

Title: Petz map and Python's lunch

Speakers: Ying Zhao

Series: Quantum Fields and Strings

Date: April 14, 2020 - 2:30 PM

URL: <http://pirsa.org/20040085>

Abstract: We look at the interior operator reconstruction from the point of view of Petz map and study its complexity. We show that Petz maps can be written as precursors under the condition of perfect recovery. When we have the entire boundary system its complexity is related to the volume / action of the wormhole from the bulk operator to the boundary. When we only have access to part of the system, Python's lunch appears and its restricted complexity depends exponentially on the size of the subsystem one loses access to.



Petz map and Python's lunch

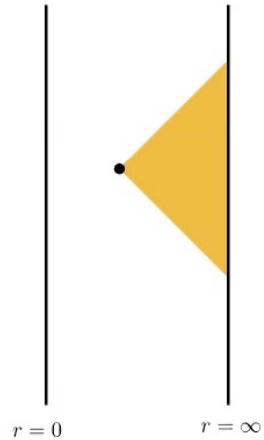
[Ying Zhao arXiv:2003.03406](#)

Operator reconstruction inside causal wedge



- HKLL reconstruction of bulk operator

A. Hamilton, D. Kabat, G. Lifschytz, D. Lowe [arXiv:hep-th/0606141](https://arxiv.org/abs/hep-th/0606141)

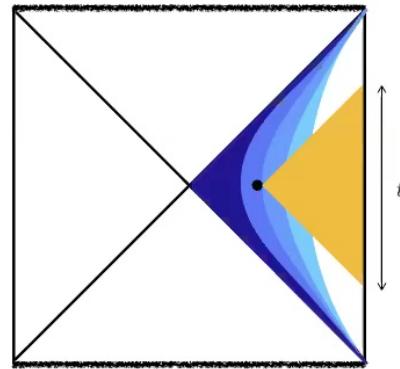
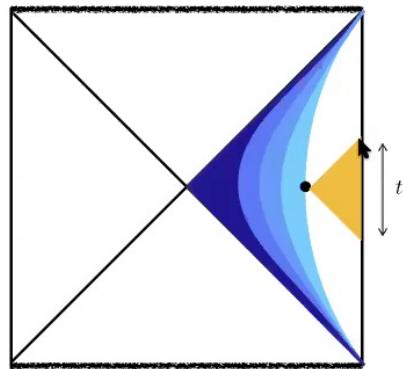


Complexity of a bulk operator

L. Susskind arXiv:1402.5674v2



- Complexity of an operator: The number of simple gates needed to apply this operator
- Layered structure

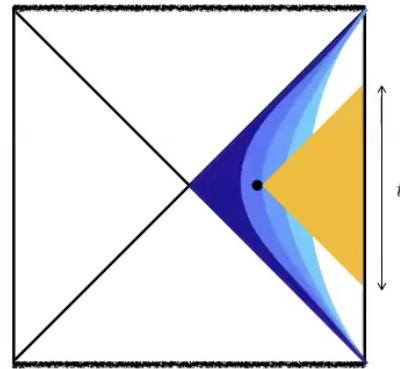
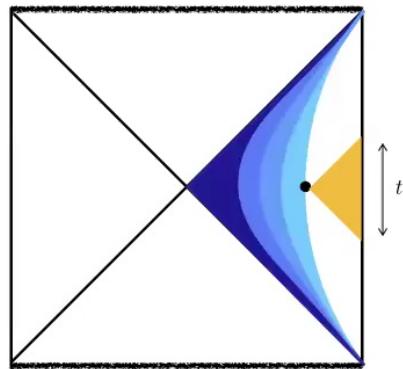


$$t \sim \frac{\beta}{2\pi} \log\left(\frac{A}{\Delta A}\right)$$

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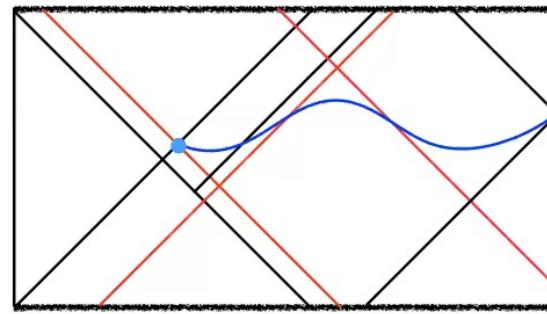
$$t \sim \frac{\beta}{2\pi} \log\left(\frac{A}{\Delta A}\right)$$

- Complexity blows up as one approaches horizon



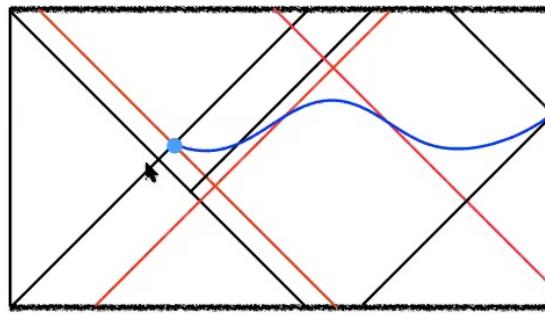
Operator reconstruction in the interior

- Operator reconstruction beyond the causal wedge is still possible
JLMS, Quantum error correction, Petz map, Modular flow
- The minimal circuit preparing the state is stored in the interior



Operator reconstruction in the interior

- Operator reconstruction beyond the causal wedge is still possible
JLMS, Quantum error correction, Petz map, Modular flow
- The minimal circuit preparing the state is stored in the interior



- To construct interior operators one needs to undo the horizontal circuit
- Operator deeper in the interior has larger complexity



Petz map

C. Chen, G. Penington, G. Salton arXiv:1902.02844

G. Penington, S. Shenker, D. Stanford, Z. Yang
arXiv:1911.11977

- $V : \mathcal{H}_{\text{code}} \rightarrow \mathcal{H}_{\text{CFT}}$
- The reconstruction of operator W on subsystem A is given by

$$W_A = \sigma_A^{-\frac{1}{2}} \text{tr}_{\bar{A}}(VWV^\dagger) \sigma_A^{-\frac{1}{2}}$$

where $\sigma_A = \text{tr}_{\bar{A}} \Pi_{\text{code}}^{\uparrow} = \text{tr}_{\bar{A}}(VV^\dagger)$

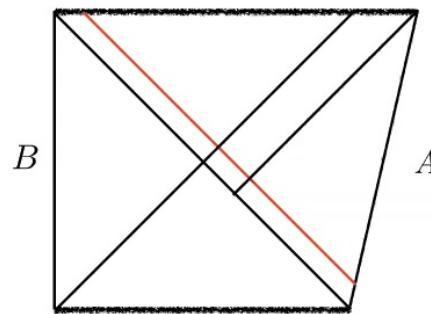


Perturbed thermofield double

- Thermofield double: $|\text{TFD}\rangle = \frac{1}{\sqrt{N}} \sum_I |I\rangle_A |I\rangle_B$

- Perturb thermofield double:

$$|\psi_0\rangle = \frac{1}{\sqrt{N}} \sum_I |a, I\rangle_A |I\rangle_B$$
$$|\psi_1\rangle = \frac{1}{\sqrt{N}} \sum_{I,i} |i\rangle_A U_{i,aI} |I\rangle_B$$



- Alice wants to flip the spin: $|a\rangle \rightarrow \sum_b |b\rangle W_{ba}$

$$V : \mathcal{H}_{\text{code}} \longrightarrow \mathcal{H}_A \times \mathcal{H}_B$$

$$|a\rangle \mapsto \frac{1}{\sqrt{N}} \sum_{I,i} |i\rangle_A U_{i,aI} |I\rangle_B$$





- Petz map: $W_A = UWU^\dagger$
- What is U ? We don't know U .

Knowledge of code subspace:

$$\text{tr}_B(V |a\rangle \langle b| V^\dagger) = \frac{1}{N} \sum_{i,j} |i\rangle_A \langle j| \sum_I U_{I,aI} U_{j,bI}^* \equiv M_{ab}$$

$$W_A = \sum_{a,b} W_{ab} M_{ab}$$



Pure state black hole



- Initial state: $|a, I\rangle$

Time evolution: $|\psi_1\rangle = \sum_{i,\alpha} |i\rangle_A |\alpha\rangle_{\bar{A}} U_{i\alpha,aI}$

We want to apply $W = \sigma_x$ on the spin.

- $V : \mathcal{H}_{\text{code}} \rightarrow \mathcal{H}$

$$|a\rangle \mapsto \sum_{i,\alpha} |i, \alpha\rangle U_{i\alpha,aI} \equiv |e_{aI}\rangle$$

Knowledge of code subspace:

$$\text{tr}_{\bar{A}}(V |a\rangle \langle b| V^\dagger) = \sum_{\alpha} \sum_{i,j} |i\rangle_A \langle j| U_{i\alpha,aI} U_{j\alpha,bI}^*$$

Keep the entire system

- Petz map: $W_A = \sigma_A^{-\frac{1}{2}} \text{tr}_{\bar{A}}(VWV^\dagger) \sigma_A^{-\frac{1}{2}}$
- $(VWV^\dagger)_{ij} = \sum_{a,b} U_{i,aI} W_{ab} U_{j,bI}^*$ Not UWU^\dagger
 $VWV^\dagger = \sum_{a,b} |e_{aI}\rangle W_{ab} \langle e_{bI}|$ No summation on I
 $\Pi_{\text{code}} = VV^\dagger = \sum_a |e_{aI}\rangle \langle e_{aI}|$ No summation on I
- $UWU^\dagger = \sum_{a,b,J} |e_{aJ}\rangle W_{ab} \langle e_{bJ}|$
 $(\Pi_{\text{code}})^{\frac{1}{2}} (UWU^\dagger) (\Pi_{\text{code}})^{\frac{1}{2}} = VWV^\dagger$
 $W_A = (\Pi_{\text{code}})^{-\frac{1}{2}} (VWV^\dagger) (\Pi_{\text{code}})^{-\frac{1}{2}} = UWU^\dagger$



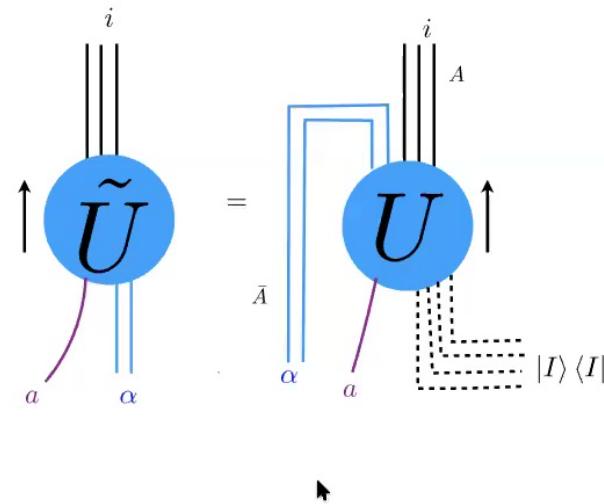
Tracing out part of the system

$$\begin{array}{c} \bar{A} \\ | \\ a \end{array} \quad \begin{array}{c} A \\ | \\ \alpha \end{array}$$
$$|a||\alpha| \leq |i|$$

P. Hayden, J. Preskill [arXiv:0708.4025v2](https://arxiv.org/abs/0708.4025v2)

$$W_A = \tilde{U} W \tilde{U}^\dagger$$

$$\tilde{U}_{i,a\alpha} \equiv d_{\bar{A}} U_{i\alpha,aI}$$



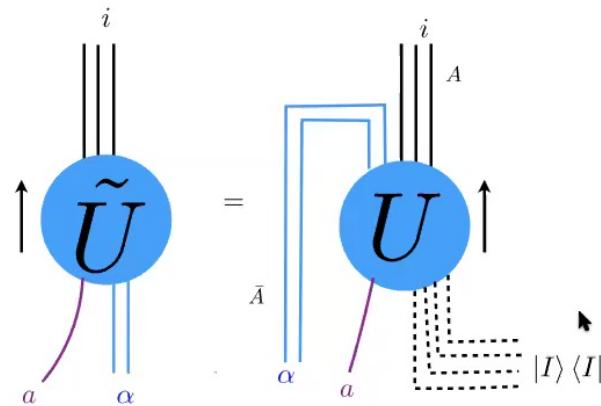
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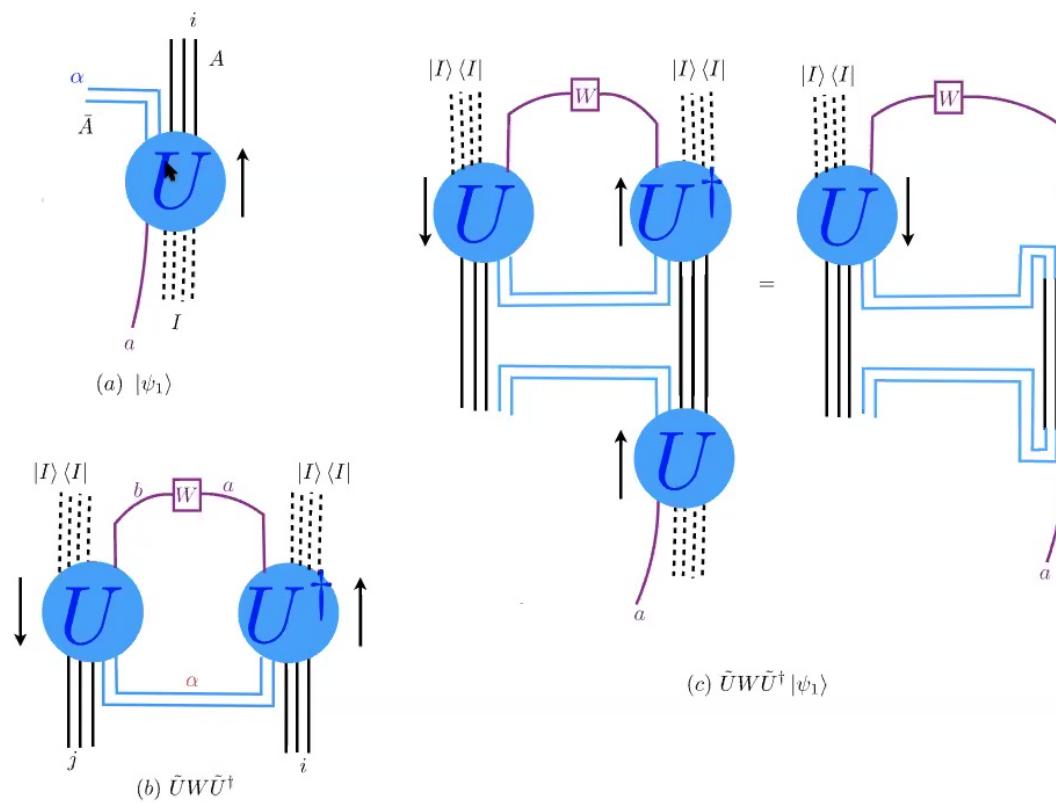
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The more qubits you lose, the more the construction depends on the initial state $|I\rangle$

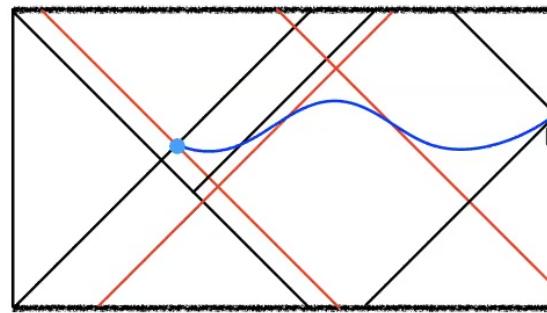




Complexity of Petz map

$$W_A = \tilde{U} W \tilde{U}^\dagger$$

- Keep the entire system, \tilde{U} is k-local



Complexity \sim volume / action from the operator to the boundary



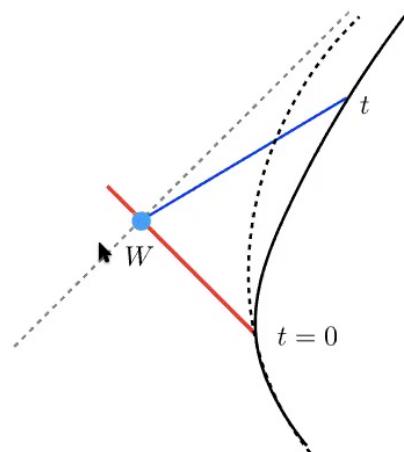
Complexity of Petz map and wormhole geometry



- SYK and JT gravity $\psi_1(t) = U(t)\psi_1U(t)^\dagger$

- $\mathcal{C}(\psi_1(t)) = 1 + 2\left(\frac{\beta\mathcal{J}}{\pi}\right)^2 \sinh^2\left(\frac{\pi}{\beta}t\right)$ $t < t^*$ [X. Qi, A. Streicher arXiv:1810.11958](#)

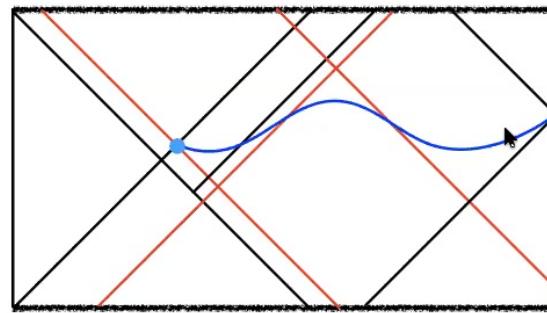
- $d(t) \approx \log\left(1 + \frac{2\delta S}{S - S_0} \sinh^2\left(\frac{\pi}{\beta}t\right)\right) + \text{const}$



Complexity of Petz map

$$W_A = \tilde{U} W \tilde{U}^\dagger$$

- Keep the entire system, \tilde{U} is k-local



Complexity \sim volume / action from the operator to the boundary



Lose part of the system: Python's lunch

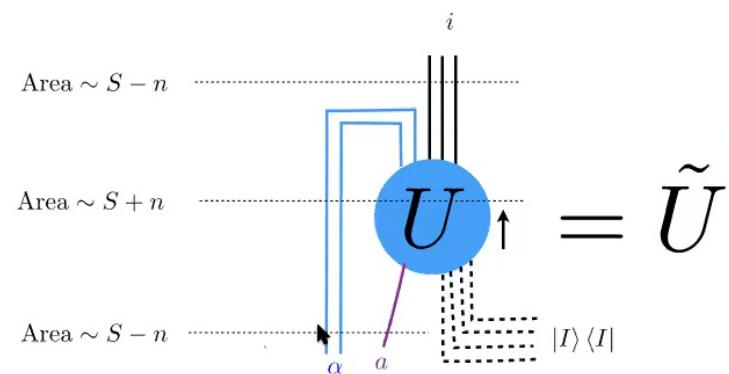
D. Harlow, P. Hayden [arXiv:1301.4504v4](#)

A. Brown, H. Gharibyan, G. Penington, L. Susskind [arXiv: 1912.00228](#)



- U is k-local, \tilde{U} is not

n : the number of qubits one loses access to



Lose part of the system: Python's lunch

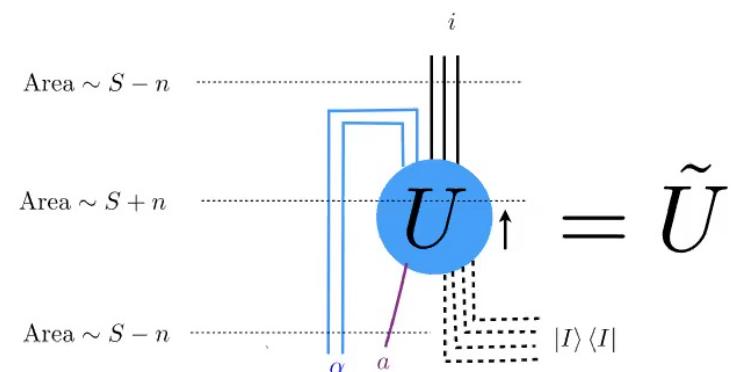
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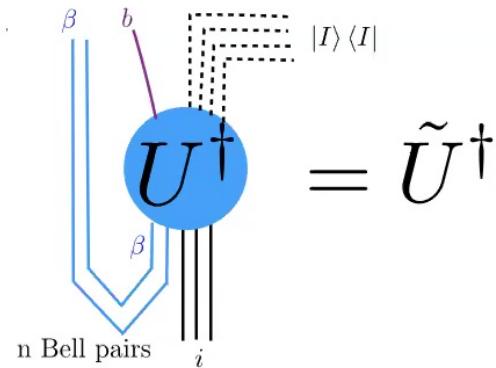
n : the number of qubits one loses access to



$$\mathcal{C}(\tilde{U}) \sim e^{\frac{1}{2}(S_{max} - S_{min})} \sim e^n$$



$$\tilde{U}^\dagger : |i\rangle \mapsto \sum_{b,\beta} |b,\beta\rangle U_{i\beta,bI}^* \quad \text{for any } i, \text{ fixed } I$$



- Allow projection, $\mathcal{C}(\tilde{U}^\dagger) \sim \mathcal{C}(U^\dagger)$
- No projection, $\mathcal{C}(\tilde{U}^\dagger) \sim e^{\frac{n}{2}} \mathcal{C}(U^\dagger)$

B. Yoshida, A. Kitaev arXiv: [arXiv:1710.03363v2](https://arxiv.org/abs/1710.03363v2)
A. Brown, H. Gharibyan, G. Penington, L. Susskind arXiv: [1912.00228](https://arxiv.org/abs/1912.00228)



Comparison with exterior operator reconstruction

- The complexity to figure out an operator
V.S.
The complexity to apply an operator
- Computational pseudorandomness: Can efficiently create ensembles of quantum states that are exponentially complex to distinguish from random

[A. Bouland, B. Fefferman, U. Vazirani arXiv: 1912.00228](#)

[L. Susskind arXiv: 2003.01807](#)

- Figuring out the horizontal circuit is hard



Thank you.