Title: JT gravity at finite cutoff

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Series: Quantum Fields and Strings

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Abstract: We compute the partition function of 2D Jackiw-Teitelboim (JT) gravity at finite cutoff in two ways: (i) via an exact evaluation of the Wheeler-DeWitt wave-functional in radial quantization and (ii) through a direct computation of the Euclidean path integral. Both methods deal with Dirichlet boundary conditions for the metric and the dilaton. In the first approach, the radial wavefunctionals are found by reducing the constraint equations to two first order functional derivative equations that can be solved exactly, including factor ordering. In the second approach we perform the path integral exactly when summing over surfaces with disk topology, to all orders in perturbation theory in the cutoff. Both results precisely match the recently derived partition function in the Schwarzian theory deformed by an operator analogous to the TTÂ¯ deformation in 2D CFTs. This equality can be seen as concrete evidence for the proposed holographic interpretation of the TTÂ¯ deformation as the movement of the AdS boundary to a finite radial distance in the bulk.

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JT gravity at Finite Cutoff

Work with:

Luca Iliesiu

Joaquin Turiaci

Herman Verlinde

ArXiv: 2004.07242

Also based on earlier work with Edgar Shaghoulian, Andrew Rolph and David Gross: 1907.04873

1912.06132

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Recently, there has been a lot of work on the so-called $T\bar{T}$-deformation of 2d QFTs:

$$\partial_\lambda S[\Phi] \sim \int d^2x \sqrt{g}\, \mathcal{O}_{T\bar{T}}$$

deformation parameter

irrelevant

Defines a flow in theory space.

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Why interesting?

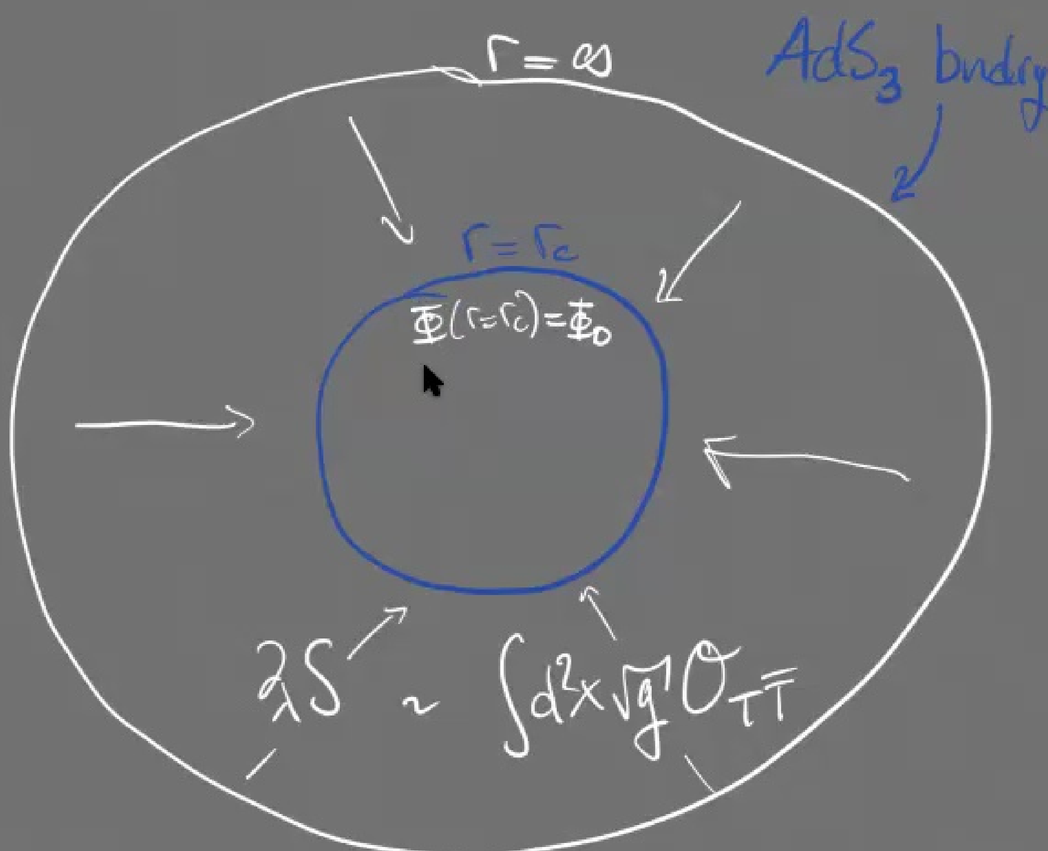
i) Despite irrelevance $\mathcal{O}_{T\bar{T}}$, still calculable observables, like the spectrum.

ii) Sneak preview into non-local theories and relations with string theory.

iii) Holography. There is a lot of evidence that the flow described above moves the CFT into the bulk:

$$\lambda \sim \frac{1}{r_c^2}$$

In a cartoon:


$r=\infty$
AdS$_3$ bndry
$r=r_c$
$\Phi(r=r_c)=\Phi_0$
$\partial_\lambda S \sim \int d^2x\sqrt{g}\,O_{T\bar{T}}$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Today I will present some more evidence for the relation with holography in AdS_2 JT gravity.

In particular, I will show that the recently proposed deformed Schwarzian theory can be obtained from a radial Wheeler-deWitt wavefunctional.

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Deformed Schwarzian Whut?

For 2d bndry it was shown that

$$\partial_\lambda S = \int_{\text{Euc. Cylinder}} d^2x \; \underbrace{\mathcal{O}_{T\bar{T}}}_{\epsilon_{ij}\epsilon_{kl} T^{ik} T^{jl}}$$

$$\Downarrow$$

$$\partial_\lambda E + 2E\partial_R E + \frac{k^2}{R} = 0$$

$$\Rightarrow \quad E(\lambda) = \frac{1}{4\lambda}\left(1 - \sqrt{1 - 8\lambda\frac{E_0}{R} + 16\lambda^2\frac{k^2}{R^2}}\right)$$

Is dual, semi-classically, to Dirichlet bndry conditions on

This connection was first noticed by McGough et al. and generalised in various directions.

Today I want to focus on finite cutoff JT gravity:

$$S = -\frac{1}{2}\int d^2x\, \phi\,(R+2) - \int_\partial du\, \phi\sqrt{g_{uu}}\,(K-1)$$

Integrating out ϕ

$$\begin{cases} g_{uu} = \frac{1}{\varepsilon^2} \\ \phi = \frac{\phi_r}{\varepsilon} \end{cases}$$

$$S_\partial = -\int du\, \phi_r(u)\,\{t(u),u\}$$

Schwarzian Theory

One can derive a similar deformation (or flow) dual to finite cutoff JT gravity

A simple way is through dim-reduction of the 2d story.

It turns out $E(\lambda)$ with $k=0$:

$$E(\lambda) = \frac{1}{4\lambda}\left(1 - \sqrt{1 - 8\lambda E_0}\right)$$

does the job!

complexifies at $E_0 > 1/8\lambda$,

This allows us to compute the partition f^n for the finite cutoff QM:

$$Z_{Sch}(\beta) = \int_0^\infty dE \, \sinh(2\pi\sqrt{E}) \, \underbrace{e^{-\beta E}}$$

$E \to E(\lambda)$

$$Z_{def.\,Sch}(\beta) = \int_0^\infty dE \, \sinh(2\pi\sqrt{E}) \, e^{\frac{\beta}{-4\lambda}\left(1-\sqrt{1-8\lambda E}\right)}$$

We want to reproduce this from gravity, beyond the semiclassical limit.

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In the paper we discussed 2 ways of doing that :

i) Through WdW wavefunctional, i.e. canonical quantisation

ii) Direct evaluation Euclidean path integral

Today I will focus on the first.

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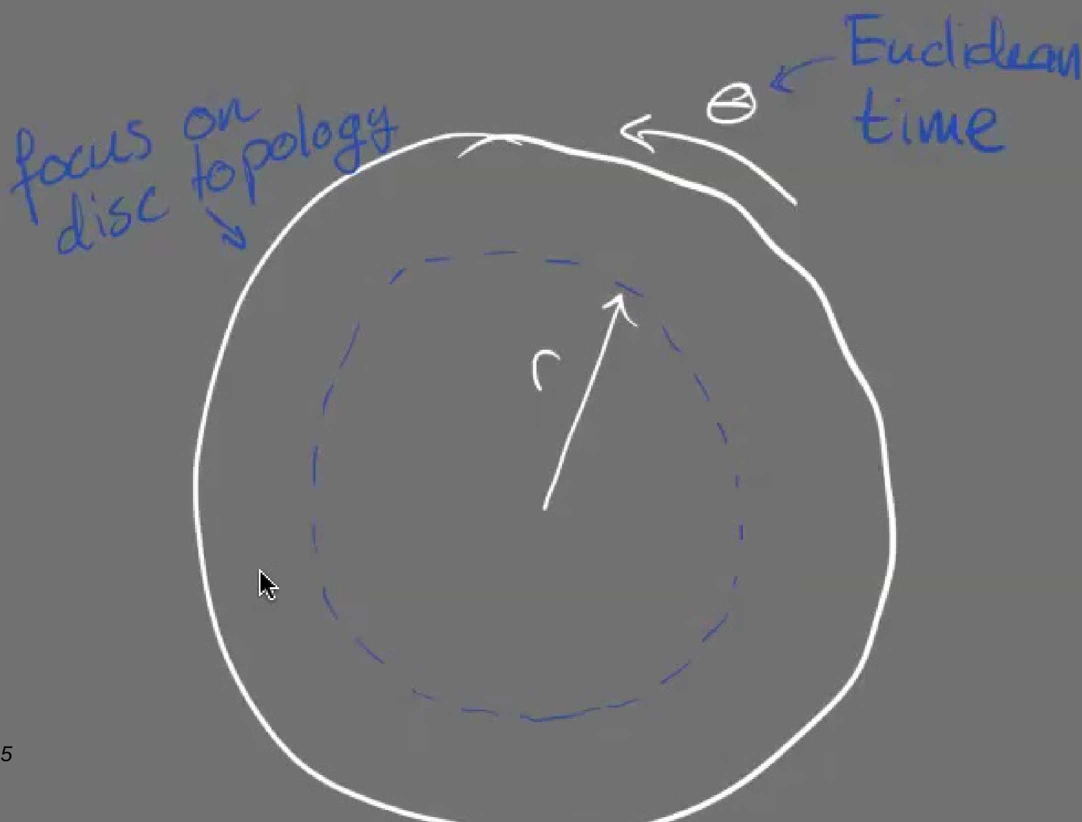
WdW analysis

$$ds^2 = N^2 dr^2 + e^{2\sigma}(d\theta + N_\perp dr)^2$$

$$\theta \sim \theta + 1$$



$$S_{JT} = -\frac{1}{2}\int d^2x \sqrt{g}\,\phi\,(R+2)$$

$$- \int du \sqrt{g_{uu}}\, K\phi$$

(counter-term will be added later)

In the ADM parametrisation,

∂_τ $\quad$ ∂_θ

$$S_{JT} = \int d^2x\, e^{\sigma}\Big[\frac{\dot\phi}{N}\left(N_\perp' - \dot\sigma + N_\perp\sigma'\right)$$

$$+ \frac{\phi'}{N}\left(N_\perp\dot\sigma - N_\perp^2\sigma' - \frac{NN'}{e^{2\sigma}} - N_\perp N_\perp'\right)$$

$$- N\phi\Big]$$

$$\Pi_\sigma = \frac{e^\sigma}{N}\left(\phi' N_\perp - \dot{\phi}\right)$$

$$\Pi_\phi = \frac{e^\sigma}{N}\left(N_\perp' - \dot{\sigma} + N_\perp \sigma'\right)$$

$$\left.\begin{aligned} \Pi_N &= 0 \\ \Pi_{N_\perp} &= 0 \end{aligned}\right\} \ N, N_\perp \text{ act as Lagrange multipliers}$$

$$\Downarrow$$

$$H = \int d\theta \left(e^{-\sigma} N \mathcal{H}_{WdW} + N_\perp P\right)$$

$$\mathcal{H}_{WdW} = -\Pi_\phi \Pi_\sigma + \sigma'\phi' - \phi'' + e^{2\sigma}\phi$$

$$P = \sigma'\Pi_\sigma + \phi'\Pi_\phi - \Pi_\sigma'$$

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Let us upgrade these constraints to quantum mechanical ones:

$$\hat{\mathcal{H}}_{WdW}\Psi = 0 \,, \quad \hat{P}\Psi = 0$$

$$\hat{\Pi}_\sigma = -i\frac{\delta}{\delta\sigma} \,, \quad \hat{\Pi}_\phi = -i\frac{\delta}{\delta\phi}$$

Usually hopeless to solve without some approximation

Here we can solve them

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This works because we can eliminate one of the momenta and write down two first order equations

ADM mass

$$\hat{\Pi}_\sigma \Psi = \pm\left(\left(\phi^2 - M\right)e^{2\sigma} - \left(\partial_\theta \phi\right)^2\right)^{1/2} \Psi$$

$$\hat{\Pi}_\phi \Psi = \pm \frac{\phi'' - \sigma'\phi' - \phi e^{2\sigma}}{\left(\left(\phi^2 - M^2\right)e^{2\sigma} - \left(\partial_\theta \phi\right)^2\right)^{1/2}} \Psi$$

These can be directly integrated (functionally)

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$$\Psi = \Psi_+ + \Psi_-$$

$$\Psi_\pm = \int dM \, \rho_\pm(M) \, \Psi_\pm(M)$$

$$\Psi_\pm(M) = \exp\left[\pm \int_0^L du \left(\sqrt{\phi_b^2 - M - (\partial_u \phi_b)^2} - \partial_u \phi_b \tan^{-1}\left(\frac{\sqrt{\phi_b^2 - M - (\partial_u \phi_b)^2}}{\partial_u \phi_b} \right) \right) \right]$$

$$du = e^\sigma d\theta, \quad L = \int_0^1 e^\sigma d\theta$$

ϕ_b profile dilaton along radial slice.

Let us focus on the large L limit. Keep ϕ_b, σ constant:

$$\Psi_+[\phi_b, \sigma] = \int dM\, \rho_+(M)\, e^{\int_0^L du \sqrt{\phi_b^2 - M}}$$

$$\Big\downarrow \; L \to \infty,\ \phi_b \to \infty,\ \frac{L}{\phi_b} \text{ fixed}$$

$$\Psi_+ \to e^{L\phi_b} \int dM\, \rho(M)\, e^{-\frac{L}{2\phi_b} M + \dots}$$

After removing $\quad \Psi_+$ should agree with

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We thus see :

$$M = E$$

$$\rho_+(M) = \sinh(2\pi\sqrt{E})$$

$$\frac{L}{2\phi_b} = \beta$$

$$\Psi_+ = \int_0^\infty dM \, \sinh(2\pi\sqrt{M})\, e^{\phi_b L\sqrt{1-\frac{M}{\phi_b^2}}}$$

$\downarrow$ including counter term $e^{-\phi_b L}$

$$Z = \int_0^\infty dM \, \sinh(2\pi\sqrt{M})\, e^{\underbrace{-\phi_b L\left(1-\sqrt{1-\frac{M}{\phi_b^2}}\right)}}$$

we have seen this before!!

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$T\bar{T}$ deformation.

Comments:

i) Result for $\underline{\Psi}_{\pm}$ can be thought of as 1d version of Freidel's result in 3d gravity.

ii) Wavefunctional also knows about Schwarzian

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theory with varying dilaton $\phi_b(u)$.

iii) Ψ is not real! Hence Z is not real...

This is not great. However, can be cured by including the Ψ_- branch, but

$$Z = Z_+ - Z_-$$

!!!

This is not an ordinary

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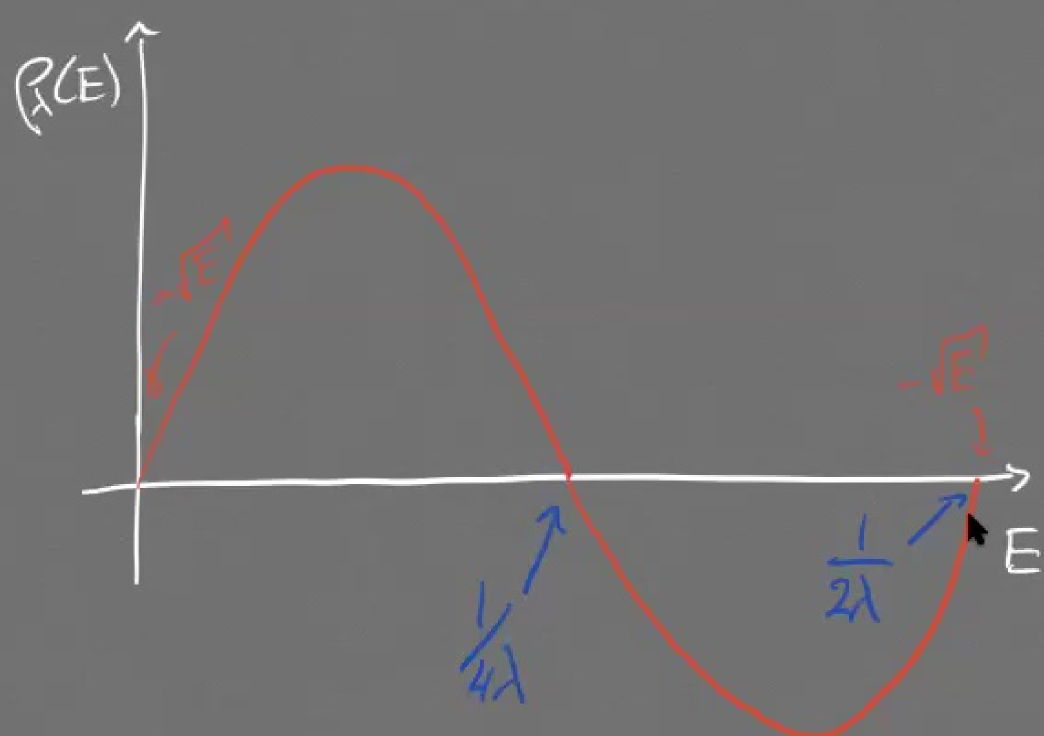
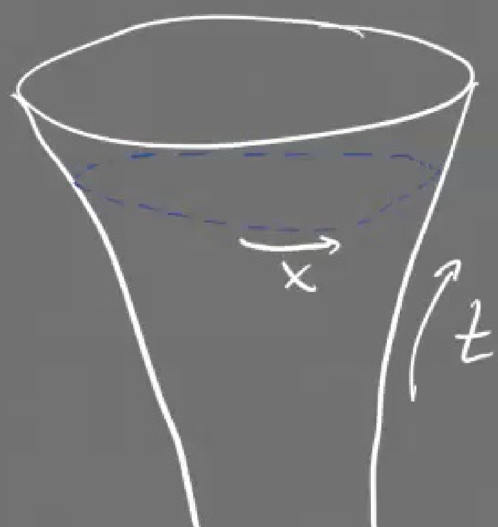


In fact, from WdW, it is consistent to add any

v) We can repeat this analysis also for dS_2 JT gravity



And obtain wavefunction at finite time.

vi) We can also compute the Euclidean path integral :

$$Z(\phi_b, L) = \int \mathcal{D}\phi \, \mathcal{D}g \; e^{-S_{JT}}$$

bndry length

$$g_{uu} = \frac{1}{\varepsilon^2}$$

$$\phi_b(u) = \frac{\phi_r}{\varepsilon}$$

reduces to PI over bndry action.

with ε finite.

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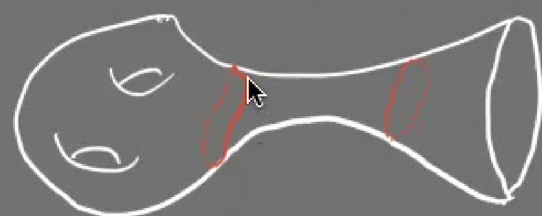
Conclusion:

We studied the partition function associated to finite cutoff JT gravity as arising from a 1d version of the $T\bar{T}$ deformation.

Using a WdW analysis we found the exact JT wavefunctional, which matched with the QM result.

Future directions:

i) Other topologies



ii) Complex energies, i.e.
how to get a unitary theory at finite cutoff with positive DOS.

iii) Coupling to matter.