Title: Resolving the H_0 tension with diffusion

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Resolving the H_0 tension with diffusion

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Perez, Sudarsky and WE, arXiv:2001.07536 [astro-ph.CO]

Perimeter Institute Seminar

Motivation

Are there microscopic geometric degrees of freedom?

The existence of atoms and molecules can be inferred by a number of different physical phenomena; one example being Brownian motion.

Another well known example is diffusion: although energy is not lost, it is transferred from macroscopic objects to microscopic degrees of freedom.

If there are microscopic geometric degrees of freedom, would it be possible for energy to be transferred to them from matter fields? And if this is possible, how would this transfer occur? Would there be any observational consequences?

My goal in this talk is to explore this possibility, within a classical framework that allows for diffusion to be present.

Diffusion in Gravity

In general relativity, there cannot be any diffusion from matter to geometric degrees of freedom since

$$\nabla^a T_{ab} = 0. \tag{1}$$

Of course, it is possible to transfer energy between different matter fields, for example

$$\nabla^a T_{ab}^{(\phi)} \neq 0, \qquad \nabla^a T_{ab}^{(\chi)} \neq 0, \tag{2}$$

but the total stress energy tensor is always conserved. In this example,

$$\nabla^a \left(T_{ab}^{(\phi)} + T_{ab}^{(\chi)} \right) = 0. \tag{3}$$

However, there is a theory for gravity that allows for diffusion with $\nabla^a T_{ab}^{(tot)} \neq 0$: unimodular gravity.

Unimodular Gravity

There is a long history to unimodular gravity, but I will focus on its formulation as the trace-free version of the Einstein equations:

$$R_{ab} - \frac{1}{4} R\, g_{ab} = 8\pi G \left(T_{ab} - \frac{1}{4} T\, g_{ab} \right). \qquad (4)$$

An important property of unimodular gravity is that in this theory vacuum energy does not couple to the metric. For a perfect fluid with energy density ρ, pressure p and 4-velocity u^a,

$$T_{ab} = (\rho + p) u_a u_b + p\, g_{ab}, \qquad (5)$$

$$\Rightarrow \quad T_{ab} - \frac{1}{4} T\, g_{ab} = (\rho + p) \left(u_a u_b + \frac{1}{4} g_{ab} \right), \qquad (6)$$

which vanishes for the vacuum energy case of $p = -\rho$. [Weinberg, 1989; Ellis, van Elst, Murugan, Uzan, 2011]

Diffusion

While it is possible to add the condition $\nabla^a T_{ab} = 0$ to the matter fields (and this is commonly done), this condition is not necessary. In full generality,

$$\nabla^b T_{ab} = \frac{1}{8\pi G} J_a, \tag{7}$$

with J_a not necessarily vanishing.

However, J_a is not entirely free: the symmetries of unimodular gravity imply that $\mathrm{d}J = 0.$ [Josset, Perez, Sudarsky, 2017]

This will allow us to study the phenomenology of diffusion in a classical framework. The question now is: how can this diffusion affect the physics? To understand this, it is helpful to rewrite the trace-free Einstein equations in a more familiar way.

Rewriting the Trace-free Einstein Equations

Taking the divergence of the trace-free Einstein equations and using the Bianchi identities $\nabla^a G_{ab} = 0$,

$$\frac{1}{4}\nabla_a \left(R + 8\pi G\, T g_{ab}\right) = 8\pi G \nabla^b T_{ab} = J_a. \tag{8}$$

This can be integrated and used to simplify the trace-free Einstein equations to [Josset, Perez, Sudarsky, 2017]

$$R_{ab} - \frac{1}{2}R\, g_{ab} + \left(\Lambda_0 + \int_\ell J\right) g_{ab} = 8\pi G\, T_{ab}. \tag{9}$$

Here the integral is evaluated along a path ℓ starting from some given initial point, and Λ_0 is a constant of integration. Note that the condition $\mathrm{d}J = 0$ implies that $\int_\ell J$ is independent of the path ℓ so $\Lambda_0 + \int_\ell J = \Lambda(x^a)$.

These have the form of the usual Einstein equations, except with a cosmological 'constant' $\Lambda(x^a)$ that depends on (space-time) position.

The Cosmological Sector

To study the implications of diffusion in cosmology, let's consider the flat FLRW metric

$$ds^2 = -dt^2 + a^2(t) \, d\vec{x}^2. \tag{10}$$

The expansion rate of the universe is given by the Hubble rate $H = \dot{a}/a$, and due to the symmetries $J_a = J(t)(dt)_a$.

Then, the Friedman equation is

$$H^2 = \frac{8\pi G}{3} \sum_i \rho_i + \frac{\Lambda(t)}{3}, \tag{11}$$

with

$$\Lambda(t) = \Lambda_0 + \int_{t_i}^{t} J(\tilde{t}) \, d\tilde{t}. \tag{12}$$

In unimodular gravity, diffusion necessarily leads to a dynamical $\Lambda(t)$. Note that this is the only effect: diffusion does not modify the equations of motion in any other way.

Effects in Cosmology

The specific functional form of $\Lambda(t)$ is determined by the rate of diffusion, and so must be fixed by some input from fundamental physics. For now, let's simply assume that $\Lambda(t)$ is increasing with time, following the argument that diffusion is a transfer of energy from matter degrees of freedom to microscopic geometric degrees of freedom.

If diffusion generates an evolving $\Lambda(t)$, what possible effects could we look for?

Since this would lead to departures from the standard GR-ΛCDM cosmological model, a promising approach is to consider apparent discrepancies between different observations that are hard to reconcile according to GR-ΛCDM.

One example is the Hubble tension.

Hubble Tension

H_0, the value of the Hubble rate today, has been measured very accurately in two different ways:

1. Supernovae observations with a distance ladder calibrated by Cepheid stars gives [Reid, Pesce, Riess, 2019]

$$H_0 = 73.5 \pm 1.4 \text{ km s}^{-1} \text{ Mpc}^{-1}, \tag{13}$$

2. CMB data $+$ GR-ΛCDM model gives [Planck Collaboration, 2018]

$$H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}. \tag{14}$$

These differences are statistically significant at the level of 4.2σ.

The recent calibration of the distance ladder for supernovae using the tip of the red giant branch gives $H_0 = 69.8 \pm 2.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which reduces the tension. [Freedman et al, 2019]

Measurement from Supernovae Observations

A rather direct measurement of H_0 can be obtained by studying the magnitude of type Ia supernovae and their redshift $z = (1 - a)/a$. Inferring the luminosity distance d_L of each supernova from its apparent and absolute magnitudes (i.e., assuming a fall-off in brightness of $1/d_L^2$) gives $d_L(z)$.

Setting $a(t_0) = 1$, for the Taylor series expansion of $a(t)$

$$a(t) = 1 + H_0 \Delta t - \frac{q_0}{2} H_0^2 (\Delta t)^2 + \frac{j_0}{6} H_0^3 (\Delta t)^3 + \ldots, \qquad (17)$$

a short calculation gives [Visser, 2004]

$$d_L = \frac{z}{H_0} \left[1 + \frac{1 - q_0}{2} z - \frac{1 - q_0 - 3q_0^2 + j_0}{6} z^2 + \ldots \right]. \qquad (18)$$

For observations at small z, the determination of H_0 is relatively insensitive to uncertainties in q_0, j_0, etc.

Measurement from Cosmic Microwave Background

The value for H_0 is inferred from CMB observations using a Boltzmann code. A simpler determination of H_0 from the CMB is based on the angular location of the acoustic peaks in the CMB,

$$\theta \approx \frac{r_{\mathrm{s}}}{R_{\mathrm{LS}}}. \tag{19}$$

The sound horizon is the distance waves can travel from the time of reheating to recombination,

$$r_{\mathrm{s}} = \int_{t_{reheating}}^{t_{CMB}} c_s \, \mathrm{d}t, \tag{20}$$

and the radius to the surface of last scattering is

$$R_{\mathrm{LS}} = \int_{t_{CMB}}^{t_0} \frac{\mathrm{d}t}{a(t)} = \int_{z_0=0}^{z_{\mathrm{CMB}}=1090} \frac{\mathrm{d}z}{H(z)}. \tag{21}$$

The Radius of Last Scattering

Note that the sound horizon r_s depends only on pre-CMB physics. In the following, we will assume that diffusion only becomes important at later times, and that r_s is not affected by any diffusion.

The radius of last scattering clearly depends on the dynamics of the universe through $H(z)$. For standard GR-ΛCDM cosmology,

$$H(z)^2 = \overline{H}_0^2 \left[\Omega_r^0(1+z)^4 + \Omega_m^0(1+z)^3 + \Omega_\Lambda^0 \right], \qquad \Omega_i^0 = \frac{\rho_i(t_0)}{\rho_{tot}(t_0)}. \quad (22)$$

But if diffusion generates a dynamical $\Lambda(t)$ this will modify $H(z)$, as compared to GR-ΛCDM cosmology, and will lead to a different inferred value for H_0.

Can this resolve the H_0 tension?

The Role of Diffusion

To determine whether diffusion can resolve the H_0 tension, we will take as inputs:

1. the value of H_0^{SN} measured by supernovae observations,
2. the values of $\rho_i(t_{CMB})$ as inferred from CMB observations,
3. the equations of motion including diffusion effects that give $\Lambda(t)$.

From the 2nd and 3rd points, we can infer $H_0^{CMB+diff}$ and then compare this with H_0^{SN}.

Is it possible to obtain a value for $H_0^{CMB+diff}$ that is in agreement with H_0^{SN} ?

Recall that some input from fundamental physics is required to determine $\Lambda(t)$. Here are two simple phenomenological models.

Example 1: Sudden Transfer

In the first phenomenological model, diffusion occurs suddenly:

$$\Lambda(z) = \Lambda_\infty + (\Delta\Lambda)\,\theta_-(z - z^\star). \tag{23}$$

In calculations it is convenient to use the quantity

$$\alpha = \frac{\Omega_m^0 H_0^2}{\overline{\Omega}_m^0 \overline{H}_0^2}, \tag{24}$$

and $1 - \alpha$ gives the fraction of energy lost to diffusion. Here Ω_m^0 and H_0 are the values inferred from the CMB including diffusion effects, while $\overline{\Omega}_m^0$ and $\overline{H}_0$ are the values inferred for GR-ΛCDM.
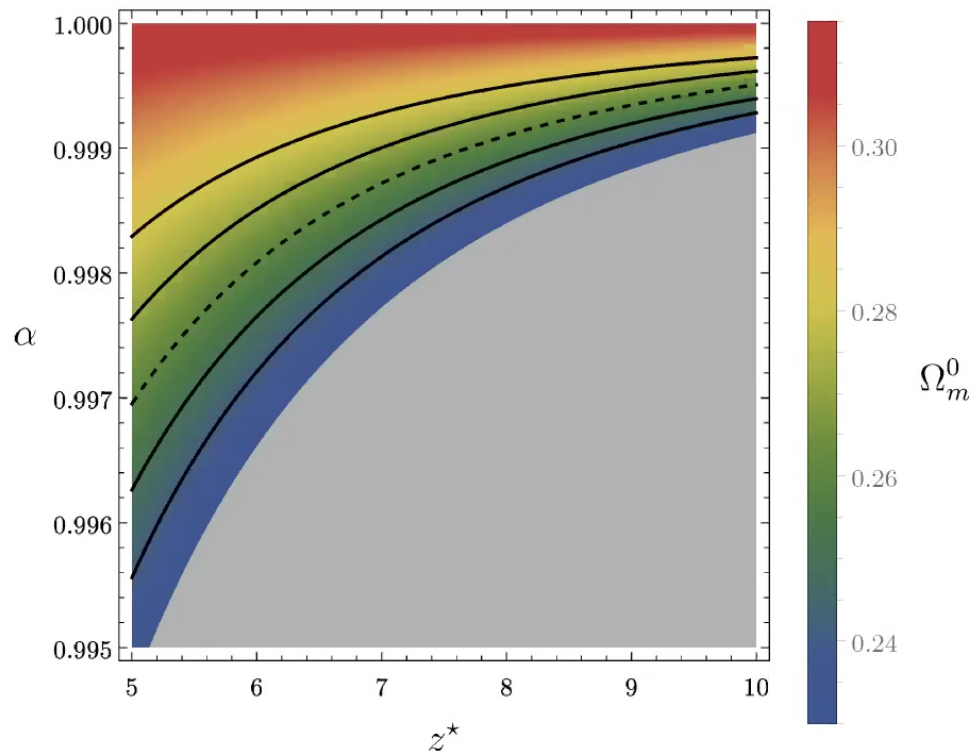
In terms of these quantities, a short calculation gives

$$\Lambda(z) = \Lambda_\infty + 3(1 - \alpha)(1 + z^\star)^3\,\overline{\Omega}_m^0 \overline{H}_0^2\,\theta_-(z - z^\star). \tag{25}$$

There are 3 free parameters: $\Lambda_\infty, \alpha, z^\star$.

Sudden Transfer Results 1

For Λ_∞ set to the preferred value in GR-ΛCDM, this plot shows for what region of parameter space the Hubble tension is resolved: $H_0^{CMB+diff} = H_0^{SN}$ on the dashed line.

In the best fit region where $H_0^{CMB+diff} \sim H_0^{SN}$, we find $\Omega_m^0 \sim 0.26$.

Sudden Transfer Results 2

For Λ_∞ set to the preferred value in GR-ΛCDM, this plot shows for what region of parameter space the Hubble tension is resolved: $H_0^{CMB+diff} = H_0^{SN}$ on the dashed line.
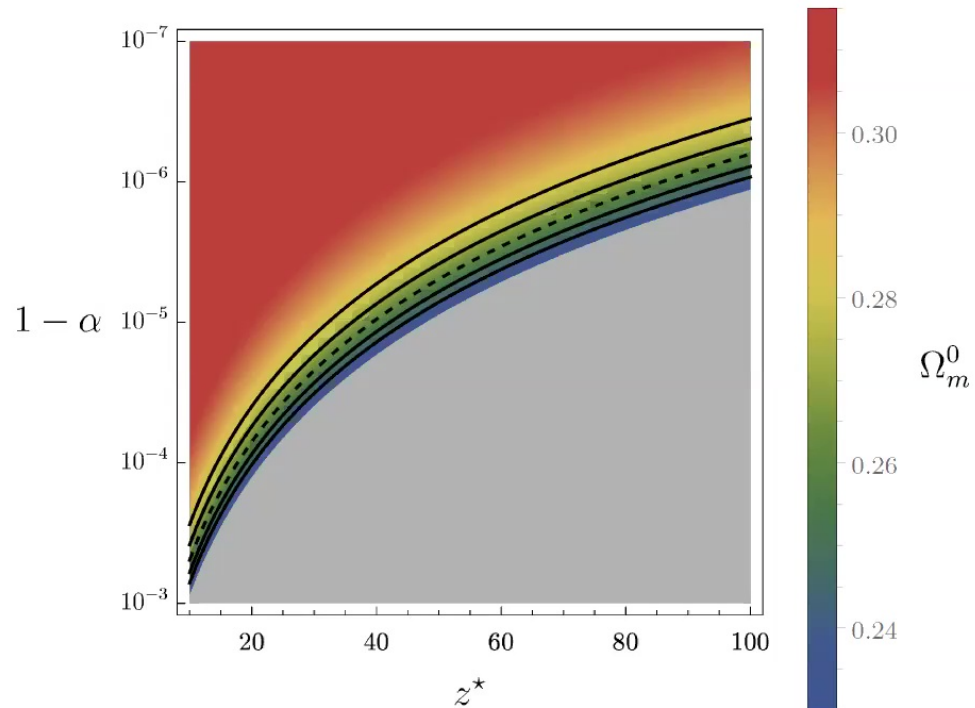
In the best fit region where $H_0^{CMB+diff} \sim H_0^{SN}$, we find $\Omega_m^0 \sim 0.26$.

Example 2: Anomalous Decay of Dark Matter

In the second phenomenological model, we assume that for $z < z^\star$, dark matter decays in an anomalous fashion in an expanding universe:

$$\rho_m \propto \frac{1}{a^{3+\gamma}}, \tag{26}$$

rather than $\rho_m \propto a^{-3}$. Then, again for $z < z^\star$,

$$\Lambda(t) = \Lambda_\infty + A\left((z^\star + 1)^{3+\gamma} - (z + 1)^{3+\gamma} \right), \tag{27}$$

where A is determined by γ and $z^\star$. Note that it is possible to set $z^\star = z_{CMB} = 1090$ in order for diffusion to occur continuously from recombination on.

In this model, $\Lambda(t)$ is continuous. Again, there are three parameters: $\Lambda_\infty, \gamma, z^\star$.

Anomalous Decay Results

For Λ_∞ set to the preferred value in GR-ΛCDM, this plot shows for what region of parameter space the Hubble tension is resolved: $H_0^{CMB+diff} = H_0^{SN}$ on the dashed line.
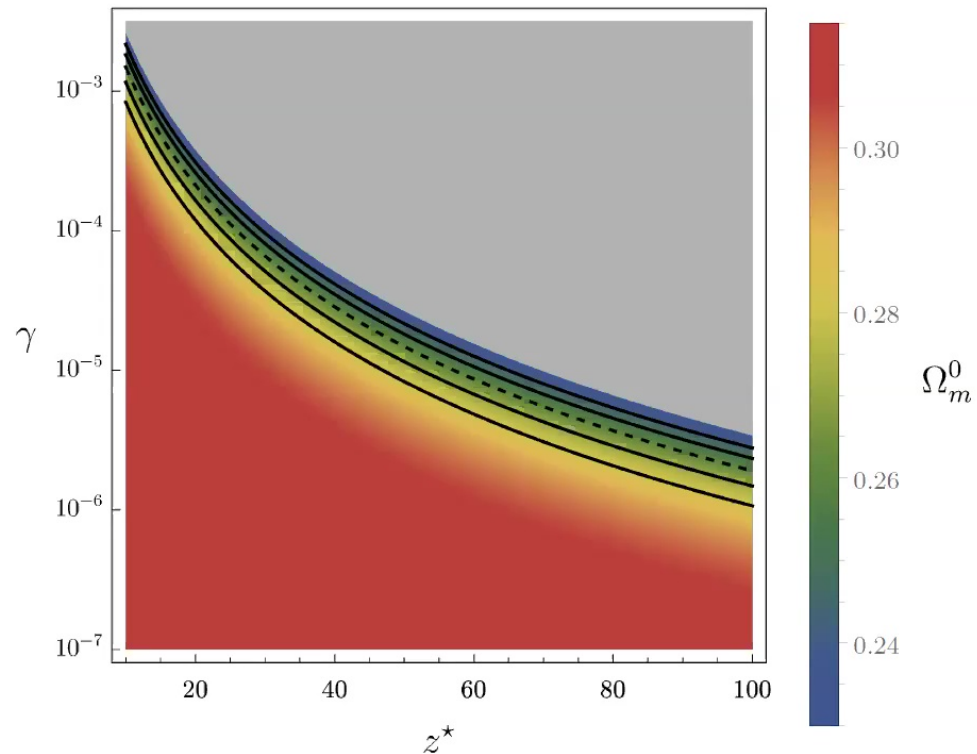
In the best fit region where $H_0^{CMB+diff} \sim H_0^{SN}$, we find $\Omega_m^0 \sim 0.26$.

Discussion

Summary:

- Diffusion effects can be included in unimodular gravity, they generate $\Lambda(t)$.
- As explored in two phenomenological models, for some $\Lambda(t)$ it is possible to resolve the H_0 tension.

Future Work:

- The specific form of $\Lambda(t)$ depends on fundamental physics. The next step is to develop specific models motivated by fundamental physics, determine the resulting $\Lambda(t)$, and extract the resulting physics. Are there specific realizations that can resolve the H_0 tension? [Bjorken, Perez, Sudarsky, 2018; Perez, Sudarsky, 2019]
- It is important to use a Boltzmann code to obtain a more accurate constraint on these models.

Thank you for your attention!