Title: Why standard entanglement theory is inappropriate for the study of Bell scenarios

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Abstract: A standard approach to quantifying resources is to determine which operations on the resources are freely available and to deduce the ordering relation among the resources that these operations induce. If the resource of interest is the nonclassicality of the correlations embodied in a quantum state, that is, entanglement, then it is typically presumed that the appropriate choice of free operations is local operations and classical communication (LOCC). We here argue that, in spite of the near-universal endorsement of the LOCC paradigm by the quantum information community, this is the wrong choice for one of the most prominent applications of entanglement theory, namely, the study of Bell scenarios. The nonclassicality of correlations in such scenarios, we argue, should be quantified instead by local operations and shared randomness (LOSR). We support this thesis by showing that various perverse features of the interplay between entanglement and nonlocality are merely an artifact of the use of LOCC-entanglement and that the interplay between LOSR-entanglement and nonlocality is natural and intuitive. Specifically, we show that the LOSR paradigm (i) provides a resolution of the "anomaly of nonlocality", wherein partially entangled states exhibit more nonlocality than maximally entangled states, (ii) entails a notion of genuine multipartite entanglement that is distinct from the conventional one and which is free of several of its pathological features, and (iii) makes possible a resource-theoretic account of the self-testing of entangled states which simplifies and generalizes prior results. Along the way, we derive some fundamental results concerning the necessary and sufficient conditions for convertibility between pure entangled states under LOSR and highlight some of their consequences, such as the impossibility of catalysis for bipartite pure states.

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Why standard entanglement theory is inappropriate for the study of Bell scenarios

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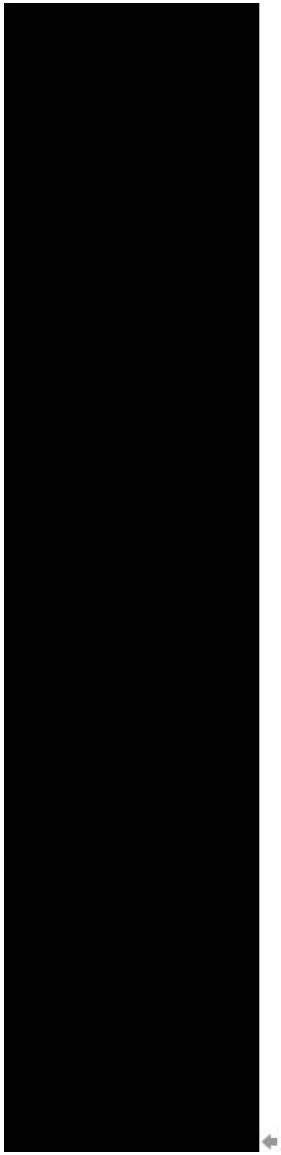
arXiv:2004.09194



What is entanglement?

A pure state is entangled if it is not a tensor product of two components
—Schrodinger

A pure state is entangled if it is not a tensor product of two components —Schrodinger

A mixed state is entangled if it is not separable (a mixture of product states)

Entanglement was then studied as a **resource** for communication tasks, such as teleportation…

…where the parties have access to arbitrary local operations and classical communication (LOCC).

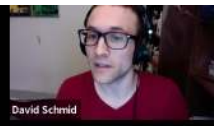
Over time, entanglement came to be understood as “the resource which cannot be generated by LOCC operations”.

Entanglement can be defined quantitatively as the resource characterized by convertibility under local operations and classical communication

ρ_1 is at least as entangled as ρ_2 iff

$$\rho_1 \rightarrow \rho_2$$

using LOCC operations



Entanglement can be defined quantitatively as the resource characterized by convertibility under local operations ~~and classical communication~~ and shared randomness

ρ_1 is at least as entangled as ρ_2 iff

$$\rho_1 \to \rho_2$$

using LOCC operations

But this isn't the only possible definition consistent with qualitative divide between separable and nonseparable!

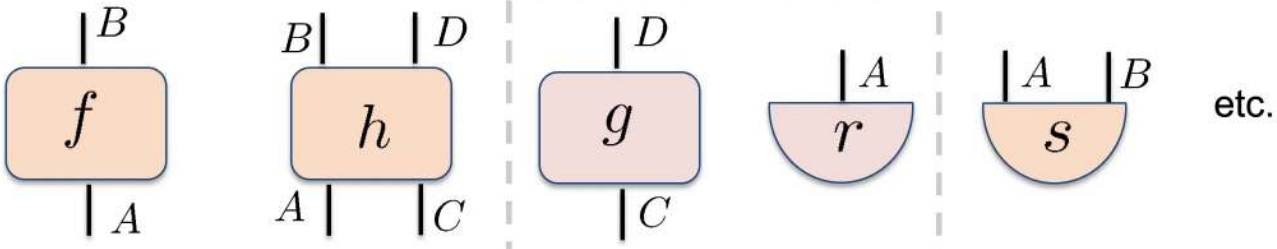
LOCC-entanglement vs LOSR-entanglement

?

Process theory T: Systems A, B, C, D, …

Processes f, g, h, r, s, … (the resources in question)

Closed under parallel and sequential composition

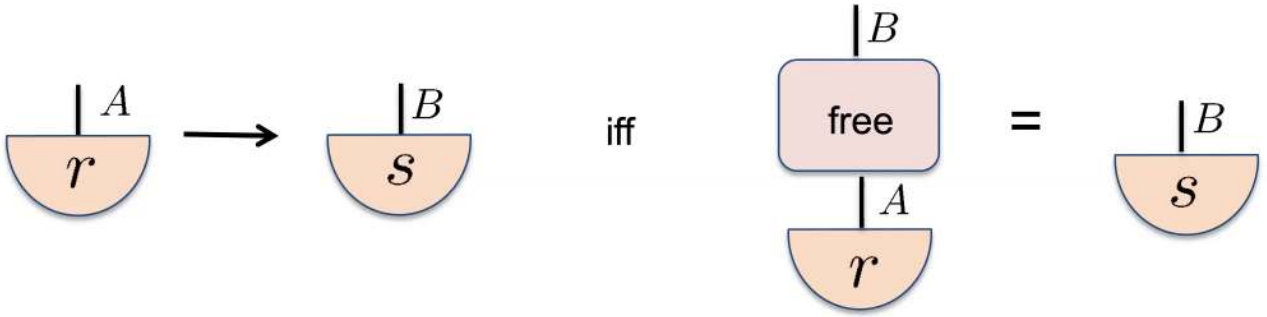


Subtheory T_{free} of *free* processes

A resource theory is a partitioned process theory (T, T_{free})

Conversion of state resources:

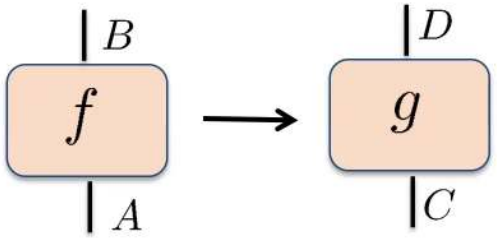
We say *r* can be converted into *s*:



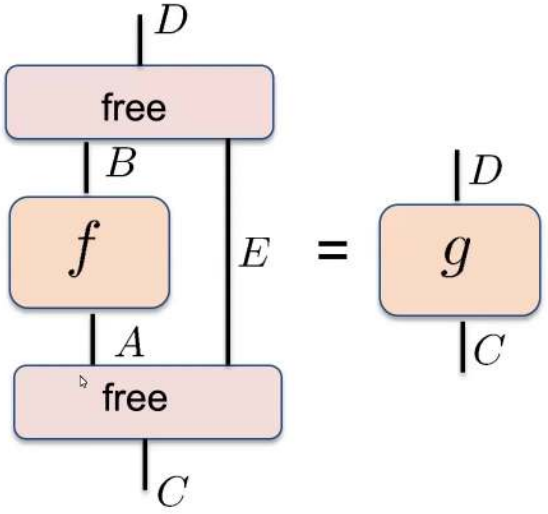
Then, *r* is **at least as valuable** as *s*.

Conversion of channel resources:

We say *f* can be converted into *g*:



iff



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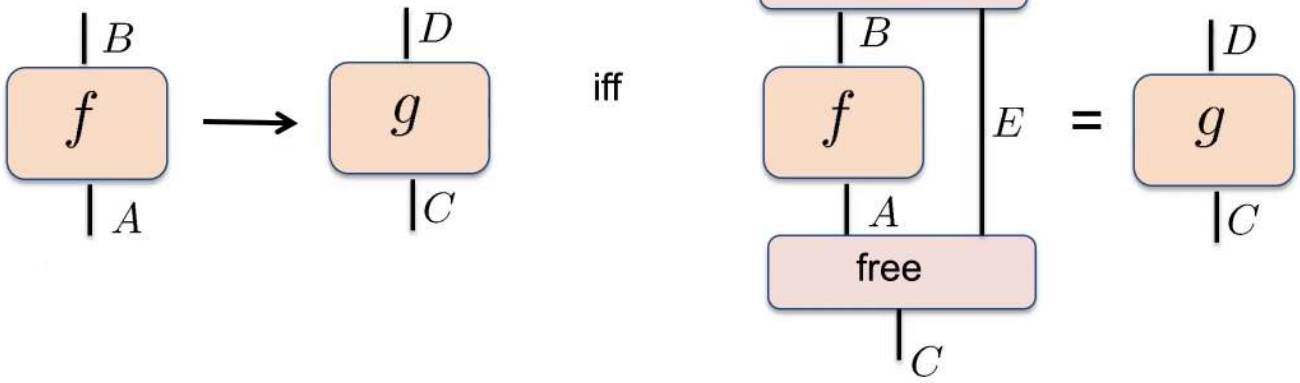
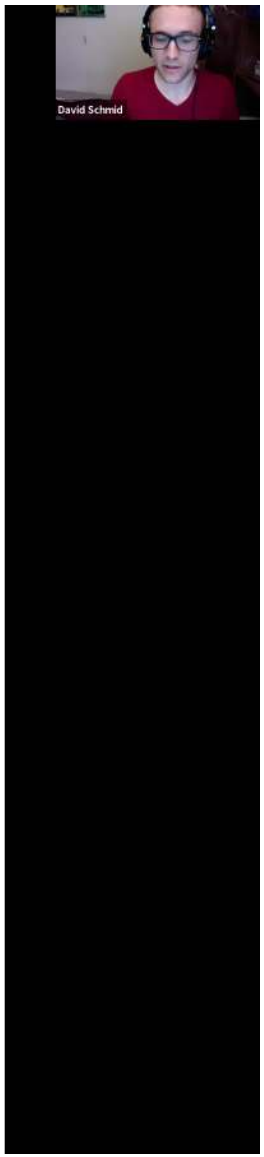
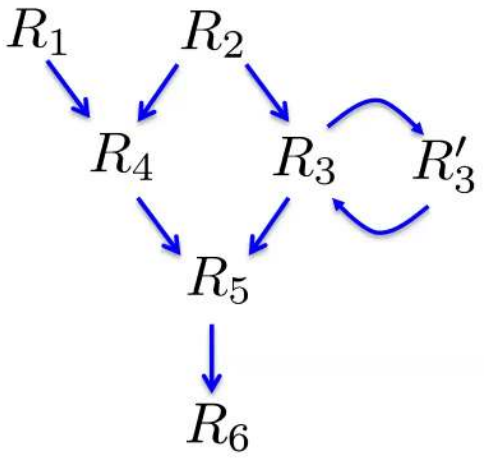


Then, *f* is **at least as valuable** as *g*.

(can also consider conversions from one type to another)

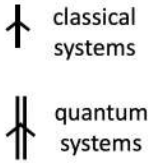


Conversion relations induce a preorder of resources



key object of interest in a resource theory

R_3 is equally as valuable as R_3' (in the same **equivalence class**)
R_1 is **strictly** more valuable than R_4
R_1 and R_2 are **incomparable**
R_1 and R_2 are *both* "maximally" valuable

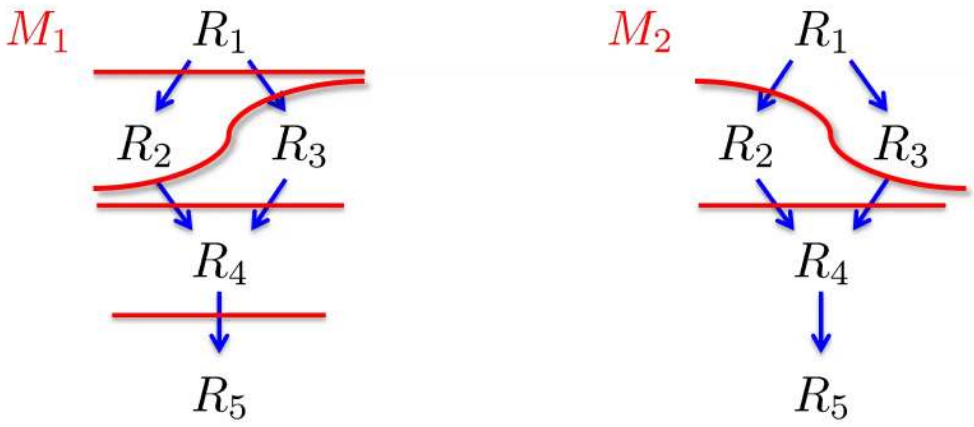


The two relevant resource types

Measures of a resource

Def'n: A function *M* from resources to the real numbers is a resource monotone if it is nonincreasing under the free operations.

e.g. negativity, von Neumann entropy, entanglement of formation/distillation, relative entropy of entanglement



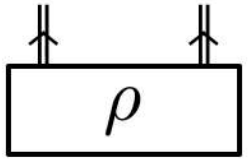
If it is not a total order, there cannot be "one measure to rule them all"

classical systems

quantum systems

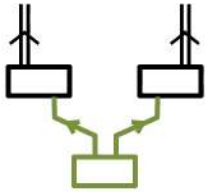
The two relevant resource types

State



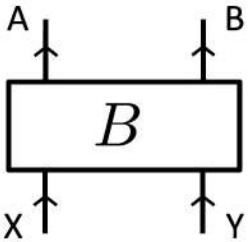
density operator

is separable if it decomposes as



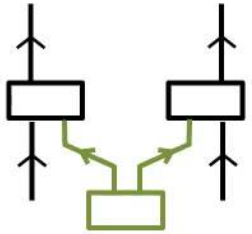
otherwise, it “is entangled”

Box



conditional probability distribution P(AB|XY)

has a hidden variable model if it decomposes as



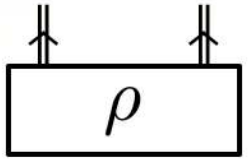
otherwise, it "is nonlocal”

classical systems

quantum systems

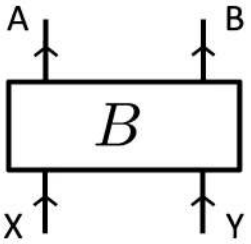
The two relevant resource types

State



density operator

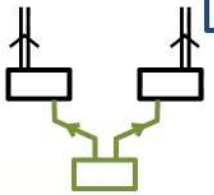
Box



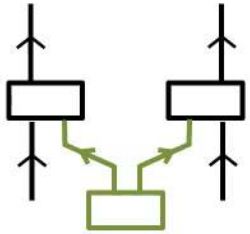
conditional probability distribution P(AB|XY)

LOSR-free!

is separable if it decomposes as



has a hidden variable model if it decomposes as

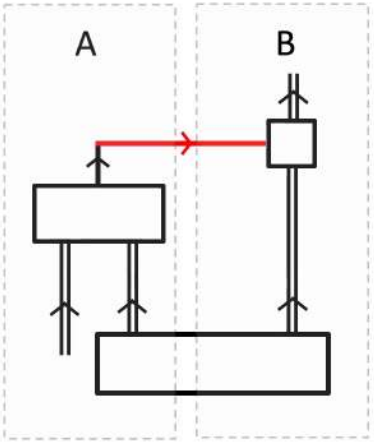


otherwise, it “is entangled”

LOSR-nonfree!

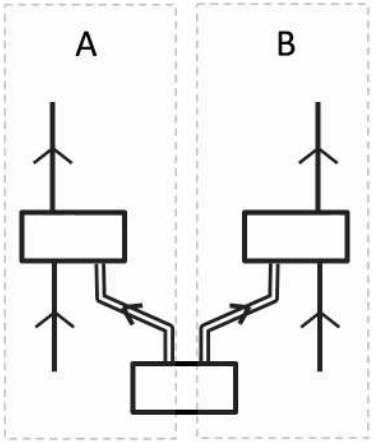
otherwise, it "is nonlocal”

We argue that LOSR-entanglement is the appropriate one for Bell scenarios
(and hence for key distribution and randomness generation).
LOCC-entanglement is appropriate in some other contexts, e.g. for teleportation.



Teleportation: LOCC

the task inherently requires the agents to have access to classical communication



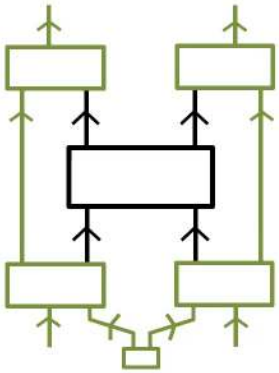
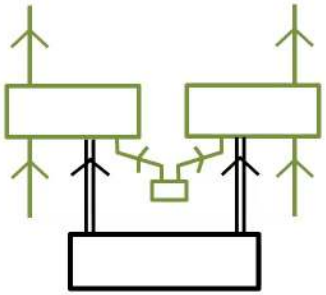
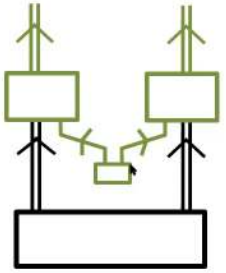
Bell scenario: LOSR

-all operations are at space-like separation
-causal structure is a common cause

→ no classical communication

if one wants to study *nonclassicality*, then one takes only the *classical* common causes as free

Assume one seeks a unified resource theory of entanglement and nonlocality.

box-to-box conversions		state-to-box conversions		state-to-state conversions
nonlocality		interplay between nonlocality and entanglement		entanglement theory
				
the free operations are LOSR	transitivity $\Longrightarrow$	the free operations are LOSR	transitivity $\Longrightarrow$	the free operations are LOSR
arXiv:1903.06311 see also de Vicente (2014), Gallego and Aolita, (2017), etc…		(not LO!)		(not LOCC!)

So in Bell scenarios, the relevant notions of entanglement, nonlocality, and their interplay are all captured within a single resource theory: that defined by taking LOSR operations to be free.

Three dividends of studying resource theory defined by LOSR:

1. resolving the “anomalies of nonlocality”
2. providing a better notion of genuine multipartite entanglement (and nonlocality)
3. clarifying and extending the notion of self-testing

Resolving the “anomalies of nonlocality”

The anomalies of nonlocality:

There exist measures of nonlocality which can be maximized by a partially entangled state, but *not* by a maximally entangled state.

[20] A. A. Methot and V. Scarani, "An Anomaly of Nonlocality," Quantum Info. Comput. **7**, 157 (2007).

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[28] E. A. Fonseca and F. Parisio, "Measure of nonlocality which is maximal for maximally entangled qutrits," Phys. Rev. A **92**, 030101 (2015).

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[30] V. Kabel, *Exploring the Interplay between Entanglement and Nonlocality: A novel perspective on the Peres Conjecture*, Ph.D. thesis, Ludwig Maximilians Universität München (2017).

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[34] V. Lipinska, F. J. Curchod, A. Máttar, and A. Acín, "Towards an equivalence between maximal entanglement and maximal quantum nonlocality," New J. Phys. **20**, 063043 (2018).

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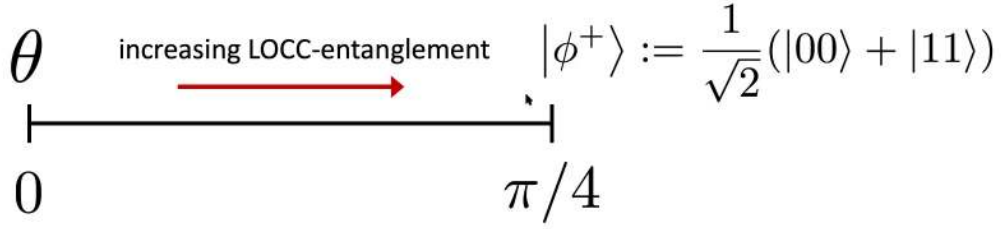
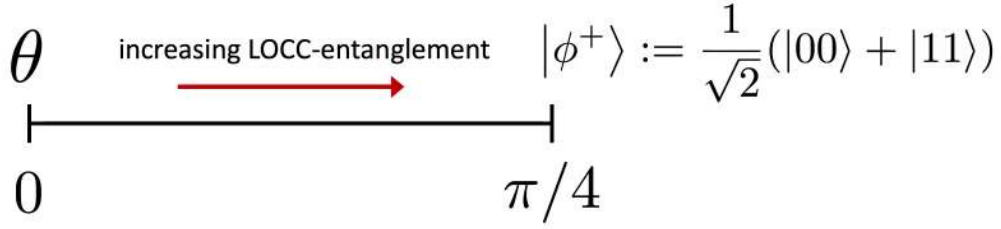
The anomalies of nonlocality:

Consider the family of states given by $\cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$



The anomalies of nonlocality:

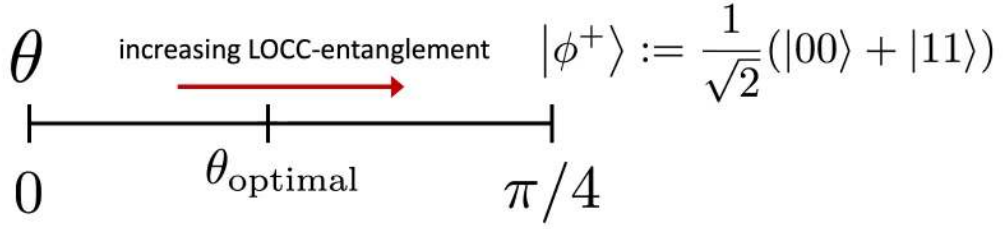
Consider using these states to generate nonlocality, as measured by:



The anomalies of nonlocality:

Consider using these states to generate nonlocality, as measured by:

1. probability of running a Hardy proof of nonlocality
2. violation of a tilted Bell inequality
3. extractable secret key rate
4. relative entropy distance from the local set



The optimum is different in each case

Lucien Hardy's proof of nonlocality

$$\Pi_x^{(Alice)} = \begin{cases} \sqrt{\frac{1}{25}}S_Z - \sqrt{\frac{24}{25}}S_X & \text{if } x = 0 \\ S_Z \text{ if } x = 1 \end{cases}$$

$$\Pi_y^{(Bob)} = \Pi_{x \to y}^{(Alice)}$$

$$|\psi_{\text{Hardy}}\rangle = \frac{1}{2}|00\rangle - \sqrt{\frac{3}{8}}(|01\rangle + |10\rangle) \longrightarrow B_{\text{Hardy}} := p(ab|xy) =$$

$$\begin{array}{c} \\ (y,b) \end{array} \begin{array}{c} \\ \begin{matrix} (0,0) \\ (0,1) \\ (1,0) \\ (1,1) \end{matrix} \end{array} \begin{array}{c} (x,a) \\ \begin{matrix} (0,0) & (0,1) & (1,0) & (1,1) \end{matrix} \\ \left[\begin{array}{cc|cc} 0.64 & 0.135 & 0.15 & 0.625 \\ 0.135 & 0.09 & 0.225 & 0 \\ \hline 0.15 & 0.225 & 0 & 0.375 \\ 0.625 & 0 & 0.375 & 0.25 \end{array}\right] \end{array}$$

Under the assumption of local hidden variables, these particular correlations allow one to run a chain of counterfactual inferences that generate a contradiction.

Hardy showed one cannot generate such a proof using $|\phi^+\rangle$

(a resource-theoretic spin on the anomaly)

Entanglement theory says

LOCC $\quad |\phi^+\rangle \rightarrow |\psi_{\text{Hardy}}\rangle$

By definition

LO $\quad |\psi_{\text{Hardy}}\rangle \rightarrow B_{\text{Hardy}}$

But

LO $\quad |\phi^+\rangle \not\rightarrow B_{\text{Hardy}}$

Apparent inconsistency

Because in a resource theory, conversion is a transitive relation…

But we have argued that one must take all three of these relative to LOSR

Under LOSR operations

$$|\phi^+\rangle \not\to |\psi_{\text{Hardy}}\rangle$$

Under LOSR operations

$$|\psi_{\text{Hardy}}\rangle \to B_{\text{Hardy}}$$

Under LOSR operations

$$|\phi^+\rangle \not\to B_{\text{Hardy}}$$

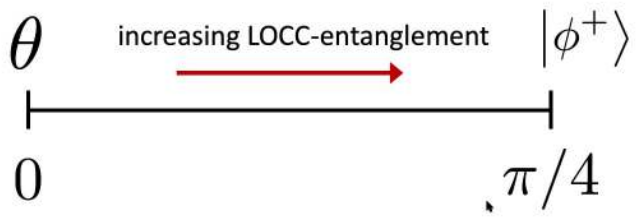
Consistent!

Relative to LOSR

$|\phi^+\rangle$ incomparable to $|\psi_{\mathrm{Hardy}}\rangle$

Hence the terms “maximally entangled” and “partially entangled” are *not appropriate* for LOSR-entanglement

Consider again the family of states given by $\cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle$



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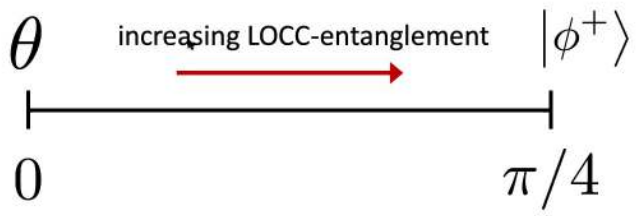

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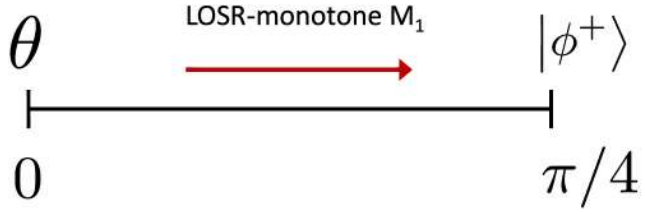

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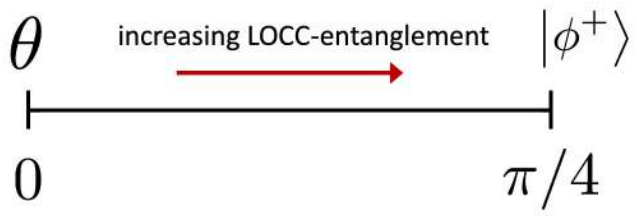

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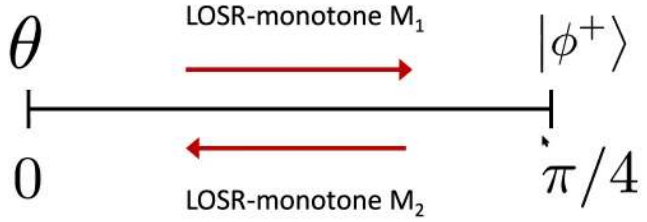

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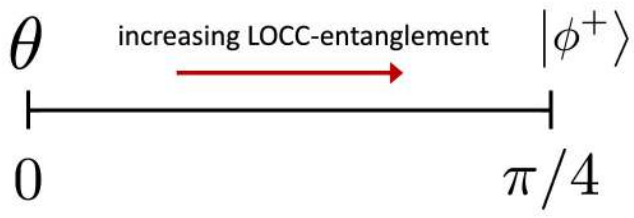

For each anomaly, the associated task (generating a Hardy paradox, generating a secret key, etc) has its own optimal state, and defines a monotone which is peaked at that state!

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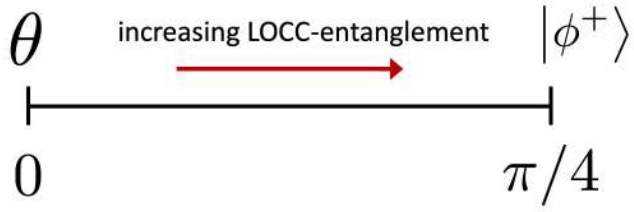

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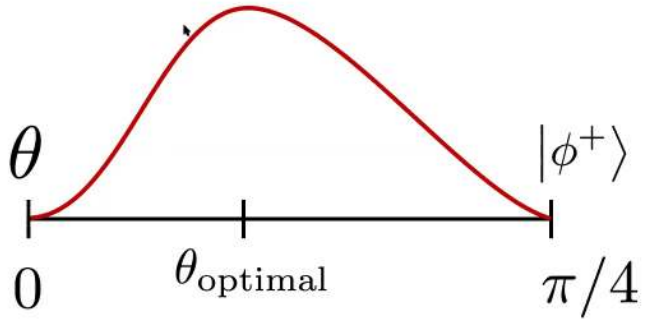

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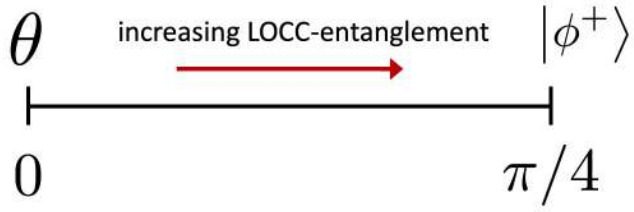

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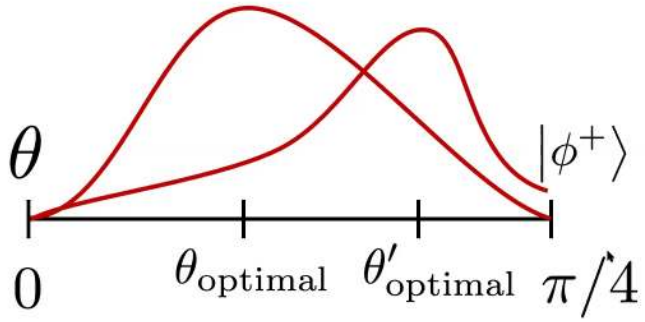

Standard conclusion from the anomalies:
Nonlocality and entanglement are "different resources"

Our conclusion:

Nonlocality and LOCC-entanglement are different resources
(but there was no reason to expect these two to be related anyway!)

Nonlocality and LOSR-entanglement ARE manifestations of the same resource (nonclassicality of common cause)

Indeed, there is no anomaly of nonlocality when one quantifies entanglement with respect to LOSR rather than LOCC

$|\psi_{\mathrm{GHZ}}\rangle := \frac{1}{\sqrt{2}}(|000\rangle_{ABC} + |111\rangle_{ABC})$ intuitively, this is genuinely 3-way entangled

$|\psi_{\mathrm{2Bell}}\rangle := |\phi^+\rangle_{A_1B} \otimes |\phi^+\rangle_{A_2C}$ intuitively, this is NOT genuinely 3-way entangled

Traditional definition: a tripartite state is *genuinely 3-way entangled* iff it is not a mixture of states that are separable with respect to partitionings of the parties into two groups

Pathology 1: this deems *both* of the above states to be genuinely 3-way entangled!

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Pathology 1: this deems *both* of the above states to be genuinely 3-way entangled!
(can be cast as an anomaly of 3-way nonlocality)

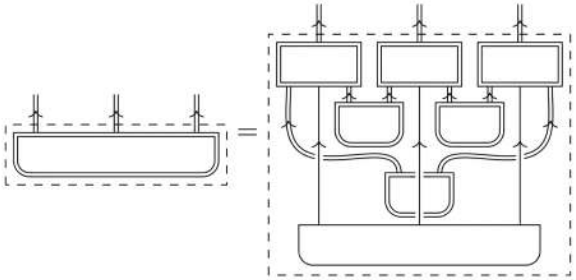
Pathology 2: the set of states which are *not* G3WE is not closed under tensor product
-violates the assumption that the free set in any resource theory be closed

Note that with respect to LOCC-entanglement, $|\psi_{\mathrm{2Bell}}\rangle \to |\psi_{\mathrm{GHZ}}\rangle$

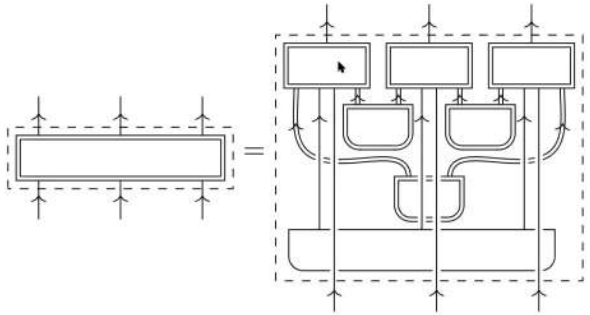
Start fresh: how *should* one define genuine 3-way entanglement?

"That which cannot be generated by arbitrary bipartite entanglement (and 3-way shared randomness)"

a state is genuinely 3-way entangled iff it is not decomposable as



a box is genuinely 3-way nonlocal iff it is not decomposable as



Clarifying and extending self-testing

Motivation: learn something about the internal components of a physical process (or a quantum device you bought at Canadian Tire) without opening it up and testing individual components

Example: If you measure some bipartite system and observe some box (at space-like separation) which achieves the quantumly maximal violation of the CHSH inequality, then the state that was used must have been the (LOCC)-maximally entangled state

…but not quite: there is *never* a unique state that can generate a given box

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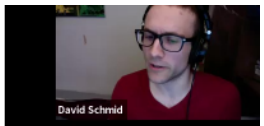
So self-testing is defined “up to some freedom”

Traditional definition of self-testing:

To say that $|\psi\rangle$ is self-tested by B is to say that B can be obtained by local measurements on a state $|\phi\rangle$ only if $|\phi\rangle$ can be mapped to $|\psi\rangle|\zeta\rangle$ for some $|\zeta\rangle$ by a local isometry.

Note:
-defined only for pure states
-in fact, it has been claimed that mixed states cannot be self-tested
-there is no a priori justification for choosing local isometries

Two views of self-testing:
(1) the goal is to infer that whatever state was used to generate B, one could *extract* ψ from it
(2) the goal is to *uniquely identify* the state ψ that was used to generate B



View (1) on ρ being self-tested by B:

The goal is to infer that one can extract ρ from the state that was used to generate B

We take this idea on board.
But, we already argued that the appropriate set of operations for both state-to-state and state-to-box conversions is LOSR!

Our resource-theoretic definition of self-testing:

Definition 2. We say that a density operator ρ *is self-tested by a box* B if it holds that

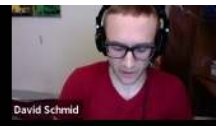
$$\rho \mapsto B$$

and

$$\forall \sigma : \text{if } \sigma \mapsto B \text{ then } \sigma \mapsto \rho,$$

where all conversions are evaluated relative to LOSR.

Very simply: ρ is the *least LOSR-entangled state* that can generate B



Coincides with the traditional definition of self-testing for pure states.

So the articulation of the freedom of a state in the conventional definition of self-testing is nothing more than the condition for LOSR-convertibility for pure states.

But our definition also applies to mixed states.

It is easy to see that *some* mixed states can be self-tested, e.g.

$$\rho_{AB} := \sum_{ij} p(ij) \left(U_A^{(i)} \otimes U_B^{(j)}\right) |\psi\rangle\langle\psi|_{AB} \left(U_A^{(i)\dagger} \otimes U_B^{(j)\dagger}\right) \otimes |i\rangle\langle i|_A \otimes |j\rangle\langle j|_B$$

What about more generic mixed states?
Implications for robust self-testing?

View (2) on self-testing

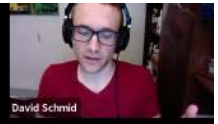
The goal is to uniquely identify the state ρ that was used to generate B

Problem: If ρ can generate B, then so can any state that is more LOSR-entangled

When is unique identification possible?

If one is promised that the set from which ρ is being drawn contains states that are all pairwise incomparable (or equivalent), *then* one can hope to identify a unique LOSR-equivalence class that could have generated B

This motivates the search for sets of states which are pairwise incomparable



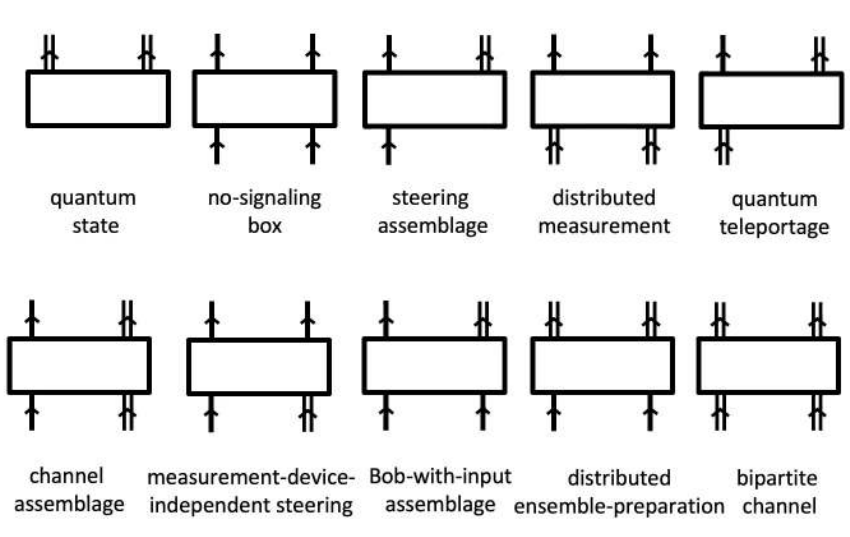
E.g. all two-qubit pure states: $\cos(\theta)\,|00\rangle + \sin(\theta)\,|11\rangle \qquad 0 \leq \theta \leq \pi/4$

In fact, any two bipartite pure states of equal Schmidt rank are LOSR-incomparable (or equivalent).

This is why all bipartite pure states (of a given Schmidt rank) can be not only self-tested, but also *uniquely identified* from the boxes they generate

Further generalizations of self-testing

- self-testing of steering assemblages, channels, and any other type of LOSR resource



All these are unified within the resource theory of LOSR

arXiv:1909.04065

- self-testing of resources in other resource theories

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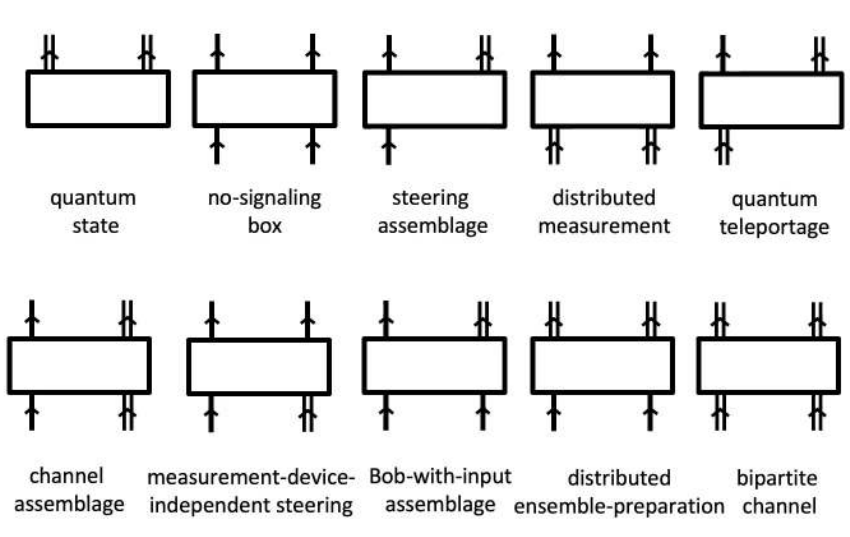

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- self-testing of resources in other resource theories
- a weaker notion of self-testing
  (that always succeeds, but is more or less informative depending on what box is observed)

Other results about LOSR-entanglement theory

- an analogue of Nielsen's theorem: a necessary and sufficient condition for when LOSR-conversions between two bipartite pure states are possible
- proof that there are no catalytic conversions between bipartite pure states
- results relating the LOSR-preorder to equivalence under local unitaries
- a necessary condition for conversions among n-partite states
  (an instance of the spectral quantum marginals problem)
- some operationally-motivated monotones (that are not LOCC-monotones)
- a more abstract resource-theoretic characterization of self-testing

arXiv:2004.09194

The fact that researchers have previously taken LOCC-entanglement, rather than LOSR-entanglement, as the appropriate notion in Bell scenarios has led to the view that the interplay between entanglement and nonlocality is rather perverse. We have shown, however, that this perversity is merely an artifact of considering the wrong notion of entanglement. Once one focusses on LOSR-entanglement, the interplay with nonlocality is found to be quite natural.

This motivates a new branch of entanglement theory, where the free operations are LOSR rather than LOCC.

Revisit other related results:
-Peres conjecture, distillation, Werner states, entanglement in networks, etc.

Why standard entanglement theory is inappropriate for Bell scenarios
arXiv:2004.09194

Unifying many other resource types into the LOSR resource theory
arXiv:1909.04065

The resource theory of nonlocality
arXiv:1903.06311

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Thank you!