Title: A quantum circuit interpretation of evaporating black hole geometry

Speakers: Ying Zhao

Date: April 21, 2020 - 9:30 AM

URL: http://pirsa.org/20040092

Abstract: When Alice shares thermofield double with Bob, her time evolution can make the wormhole grow. We identify different kinds of operations Alice can do as being responsible for the growth of different parts of spacetime and see how it fits together with subregion duality. With this, we give a quantum circuit interpretation of evaporating black hole geometry. We make an analogy between the appearance of island for evaporating black hole and the transition from two-sided to one-sided black hole in the familiar example of perturbed thermofield double. If Alice perturbs thermofield double and waits for scrambling time, she will have a one-sided black hole with interior of her own. We argue that by similar mechanism the radiation gets access to the interior (island forms) after Page time. The growth of the island happens as a result of the constant transitions from two-sided to one-sided black holes.

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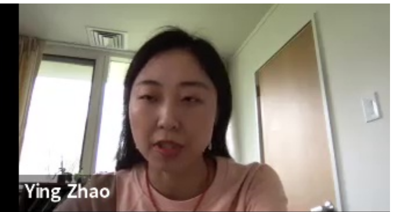


A quantum circuit interpretation of Evaporating black hole geometry

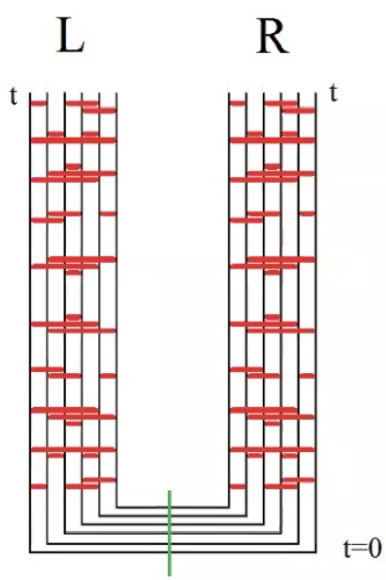
Ying Zhao

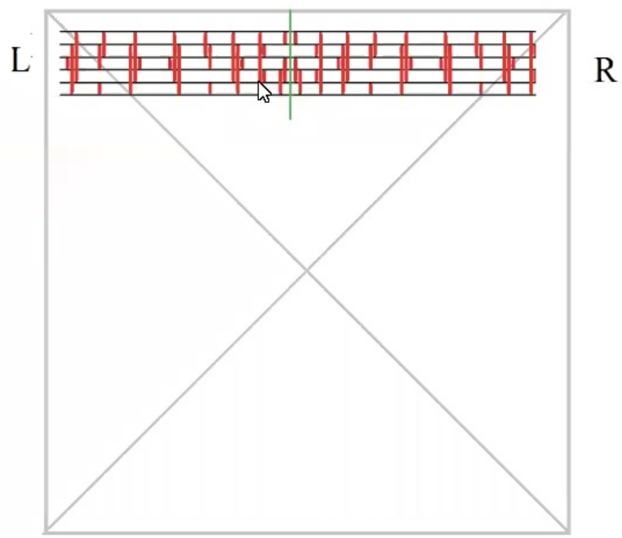
arXiv:1711.03125
arXiv: 1912.00909v2

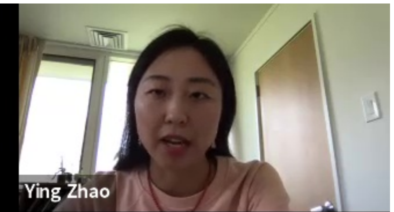
Bulk tensor network and quantum circuit

B. Swingle arXiv:1209.3304
T. Hartman, J. Maldacena arXiv:1303.1080
L. Susskind arXiv:1411.0690

- The bulk geometry reflects the minimal circuit preparing the state.

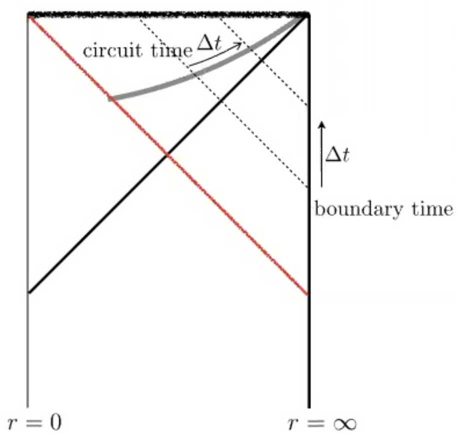


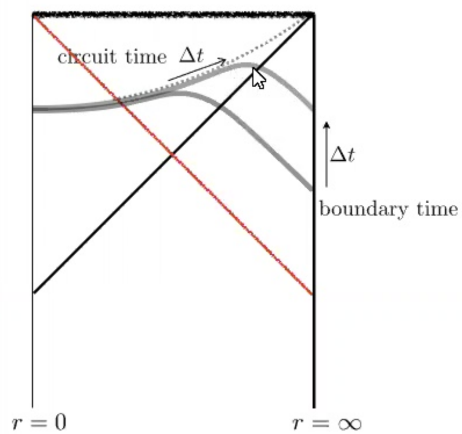




Pure state black hole

L. Susskind, Y. Zhao arXiv:1408.2823





- As we apply unitary time evolution to the circuit, the state gets more complex, the minimal circuit gets longer, and the Einstein-Rosen bridge also gets longer.

- Identify circuit time with boundary time: $d\tau = \frac{2\pi}{\beta} dt$

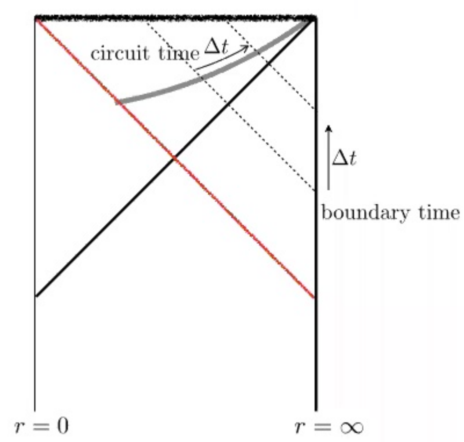
Digression: Uncomplexity

- Uncomplexity of a pure state: A. Brown, L. Susskind arXiv:1701.01107

$$\mathcal{UC}(|\psi\rangle) = \max_U \; \mathcal{C}(U|\psi\rangle) - \mathcal{C}(|\psi\rangle)$$

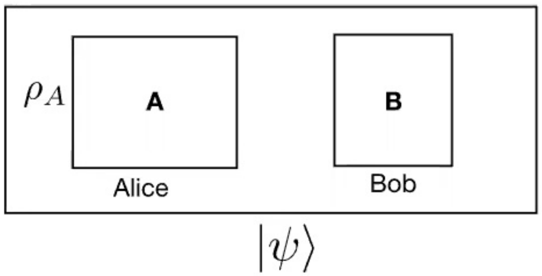
- The growth of wormhole consumes uncomplexity. L. Susskind arXiv:1507.02287

Uncomplexity decreases. Wormhole gets longer.



Mixed state

One can apply unitary operators U: $\rho \to U\rho U^\dagger$. But not all U can affect the density matrix ρ. We need to identify those operators that affect ρ.



Question:

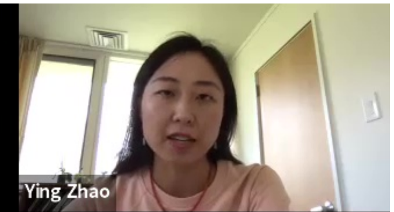
How much computational power can Alice get out of the state $|\psi\rangle$ when she only has subsystem A?

- Uncomplexity of a mixed state:

Y. Zhao arXiv:1711.03125

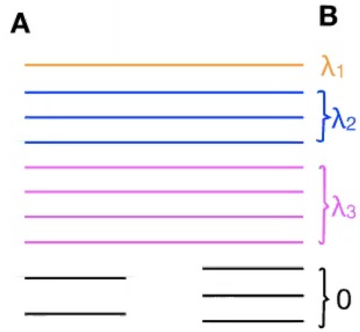
$$\mathcal{UC}(\rho_A) = \max_{U_A} \mathcal{C}(U_A |\psi\rangle) - \max_{\substack{U_A \text{ does not} \\ \text{change } \rho_A}} \mathcal{C}(U_A |\psi\rangle)$$

- We need to exclude those unitaries that do not change the density matrix ρ_A. They do not contribute to uncomplexity of ρ_A.



Claim: If an unitary operator U_A does not change the density matrix ρ_A, then U_A can be undone by some unitary operator on B, i.e. there exists U_B, s.t. $U_A\,|\psi\rangle = U_B\,|\psi\rangle$.

$$\rho_A = \begin{array}{c} \left(\begin{array}{ccccc} \lambda_1\mathbb{1} & 0 & \dots & 0 & 0 \\ 0 & \lambda_2\mathbb{1} & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & \lambda_k\mathbb{1} & 0 \\ 0 & 0 & \dots & 0 & 0 \end{array}\right) \\ \begin{array}{ccccc} \underbrace{\quad}_{N_1} & \underbrace{\quad}_{N_2} & \dots & \underbrace{\quad}_{N_k} & \underbrace{\quad}_{N_0} \end{array} \end{array} \begin{array}{l} \}N_1 \\ \}N_2 \\ \vdots \\ \}N_k \\ \}N_0 \\ \\ \end{array}$$



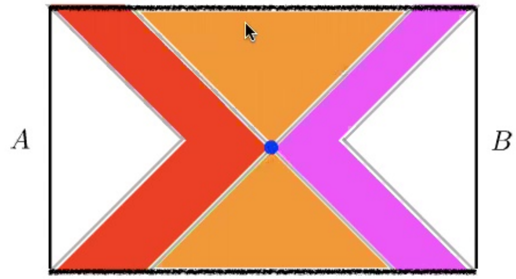
Relative Schmidt basis rotations

$$SU(N_1)\times SU(N_2)\times \dots \times SU(N_k)\times SU(N_0)$$

- Two kinds of operations Alice can apply:

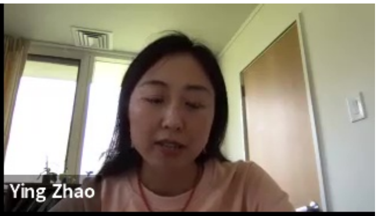
1. Relative Schmidt basis rotations between two subsystems A and B. These can be undone from side B. They do not belong to either side alone.
2. Unitary operations U_A that cannot be undone from side B. They belong to subsystem A.

- Relation to subregion duality



The first kind of gates are responsible for the growth of the orange region. The first kind of gates are stored in the orange region, while the second kind of gates are stored in the red region.

When we say some gates are stored in some region, what we really mean is the growth of certain spacetime region accompanies the growth of certain parts of the quantum circuit.

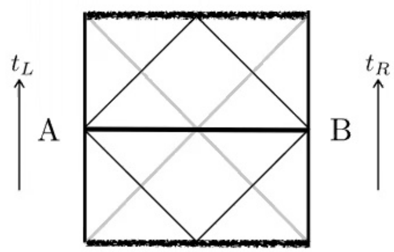


Examples

Two-sided black hole

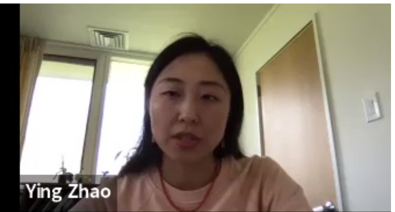
Thermofield Double

(a) Quantum circuit

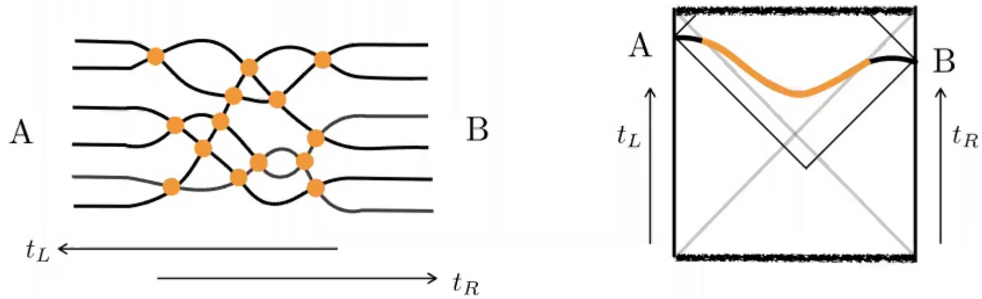


(b) Wormhole geometry

We represent thermofield double by S Bell pairs. The corresponding wormhole geometry has minimal length.



Time-evolved thermofield Double



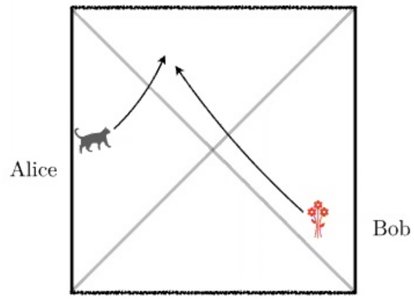
(a) Quantum circuit (b) Wormhole geometry

- The orange gates can be undone from either side. They do not belong to subsystem A or B alone.
- The wormhole gets longer but the part that gets longer is outside A's entanglement wedge and also outside B's entanglement wedge.

Compare one-sided and two-sided black holes

Two-sided black hole

- When Alice has a two-sided black hole, the interior is outside her entanglement wedge.
  Alice cannot predict an infalling observer's experience

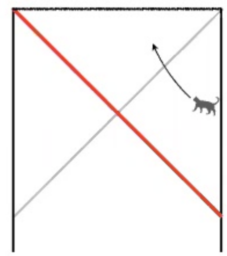


- The expansion of the interior is fueled by the relative rotation of the Schmidt basis, not Alice's uncomplexity.

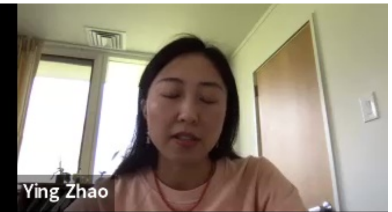
  As the quantum circuit gets longer, the gates in the circuit does not change Alice's density matrix. The growth of the wormhole does not consume Alice's uncomplexity.

One-sided black hole

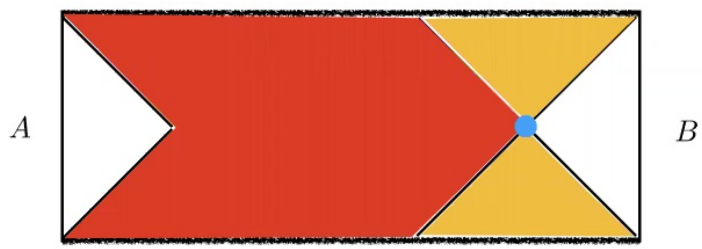
- When Alice has a one-sided black hole, she can predict an infalling observer's experience.



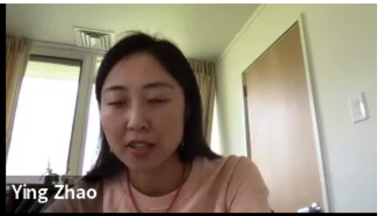
- The expansion of the interior is fueled by Alice's uncomplexity.

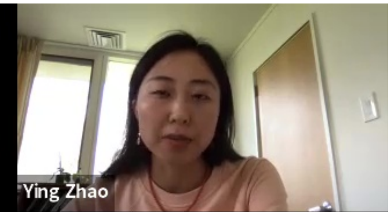


In this talk, when we say Alice has one-sided black hole, we mean that the interior region accessible to her is determined by her subsystem. She can in principle predict the infalling observer's experience. The property of a black hole being one-sided or two-sided can be time-dependent. We do NOT mean counting the number of boudaries.

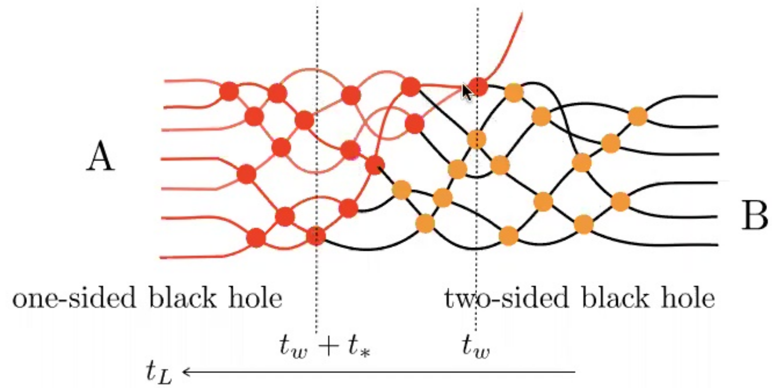


Alice's black hole is effectively one-sided.





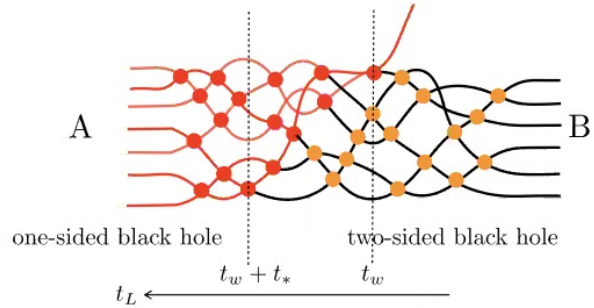
Quantum circuit picture

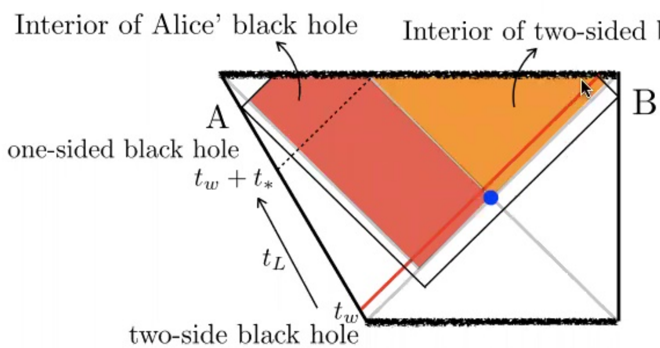


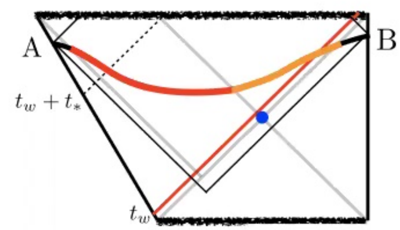
Initially, we started from S Bell pairs and the orange gates can be done from either side. Then an extra qubit comes in to Alice's side. It will affect the entire system after scrambling time. The red gates can be undone by Alice but can no longer be undone by Bob.

Quantum circuit and Black hole geometry

Different kinds of gates are stored in different spacetime regions.

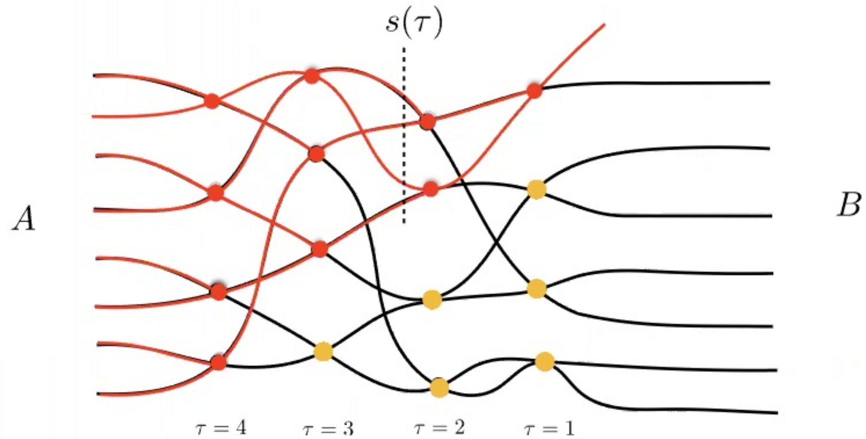






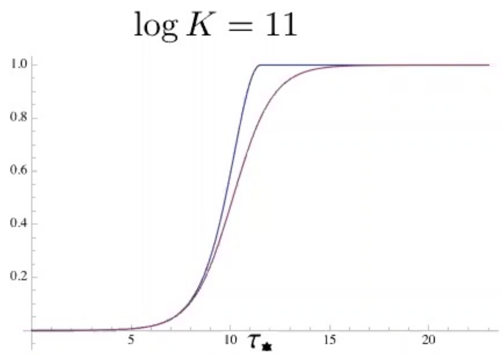
Epidemic model: the growth of an operator

P. Hayden, J. Preskill arXiv:0708.4025v2
L. Susskind, Y.Z. arXiv:1408.2823
A. Brown, L. Susskind, Y.Z. arXiv:1608.02612

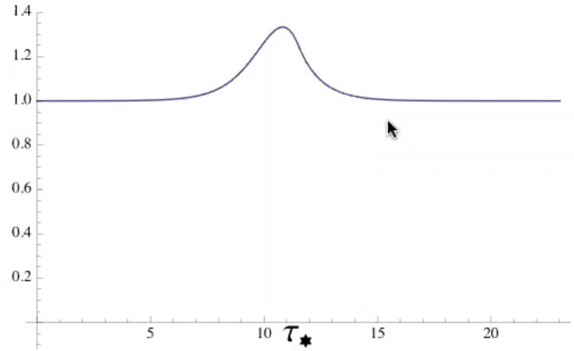


$$\frac{ds}{d\tau} = (K+1-s)\cdot\frac{s}{K}, \qquad \frac{s(\tau)}{K+1} = \frac{\frac{1}{K}e^{\tau}}{1+\frac{1}{K}e^{\tau}}, \quad s(0)=1$$

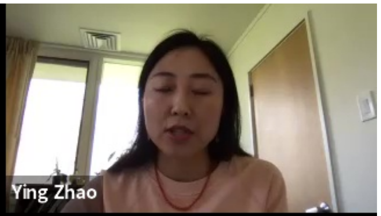
$$\frac{dN_{\text{cannot be undone}}(\tau)}{d\tau} = s(\tau), \qquad \frac{dN_{\text{can be undone}}(\tau)}{d\tau} = K - s(\tau)$$



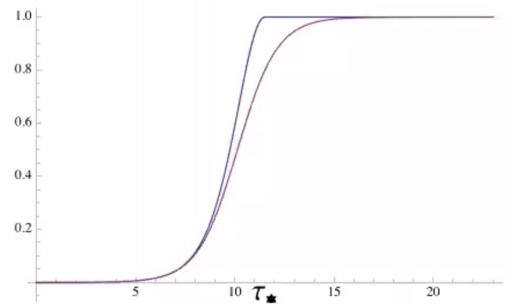
blue line: increase of spacetime volume in Alice's entanglement wedge every thermal time
pink line: increase of the number of gates that cannot be undone by Bob

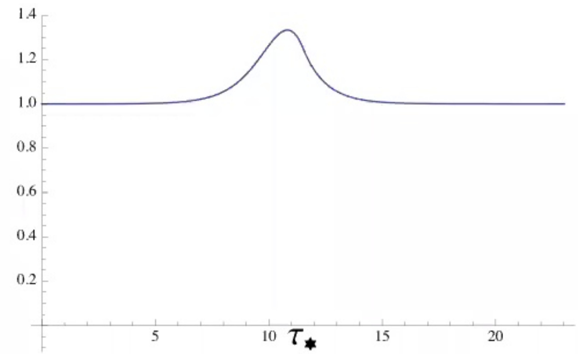


Ratio of the results from black hole calculation and from the circuit picture analysis.



$\log K = 11$

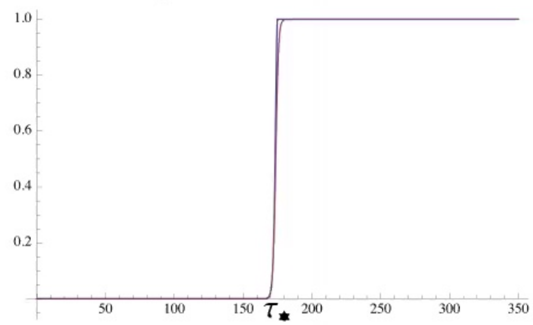


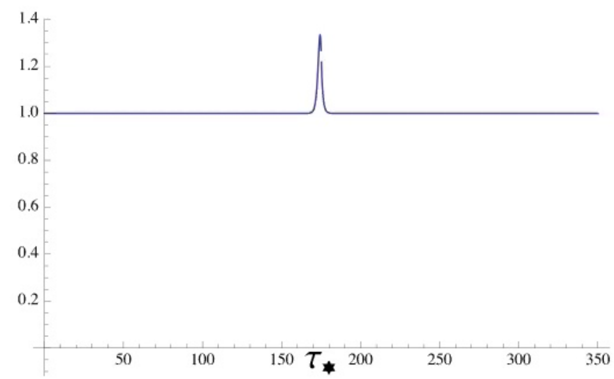


blue line: increase of spacetime volume in Alice's entanglement wedge every thermal time
pink line: increase of the number of gates that cannot be undone by Bob

Ratio of the results from black hole calculation and from the circuit picture analysis.

$\log K = 175$



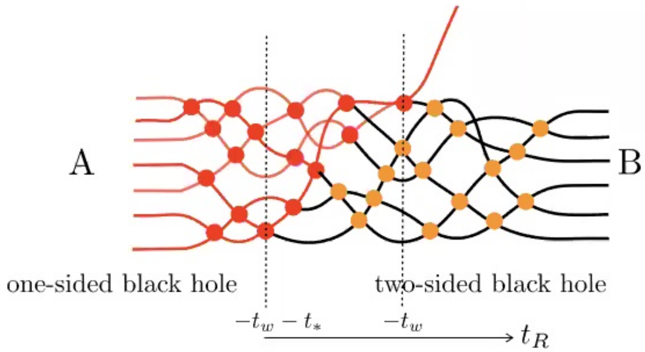


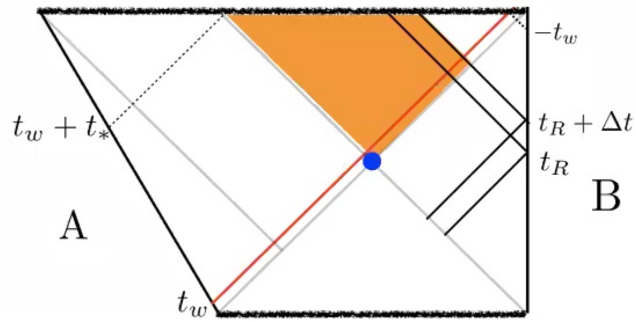
The spacetime growth (both inside and outside Alice's entanglement wedge) and the qubit model share the same key features:

- Exponential growth
- Saturation after scrambling time

Different kinds of gates Alice can apply are responsible for the growth of different parts of spacetime.

From the point of view of Bob

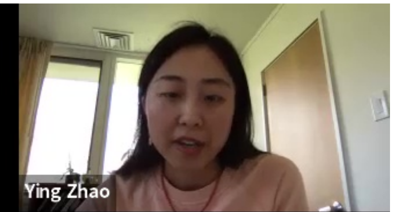




$$\frac{d\mathrm{Vol}_{\mathrm{orange}}(t_R)}{dt_R} = \pi r_H^2 \left[1 - \frac{\frac{2\delta S}{S}\cosh^2\left(\frac{\pi}{\beta}(-t_w - t_R)\right)}{1 + \frac{2\delta S}{S}\cosh^2\left(\frac{\pi}{\beta}(-t_w - t_R)\right)} \right]$$

$$\approx \pi r_H^2 \left(1 - \frac{\mathrm{size}\left(\frac{2\pi}{\beta}(-t_w - t_R)\right)}{S} \right)$$

Bob can only undo the orange gates in the circuit.



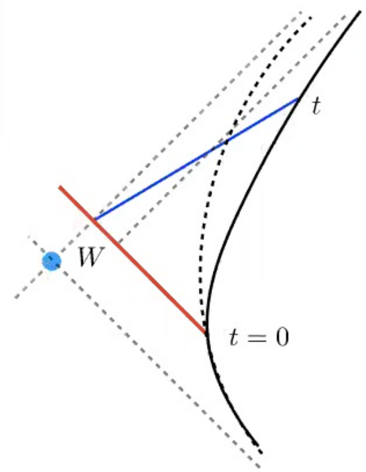
Another example: SYK and JT gravity

- Alice applies $\psi_1(t) = U(t)\psi_1 U(t)^\dagger$

- $$\text{size}(\psi_1(t)) = 1 + 2\left(\frac{\beta\mathcal{J}}{\pi}\right)^2 \sinh^2\left(\frac{\pi}{\beta}t\right) \qquad t < t_*$$
  $$\mathcal{C}(\psi_1(t)) \sim 2\left(\frac{\beta\mathcal{J}}{\pi}\right)^2 \sinh^2\left(\frac{\pi}{\beta}t\right) \qquad t < t^*$$

- $$d(t) \approx \log\left(1 + \frac{2\delta S}{S - S_0}\sinh^2\left(\frac{\pi}{\beta}t\right)\right) + \text{const}$$

X. Qi, A. Streicher arXiv:1810.11958



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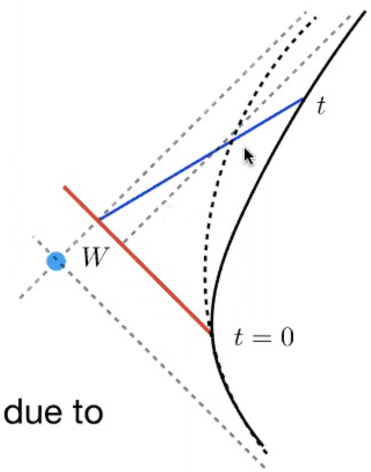
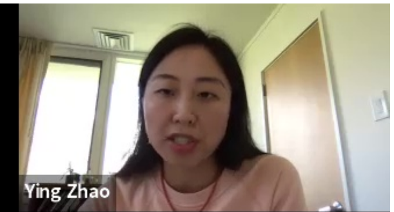
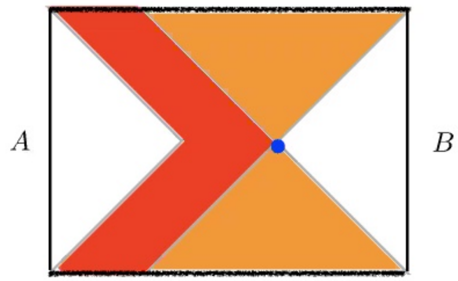


The growth of Alice's entanglement wedge is due to computations involving $\psi_1(t)$

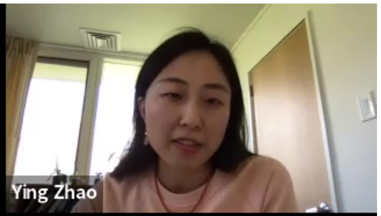


Summary



Entanglement region (orange): Relative Schmidt basis rotations

Alice's entanglement wedge (red): Unitary operations that can be undone by Alice but not by Bob



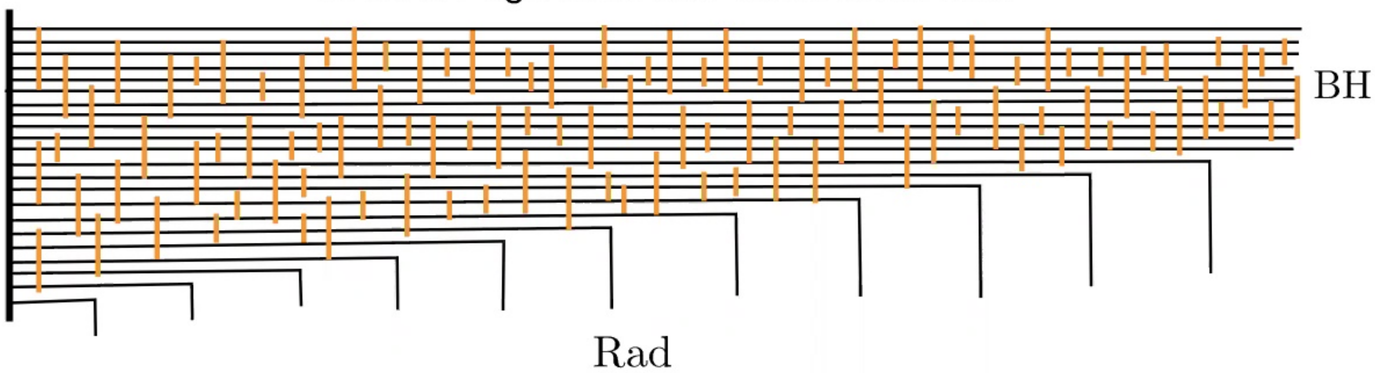
Evaporating black hole

G. Penington arXiv: 1905.08255v2
A.Almheiri, N. Engelhardt, D. Marolf, H. Maxfield arXiv: 1905.08762v3

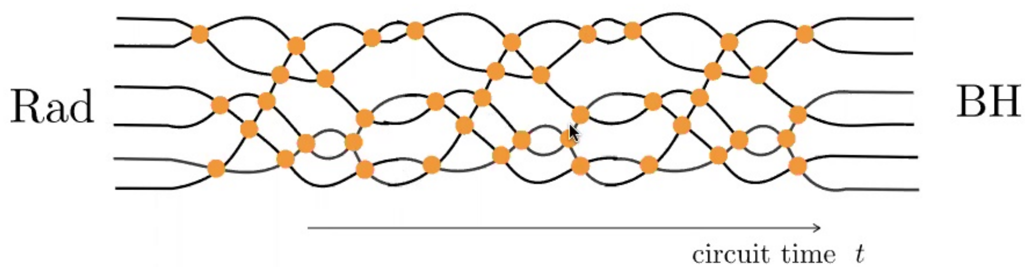
- Why is it that adding a few more qubits to the radiation give rises to a big jump in the size of the region contained in its entanglement wedge?

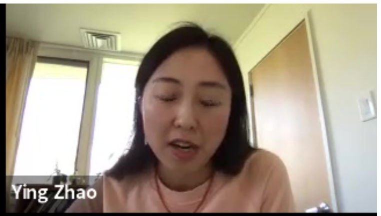
- Why does the bulk region belonging to the radiation (island) keep growing after Page time while the radiation doesn't do any computations?

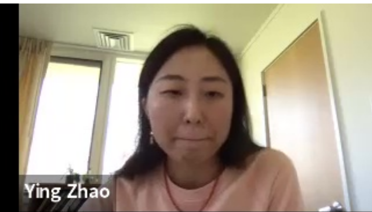
Around Page time: Two-sided black hole



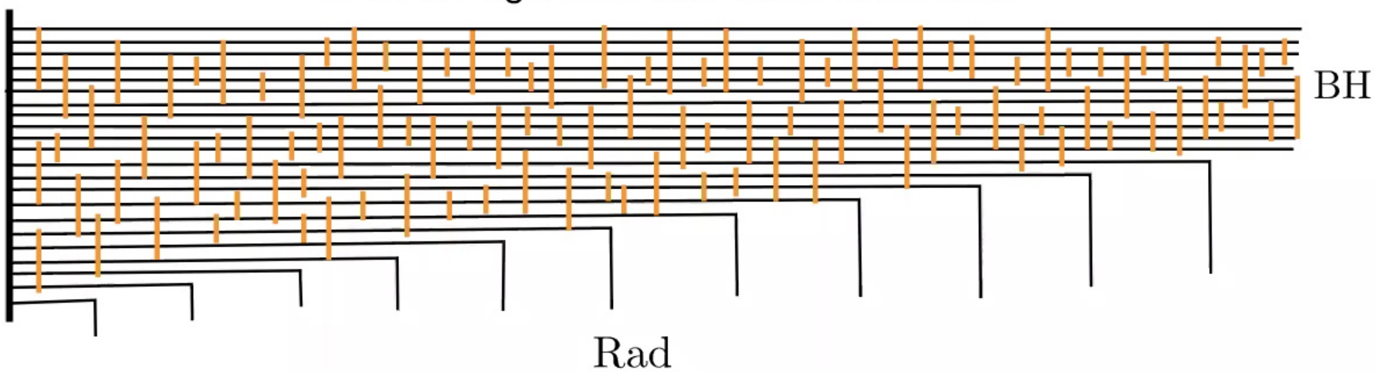
At Page time, the black hole is maximally entangled with the radiation. There is a circuit connecting them. The gates can be undone from both sides.







Around Page time: Two-sided black hole



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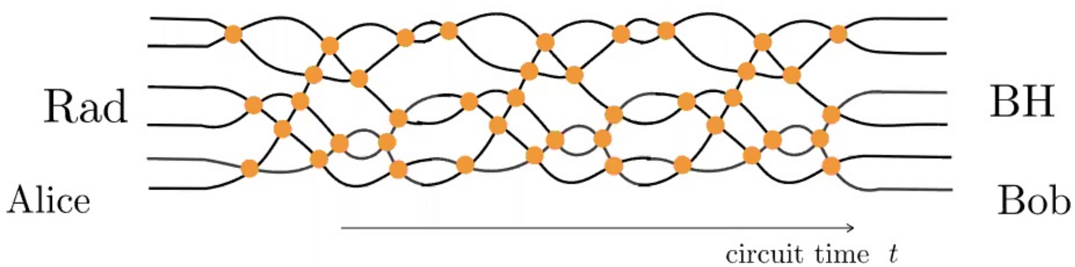
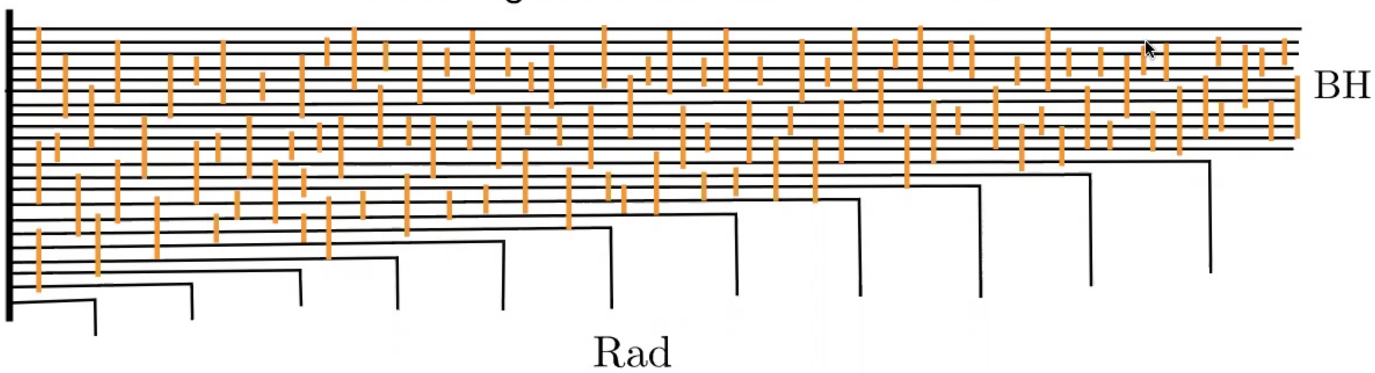
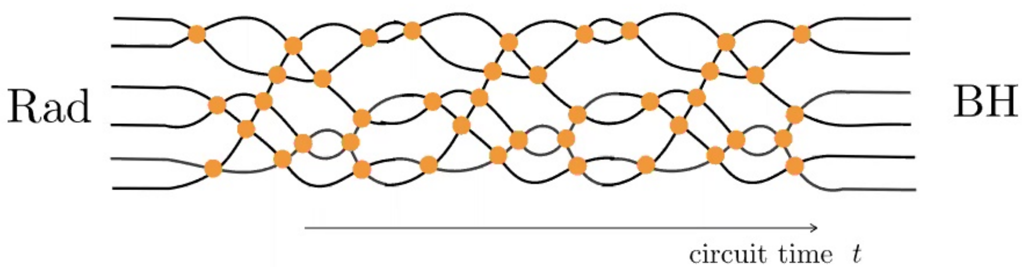


Alice has the radiation while Bob has the remaining black hole.

Around Page time: Two-sided black hole



At Page time, the black hole is maximally entangled with the radiation. There is a circuit connecting them. The gates can be undone from both sides.



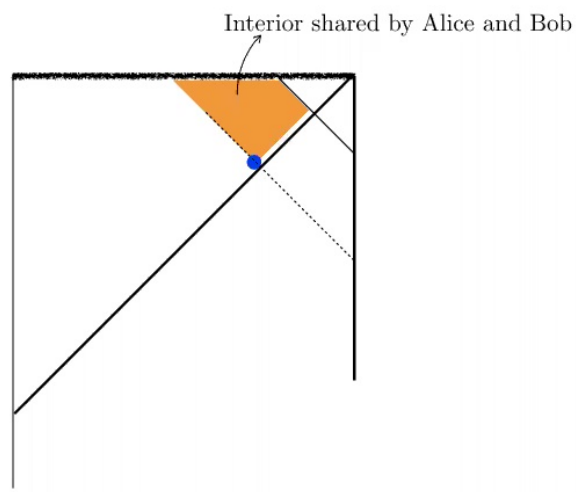
The remaining black hole is maximally entangled with the radiation

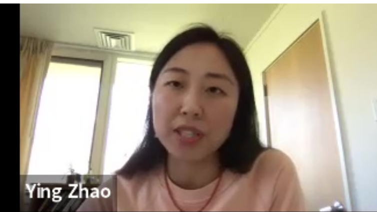
$\Longrightarrow$ Bob's computations can be undone by Alice

$\Longrightarrow$ Bob doesn't have his own computations.
He has a two-sided black hole.

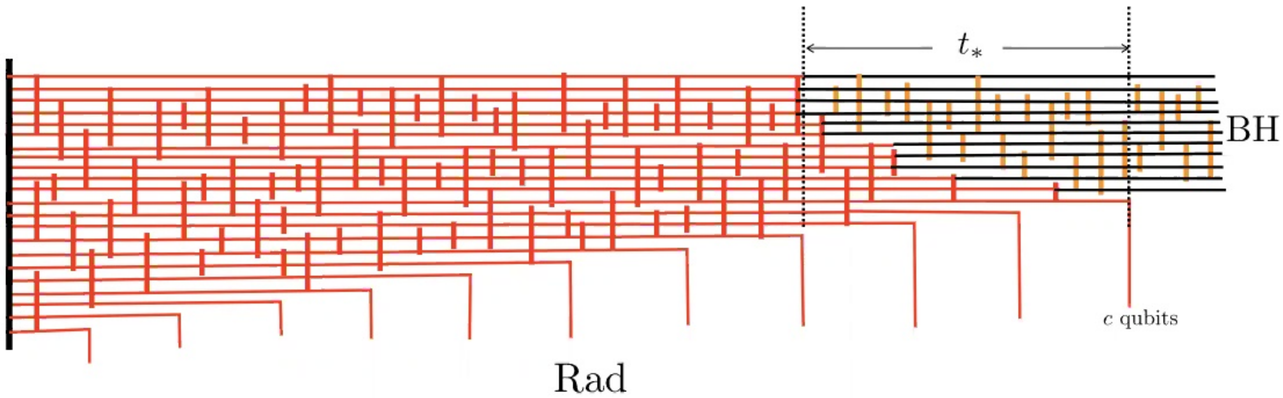
$\Longrightarrow$ Bob shares interior with Alice. He doesn't have his own interior.

$\Longrightarrow$ The RT surface lies at the horizon.

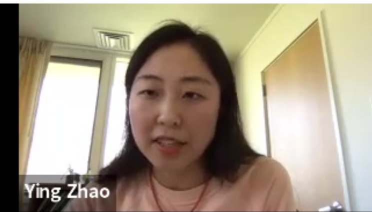


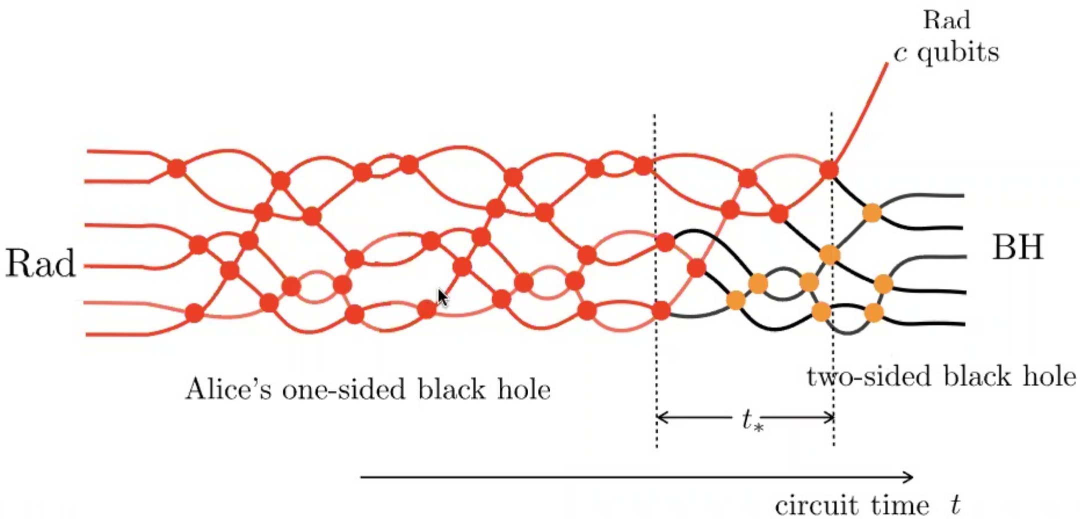


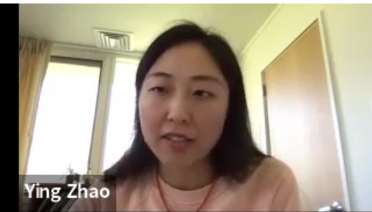
The appearance of island



- The circuit stored in the wormhole does scrambling. $t_* = \frac{\beta}{2\pi} \log \frac{S}{c}$







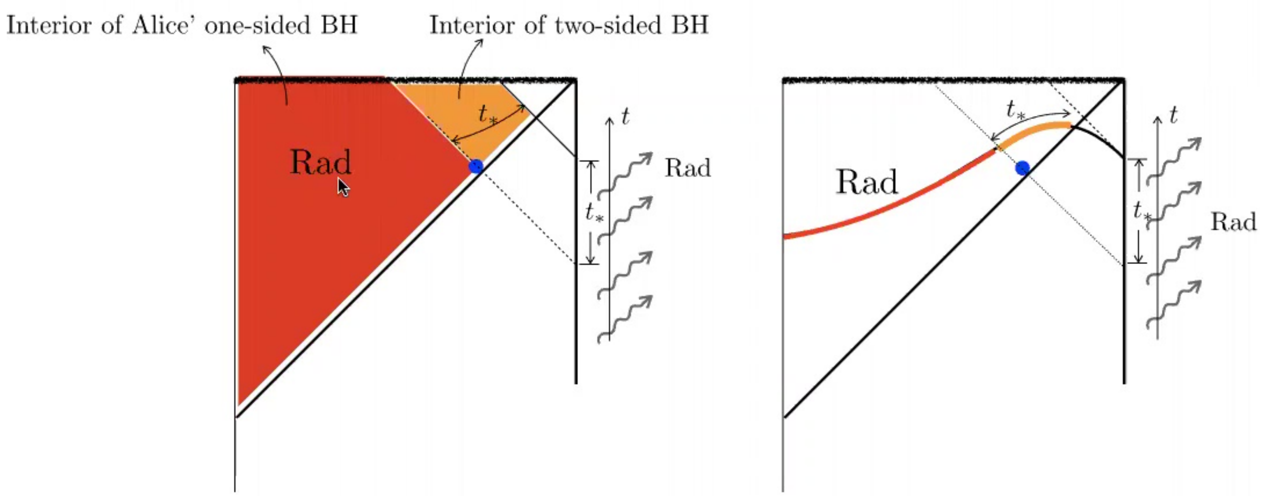
- Alice gets her own gates even though the computations were done by Bob.

- The red gates form the island: interior of Alice's one-sided Black hole.

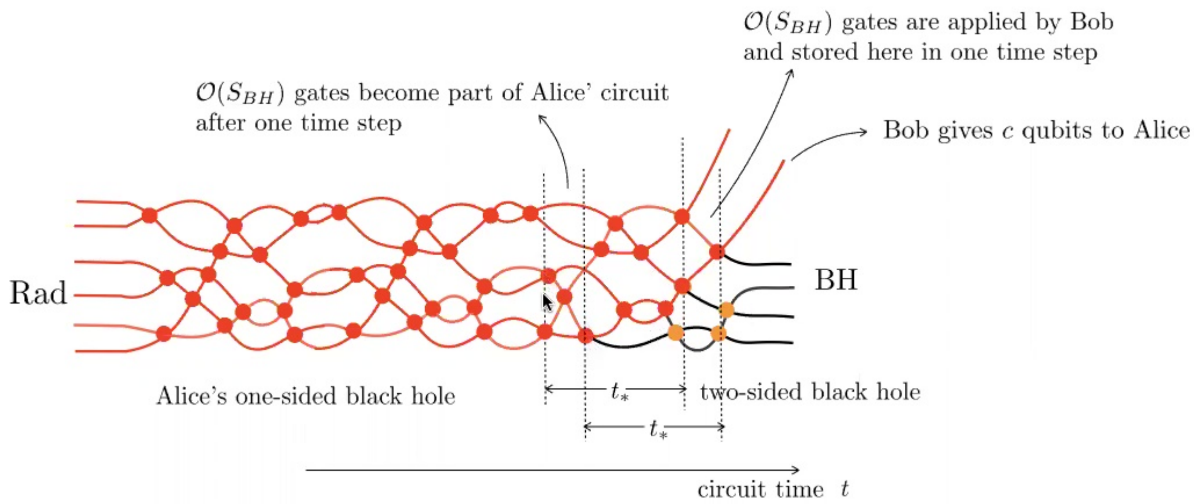
The circuit stored in the interior takes time $t_* = \frac{\beta}{2\pi} \log \frac{S}{c}$ to scramble the c qubits.

$\Longrightarrow$ Gates applied t_* earlier can no longer be undone by Bob.

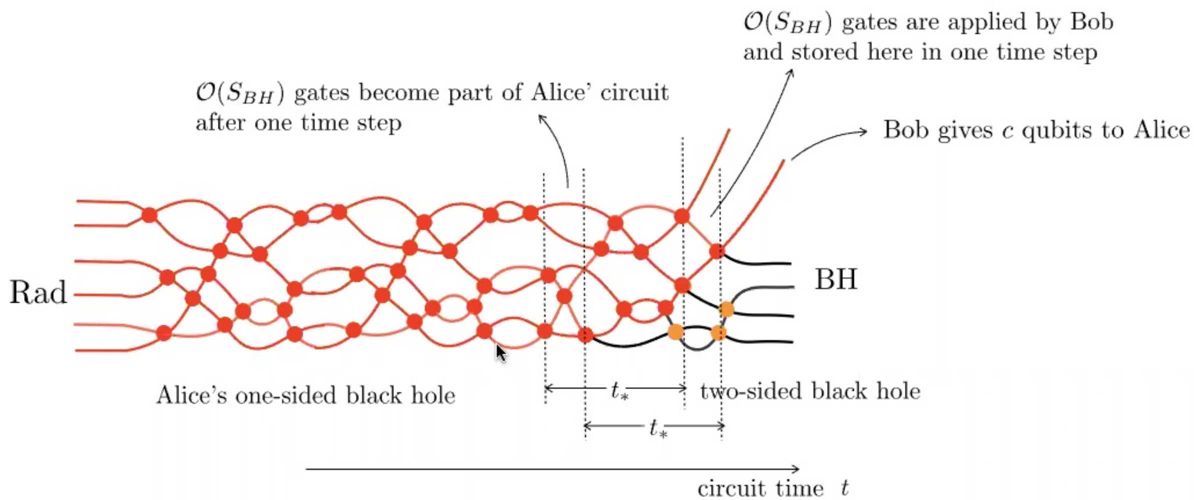
$\Longrightarrow$ The RT surface is $t_* = \frac{\beta}{2\pi} \log \frac{S}{c}$ behind.



The growth of island



The growth of island



Bob has a two-sided black hole and he does computations. As the interior of two-sided black hole grows at one end of the circuit, the transition from two-sided to one-sided black hole also constantly happens.

Thank you.