Title: Black hole information, spacetime wormholes and baby universes

Speakers: Henry Maxfield

Series: Quantum Fields and Strings

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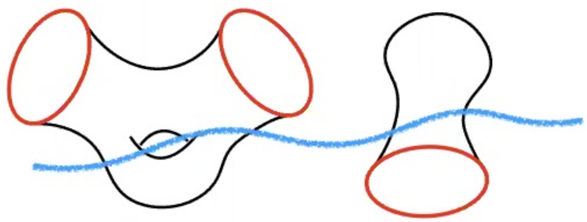
Abstract: Hawking famously observed that the formation and evaporation of black holes appears to violate the unitary evolution of quantum mechanics. Nonetheless, it has been recently discovered that a signature of unitarity, namely the "Page curve" describing the evolution of entropy, can be recovered from semiclassical gravity. This result relies on "replica wormholes" appearing in the gravitational path integral, which are examples of spacetime wormholes studied more than 30 years ago and related to interactions with closed "baby" universes. I will discuss the connection between these new and old ideas and the consequences for the black hole information problem. A central lesson is that a Hilbert space in quantum gravity can be dramatically modified by nonperturbative topology-changing processes.

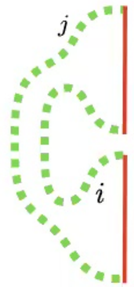
Black hole information, spacetime wormholes and baby universes



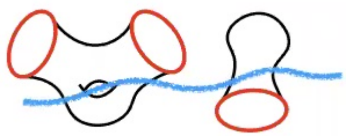
Henry Maxfield, UCSB

[2002.08950] & [2005(1).xxxxx] with Don Marolf

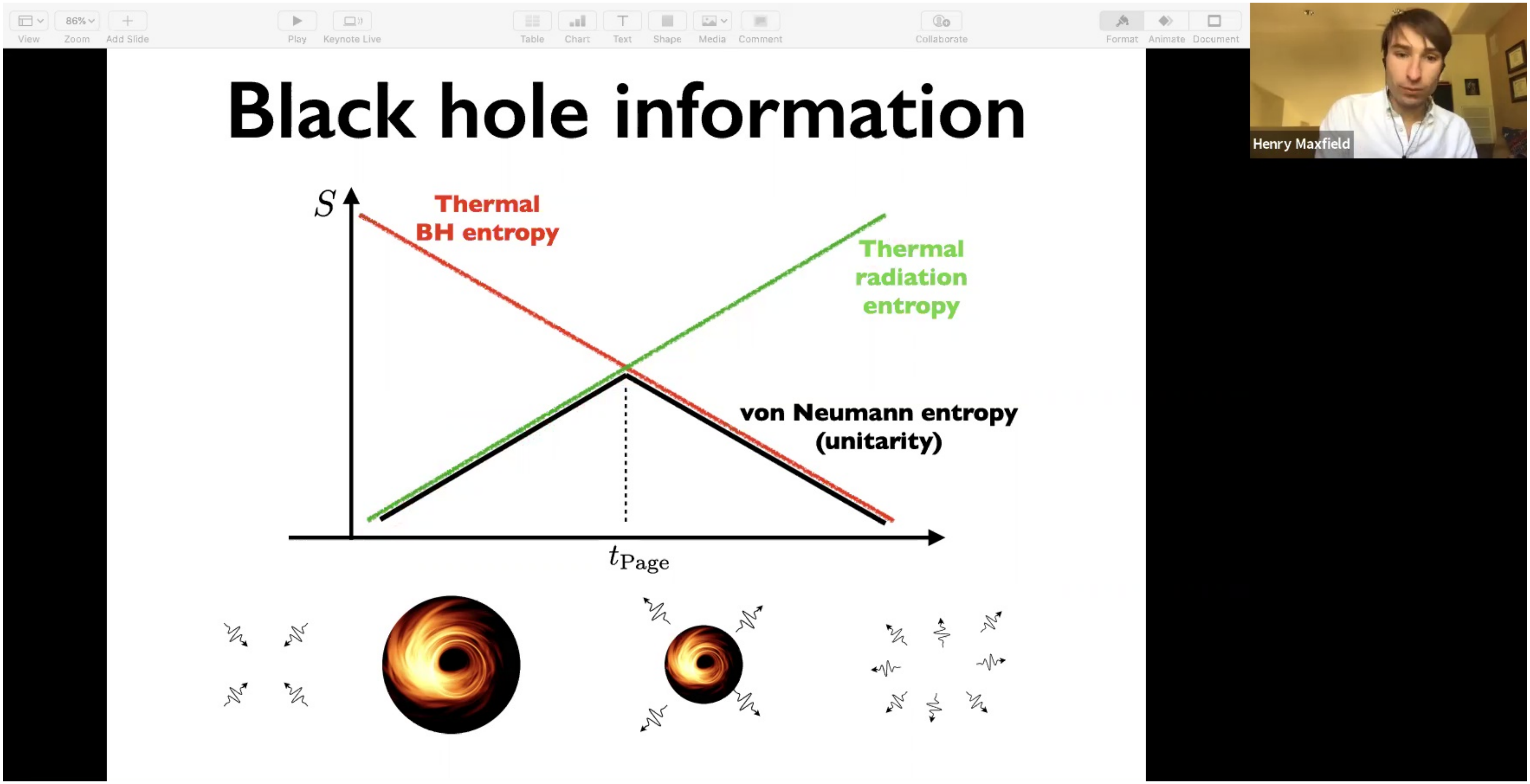
1. Page curve and replica wormholes



2. Spacetime wormholes and baby universes



3. Semiclassical predictions for black hole evaporation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S
Thermal
BH entropy
Thermal
radiation
entropy
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(unitarity)
t_{Page}

Black hole information

[Almheri,Engelhardt,Marolf,HM][Penington]: **semiclassical calculation of Page curve**

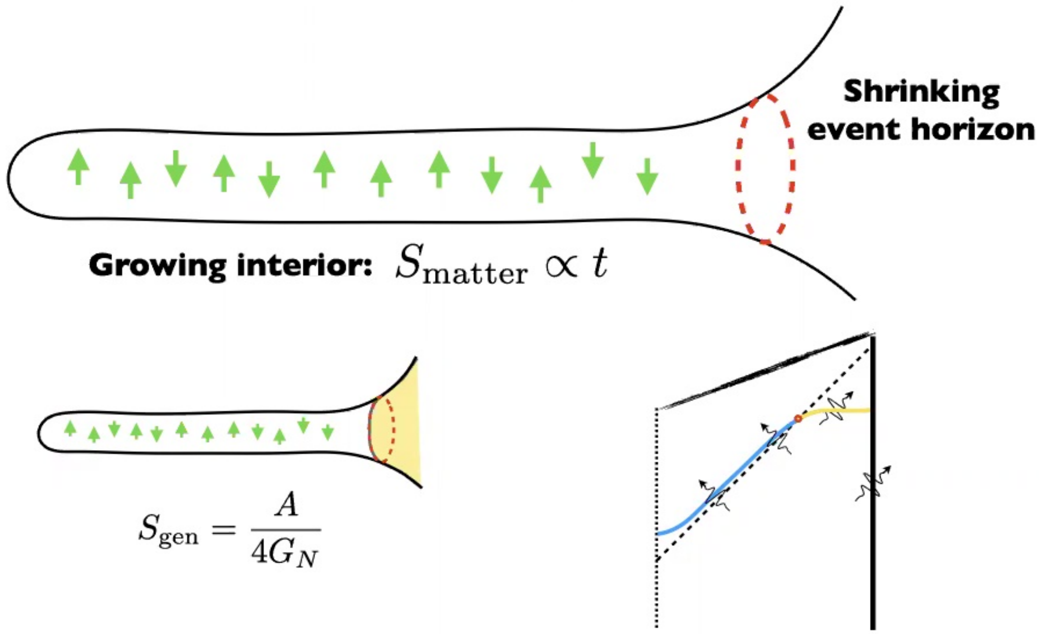
Used "quantum RT formula": $S(\rho) = -\operatorname{Tr}(\rho \log \rho) = \operatorname*{extr}_{\Sigma} S_{\text{gen}}(\Sigma)$

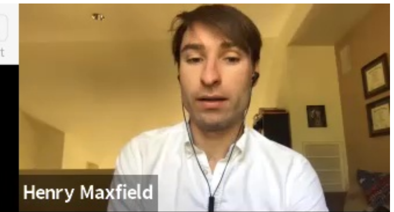
[Ryu,Takayanagi][HubenyRangamaniTakayanagi][EngelhardtWall]

Generalised entropy $S_{\text{gen}}(\Sigma) = \dfrac{A(\partial\Sigma)}{4G_N} + S_{\text{matter}}(\Sigma)$

Extremise over partial Cauchy slices Σ

Quantum extremal surface in an evaporating black hole





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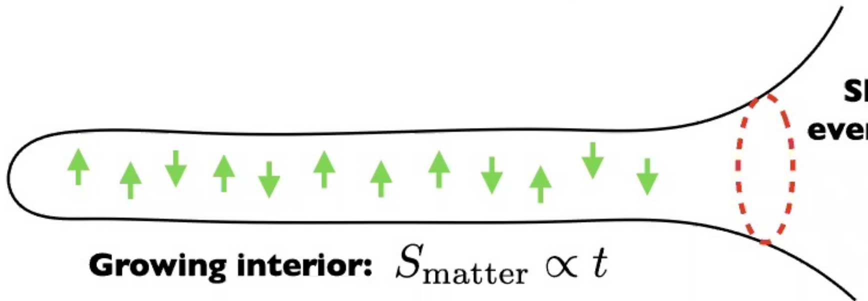
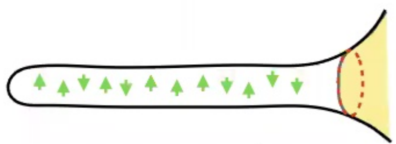
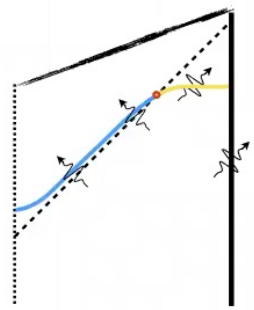


Shrinking event horizon

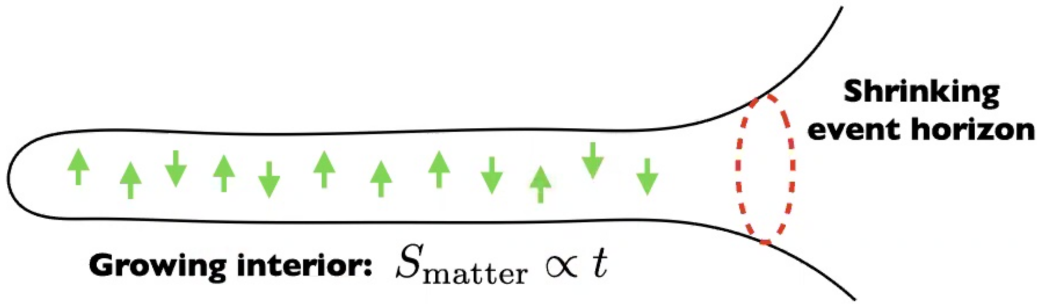
Growing interior: $S_{\text{matter}} \propto t$



$$S_{\text{gen}} = \frac{A}{4G_N}$$



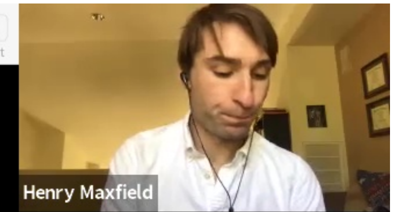
1. How is it possible that $S < S_{\mathrm{matter}}$?



Answer: surprising relationships in the gravitational Hilbert space

$$\sum_n c_n \left|\psi_n\right\rangle = 0$$

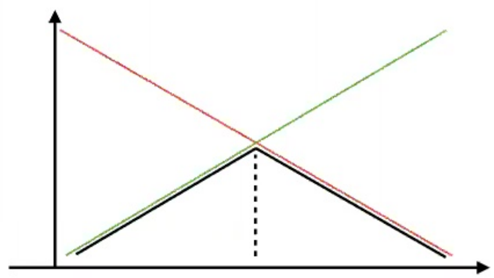
Gauge equivalences arising from topology change



2. Revisit: what does semiclassical gravity predict for an observer?

Answer: a pure state of Hawking radiation, *[PolchinskiStrominger'94]*

following the Page curve



But it doesn't predict any specific state…

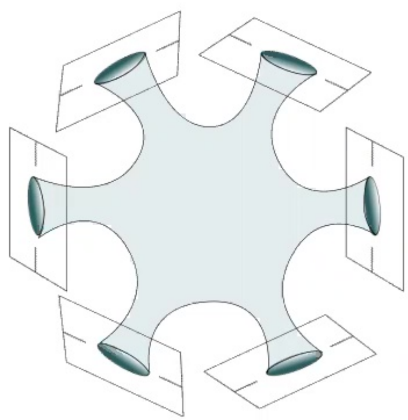
Replica wormholes

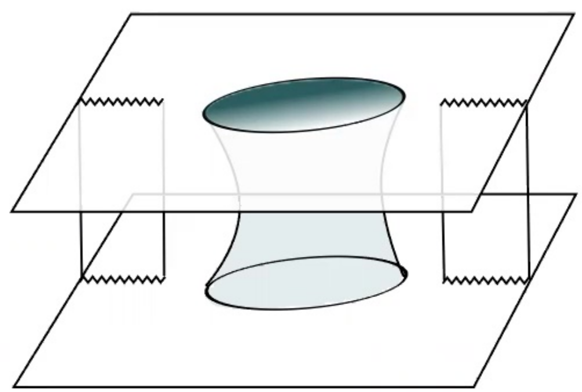
Where does RT come from?

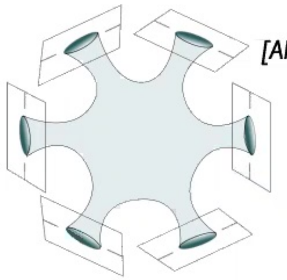
Replica path integral for $\mathrm{Tr}\,\rho^n$

[Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini]

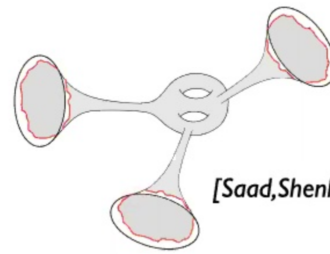
[Penington, Shenker, Stanford, Yang]





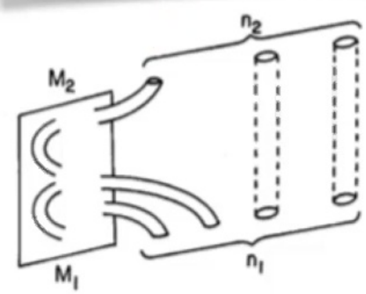


[Almheiri,Hartman,Maldacena,
Shaghoulian,Tajdini]



[Saad,Shenker,Stanford]

Spacetime wormholes & baby universes



[Coleman]

Figure 3.2. A topologically nontrivial process involving joining and splitting universes.

[Giddings,Strominger]

Asymptotically AdS path integrals

The AdS/CFT dictionary: *[Gubser,Klebanov,Polyakov][Witten]*

$$Z[J] = \int_{\Phi \sim J} \mathcal{D}\Phi \; e^{-S[\Phi]}$$

Dual CFT partition function — **Path integral over bulk fields** $\Phi \supseteq g$

Sources J — **Boundary conditions** $\Phi \sim J$

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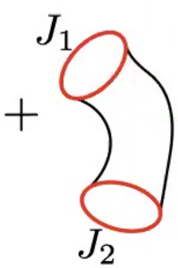
Spacetime wormholes: a factorisation problem

$$Z[J_1 \sqcup J_2] =$$

$$+$$

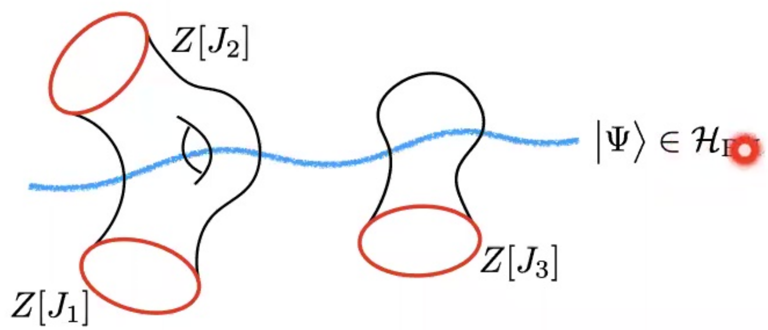


$$\neq Z[J_1]Z[J_2]$$

aAdS gravity with wormholes

Notation for path integral:

$$\left\langle Z[J_1]\cdots Z[J_n]\right\rangle = \int_{\Phi\sim J}\mathcal{D}\Phi\, e^{-S[\Phi]}$$



Aim: give a Hilbert space interpretation in terms of intermediate states of closed "baby" universes

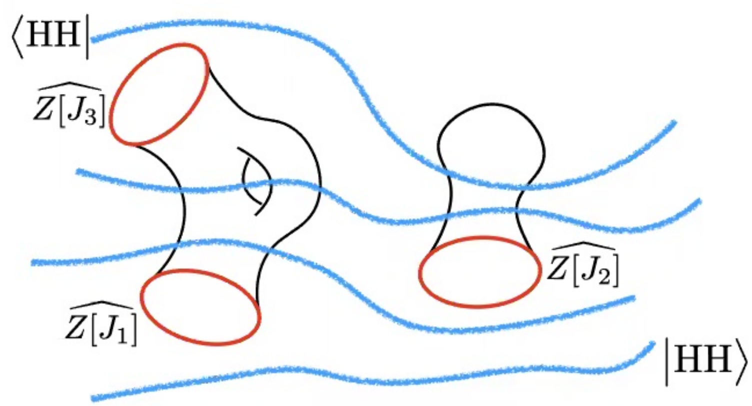
An axiomatic approach

Construct the Hilbert space from amplitudes

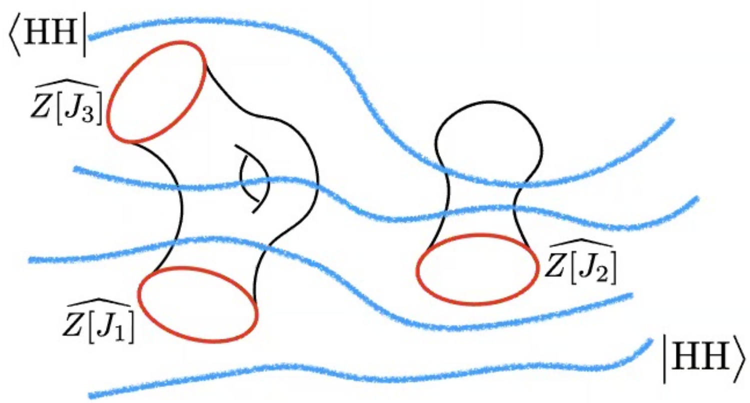
$$\left\langle Z[J_1] \cdots Z[J_n] \right\rangle = \int_{\Phi \sim J} \mathcal{D}\Phi \, e^{-S[\Phi]}$$

Build from:

- A Hartle-Hawking "no boundary" state $|\mathrm{HH}\rangle \in \mathcal{H}_{\mathrm{BU}}$
- aAdS boundaries as operators on $\mathcal{H}_{\mathrm{BU}}$: $\widehat{Z[J]}$



An axiomatic approach



$$\left\langle Z[J_1]Z[J_2]Z[J_3]\right\rangle = \left\langle \mathrm{HH}\middle|\widehat{Z[J_3]}\,\widehat{Z[J_2]}\,\widehat{Z[J_1]}\middle|\mathrm{HH}\right\rangle$$

Hermitian conjugation = CPT conjugate on sources: $\widehat{Z[J]}^{\dagger} = \widehat{Z[J^*]}$

States formed by acting with $\widehat{Z[J]}$ on $|\mathrm{HH}\rangle$ are dense in $\mathcal{H}_{\mathrm{BU}}$

This is sufficient to define $\mathcal{H}_{\mathrm{BU}}$

Reflection positivity

One axiom necessary for a quantum mechanics of closed universes

States $\Big|Z[J_1]\cdots Z[J_n]\Big\rangle := \widehat{Z[J_1]}\cdots\widehat{Z[J_n]}\Big|\mathrm{HH}\Big\rangle$, with inner product

$$\Big\langle Z[\tilde{J}_1]\cdots Z[\tilde{J}_m]\Big|Z[J_1]\cdots Z[J_n]\Big\rangle = \langle Z[\tilde{J}_1^*]\cdots Z[\tilde{J}_m^*]Z[J_1]\cdots Z[J_n]\rangle$$

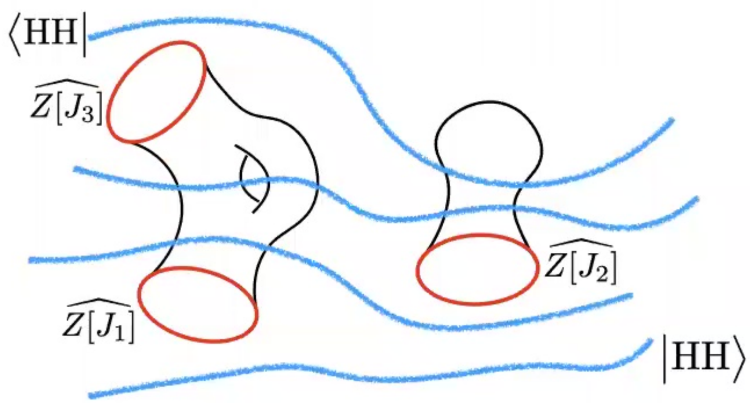
Require that this is positive semidefinite: $\Big\|\,|\Psi\rangle\,\Big\|^2 = \langle\Psi|\Psi\rangle \geq 0$

Then, $\mathcal{H}_{\mathrm{BU}}$ is a Hilbert space

$$\mathcal{H}_{\mathrm{BU}} = \text{completion of span}\left\{\Big|Z[J_1]\cdots Z[J_n]\Big\rangle\right\}$$

C.f.: Wightman/Osterwalder-Schrader constructions of QFT Hilbert space

An axiomatic approach



$$\langle Z[J_1]Z[J_2]Z[J_3]\rangle = \left\langle \mathrm{HH}\middle|\widehat{Z[J_3]}\,\widehat{Z[J_2]}\,\widehat{Z[J_1]}\middle|\mathrm{HH}\right\rangle$$

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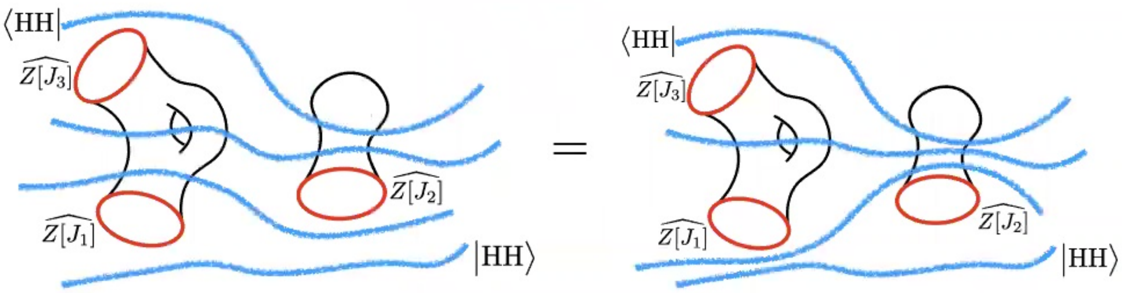
The alpha eigenbasis

$\widehat{Z[J]}$ **operators mutually commute:** $\left[\widehat{Z[J_1]}, \widehat{Z[J_2]}\right] = 0$



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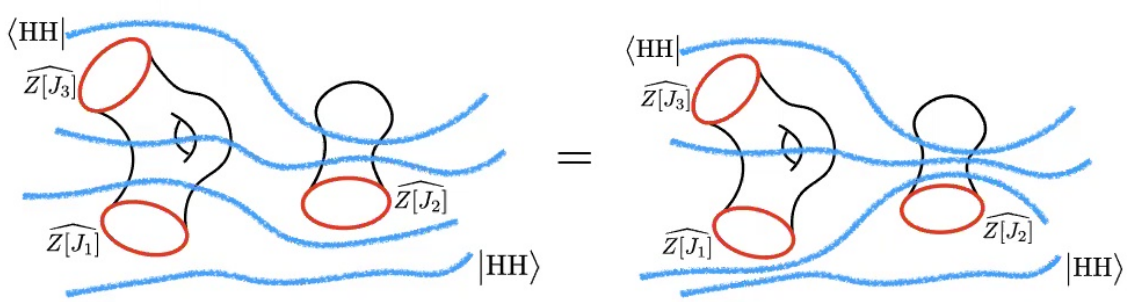


Diagonalise! **Simultaneous eigenstates of all** $\widehat{Z[J]}$**:**

$$\widehat{Z[J]}|\alpha\rangle = Z_\alpha[J]|\alpha\rangle \quad \forall J$$

States $|\alpha\rangle$ **form orthonormal basis of** $\mathcal{H}_{\mathrm{BU}}$

A dual CFT ensemble

Compute amplitudes by inserting complete sets of alpha states:

$$\left\langle Z[J_1]\cdots Z[J_m]\right\rangle = \langle \mathrm{HH}|\widehat{Z[J_1]}\cdots\widehat{Z[J_m]}|\mathrm{HH}\rangle$$

$$= \sum_\alpha p_\alpha Z_\alpha[J_1]\cdots Z_\alpha[J_m]$$

$Z[J]$ **are random variables,** $\langle\cdot\rangle$ **is an ensemble average**

Orthonormal basis $|\alpha\rangle$ **for** $\mathcal{H}_{\mathrm{BU}}$ $\longleftrightarrow$ **CFTs** $\mathcal{C}_\alpha$ **in the ensemble**

Probability of each theory is $p_\alpha = |\langle \mathrm{HH}|\alpha\rangle|^2$

C.f.: *[Coleman][GiddingsStrominger]*

Example: Jackiw-Teitelboim gravity *[SaadShenkerStanford]*

Null states

An important feature of $\mathcal{H}_{\mathrm{BU}}$: surprising null states!

$$\sum_i c_i \Big| Z[J_{i,1}] \cdots Z[J_{i,m_i}] \Big\rangle = 0$$

Interpret as gauge equivalence under diffeomorphisms

See toy model: Sec 3, or Don's IFQ seminar

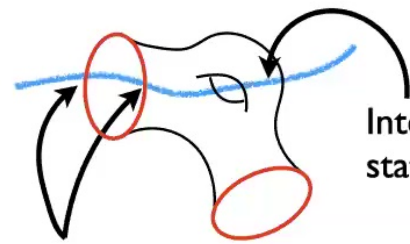
Doubly nonperturbative effect in G_N expansion

$$\exp\left(\# e^{-\frac{\#}{G_N}}\right)$$

Hilbert spaces with boundaries

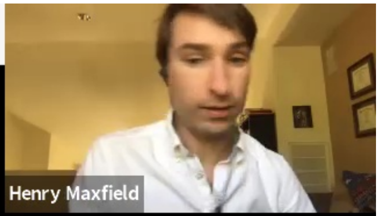
So far, studied $\mathcal{H}_{\mathrm{BU}}$, the Hilbert space of closed universes

Can also consider Hilbert spaces with boundaries:



States created with one-sided boundary conditions: $\left| \; i \; \right\rangle \in \mathcal{H}_\Sigma$

Can also include closed boundaries: $\left| \; i \; ; Z[J] \right\rangle \in \mathcal{H}_\Sigma$



Entropy bounds

A state in a two-boundary gravitational Hilbert space $\mathcal{H}_{\Sigma \sqcup \Sigma}$**:**

$$|\Delta\rangle = \Big| \quad \Big\rangle - \sum_{i=1}^{k} \Big| \quad {}_{i} \quad {}_{i} \quad \Big\rangle$$

After choosing a basis for $\mathcal{H}_{\Sigma}$: $\Big\langle \quad {}^{j}_{i} \quad \Big\rangle = \delta_{ij}$

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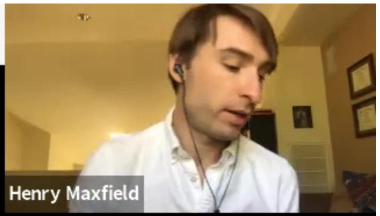
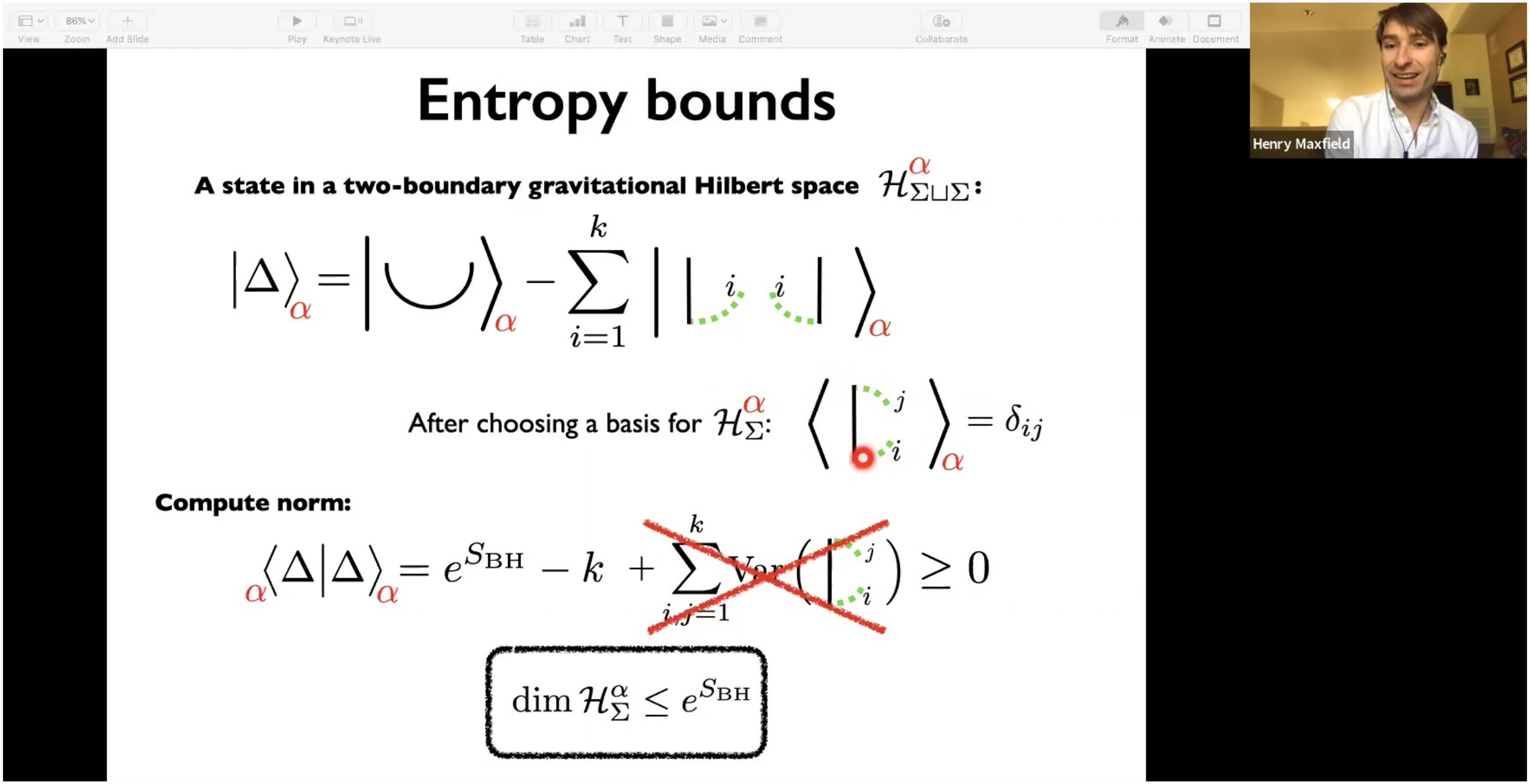
After choosing a basis for $\mathcal{H}_{\Sigma}$: $\left\langle\ \right\rangle = \delta_{ij}$

Compute norm:

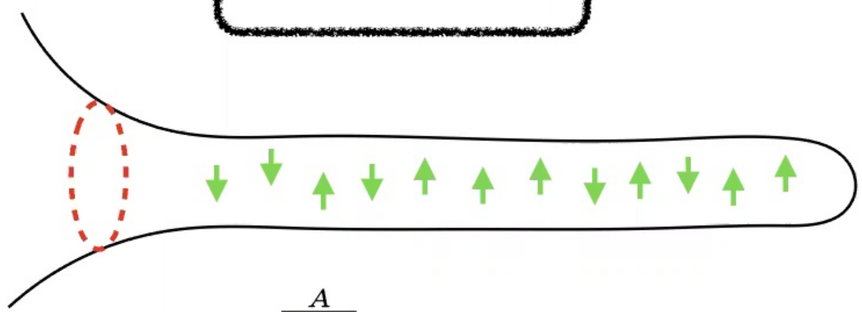
$$\langle \Delta | \Delta \rangle = e^{S_{\mathrm{BH}}} - k + \sum_{i,j=1}^{k} \mathrm{Var}\left(\ \right) \geq 0$$




Entropy bounds
A state in a two-boundary gravitational Hilbert space $\mathcal{H}^{\alpha}_{\Sigma\sqcup\Sigma}$:
$|\Delta\rangle_\alpha = |\;\rangle_\alpha - \sum_{i=1}^{k} |\;\rangle_\alpha$
After choosing a basis for $\mathcal{H}^{\alpha}_{\Sigma}$: $\langle\;\rangle_\alpha = \delta_{ij}$
Compute norm:
${}_\alpha\langle\Delta|\Delta\rangle_\alpha = e^{S_{\rm BH}} - k + \sum_{i,j=1}^{k} \mathrm{Var}(\;) \geq 0$
$\dim \mathcal{H}^{\alpha}_{\Sigma} \leq e^{S_{\rm BH}}$
Henry Maxfield

The Page curve

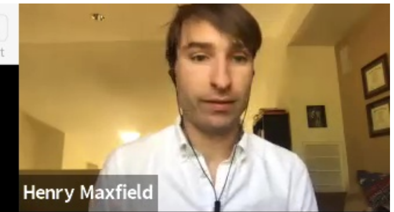
Entropy bound: $\boxed{\dim \mathcal{H}^{\alpha}_{\Sigma} \leq e^{S_{\mathrm{BH}}}}$



At most $e^{\frac{A}{4G_N}}$ **linearly independent states**

Guaranteed in alpha states by reflection positivity

Requires many surprising null states: $\sum_i c_i \left| \; {}^{i} \; \right\rangle_\alpha = 0$



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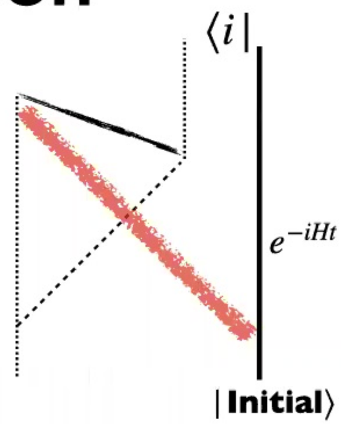
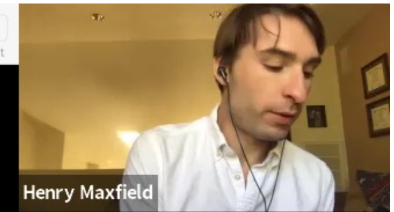
The state of radiation

Wavefunction of Hawking radiation $|\psi_R\rangle$**:**

boundary condition for path integral $\psi_R^i = \langle i | \psi_R \rangle$





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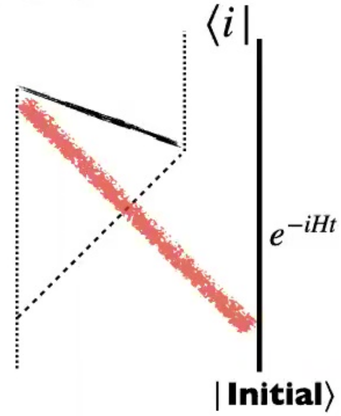
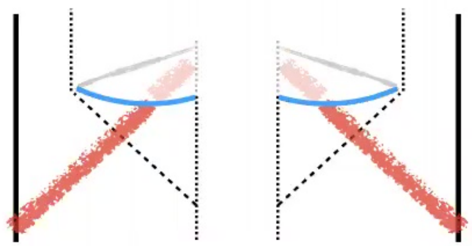


Instead, use "in-in formalism": $\rho_R^{ij} = \psi_R^i \bar{\psi}_R^j$



$\left\langle \rho_R^{ij} \right\rangle = \rho^{ij}_{\textbf{Hawking}}$

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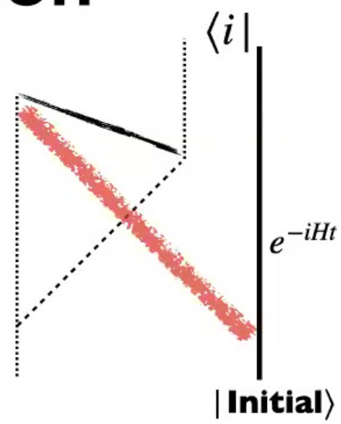

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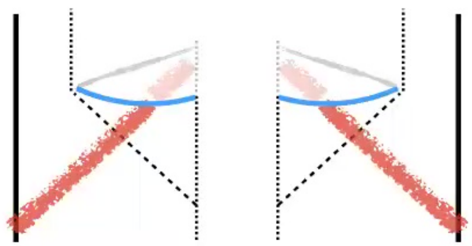
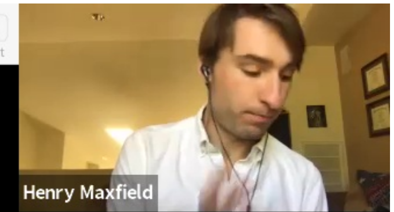


$\left\langle \rho_R^{ij} \right\rangle = \rho^{ij}_{\textbf{Hawking}}$

Hawking's calculation is "two-replica", and involves a spacetime wormhole!

Information paradox = factorisation problem

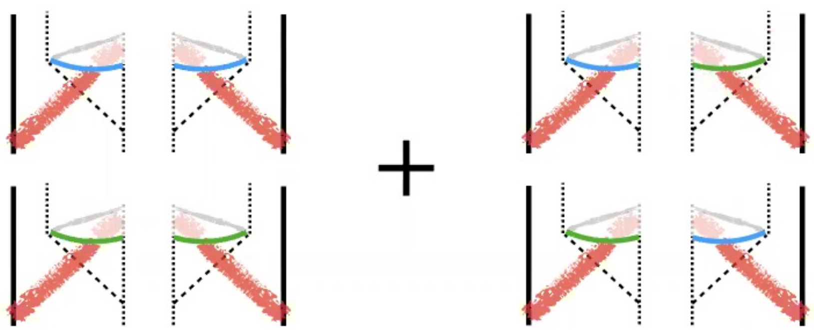


The state of radiation

Try to verify mixedness experimentally: swap test!

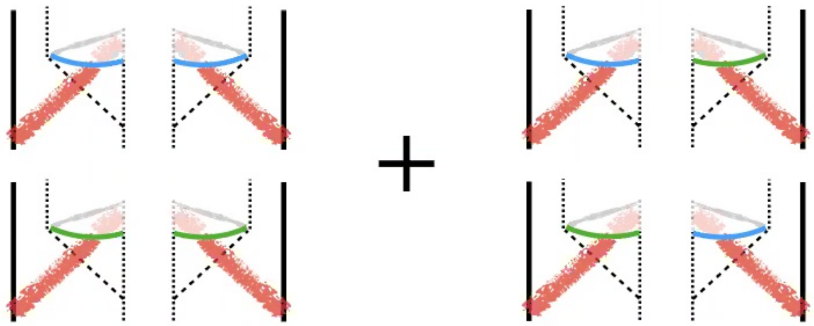
Prepare two sets of radiation. $\mathrm{Tr}(\rho^2) = \mathrm{Tr}(S\rho \otimes \rho) = e^{-S_2(\rho)}$

$$\left\langle \rho_R^{i_1 j_1} \rho_R^{i_2 j_2} \right\rangle = \rho_{\mathbf{H}}^{i_1 j_1}\rho_{\mathbf{H}}^{i_2 j_2} + \rho_{\mathbf{H}}^{i_1 j_2}\rho_{\mathbf{H}}^{i_2 j_1}$$



[PolchinskiStrominger'94]

The state of radiation



[PolchinskiStrominger'94]

Semiclassical result: $\rho_R^{(n)} = P_{\text{sym}}\, \rho_{\mathbf{H}}^{\otimes n}$

From general considerations: $\sum_\alpha p_\alpha |\psi_\alpha\rangle_R^{\otimes n} \otimes |\alpha\rangle_{BU}$

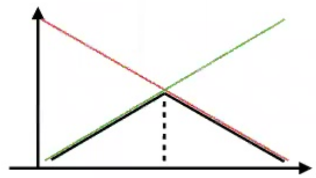
$\cancel{\left(\sum_\alpha p_\alpha |\psi_\alpha\rangle_R \otimes |\alpha\rangle_{BU}\right)^{\otimes n}}$

Information loss is unobservable!

Summary

Semiclassical gravity predicts:

- A **mixed** state of Hawking radiation
- But mixedness is in principle unobservable
- A Page curve (replica wormholes) for "observed" entropy:
  - Expectation value of swap operator
  - Or "ensemble average" of entropy
  - RWs don't count entanglement with BUs



Consistent quantum description implies:

- Surprising relations between states, which ensure
- Number of independent states bounded by $e^{S_{\mathrm{BH}}}$