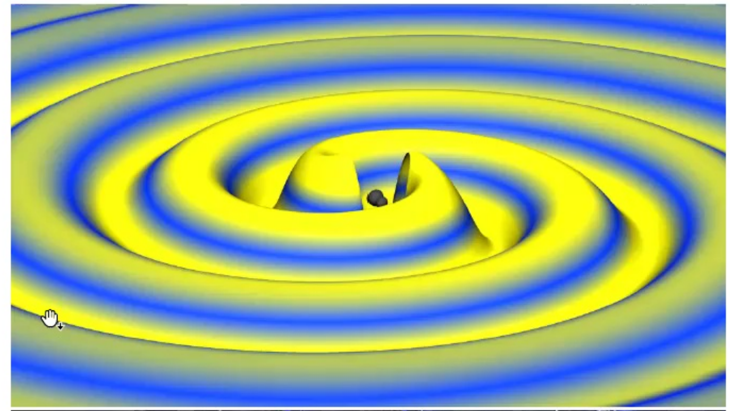
Strategy

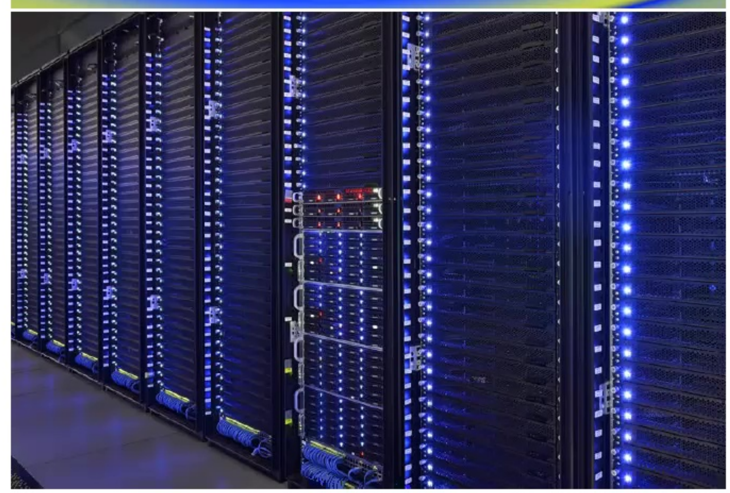
"Put Einstein's equation on a grid and evolve on a supercomputer."

- First attempts in the 1960s [Hahn&Lindquist, 1964]
- First success in 2005 [Pretorius, 2005]
- Now binary simulations are "routine".

Limitations

- Expensive (single simulations take months to years)
- Can run only for limited number of orbits $\mathcal{O}(10^2)$
- Costs scale with (at least) $(M_1/M_2)^2$. Practically limited to $M_2/M_1 \gtrsim 1/10$





Post-Newtonian (PN) theory

$$\underset{0}{H}_+ = -(1+c_i^2)\cos 2\psi - \frac{1}{96}\,s_i^2\,(17+c_i^2)\,, \tag{322a}$$

$$\underset{1/2}{H}_+ = -\,s_i\,\Delta\left[\cos\psi\left(\frac{5}{8}+\frac{1}{8}\,c_i^2\right) - \cos 3\psi\left(\frac{9}{8}+\frac{9}{8}\,c_i^2\right)\right], \tag{322b}$$

$$\begin{aligned}\underset{1}{H}_+ = {} & \cos 2\psi\left[\frac{19}{6}+\frac{3}{2}\,c_i^2-\frac{1}{3}\,c_i^4+\nu\left(-\frac{19}{6}+\frac{11}{6}\,c_i^2+c_i^4\right)\right] \\ & -\cos 4\psi\left[\frac{4}{3}\,s_i^2(1+c_i^2)(1-3\nu)\right],\end{aligned} \tag{322c}$$

$$\begin{aligned}\underset{3/2}{H}_+ = {} & s_i\,\Delta\cos\psi\left[\frac{19}{64}+\frac{5}{16}\,c_i^2-\frac{1}{192}\,c_i^4+\nu\left(-\frac{49}{96}+\frac{1}{8}\,c_i^2+\frac{1}{96}\,c_i^4\right)\right] \\ & +\cos 2\psi\left[-2\pi(1+c_i^2)\right] \\ & + s_i\,\Delta\cos 3\psi\left[-\frac{657}{128}-\frac{45}{16}\,c_i^2+\frac{81}{128}\,c_i^4+\nu\left(\frac{225}{64}-\frac{9}{8}\,c_i^2-\frac{81}{64}\,c_i^4\right)\right] \\ & + s_i\,\Delta\cos 5\psi\left[\frac{625}{384}\,s_i^2(1+c_i^2)(1-2\nu)\right],\end{aligned} \tag{322d}$$

$$\begin{aligned}\underset{2}{H}_+ = {} & \pi\,s_i\,\Delta\cos\psi\left[-\frac{5}{8}-\frac{1}{8}\,c_i^2\right] \\ & +\cos 2\psi\left[\frac{11}{60}+\frac{33}{10}\,c_i^2+\frac{29}{24}\,c_i^4-\frac{1}{24}\,c_i^6+\nu\left(\frac{353}{36}-3\,c_i^2-\frac{251}{72}\,c_i^4+\frac{5}{24}\,c_i^6\right)\right. \\ & \qquad \left.+\nu^2\left(-\frac{49}{12}+\frac{9}{2}\,c_i^2-\frac{7}{24}\,c_i^4-\frac{5}{24}\,c_i^6\right)\right] \\ & +\pi\,s_i\,\Delta\cos 3\psi\left[\frac{27}{8}(1+c_i^2)\right] \\ & +\frac{2}{15}\,s_i^2\cos 4\psi\left[59+35\,c_i^2-8\,c_i^4-\frac{5}{3}\,\nu\,(131+59\,c_i^2-24\,c_i^4)+5\,\nu^2\,(21-3\,c_i^2-8\,c_i^4)\right] \\ & +\cos 6\psi\left[-\frac{81}{40}\,s_i^4(1+c_i^2)\,(1-5\nu+5\nu^2)\right] \\ & + s_i\,\Delta\sin\psi\left[\frac{11}{40}+\frac{5\ln 2}{4}+c_i^2\left(\frac{7}{40}+\frac{\ln 2}{4}\right)\right] \\ & + s_i\,\Delta\sin 3\psi\left[\left(-\frac{189}{40}+\frac{27}{4}\ln(3/2)\right)(1+c_i^2)\right],\end{aligned} \tag{322e}$$

[Source: Blanchet, LRR]

Strategy

"Expand dynamics in powers $1/r \propto v^2/c^2$ of the separation."

- Leading contribution Newtonian dynamics (Kepler's laws)
- Full results up to 4PN currently available.
- Some partial results at higher orders.

Limitations

The post-Newtonian approximation in an asymptotic series, with poor convergence in the strong field (r/M order few) regime.

Effective waveform models

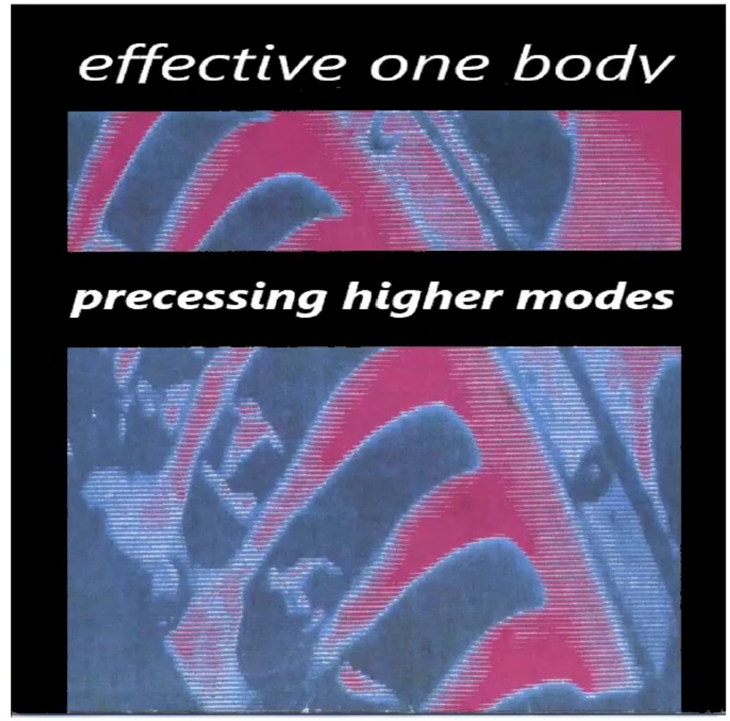
Strategy

"Attain best of both worlds by combining results from both post-Newtonian theory and Numerical Relativity."

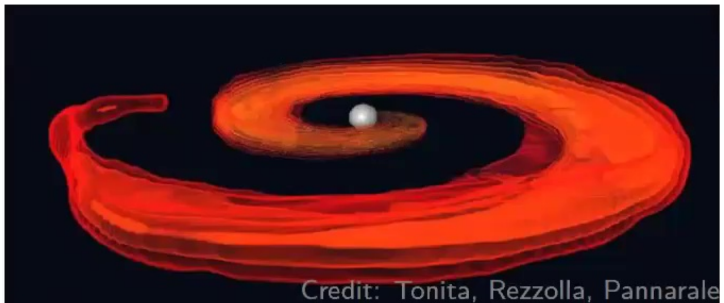
- Used for generating waveforms for analysis of LIGO/Virgo/KAGRA data
- Effective-One-Body: e.g. `SEOBNRv4PHM`
- Phenom family: e.g. `IMRPhenomXHM`

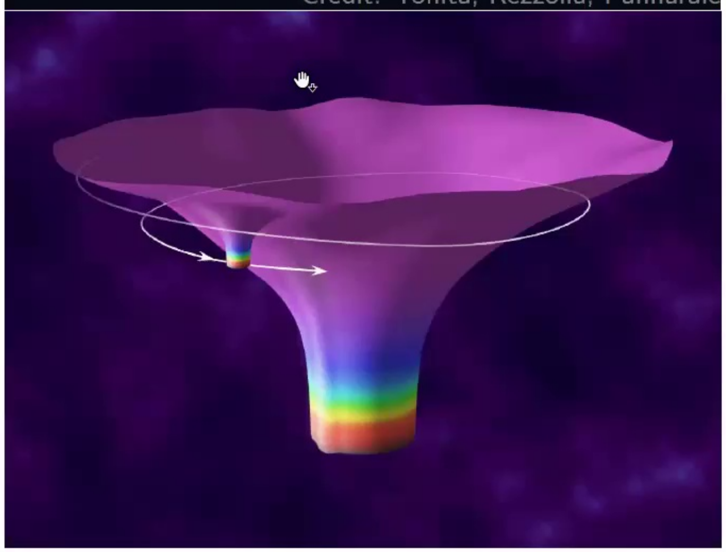
Limitations

Inherits common parameter space limitations of NR and PN. In particular, can't produce faithful waveforms for mass-ratios $q = M_2/M_1 \lesssim 1/10$.



Why care about small mass-ratios?





Neutron Star-Black hole binaries

Neutron star black hole binaries, could have mass-ratios as low as 1:30.

SMBH binaries

Supermassive black hole binaries can have a much wider range of mass-ratios. Some possibly as low as 1:100.

EMRIs

Extreme mass-ratio binaries consisting of a stellar mass object orbiting a supermassive black hole have mass-ratios $\mathcal{O}(10^{-5})$, and can be observed by LISA, and would act as highly sensitive probes of black hole physics.

IMRIs

Consisting of either a SMBH+IMBH (LISA) or IMBH + compact object (Einstein Telescope). Can be used to search for intermediate mass black holes.

Small mass-ratio perturbation theory

"Elk nadeel hep z'n voordeel."
[Johan Cruijff]

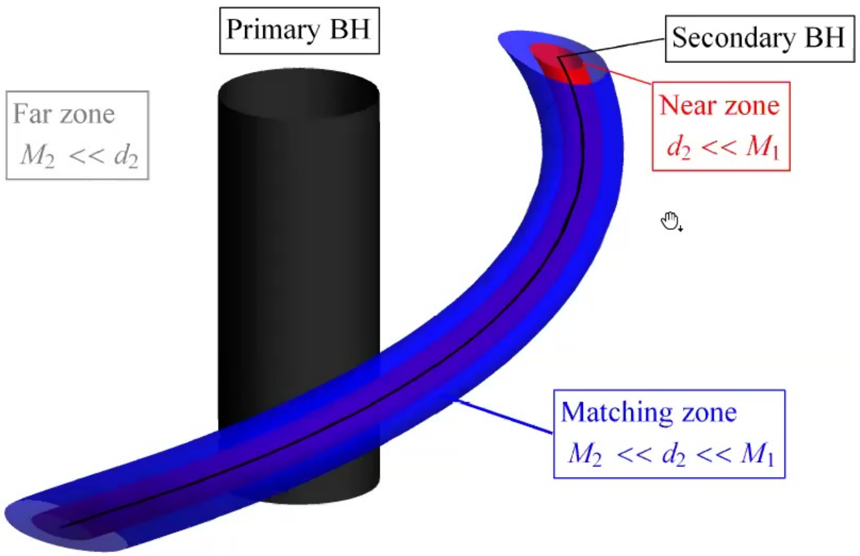
Strategy

Use the smallness of the mass-ratio $q := \frac{M_2}{M_1}$ (or $\nu := \frac{M_1 M_2}{(M_1+M_2)^2}$) to our advantage and use it as an expansion parameter using:

- Black hole perturbation theory
- Multi length scale analysis (matched asymptotic expansions)
- Multi time scale analysis

Equations of Motion: Matched asymptotic expansions

[Mino, Sasaki & Tanaka, 1997] [Poisson, 2003][Pound, 2008-]



far zone

Kerr geometry of primary plus perturbation generated by secondary.

near zone

Kerr geometry of secondary (in rest frame) plus perturbation generated by primary.

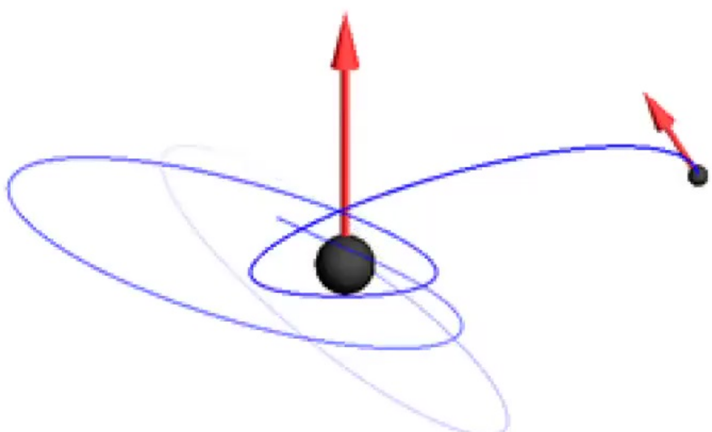
Strategy:

Write down most general peturbative solution in each zone, and match unknown coefficients in overlap region, resulting in a coupled set of equations for metric and trajectory in the far zone.

Leading order results: geodesics and parallel transport

Metric

The leading metric is given by the Kerr geometry $g_{\mu\nu}^{\text{Kerr}}$ generated by the primary object.



Secondary trajectory

The secondary follows a geodesic in $g_{\mu\nu}^{\text{Kerr}}$.

$$\frac{D^2}{d\tau^2}x^\mu = 0$$

Complete closed form solutions are available. [Carter, 1968][Schmidt,2002][Fujita&Hikida,2009]

Secondary spin

The secondary's spin is parallel transported along the trajectory x^μ.

$$\frac{D}{d\tau}a_2^\mu = 0$$

Complete closed form solutions are available [Marck,1983][MvdM,2019]

1st order: Overview

Metric

The leading order metric perturbation $h^1_{\mu\nu}$ is given by solving the linearized Einstein equation on the Kerr background sourced by a point mass M_2 following the trajectory x^μ.

Trajectory

The geodesic equation for the trajectory is modified by an effective $\mathcal{O}(q)$ force term with two contributions:

- The backreaction of h^1 on the worldline, the gravitational self-force (GSF).
- The coupling of the secondary spin to the background metric $g^{\mathrm{Kerr}}_{\mu\nu}$, the Mathisson-Papapetrou-Dixon (MPD) force.

Spin

The parallel transport equation for the secondary spin is modified by an effective $\mathcal{O}(q)$ torque term with two contributions:

- The backreaction of h^1 on the worldline, the gravitational self-torque.
- An MPD coupling to background (depending on a spin supplementary condition).

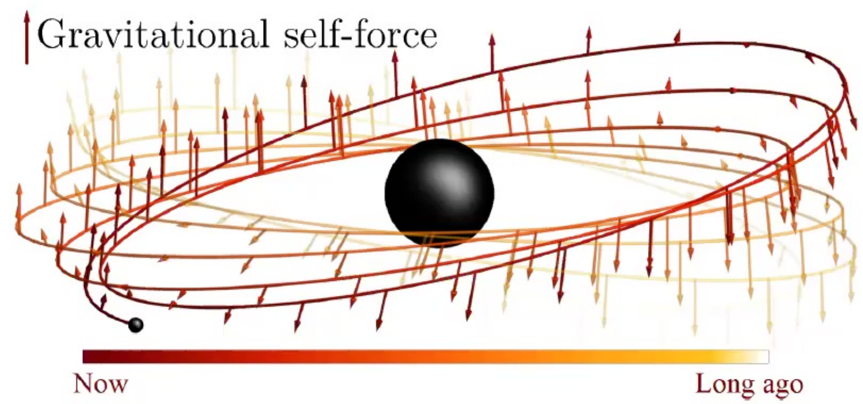
Key steps:

- Calculate metric perturbation
- Find "regular" part
- Calculate GSF

- First calculation for circular orbits in Schwarzschild in 2007 [Barack&Sago, 2007]
- Steady progress since then, improving efficiency, accuracy, and extending the parameter range and gauges used.

State-of-the-art

The first order GSF can now be calculated for any bound geodesic on a Kerr background. [MvdM, 2018]



1st order: Self-torque

Spin precession correction

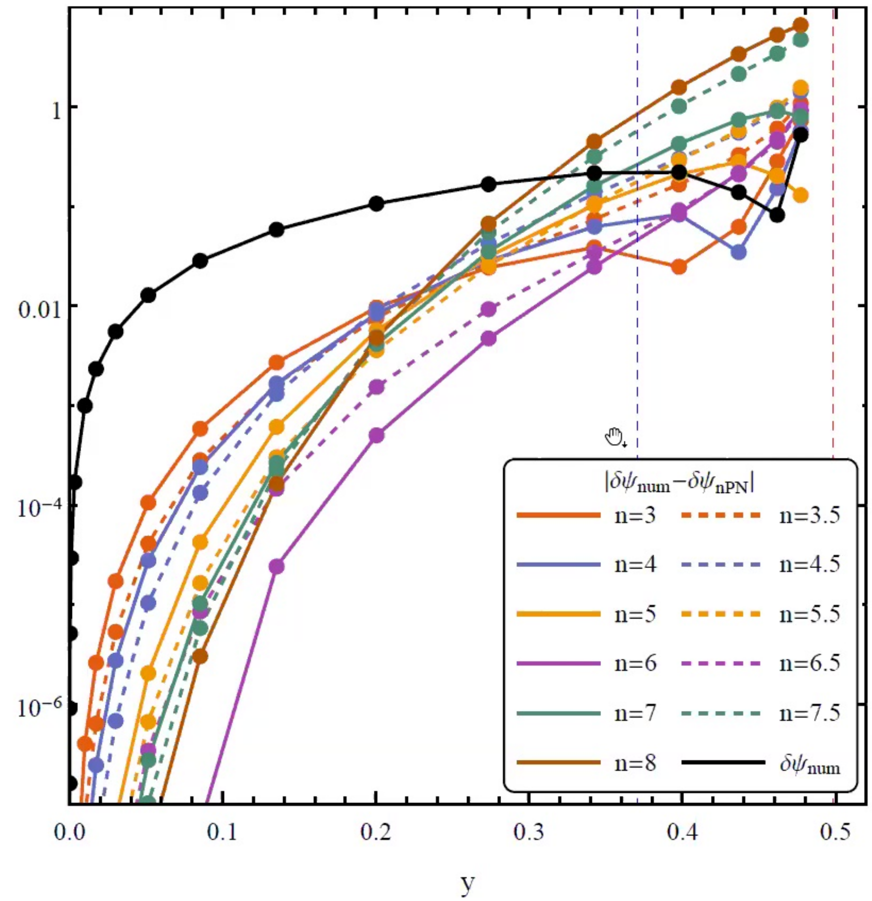
The gravitational self-torque leads to a correction to rate at which the secondary spin precesses.

Calculated for:

- Circular orbits in Schwarzschild [Dolan et al., 2013]
- Eccentric orbits in Schwarzschild [Akcay et al. 2016]
- Circular orbits in Kerr [Bini et al.+(MvdM),2018]

To do

Eccentric and inclined orbits in Kerr.



2nd order: Overview & results

Formalism

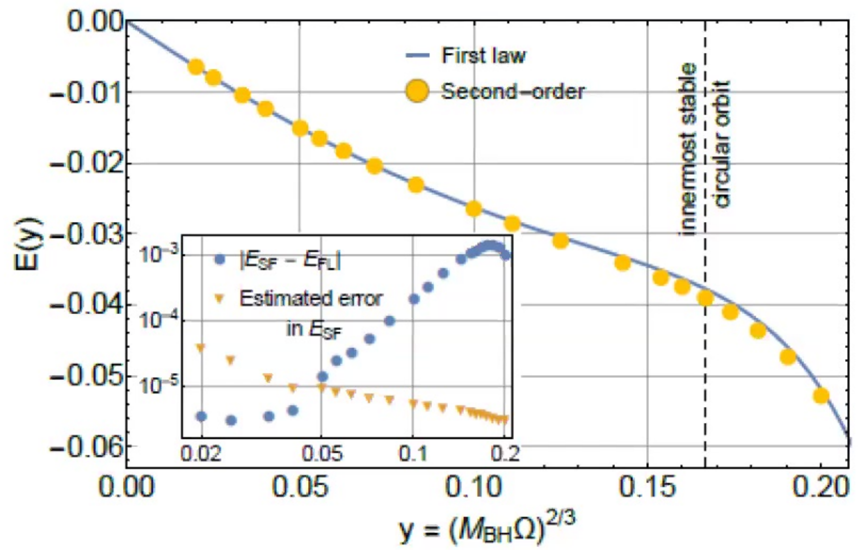
Second order perturbation theory is much more involved an subtle than its linear counterpart. Most conceptual issues have now been addressed and the formalism for calculating the second order GSF is in place.

First results

The correction to the orbital binding energy of quasicircular orbits in Schwarzschild has been calculated

Soon$^{\mathrm{TM}}$...

2GSF for quasicircular orbits in Schwarzschild



Challenge

2^{nd} calculations in Kerr

Inspiral evolution

Equations of motion:

$$\frac{D^2}{d\tau^2}x^\mu = 0 + qF_1^\mu[\tau,\gamma,a^\mu] + q^2F_2^\mu[\tau,\gamma,a^\mu] + \mathcal{O}(q^3)$$
$$\frac{D}{d\tau}a^\mu = 0 + qT_1^{\mu\alpha}[\tau,\gamma,a^\mu]a_\alpha + q^2T_2^{\mu\alpha}[\tau,\gamma,a^\mu]a_\alpha + \mathcal{O}(q^3)$$

The GSF and GST depend on the metric perturbation which depends on the entire history of the particle trajectory γ.

Two options:

- Solve equations for metric peturbation and trajectory simultaneously in the time domain. Problem: slow, many of the issues of full Numerical Relativity.
- Order reduction. For each pair (x^μ, u^ν) there is a unique trajectory $\gamma(x^\mu, u^\nu)$. Substituting this gives a closed system of equations for $(x^\mu, u^\nu, a^\lambda)$.

$$\frac{D}{d\tau}u^\mu = 0 + qF_1^\mu[x^\mu,p^\nu,a^\lambda] + q^2F_2^\mu[x^\mu,p^\nu,a^\lambda] + \mathcal{O}(q^3)$$
$$\frac{D}{d\tau}a^\mu = 0 + qT_1^{\mu\alpha}[x^\mu,p^\nu,a^\lambda]a_\alpha + q^2T_2^{\mu\alpha}[x^\mu,p^\nu,a^\lambda]a_\alpha + \mathcal{O}(q^3)$$

Near-identity (averaging) transform [MvdM&Warburton,2018]

The leading order equations of motion (geodesic+parallel transport) are integrable. It is therefore convenient to pass to action-angle variables $(\vec{J}, \vec{q})$: (It is also convenient to reexpress the expansion in terms of the symmetric mass ratio $\nu = \frac{q}{(1+q)^2}$, if only not to have two types of q)

$$\frac{d\vec{J}}{d\tau} = 0 \qquad + \nu \vec{F}_1(\vec{J}, \vec{q}) + \nu^2 \vec{F}_2(\vec{J}, \vec{q}) + \mathcal{O}(\nu^3)$$

$$\frac{d\vec{q}}{d\tau} = \vec{\Omega}(\vec{J}) + \nu \vec{f}_1(\vec{J}, \vec{q}) + \nu^2 \vec{f}_2(\vec{J}, \vec{q}) + \mathcal{O}(\nu^3)$$

This can be further simplified by applying a small (near identity) transformation $\vec{J} \mapsto \vec{J}\nu X_1(\vec{J}, \vec{q}) + \nu^2 X_2(\vec{J}, \vec{q})$, $\vec{q} \mapsto \vec{q}\nu Y_1(\vec{J}, \vec{q}) + \nu^2 Y_2(\vec{J}, \vec{q})$, which can be used to eliminate the dependence of the RHS. But, beware of resonances!

$$\frac{d\vec{J}}{d\tau} = 0 \qquad + \nu \vec{G}_1(\vec{J}) + \nu^2 \vec{G}_2(\vec{J}) + \mathcal{O}(\nu^3)$$

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$\vec{G}_1 = \langle \vec{F}_1 \rangle$ and $\vec{g}_1 = \langle \vec{f}_1 \rangle$, but (schematically)
$\vec{G}_2 = \langle \vec{F}_2 \rangle + \langle f_1 F_1 \rangle$, and $\vec{g}_2 = \langle \vec{f}_2 \rangle + \langle f_1 F_1 \rangle$

Two-timescale expansion [Hinderer&Flanagan, 2008]

The two sets of equations evolve on different timescales, The actions $\vec{J}$ evolve slowly changing only on the inspiral $\mathcal{O}(\nu^{-1})$ timescale, while the actions/phases $\vec{q}$ change on the orbital time scale $\mathcal{O}(\nu^{0})$. This hierarchy of timescale can be address by introducing a "slow time" $\tilde{t} = \nu\tau$, and doing a two-timescale expansion resulting in,[Hinderer&Flanagan, 2008]

$$
\begin{aligned}
\vec{J}(\tilde{t},\nu) &= \vec{J}_0(\tilde{t}) \quad + \nu \vec{J}_1(\tilde{t}) + \nu^2 \vec{J}_2(\tilde{t}) + \mathcal{O}(\nu^3) \\
\vec{q}(\tilde{t},\nu) &= \frac{1}{\nu}\vec{q}_0(\tilde{t}) + \vec{q}_1(\tilde{t}) \quad + \nu \vec{q}_2(\tilde{t}) \quad + \mathcal{O}(\nu^3)
\end{aligned}
$$

Leading (adiabatic) order

$\vec{J}_0(\tilde{t})$ and $\vec{q}_0(\tilde{t})$ require:

- $\vec{\Omega}(\vec{J})$ (geodesic dynamics)
- $\langle F_1 \rangle(\vec{J})$ (average dissipative GSF)

1-post-Adiabatic order

$\vec{J}_1(\tilde{t})$ and $\vec{q}_1(\tilde{t})$ additionally require:

- $F_1(\vec{J})$ and $f_1(\vec{J})$ (full first order GSF and GST)
- $\langle F_2 \rangle(\vec{J})$ (average dissipative 2GSF)

2-post-Adiabatic order

$\vec{J}_2(\tilde{t})$ and $\vec{q}_2(\tilde{t})$ additionally require:

- $F_2(\vec{J})$ and $f_2(\vec{J})$ (full second order GSF and GST)
- $\langle F_3 \rangle(\vec{J})$ (average dissipative 3GSF)

When is a small mass-ratio small?

Question(s):

- How far do we have to go in the SMR expansion to obtain result that are accurate enough to for use in gravitational wave observations?
- How small does ν have to be? Is $\nu = 1/4$ (equal mass) "small"?

$$\vec{J}(\tilde{t},\nu) = \vec{J}_0(\tilde{t}) \quad + \nu \vec{J}_1(\tilde{t}) + \nu^2 \vec{J}_2(\tilde{t}) + \mathcal{O}(\nu^3)$$
$$\vec{q}(\tilde{t},\nu) = \frac{1}{\nu}\vec{q}_0(\tilde{t}) + \vec{q}_1(\tilde{t}) \quad + \nu \vec{q}_2(\tilde{t}) \quad + \mathcal{O}(\nu^3)$$

Naive argument

The phase of the gravitational waves generated by a binary depends directly on $\vec{q}$. In order for the waveform template to be useful we need the error in phase to be $\lesssim 1 rad$ (at least). Truncating the series at adiabatic order we make an error that is $\mathcal{O}(\nu^0)$, that is probably too much (for any mass-ratio). If we go one order beyond that the error become $\mathcal{O}(\nu^0)$. For sufficiently small ν this probably enough.

Can we do better?

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2-post-Adiabatic order

$\vec{J}_2(\tilde{t})$ and $\vec{q}_2(\tilde{t})$ additionally require:

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Can we do better?

Previous comparisons between GSF and NR results

Previous comparisons between NR and SMR

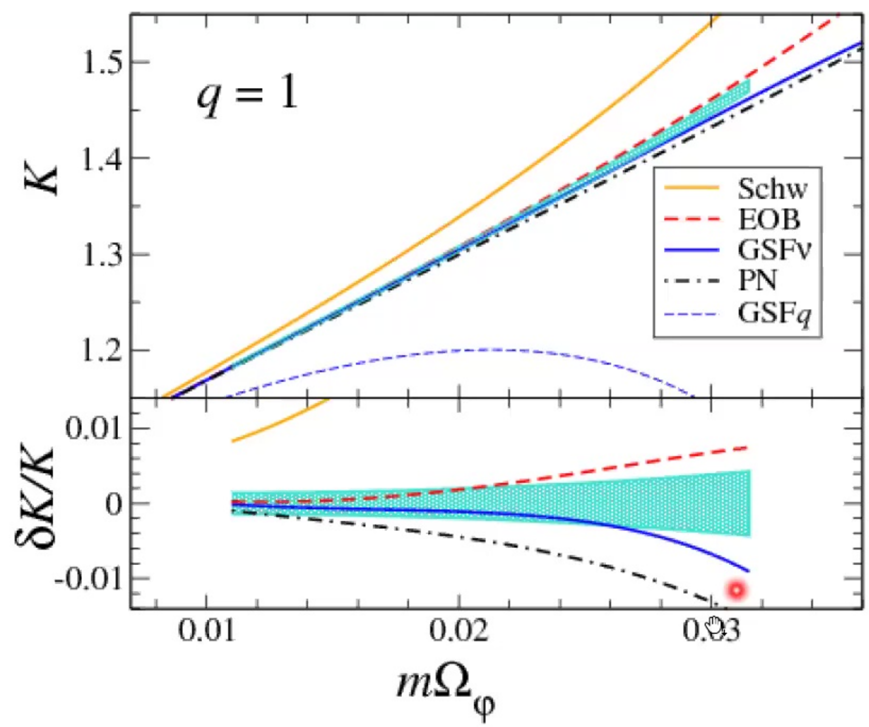
Focussed on conservative quantities:

- Periapsis shift [Le Tiec+, 2011, 2013][MvdM, 2016]
- Redshift [Zimmerman+, 2016]

Found suprisingly good agreement at equal mass.
Limited by ability to isolate conservative effects in NR

Limitations

NR results cannot be unambiguously split in a conservative and dissipative part. This limits the accuracy with which conservative effects can be extracted.



[Le Tiec+, 2011]

Lets look at the GW phase of quasicircular non-spinning inspirals.

SMR

Much simpler... (use orbital frequency $M\Omega$ as "slow time")

$$\phi(M\Omega, \nu) = \frac{1}{\nu}\phi_0(M\Omega) + \phi_1(M\Omega) + \nu\phi_2(M\Omega) + \mathcal{O}(\nu^3)$$

- ϕ_0 needs orbital frequency and first order energy flux (easy)
- ϕ_1 needs first order self-force (available) + second order fluxes (soon)
- ϕ_2 needs second order self-force (soonish) + third order fluxes (not soon)

NR

Plenty of NR simulations available.

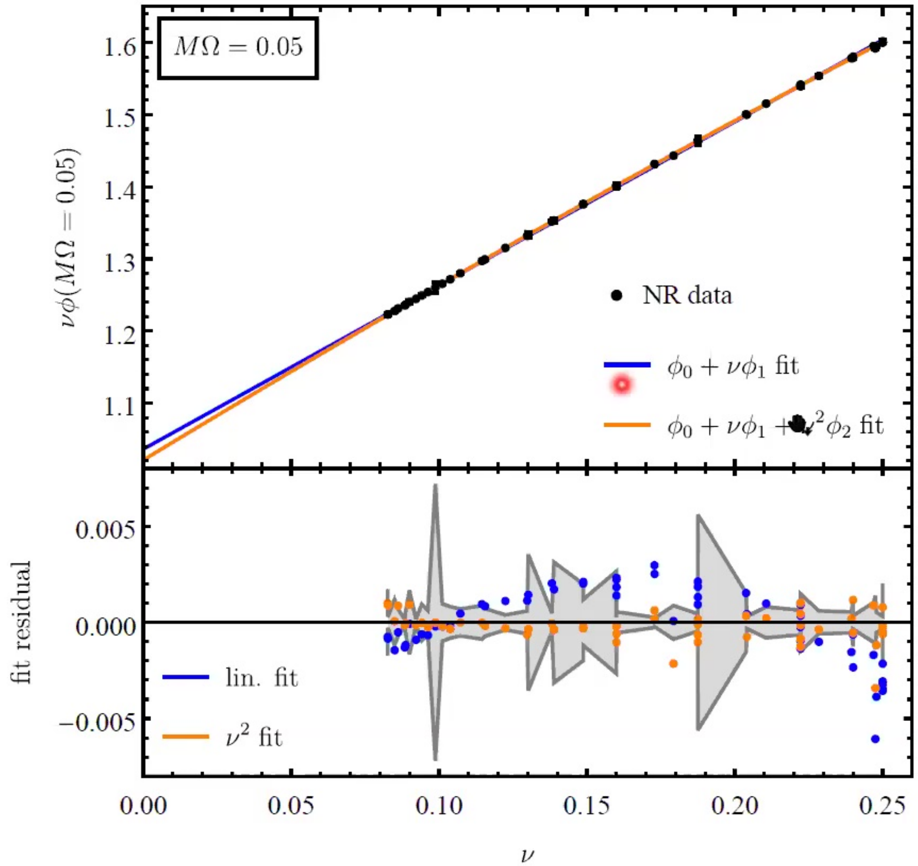
- 55 BHB simulations from SXS collaboration.
- quasi-circular
- non-spinning
- mass-ratio q between $\frac{1}{10}$ and 1

Idea

Can we extract the coefficients of the PA expansion from the NR data?

Fitting procedure

Maarten van de Mee...



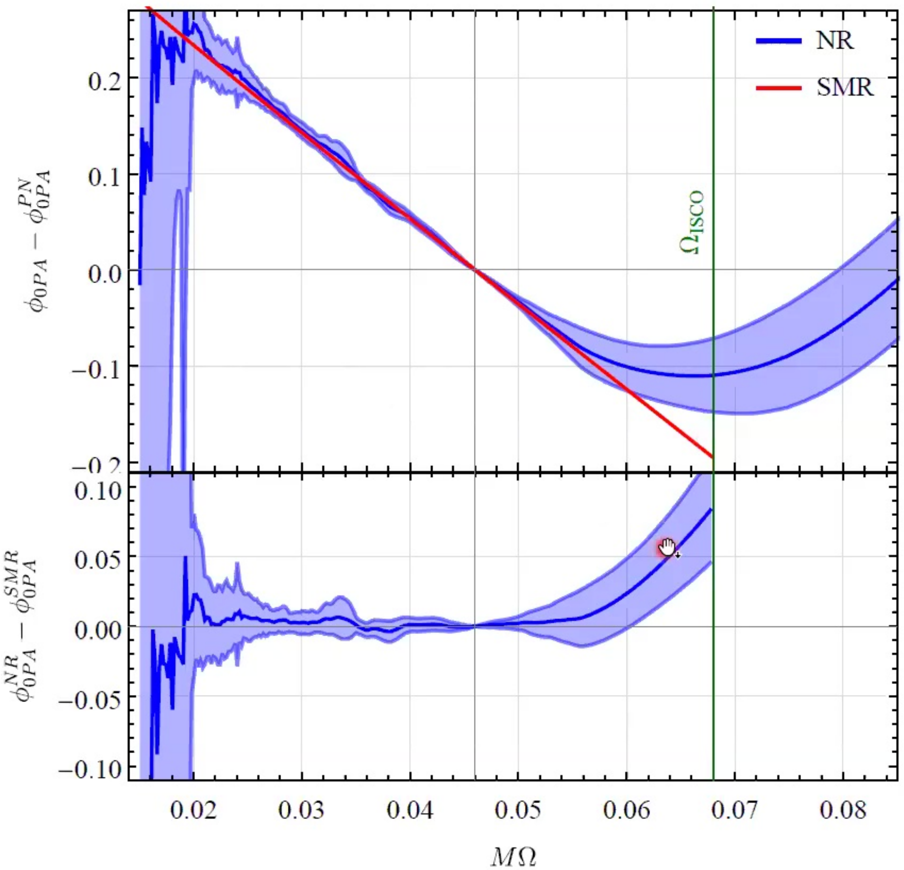
Procedure

- Extract $\phi(\Omega)$

- Linear fit in ν capture most variability in $\nu\phi$ ($1 - \bar{R}^2 \approx 1.8 \cdot 10^{-4}$)
- Adding ν^2 (i.e. 2PA ϕ_2) term captures remaining variability ($1 - \bar{R}^2 \approx 1.8 \cdot 10^{-5}$).
- Below $M\Omega \lesssim 0.05$ little or no evidence for 2PA terms and beyond in NR data.

Adiabatic order (0PA)

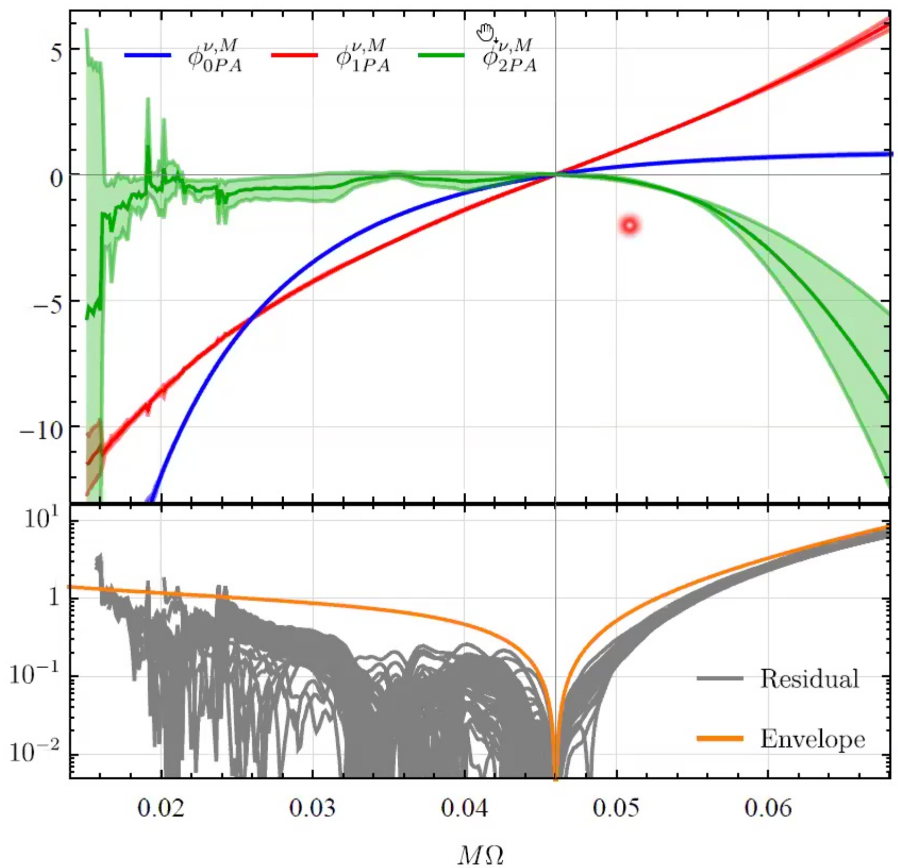
Maarten van de Mee...



[MvdM&Pfeiffer, in prep]

- Leading order 0PA term extract from NR agrees (almost) perfectly with SMR predicition.
- Slight deviation near ISCO frequency likely due to onset of transition to plunge. (Leading to appearance $\nu^{-1/5}$ terms. [Buonanno & Damour, 2000][Ori & Thorne, 2000])
- From here on, will fix ϕ_0 to SMR value.
- Recovery of leading 0PA term signals that the PA expansion is well behaved for comparable masses. (Compare difficulty of extracting PN terms from NR data.)

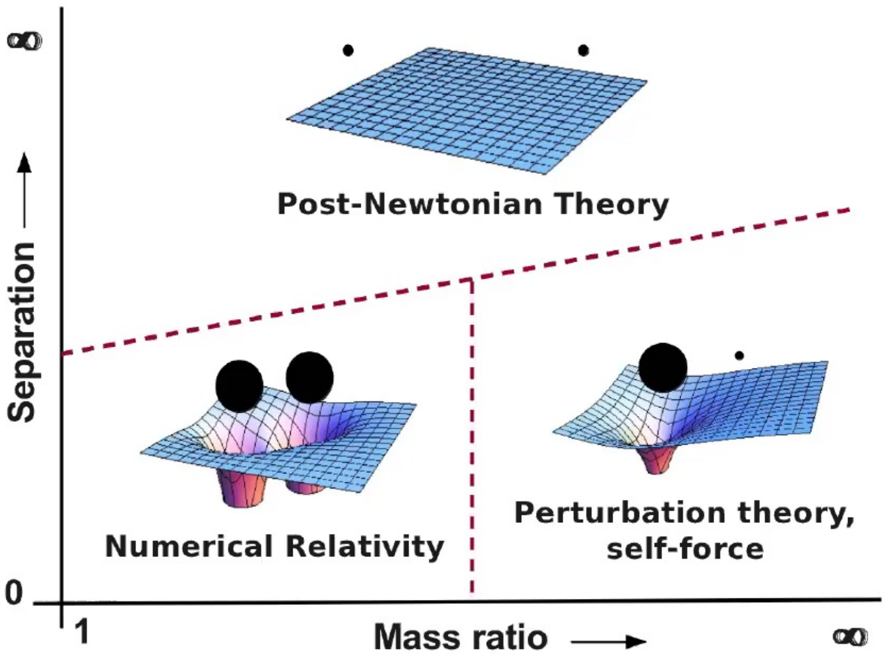
Post-adiabatic corrections (1PA & 2PA)



[MvdM&Pfeiffer, in prep]

- ϕ_{1PA} accumulates $\mathcal{O}(10)$ radians over the relevant frequency range. $\rightarrow$ The adiabatic approximation is insufficient at <u>any</u> mass-ratio.
- ϕ_{2PA} remains $\lesssim \mathcal{O}(10)$ over the entire range. Below $M\Omega \lesssim 0.05$ ϕ_{2PA} is consistent with 0.
- Expansion is dominated by first two terms at (almost) all mass-ratios.

Regimes of validity



In discussing the differences between various approaches to the 2-body problem one often encounters diagrams like the one here by Leor Barack. The results obtained by comparing NR results to the SMR expansion (and PN results), allow us to make an empirically informed version of these diagrams.

Setup

The red (blue) contours give the region where 1PA (3.5PN) results are accurate up to a residual error of $\frac{\pi}{4}$, $\frac{\pi}{8}$, or $\frac{\pi}{16}$. The red contours have a lower and upper bound since, the 2PA result diverges at $\Omega = 0$.

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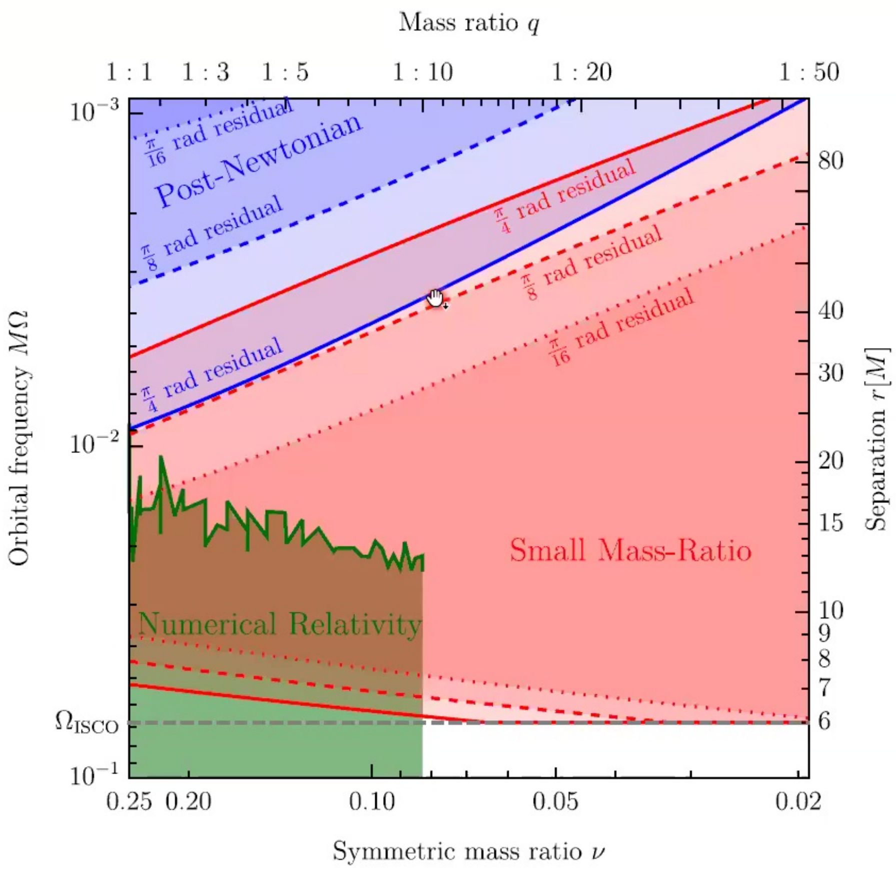

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Incorporating SMR results in EOB [Antonelli+,2019]

Can we include SMR results in EOB models?

Light ring problem

The standard formulation of EOB leads (coordinate) divergences at $r = 3M$ when including conservative GSF results.

Solution

Reformulate EOB in a different gauge/formulation tailored to the inclusion of SMR results.