Title: Baby Universes and Black Hole Information

Speakers: Donald Marolf
Date: April 09, 2020 - 9:30 AM

URL: http://pirsa.org/20040091

Abstract: In the 1980â€™s, work by Coleman and by Giddings and Strominger linked the physics of spacetime wormholes to â€˜baby universesâ€™ and an ensemble of theories. We revisit such ideas, using features associated with a negative cosmological constant and asymptotically AdS boundaries to strengthen the results, introduce a change in perspective, and connect with recent replica wormhole discussions of the Page curve. A key new feature is an emphasis on the role of null states. We explore this structure in detail in simple topological models of the bulk that allow us to compute the full spectrum of associated boundary theories. We also argue that similar properties must hold in any consistent gravitational path integral.

Hosted By: Stanford University Footage Courtesy: Simons Foundation for Mathematics and Physical Sciences

Baby Universes and Black Hole Information

Don Marolf, UBC via Zoom

April 7, 2020

Based on arxiv:2002.08950

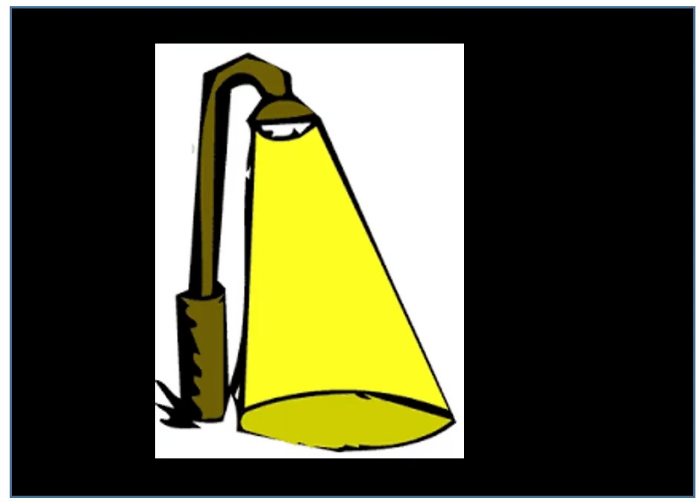
with Henry Maxfield

Motivation: Recent progress on black hole information

- As shown in arxiv:1905.08255 (G. Penington) and arxiv:1905.08762 (A. Almheiri et al) , computing S_{gen} on quantum extremal surfaces in an evaporating black hole spacetime gives a Page curve consistent with "unitarity" and the emission of information in Hawking radiation.

- It was then shown in arxiv:1911.12333 (A. Almheiri et al, aka "the East Coast paper") and arxiv:1911.11977 (G Penington et al, aka "the West Coast paper") that the above apparently-holographic recipe in fact follows from performing the bulk gravitational path integral.

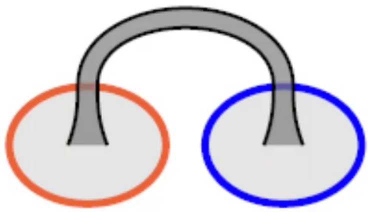
A lamp post in the dark: This suggests that non-perturbative – but still geometric! -- effects in the gravitational path integral are responsible for getting information out of black holes.



Our goal: Tear open the above calculation to understand better how it works, and what it means.

At least within a simple model, we will indeed be able to do so.

Spacetime (Euclidean) Wormholes

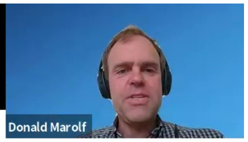


- Connected geometries with a disconnected boundary (need not be a solution!)
- Critical role in recent study of JT-gravity/random matrix dualities by Stanford group (including features related to BH info)
- Also played a prominent role in West Coast BH info paper.
- Ensemble story from 1980’s (Coleman, Giddings & Strominger, etc.)

Our work returns to such issues, addressing them in general and in a simple exactly-solvable model.

Motto: *Do the gravitational path integral.* Sum over all topologies (including disconnected spacetimes)!

This talk: The simple, solvable model with brief comments about general arguments.



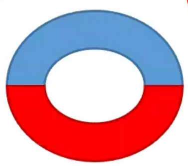
Background for model:

Intended to model Euclidean 2d asymptotically-AdS gravity (i.e., a JT-like system, perhaps with EOW branes)

Reduce to topology only; remove metric from model.

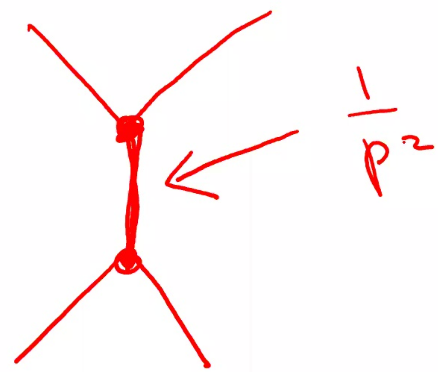
Take our grav path integral to compute the inner product.

This is precisely the way one treats a field theory path integral, but is *not* the way one treats a “worldline path integral” associated with Feynmann diagrams.



$$= < blue|red >$$

NOT a “Green’s function”



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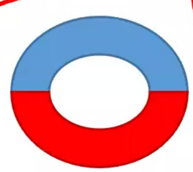

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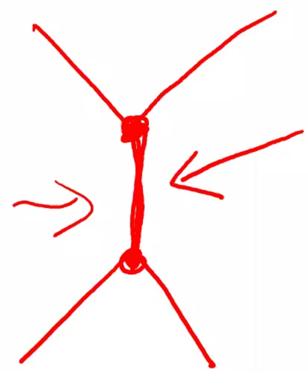

$$\frac{1}{p^2+m^2}$$

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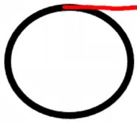
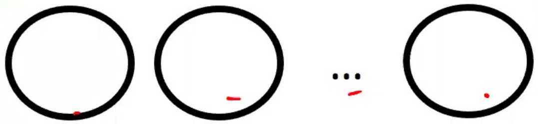
Path Integral: Boundary conditions → $\mathbb{C}$

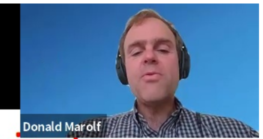
Example:  Z

$$n\ circles = Z^n$$

Simplest "Z-only" Model allowed BCs: 

Define PI by summing e^{-S} over *all* 2-manifolds with n boundaries with topological action

$$S \sim -S_0\chi = -\frac{S_0}{2\pi}\left(\int_M R + \int_{\partial M} K\right) = -S_0\left(\sum_{components}(2-2g) - n\right)$$



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Fine Points:

$$\int \mathcal{D}\Phi\, e^{-S[\Phi]} := \sum_{\text{Surfaces } M} \mu(M)\, e^{-S[M]}, \qquad \mu(M) = \frac{1}{\prod_g m_g!}.$$

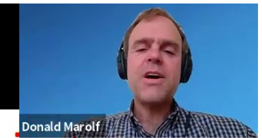
where M has components of genus g.

Also, choose

$$S = -S_0\tilde{\chi} = -S_0(\chi + \underline{n}) = -S_0\left(\sum_{components} (2-2g)\right)$$

no n

Could have given n-term an arbitrary coefficient S_∂, but take $S_\partial = S_0$ for simplicity and later convenience. Corresponds to coupling the more naïve model with an additional "boundary sector" (and perhaps integrating out the boundary sector). Note: effectively rescales Z by e^{S_∂}.



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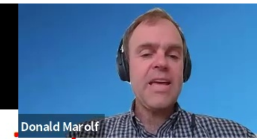

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Z^5

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$$-(S_0\chi + S_\partial n)$$

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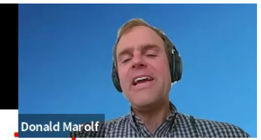

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$Z = 0$

where M has components of genus g.

$\langle Z \rangle = PI[0]$

$\langle Z^5 \rangle = PI[00000] = \langle Z \rangle^5 = (PI[0])^5$

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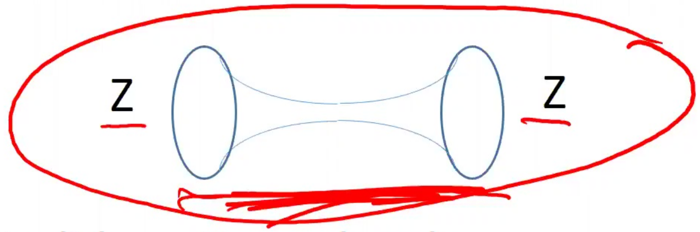
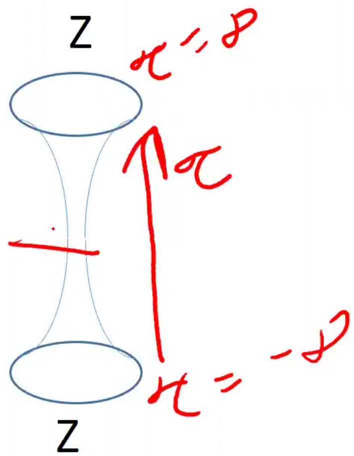
What physics will we study?

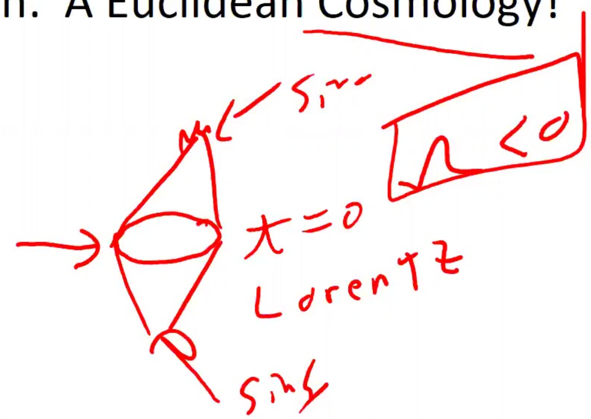
As previously advertised, we will be interested in Euclidean wormholes:

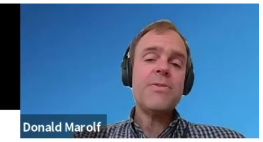


$(\langle Z \rangle)^2 \neq \langle Z^2 \rangle$

Alternate Interpretation: A Euclidean Cosmology!







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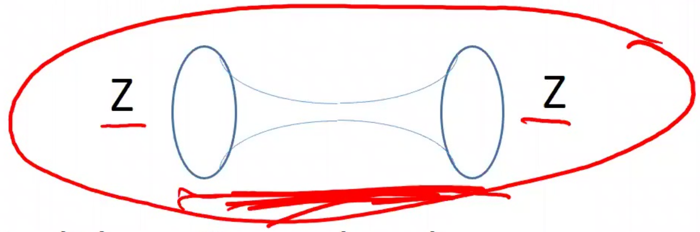

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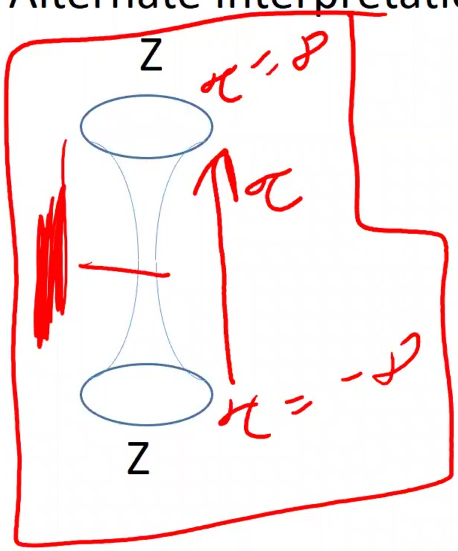
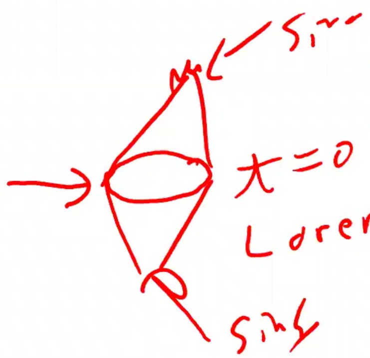
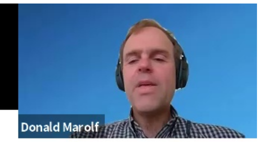




$\boxed{\Lambda < 0}$



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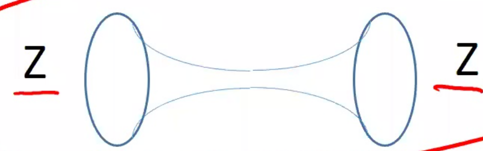

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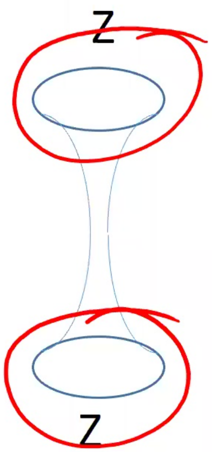

So Z is also a "source" that creates a closed universe (aka "baby universe") state.

Suggests that we define more general states $|Z^n > \in H_{BU}$ as whatever is created by the source Z^n for $n \geq 0$; i.e., include $|HH> = |Z^0> = |1>$.

We will then use our path integral to compute

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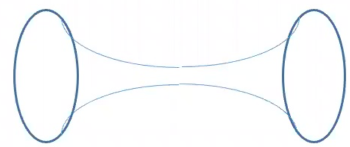
Z  Z

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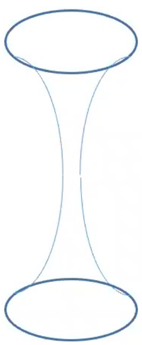

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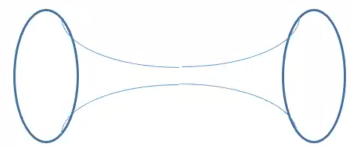
Z  Z

$$\overbrace{0\,0\,0\,\cdots\,0\,0}^{n}$$

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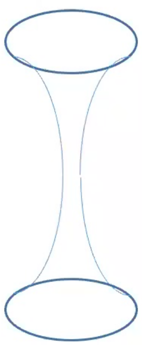

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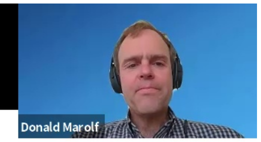

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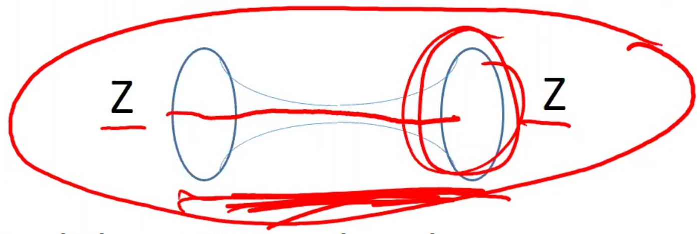

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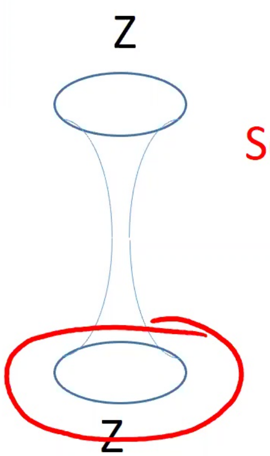

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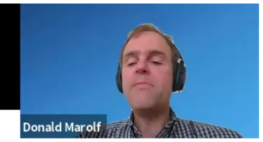

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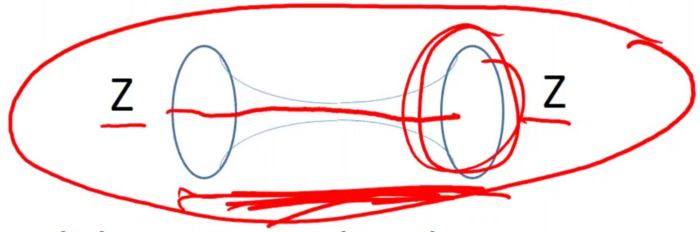

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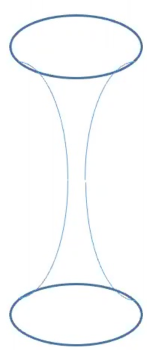

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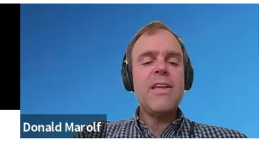
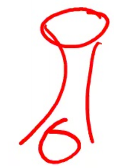
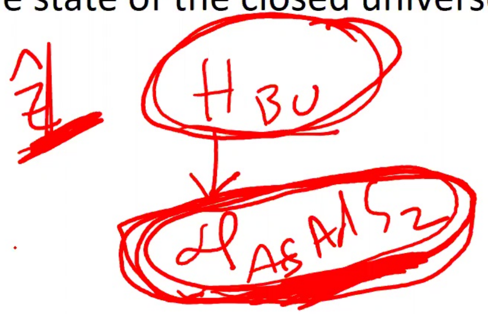


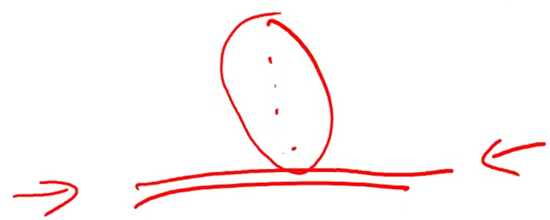
What physics does the operator $\hat{Z}$ encode? (heuristics)

Well, in more familiar contexts the path integral with a single asymptotically AdS2 circular boundary would be interpreted as an AdS2 partition function $Tr\, e^{-\beta H}$. Here the theory is topological, and H=0 (nothing depends on boundary length), so might expect Z ~ dimension of Hilbert space.



Here we find that the above BC gives $\langle Z \rangle = < HH|\hat{Z}|HH >$. So it is tempting to think that 1) there is a new class of states in the theory which are asymptotically AdS2 in the usual sense (i.e., where space is infinite but approaches the asymptotically AdS2 boundary) and 2) the dimension of the associated Hilbert space is somehow not a fixed number, but is somehow an operator $\hat{Z}$ on the Hilbert space of closed universes. I.e., that the dimension of the dual CFT Hilbert space depends on the state of the closed universes.





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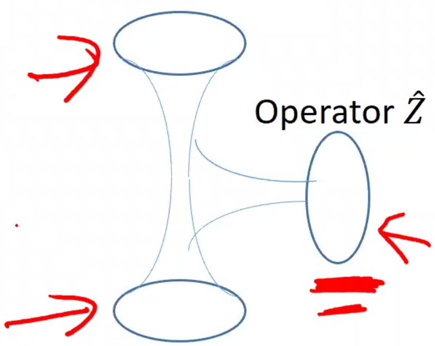
$\hat{Z} \sim H_{BU}$

$\hat{Z}^5 = (\hat{Z})^5$

In particular, we can get different expectation values for $\hat{Z}$ when we turn on sources for the BU states.

$O = (HH)$

We will see below that this is the case, and what this all means.



$\boxed{<1>=<Z^0> = e^{\lambda}}$

Now introduce boundaries and compute $< Z^n >$ via the generating function $< e^{uZ} >$

Note: For <1>, we found that the full sum over disconnected diagrams is the exponential of the sum over connected diagrams. This is a familiar result from e.g. bosonic QFT. And indeed, it happened there because the symmetry factors are identical to those in bosonic QFT. So this result is general, and we also find:

$$\log \left\langle e^{uZ} \right\rangle = \sum_{n=0}^{\infty} \sum_{\substack{\text{Connected } M \\ n \text{ boundaries}}} \frac{u^n}{n!} e^{S_0 \tilde{\chi}(M)}.$$

$$\log \left\langle e^{uZ} \right\rangle = \left(\sum_{g=0}^{\infty} e^{S_0 \chi(\tilde{M})} \right) \left(\sum_{n=0}^{\infty} \frac{u^n}{n!} \right) = \lambda e^u.$$

$\frac{\langle Z^n \rangle}{\langle 1 \rangle} = \sum_{d=0}^{\infty} d^n p_d(\lambda),$ w/ $p_d(\lambda) = e^{-\lambda} \frac{\lambda^d}{d!}$

Poisson distribution with mean λ!
Supported on positive integers!

Suggests that $\hat{Z}$ is a self-adjoint operator with spectrum {0,1,2,....}

And it is! Fine points below.

$$\frac{\langle Z^n \rangle}{\langle 1 \rangle} = \sum_{d=0}^{\infty} d^n p_d(\lambda), \qquad \frac{\langle f(Z) \rangle}{\langle 1 \rangle} = \sum_{d=0}^{\infty} f(d) p_d(\lambda)$$

1) Note: We defined some states $|Z^n>$. But it remains to take linear superpositions and then to also complete the Hilbert space. Doing so can create a space $\mathscr{N}$ of null states

$$|N> = \sum_{n=0}^{\infty} c_n |Z^n> \quad \text{w/} \quad <\phi|N> = 0 \text{ for all } |\phi>.$$

2) The space $\mathscr{N}$ is not empty! E.g., $|\sin(\pi Z)\rangle = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!} |Z^{2n+1}\rangle$

$$<Z^n|\sin(\pi Z)> = <Z^n \sin(\pi Z)> = \sum_{d=0}^{\infty} d^n \sin(\pi d)\, p_d(\lambda) = 0$$

3) So we need to quotient by $\mathscr{N}$. Nevertheless, $\hat{Z}$ really is well-defined on the quotient (i.e., it preserves $\mathscr{N}$).

Proof: We can show that $\hat{Z}|N> \in \mathscr{N}$ since

$$<\phi|(\hat{Z}|N>) = (<\phi|\hat{Z})|N> = 0$$

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So, $\widehat{Z}$ is a well-defined operator. Self-adjoint? Spectrum?

Recall: $|\sin(\pi Z) > = 0$. Similarly, for any function f that vanishes at non-negative integers,

$$|f(Z) > := \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} |Z^n > = 0.$$

So equivalence classes of states $|f(Z) >$ are defined by the values of f at non-negative integers. Indeed, we can introduce a basis $|Z = d>$ for the non-trivial states by choosing functions that vanish at all non-negative integers except some one value d; e.g.,

$$\left|Z = d\right\rangle = \left(\frac{\lambda^d}{d!}\right)^{-1/2} \left|\frac{\sin(\pi Z)}{\pi(Z - d)}\right\rangle \qquad \hat{Z}|Z = d > \; = \; d|Z = d >$$

Here the coefficient is for normalization: $\left\langle Z = d' \middle| Z = d \right\rangle = \delta_{dd'}$

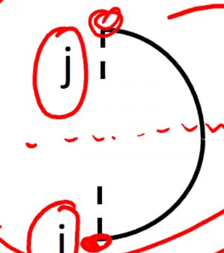
We have thus diagonalized $\widehat{Z}$ and found its full spectrum (non-negative integers, consistent with interpretation as the dimension of some [dual] Hilbert space).

Summary thus far

- Euclidean wormholes lead to a baby universe Hilbert space with arbitrary numbers of (perhaps) disconnected closed universe.
- H_{BU} has a non-trivial inner product and its construction involves a quotient by a vast null space.
- Partition-function-like quantities (Z) that one expects to be associated with asymptotically AdS states (i.e., NOT with closed universes) define operators on H_{BU} with a spectrum that must be computed. In our simple model (with $S_\partial = S_0$), where $\widehat{Z}$ should be the dimension of a dual CFT Hilbert space, the spectrum is non-negative integers.
- However, the model thus far is too trivial to actually create interesting asymptotically AdS states. Need more allowed BCs!

EOW branes

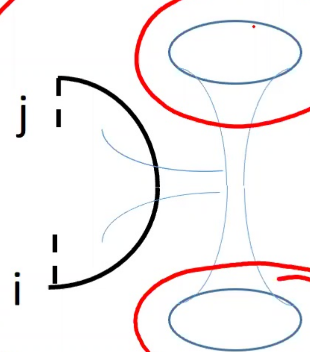
Introduce new BCs with sources for EOW branes

Asymptotically Euclidean AdS2 BCs: 

$\sim Z$

In a dual CFT, this would be an inner product. So write it as (ψ_i, ψ_i). Cutting open such path integrals will define new asymptotically AdS2 states w/ EOW branes.

But note: When cutting open the PI, there is implicitly a choice of baby universe state as well!



I.e., this inner product depends on BU state.

Like $\widehat{Z}$, the BC (ψ_j, ψ_i) defines an operator $\widehat{(\psi_j, \psi_i)}$ on H_{BU} – now also with EOW branes!

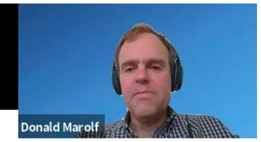
Results

- Story for $\widehat{Z}$ is unchanged. With general S_∂, find $e^{S_\partial - S_0}$ times non-negative integers.
- However, if any eigenvalue of $\widehat{Z}$ is *not* a non-negative integer, the Hilbert space IP fails to be positive definite on that sector. So only non-negative integers are allowed.
- The H_{BU} operators $\widehat{Z}$, $(\widehat{\psi_j, \psi_i})$ all commute with each other. The same is true for matrix elements of operators on the asympt. AdS2 states. Simultaneous eigenstates on H_{BU} define superselection sectors for asympt AdS2 op algebra in full theory.
- In sector with $Z = d$, find "random" inner product on asympt AdS2 states with

$$\text{Rank}\,(\psi_j, \psi_i) \leq d$$

- I.e., if many flavors k of EOW branes, IP is massively degenerate for $d \ll k$.
- This actually follows from a general argument, independent of the details of the particular model.

This large null space (gauge invariance) seems to be driving the solution of the BH info problem.
Bndy Renyis S_n are like PI BCs like Z and become operators on H_{BU}.
In our model (and also more generally) $\langle \widehat{S_n} \rangle = \langle HH | \widehat{S_n} | HH \rangle$ computes an average over BU-sectors, and S_n is bounded by the Rank in each sector.



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$\sum |\psi_i\rangle\langle\psi_i|$

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AdS_3/CFT_2

$\mathcal{N}=4$

$d=4$

AdS_5/SYM

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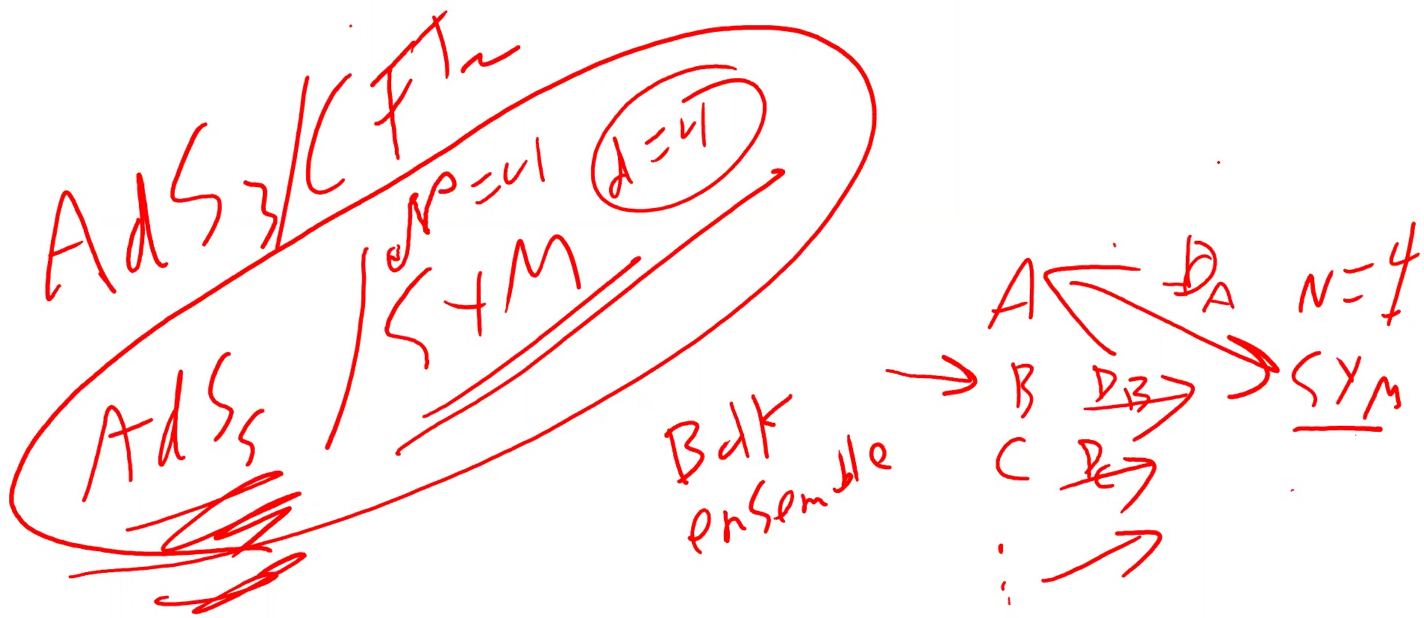

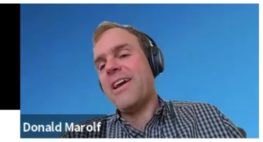

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