Title: Gravitational edge modes: From Kac-Moody charges to PoincarÃ© networks

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Abstract: In this talk I revisit the canonical framework for general relativity in its connection-frame field formulation, exploiting its local holographic nature. I will show how we can understand the Gauss law, the Bianchi identity and the space diffeomorphism constraints as conservation laws for local surface charges. These charges being respectively the electric flux, the dual magnetic flux and momentum charges. Quantization of the surface charge algebra can be done in terms of Kac-Moody edge modes. This leads to an enhanced theory upgrading spin networks to tube networks carrying Virasoro representations. Taking a finite dimensional truncation of this quantization yields states of quantum geometry, dubbed `PoincarÃ© charge networksâ€™, which carry a representation of the 3D diffeomorphism boundary charges on top of the SU(2) fluxes and gauge transformations. This opens the possibility to have for the first time a framework where spatial diffeomorphism are represented at the quantum level. Moreover, our construction leads naturally to the picture that the relevant geometrical degrees of freedom live on boundaries, that their dynamics and the fabric of quantum space itself is encoded into their entanglement, and it is designed to offer a new setting to study the coarse-graining of gravity both at the classical and the quantum levels.

Gravitational edge modes:
from Kac—Moody charges to Poincaré networks

Daniele Pranzetti

Based on work in collaboration with Laurent Freidel, Etera Livine and Alejandro Perez

- Phys. Rev. D 101, 024012 (2020), [gr-qc/1910.05642];
- Class. Quant. Grav. 36, no. 19, 195014 (2019), [hep-th/1906.07876];
- Phys. Rev. D 98, 116008 (2018), [hep-th/1806.03161];
- Phys. Rev. D 95, no. 10, 106002 (2017), [gr-qc/1611.03668].

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Central perspective

In our quest for quantum gravity, an essential task is to reach a proper understanding of the degrees of freedom and of the symmetries of gravity associated with local subregions.

★ Three interlaced insights:

- Local holographic behavior of general relativity: Black hole entropy

  -> The total number of microscopic DOF scale with the horizon area.

- Canonical gravity: General relativity is a constraint system

  -> The total energy associated to any space like region is entirely encoded in its boundary.

- Gravitational edge modes can be assigned to any boundary surface in space: Boundary symmetry algebra

  -> Gravitational edge modes as boundary symmetry algebra representation states. [Donnelly, Freidel, JHEP 2016]

The relevant geometrical DOF live on boundaries:
Their dynamics and the fabric of quantum space itself is encoded into their entanglement

This entanglement is derived from the fusion properties of the gravitational boundary symmetry algebra

A brief history of gravitational edge modes

It all started with trying to understand the nature of black hole thermodynamics and/or infinity…

- Introduction of new boundary DOF in order to restore the differentiability of the Hamiltonian constraints (canonical formalism): [Regge, Teitelboim, Annals Phys. 1974], [Carlip, PRD 1997, PRL 1999].
- Understanding of Kac-Moody symmetry as an edge mode symmetry in 3D Chern-Simons theory: [Elitzur, Moore, Schwimmer, Seiberg, NPB 1989], [Balachandran, Bimonte, Gupta, Stern, IJMPA 1992], [Banados, PRD 1996].
- Covariant formalism and "membrane paradigm" representation of black holes: [Brown, York, PRD 1993], [Thorne, Price, Macdonald, 1986].
- Black hole entropy in Quantum Gravity (counting a set of boundary Chern-Simons edge modes): [Smolin, J. Math. Phys. 1995], [Ashtekar, Baez, Krasnov, Adv. Theor. Math. Phys. 2000], [Engle, Noui, Perez, DP, PRD 2010].
- Deep connection with the renewed understanding of the meaning and importance of the asymptotic symmetry group and the corresponding soft modes in the study of the S-matrix: [Barnich, Troessaert, PRL 2015], [Strominger, 2017 and references therein], [Hawking, Perry, Strominger, PRL 2016, JHEP 2017], [Averin, Dvali, Gomez, Lüst, JHEP 2016, Mod.Phys.Lett. A 2016].
- Renewed interest in understanding the nature and dynamics of gravitational edge modes for finite boundaries: [Freidel, Perez, Universe 2018], [Freidel, Perez, DP, PRD 2017], [Geiller, NPB 2017], [Wieland, CQG 2017], [Camps, JHEP 2019], [Freidel, Livine, DP, CQG 2019, PRD 2020], [Asante, Dittrich, Girelli, Riello, Tsimiklis, 2019].
- 2D Jackiw-Teitelboim gravity and black hole states: [Blommaert, Mertens, Verschelde, JHEP 2019].

But what is an edge mode?

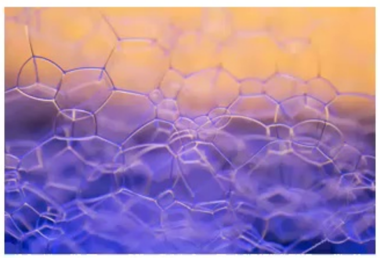
Well, edge modes are:

1. Conjugate momenta to the boundary charges;
2. Representation states for the boundary symmetry algebra;

Operator - State correspondence

From the perspective of canonical general relativity, the edge modes live on corners, that is the 2D boundary surfaces of 3D regions of space-like hypersurfaces. In the context of holography, they are supposed to reflect and represent the DOF of the 3D geometry.

Since classical spacetime is thought as a manifold, a.k.a. atlas of charts (union of bound sets), space can also be modeled as the union of 3D regions glued together through their interfaces.



Gravity = theory of edge modes living on the boundary of these patches of 3D geometry

Space = network of "bubbles"

Quantum gravity = Dynamical networks of quantum edge modes

Coarse-graining of gravity as fusion of boundary symmetry charges (see, e.g. [Dittrich et al.])

Our framework in a nutshell

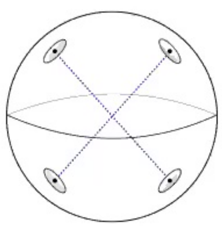
Two main ingredients:

1. Reformulation of the (kinematical) Hamiltonian constraints of general relativity as conservation laws for boundary charges on 2D surface (the corners);

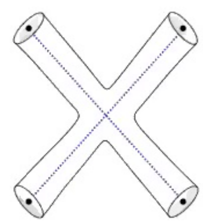
2. Introduction of distributional sources of curvature and torsion on the boundary surfaces and description of the structure and algebra of edge modes that they induce.

Started with the 'loop gravity string' framework introduced in [Freidel, Perez, DP, PRD 2017] and it resonates with ideas of [Wieland, Annales Henri Poincare 2017], [Asante, Dittrich, Girelli, Riello, Tsimiklis, 2019]

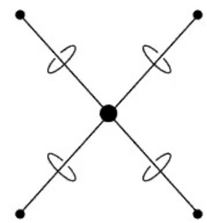
We aim to study the structure of kinematical edge modes living on the punctured 2D boundary surface



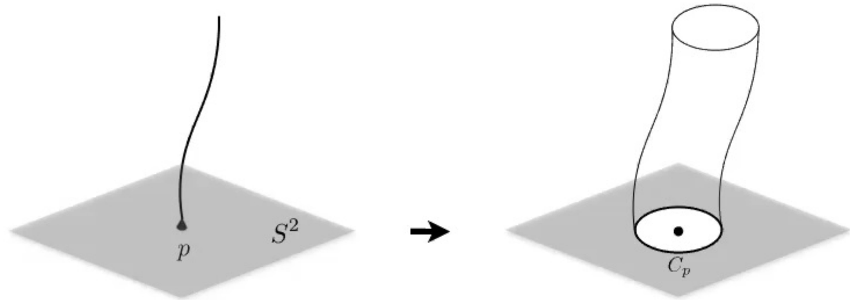
A 3D region with 3-ball topology and its boundary surface punctured by disks representing its interfaces with the neighboring 3D cells.



The punctures are sources of curvature and torsion on the boundary, assuming that the geometry is flat everywhere else. These defects propagate to tubes in the 3D bulk.



The 1-skeleton reduction of the tubular structure encoding the edge modes on the punctured surface.



Upgrade spin networks to tube networks in LQG:

- Representation of spin networks for a q-deformed SU(2) gauge group in terms of conformal blocks [Markopoulou, Smolin, PRD 1998] or in terms of the moduli space of flat SL(2, C) connections on the tube surfaces [Haggard, Han, Kaminski, Riello, Adv. Theor. Math. Phys. 2015].

- To take into account both SU(2) holonomies about and around graph edges as double spin networks [Charles, Livine, GRG 2017] or to use Drinfeld tubes [Dittrich, JHEP 2017], [Delcamp, Dittrich, JHEP 2018].

Loop gravity string framework

We derive the algebraic structures living on the tubular network directly from an analysis of the symmetries of general relativity in the presence of boundaries and we dress the surface networks with actual gravitational edge modes.

New observables (that cannot be defined in the standard LQG discretization):

- String vibration modes of the punctures, as derived in the 'loop gravity string' framework introduced in [Freidel, Perez, DP, PRD 2017]
- New momentum observable defining the boundary charge induced by the bulk invariance under 3D diffeomorphism

➡ Quantum states of geometry carrying a representation of the 3D diffeomorphisms of general relativity

The setting: First order gravity

Considering a space-like Cauchy hypersurface M in a 3+1 foliation of spacetime, we focus on a bounded 3D region $\mathcal{B}$ and its 2D boundary S:

* Time gauge: $dn^I = 0\,,\quad n = e^I n_I = e^0$ (normal to M)

Ashtekar-Barbero connection: $A^i = \Gamma^i + \gamma K^i\,,\quad \kappa = 8\pi G$

Flux 2-form (through the simplicity constraint): $\Sigma_i = \frac{1}{2\kappa\gamma}\epsilon_{ijk}\, e^j \wedge e^k := \frac{1}{2\kappa\gamma}(e\times e)_i$ (coframe fields tangent to M)

Symplectic 2-form: $\Omega = \Omega_{\mathcal{B}} + \Omega_S = \int_{\mathcal{B}}(\delta A^i \wedge \delta\Sigma_i) + \frac{1}{2\kappa\gamma}\int_S(\delta e_i \wedge \delta e^i)$

The extended phase space

Poisson brackets: $\{A^i_a(x), \Sigma^j_{bc}(y)\} = \kappa\gamma\delta^{ij}\epsilon_{abc}\delta^3(x-y)$ (bulk phase space) $\quad \{e^i_a(x), e^j_b(y)\} = \kappa\gamma\,\delta^{ij}\epsilon_{ab}\delta^2(x-y)$ (boundary phase space)

The bulk fields are given by an SU(2) valued flux 2-form Σ^i and an SU(2) valued connection A^i

- Kinematical constraints:
  - $\mathrm{d}_A\Sigma_i = 0,$ Gauss-law
  - $F^i \wedge \imath_\xi \Sigma_i = 0$ 3D diffeomorphism constraint, $\xi \in TM\,,$ $\imath_\xi$ = interior product

$\mathrm{d}_A F_i = 0$

Bianchi identity -> Conservation law for SU(2) magnetic charge $\int_D F\,.$

Duality between electric and magnetic modes in gravity (in analogy with EM [Freidel, DP, PRD 2018])

A puzzle

In loop quantum gravity we have an asymmetric treatment of the Gauss law and of the 3D diffeo constraint.

- The Gauss Law is is discretized on graphs into the closure constraint. One construct then an Hilbert space in which it acts covariantly and SU(2) gauge invariance is then implemented at the quantum level by restricting to spin network states.
- The diffeomorphism doesn't admit an infinitesimal representation. There is no 3D diffeomorphism constraint at the discrete level, thus no 3D diffeomorphism constraint operator at the quantum level. One represents finite diffeomorphism by their action on the support of the state and imposes the diffeo constraint by taking equivalence of graphs embedded in the 3D hypersurface M (group averaging).

1. The 3D diffeomorphism invariance is not treated on the same footing as the SU(2) gauge invariance;

2. It is not clear how matter fields, which enters the Einstein equations as a source for the diffeomorphism constraints, should be discretized and quantized;

$$D_a = T_{ab} n^b \longleftarrow \text{Matter momenta density}$$

3.It is not clear how boundaries, which break 3D diffeomorphism invariance, should be treated and how the resulting charges should be defined.

Towards democracy: Constraint as conservation law

The key idea is to understand the constraints as conservation law for "charge aspects"

- Gauss Law: $\mathrm{d}_A \Sigma_i = 0$ is a conservation law for the "SU(2) electric charge" or flux $\Sigma_\alpha = \int_S \alpha^i \Sigma_i$

- 3D diffeomorphism can also be written as a conservation law for the "SU(2) momentum charge" $P_\xi = \int_S \xi^i P_i$

where $P_i := \frac{\mathrm{d}_A e_i}{\sqrt{\kappa\gamma}}, \quad \xi^i = \frac{\iota_\xi e^i}{\sqrt{\kappa\gamma}}$

Proof:

$$D_\xi = -\int_{\mathcal{B}} F^i \wedge \iota_\xi \Sigma_i$$

simplicity constraint
$\kappa\gamma\iota_\xi\Sigma_i = (\iota_\xi e \times e)_i$ ⟶

$$= \frac{1}{\sqrt{\kappa\gamma}} \int_{\mathcal{B}} F^i \wedge (e \times \iota_\xi e)_i$$

$$= \frac{1}{\kappa\gamma} \int_{\mathcal{B}} (F \times e)_i \, \xi^i$$

Bianchi identity
$\mathrm{d}_A^2 e = (F \times e)_i$ ⟶

$$= \frac{1}{\kappa\gamma} \int_{\mathcal{B}} (\mathrm{d}_A^2 e_i) \, \xi^i$$

$$= \int_{\mathcal{B}} (\mathrm{d}_A P_i) \, \xi^i \qquad \square$$

- Kinematical constraints as charge conservation:

$$\mathrm{d}_A \Sigma_i = 0, \qquad \mathrm{d}_A F^i = 0, \qquad \mathrm{d}_A P_i = 0\,.$$

Flux conservation (Gauge invariance)	Monopole conservation	Momentum conservation (Diffeo invariance)

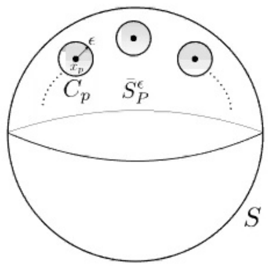
As a result, we will discover that quantum states of geometry are labelled by three quantum numbers:

j_p	k_p	P_p
Spin	Monopole charge	Momentum

In order to reveal these new charges, we need to include boundaries in our description and look at the edge modes on the boundary surface.

Edge modes on boundary surface

In the presence of boundary, the bulk constraints in $\mathcal{B}$ have to be supplemented by boundary conditions on S. These boundary conditions determine the symplectic structure for the boundary fields and reveal the edge modes living on S.



Punctured boundary sphere

$$S_P = \lim_{\epsilon \to 0} \bar{S}_P^\epsilon$$

Set of punctures

$$P \equiv \{x_p \in S \,|\, p = 1, \cdots, N\}$$

where $\bar{S}_P^\epsilon = S^2 \smallsetminus \cup_p D_p^\epsilon$ is the complement of a union of the small disks D_p^ϵ

Boundary conditions

$$\Sigma_i(x) \overset{S}{=} \frac{1}{2\kappa\gamma}(e \times e)_i, \qquad F^i(x) \overset{S}{=} 2\pi \sum_p k_p^i \, \delta_p(x)\,, \qquad P^i(x) \overset{S}{=} 2\pi \sum_p P_p^i \, \delta_p(x)\,,$$

↑ Source of curvature (k_p^i)

↑ Source of momentum (P_p^i)

We now explain how these boundary conditions naturally appear from demanding the constraints to be differentiable in the presence of a boundary

- Gauss law: $\int_{\mathcal{B}} \alpha^i \mathrm{d}_A \Sigma_i$ no longer differentiable -> add a boundary term

Bulk Gauss generator: $G^B_\alpha := -\int_B \mathrm{d}_A \alpha^i \wedge \Sigma_i = \int_B \alpha^i \mathrm{d}_A \Sigma_i - \int_S \alpha^i \Sigma_i$

but, $G^B_\alpha = 0$ if and only if $\alpha^i \Sigma_i \overset{S}{=} 0$ $\rightarrow$ gauge invariance broken by the presence of the boundary

1. The Grandma's remedy: accept this fact and promote the non-vanishing charges

$\int_S \alpha^i \Sigma_i =$ Symmetry charges

2. The LQG remedy: impose $G^B_\alpha \overset{S_P}{=} 0$ $\rightarrow$ $\Sigma^i_{\mathrm{LQG}} = \sum_p X^i_p \delta_p(x)$

Symmetry charges = loop gravity fluxes X_p associated with each puncture.

3. The loop gravity string remedy: introduce edge modes that restore the gauge symmetry

- SU(2) gauge constraint:

$$G_\alpha := -\int_B \mathrm{d}_A \alpha^i \wedge \Sigma_i + \frac{1}{2\kappa\gamma}\int_S \alpha^i (e \times e)_i$$
$$= \int_B \alpha^i \mathrm{d}_A \Sigma_i + \int_S \alpha^i \left[\frac{1}{2\kappa\gamma}(e\times e)_i - \Sigma_i\right]$$

$G_\alpha = 0 \quad \Rightarrow$ Gauss law in the bulk + Simplicity constraint on the surface

Boundary Simplicity constraint = Continuity condition

New symmetry charges

➥ $\Sigma_\alpha := (2\kappa\gamma)^{-1}\int_S \alpha^i (e\times e)_i$ act only on the edge mode variables and rotate them

★ Advantage: it becomes clear why the symmetry charges satisfy a non-commutative current algebra

$$\{e^i_A(x), e^j_B(y)\} = \kappa\gamma\, \delta^{ij}\epsilon_{AB}\delta^2(x,y) \quad \to \quad \{\Sigma_\alpha, \Sigma_\beta\} = \Sigma_{\alpha\times\beta}$$

- 3D diffeos: Differentiable generator of diffeomorphism

$$D_\xi = -\underbrace{\int_{\mathcal{B}} F^i(A)\wedge \imath_\xi \Sigma_i - \frac{1}{\kappa\gamma}\int_{S_P}(\imath_\xi e^i)\mathrm{d}_A e_i}_{gauge = D^{\mathcal{G}}_\xi} + \frac{1}{2\kappa\gamma}\underbrace{\sum_{p\in P}\oint_{C_p} e^i \imath_\xi e_i}_{symmetry}$$

$D^{\mathcal{G}}_\xi = 0$

imposes diffeomorphism constraint away from the punctures

Momentum constraint (equivalent to the diffeomorphism constraint)

$$P_\varphi := -\underbrace{\int_{\mathcal{B}}(\mathrm{d}_A\varphi)_i \wedge P^i}_{gauge = P^{\mathcal{G}}_\varphi} + \frac{1}{\sqrt{\kappa\gamma}}\underbrace{\sum_{p\in P}\oint_{C_p}\varphi^i e_i}_{symmetry}$$

$P^{\mathcal{G}}_\varphi = 0$

Translational gauge constraint, equivalent to the diffeomorphism constraint (for invertible frame)

$\varphi^i = (\kappa\gamma)^{-\frac{1}{2}}\,\imath_\xi e^i$

➥ Non-zero charges associated with each puncture:

$$D_\xi := \frac{1}{2\kappa\gamma}\oint_{C_p} e^i \imath_\xi e_i\,, \qquad P_\varphi = \frac{1}{\sqrt{\kappa\gamma}}\oint_{C_p} e^i \varphi_i$$

↙ Virasoro generators of deformations of the puncture contour

↘ $U(1)^3$ Kac-Moody currents

☆ Central charge appears already at the classical level: $\{P_\xi, P_\varphi\} \hat{=} \sum_{p\in P}\oint_{C_p}\xi_i \mathrm{d}_A\varphi^i$

Democracy ain't perfect

$$\Sigma_i(x) \stackrel{S}{=} \frac{1}{2\kappa\gamma}(e \times e)_i, \qquad F^i(x) \stackrel{S}{=} 2\pi \sum_p k_p^i \, \delta_p(x)\,, \qquad P^i(x) \stackrel{S}{=} 2\pi \sum_p P_p^i \, \delta_p(x)\,,$$

Loop gravity string remedy

Composite structure of the geometrical flux

LQG remedy

$P_p^i =$ 0-modes of the Kac-Moody currents

Mode expansion of the conserved currents requires $k_p \in \mathbb{Z}/N$.

We restrict our analysis to the case $k_p \in \mathbb{Z}$.

Holonomy of the connection around the punctures is trivial: $\exp \oint_{C_p} A = 1$

1. Although curvature is Planckian, holonomies are invisible.

2. We can treat curvature charges k_p as classical labels.

★ Introduction of magnetic edge modes may solve disparity...

The loop gravity framework

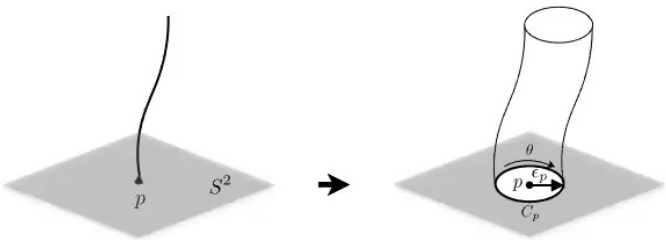
In presence of a boundary surface S, there are boundary charge densities (Σ_i, P_i)

Boundary conditions away from the punctures:

$$\Sigma_i \overset{S_P}{=} \frac{1}{2\kappa\gamma}(e\times e)_i\,, \qquad F(A)^i \overset{S_P}{=} 0\,, \qquad P^i \overset{S_P}{=} 0$$

By solving the curvature and momentum constraint on the boundary,
we can introduce currents to express the frame field around punctures.

- Current mode expansion:



$$Q^a_{p,n} := \oint_{C_p} e^{in\theta} e^a_\theta \mathrm{d}\theta$$ modes of the frame field pulled back on the circle

$$P^a_p := Q^a_{p,-k^a_p}$$ momentum

$$X^a_p$$ zero mode 'position' observable conjugated to P

Poisson brackets for the phase space associated to a single puncture

$$\{X^a_p, P^b_{p'}\} = \delta_{pp'}\delta^{a\bar{b}}\,,$$
$$\{Q^a_{p,n}, Q^b_{p',m}\} = -i\delta_{pp'}\delta^{a\bar{b}}(n + k^a_p)\delta_{n+m}$$

The structure defining each puncture becomes the 1D boundary around the puncture C_p, which can be thought as a string. This leads to a zero-mode string position and momentum (X^a, P^a), and to an infinite tower of higher/lower modes Q^a_n describing the vibration modes of the string and generating a (twisted) U(1)^3 Kac-Moody algebra.

Since we have assumed that the curvature charges k^a_p are integers, we can entirely re-absorb them as shifts of the mode expansion:

Shifted charges

$$\alpha^a_{p,n} := Q^a_{p,n-k^a_p}\,, \qquad \bar{\alpha}^a_n = \alpha^{\bar{a}}_{-n}\,,$$
$$\{\alpha^a_n\,,\,\bar{\alpha}^b_m\} = -i\,n\,\delta^{ab}\delta_{n,m}$$

- Flux as a composite object:

The total flux can be written as a sum of fluxes at each puncture: $\Sigma_S(\alpha) = \sum_p \alpha_a \Sigma^a_p$

$$\Sigma^a_p = A^a_p + S^a_p\,, \qquad A^a_p := (X_p \times P_p)^a, \qquad S^a_p := \frac{i}{2}\sum_{n\neq 0}\frac{(\alpha_{p,n}\times\alpha_{p,-n})^a}{n}$$

String (0-mode) angular momentum (in internal space) — spin encoded in the higher vibration modes

A^a_p's and S^a_p's each form a su(2) algebra and commute with each other -> su(2) Poisson algebra for the flux

The flux closure condition: $\sum_p \Sigma^i_p = 0$ implies also the momenta conservation condition $\sum_p P^i_p = 0$

(no physical relevance of the string position)

- Hilbert space of the Quantum Puncture:

Basis states labelled by the momentum $\vec{p} \in \mathbb{R}^3$ (zero mode)

and diagonalizing the oscillator energies $E^a_n = \alpha^{a\,\dagger}_n \alpha^a_n$ in terms of the numbers of quanta $N^a_n \in \mathbb{N}$ (higher modes)

$$P^a\,|\vec{p},\{N^a_n\}\rangle = p^a\,|\vec{p},\{N^a_n\}\rangle\,, \qquad E^a_{n_0}\,|\vec{p},\{N^a_n\}\rangle = n_0 N^a_{n_0}\,|\vec{p},\{N^a_n\}\rangle$$

$$\begin{cases} \alpha^a_{n_0}\,|\vec{p},\{N^a_n\}\rangle = \sqrt{n_0 N^a_{n_0}}\,|\vec{p},N^a_{n_0}-1,\{N^a_n\}_{n\neq n_0}\rangle \\ (\alpha^a_{n_0})^\dagger\,|\vec{p},\{N^a_n\}\rangle = \sqrt{n_0(N^a_{n_0}+1)}\,|\vec{p},N^a_{n_0}+1,\{N^a_n\}_{n\neq n_0}\rangle \end{cases}$$

where $|\vec{p},\{N^a_n\}\rangle = \prod_{a,n\geq 1}\frac{(\alpha^a_n\dagger)^{N^a_n}}{\sqrt{N^a_n!}}\,|\vec{p}\rangle$

Virasoro generators

Sugawara construction: $L_n = \frac{1}{2}\sum_a \sum_{m\in\mathbb{Z}} :\alpha^a_m \alpha^{\bar{a}}_{n-m}:$ where $:\alpha^a_n \alpha^b_m: = \begin{cases} \alpha^b_m \alpha^a_n & \text{if } n > 0\,, \\ \alpha^a_n \alpha^b_m & \text{if } n \leq 0\,. \end{cases}$

➡ $[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}$

On the completely algebraic level, we obtain a Virasoro algebra with $c = 3$

$L_n = \frac{1}{2\kappa\gamma}\oint e^{in\theta} e^a_\theta e_{\theta a} =$ modes of the Hamiltonian symmetry charges associated to diffeos tangent to the circle $D_\xi = \frac{1}{2\kappa\gamma}\oint_{C_p} e^i \imath_\xi e_i$

They generate reparametrization of the string around the puncture and encode all local angular deformations of the geometry

Zero mode generator = Total energy $L_0 = \frac{1}{2}\vec{P}^2 + \sum_{n\geq 1} E_n$

$|\vec{p}, \{N^a_n\}\rangle =$ States of highest weight representation of the Virasoro algebra for each fixed value of $\vec{p} \in \mathbb{R}^3$

$|\vec{p}\rangle \equiv |\vec{p}, \{N^a_n = 0\}\rangle =$ Highest weight vector: $P^a\,|\vec{p}\rangle = p^a\,|\vec{p}\rangle\,, \qquad L_0\,|\vec{p}\rangle = \frac{1}{2}\vec{p}^2|\vec{p}\rangle\,, \qquad L_{n\geq 1}\,|\vec{p}\rangle = 0$

Other states (descendants): $L_0\,|\vec{p}, \{N^a_n\}\rangle = \left(\frac{1}{2}\vec{p}^2 + \sum_n nN_n\right)|\vec{p}, \{N^a_n\}\rangle$ with $N_n = \sum_a N^a_n$

Poincaré basis

The geometrical flux $\vec{\Sigma} = \vec{A} + \vec{S}$, where $\vec{S} := \sum_{n\geq 1} \vec{S}_n$ -> su(2)-algebra part of the Poincaré isu(2)-algebra:

$$[\Sigma^a, \Sigma^b] = i\epsilon^{abc}\Sigma^c, \qquad [\Sigma^a, P^b] = i\epsilon^{abc}P^c, \qquad [P^a, P^b] = 0$$

Change of basis states adapted to the flux, i.e. diagonalizing the su(2) Casimir $\vec{\Sigma}^2$,
which diagonalizes the Poincaré Casimirs as well, namely $\vec{P}^2$, $s = \vec{\Sigma}\cdot\vec{P}$.

$$\left|\begin{array}{lcll} \vec{P}^2 & |\Delta, s, \mathcal{J}, \mathcal{M}, I\rangle & = & \Delta & |\Delta, s, \mathcal{J}, \mathcal{M}, I\rangle, \\ \vec{\Sigma}\cdot\vec{P} & |\Delta, s, \mathcal{J}, \mathcal{M}, I\rangle & = & s & |\Delta, s, \mathcal{J}, \mathcal{M}, I\rangle, \\ \vec{\Sigma}^2 & |\Delta, s, \mathcal{J}, \mathcal{M}, I\rangle & = & \mathcal{J}(\mathcal{J}+1) & |\Delta, s, \mathcal{J}, \mathcal{M}, I\rangle, \\ \Sigma^3 & |\Delta, s, \mathcal{J}, \mathcal{M}, I\rangle & = & \mathcal{M} & |\Delta, s, \mathcal{J}, \mathcal{M}, I\rangle, \end{array}\right.$$

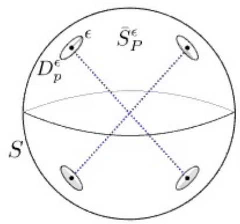
The states $|\Delta, s, \mathcal{J}, \mathcal{M}\rangle$ form a representation of the Poincaré algebra

energy carried by the 0-mode

with $\vec{S}\,|I\rangle = 0$, Δ and s are the Poincaré Casimirs, and the total spin $\mathcal{J}$ is the LQG spin.

Intertwiner between the overall S and arbitrary spins $\{j_n\}_{n\in\mathbb{N}}$, associated to the spin higher modes $\vec{S}_n$

Edge mode networks as new quantum geometry states

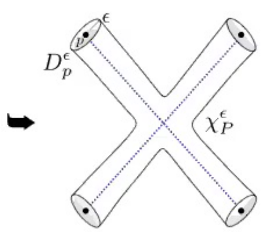


The gravity string can be understood as the quantum geometry of the local subsystem with excitations of quantum space localized at punctures on the boundary surface (point-like sources of curvature and torsion) and encoded in stringy charges forming a Kac-Moody algebra around each puncture and containing all the information about quantum geometry in $\mathcal{B}$ and its boundary S.

New CFT perspective of gravitational edge modes and the LQG discretization of geometry

(0-mode of the Kac-Moody algebra carries momentum $\vec{P} \to \Delta = \vec{P}^2$ conformal weight of the primary field vacuum state -> boundary charge associated to 3D diffeomorphisms)

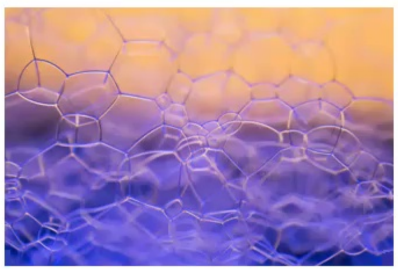
The DOF around each puncture allow to glue bounded regions together so to reconstruct the total Hilbert space associated to the whole canonical hypersurface. This gluing process can be more easily described by 'flattening' the gravity string



We extend this distributional curvature localized at the punctures to the bulk region enclosed by the 2-sphere. Consider a 2D tubular structure connecting all the circles around the punctures: Curvature propagates from one puncture to another inside the tubes, but it vanishes outside.

New quantum states of geometry in terms of this tubular structure with the tubes around each gravity string dressed with quantum puncture states, in the Kac-Moody basis with quantum numbers $(\vec{p}, \{N_n^a\})_p$ or in the recoupled spin basis with quantum numbers $(\Delta, s, \mathcal{J}, \mathcal{M}, I)_p$.

- Edge mode networks represent the portioning of space into "bubbles" bounded by surfaces.
- Each patch of the boundary surface carries algebraic data that forms the representation of the boundary group of symmetry.
- Each bubbles of space is represented by an intertwiner, a form of conservation law at the vertex, and those chunks of quantum space are then patched together by gluing the cylinders around the curvature defects through appropriately matching and propagating the quantum numbers of the puncture where the gluing happens.



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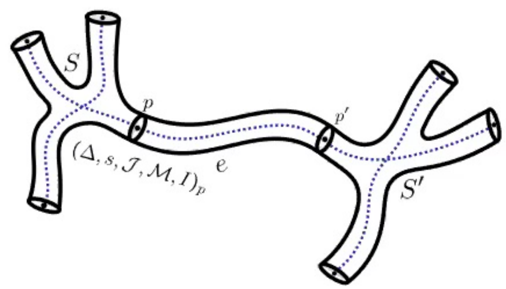
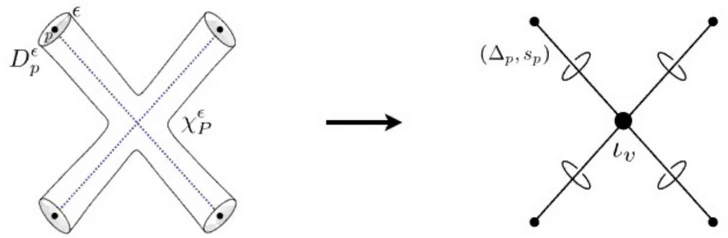


Coarse graining: Poincaré networks



- The coarse graining operation consists in keeping only the zero modes, the SU(2) flux Σ^a and the 0-mode momentum P^a, in the boundary symmetry algebra. It corresponds to a particle limit of the cylinder, with the fine structure of the Kac-Moody higher modes is effectively hidden within the flux.

- In this limit, each edge carries a representation of the Poincaré algebra isu(2). A new charge appears: The momenta P.

$$[\Sigma_p^a, \Sigma_p^b] = i\epsilon^{abc}\Sigma_p^c, \qquad [\Sigma_p^a, P_p^b] = i\epsilon^{abc}P_p^c, \qquad [P_p^a, P_p^b] = 0$$

The quantum puncture then lives in the Poincaré representation with Casimirs Δ_p, s_p and quantum basis states are further labeled by the spin $\mathcal{J}$ and the corresponding magnetic moment.

- The conservation laws of flux and momenta on the punctured surface are imposed by dressing the vertex with a Poincaré intertwiner state ι_v, implementing invariance under the 3D Poincaré group ISU(2) of the tensor product states of the quantum punctures around the vertex.

LQG corresponds to a further truncation where we simply ignore the momentum information.

Summary

- We have seen that the edge modes formalism captures the nature of local degrees of freedom involved in the division of the system into subsystems. This process is realized as a creation of boundary defect and space can be understood algebraically as the fusion of states associated with boundary symmetry algebra (generalized Kac-Moody). This understanding opens the way to a new description of quantum geometry where features of the holographic and canonical approaches are combined.

- We have exploited the appearance of a central charge in the algebra of the fundamental harmonic oscillators to construct representations of the infinite dimensional boundary symmetry algebra. This led us to the introduction of new quantum geometry states labeled by quantum numbers associated with both the geometrical flux and the diffeomorphism charges.

- We have shown how a truncation of the loop gravity string description of quantum geometry to the zero mode sector allows us to put the diffeomorphism constraint on the same footing as the Gauss law. It becomes implemented as the closure condition for the edge momenta at each node of the network. This coarse graining of the charge of symmetry leads to a representation of space as a Poincaré charge network.

Outlook

- This opens the possibility to finally have a representation of local matter inside a quantum spacetime. Now that we have a representation of space-like diffeomorphism as an operator, one can finally think about the insertion of momenta as the introduction of a topological defect inside the charge conservation law. The matter momenta will naturally appear at the vertices of our network, like in 2+1D.

- We have only discussed the kinematical constraints. Even if we now have a definition for the quasi-local momentum operator, we do not have a description of the quasi-local energy. New ideas are presumably needed to generalize the gravitational conservation laws described here to the gravitational energy and to obtain a representation of the full dynamical algebra (better understanding of coarse-graining operations and fusion / entanglement between subregions; look at null surfaces - see e.g.[Wieland, CQG 2019]...).

- Get rid of the time-gauge and repeat our construction including the boosts: Second CFT copy? Modular invariance <-> Simplicity constraint + Immirzi duality? [w.i.p. with Freidel and Geiller]

- Revise the black hole entropy calculation employing CFT techniques. [Hawking, Perry, Strominger, PRL 2016, JHEP 2017], [Averin, Dvali, Gomez, Lüst, JHEP 2016, Mod.Phys.Lett. A 2016]