

Title: Replica wormholes and the black hole information paradox

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Abstract: The information paradox can be realized in two-dimensional models of gravity. In this setting, we show that the large discrepancy between the von Neumann entropy as calculated by Hawking and the requirement of unitarity is fixed by including new saddles in the gravitational path integral. These saddles arise in the replica method as wormholes connecting different copies of the black hole. We will discuss their appearance both in asymptotically AdS and asymptotically flat theories of gravity.

Replica wormholes and the black hole information paradox

Edgar Shaghoulian
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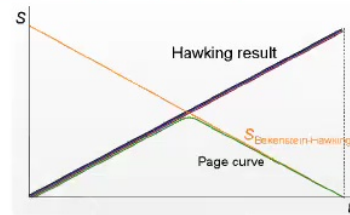
Perimeter Institute seminar
April 21, 2020

[Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini 1911.12333]

[Hartman, Shaghoulian, Strominger xxxx.yyyyy]



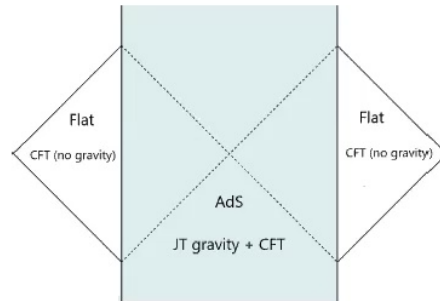
Black hole information paradox



- ▶ Hawking performed (what seemed to be) a reliable semiclassical calculation, and found pure states \rightarrow mixed states.
- ▶Possibly the most inspirational crisis in the history of quantum gravity.....
- ▶ Page provided a target of what a correct calculation in a quantum-mechanical theory should look like.
- ▶ In the past year, several authors provided a sharper target of what a correct calculation in a holographic theory should look like.
- ▶ We will discuss a general mechanism for getting these detailed answers directly in a gravitational theory – without assuming holography – and flesh it out in a particular example.

Edgar Shaghoulian

Jackiw-Teitelboim (JT) gravity coupled to CFT with wings



Take JT gravity coupled to a CFT:

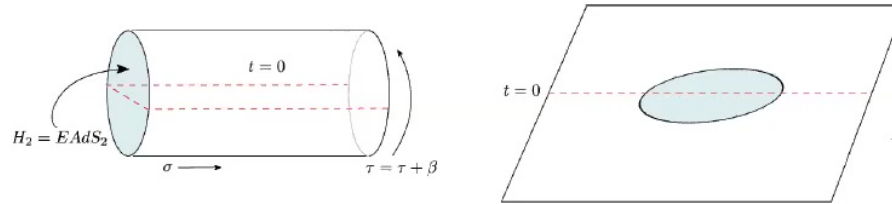
$$I = -\frac{S_0}{4\pi} \left[\int_{\Sigma_2} R + \int_{\partial\Sigma_2} 2K \right] - \int_{\Sigma_2} \frac{\phi}{4\pi} (R + 2) - \frac{\phi_b}{4\pi} \int_{\partial\Sigma_2} 2K + I_{CFT}[g]$$

- ▶ CFT has transparent boundary conditions at AdS_2 boundary, where $\phi \rightarrow \infty$. [Rocha; Anninos, Hofman, Kruthoff; Penington; Almheiri, Engelhardt, Marolf, Maxfield]
- ▶ Can choose contour to integrate out dilaton and leave Schwarzian boundary mode; not necessary, saddle point analysis sufficient!
- ▶ Topological term will lead to suppression of higher-genus saddles.



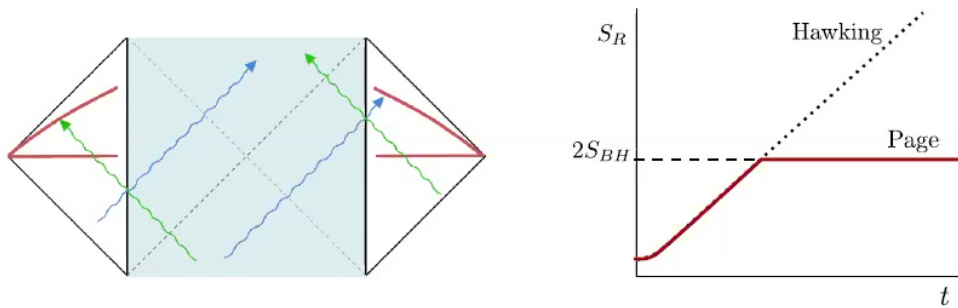
Black hole information paradox

Set up thermofield double state for combined AdS₂ and flat space regions



Finite temperature flat space is Euclidean cylinder, the AdS₂ region cuts it and glues on a cap.

Evolve upward from $t = 0$ as [Almheiri, Mahajan, Maldacena]



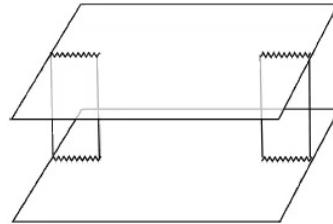
Replica trick in field theory

$$S_R = -\text{Tr } \rho_R \log \rho_R$$

through replica trick

$$Z_n = \text{Tr } \rho_R^n, \quad S_R^{(n)} = \frac{1}{1-n} \log \frac{Z_n}{(Z_1)^n} \implies S_R = -\partial_n \left(\frac{\log Z_n}{n} \right) \Big|_{n=1}.$$

Z_n is partition function on a topologically nontrivial manifold, e.g. Z_2 :

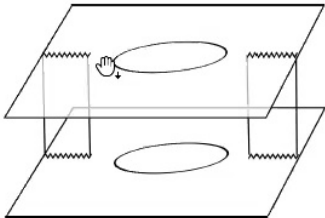


$Z_n \propto e^{-(n-1)S_R^{(n)}}$, so large entropy implies (exponentially) small Z_n .



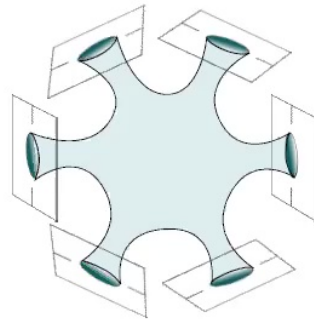
Replica trick in gravity

For gravitational path integral, fix the boundary conditions at the edge and let gravity dynamically fill in the geometry.



Replica trick in gravity

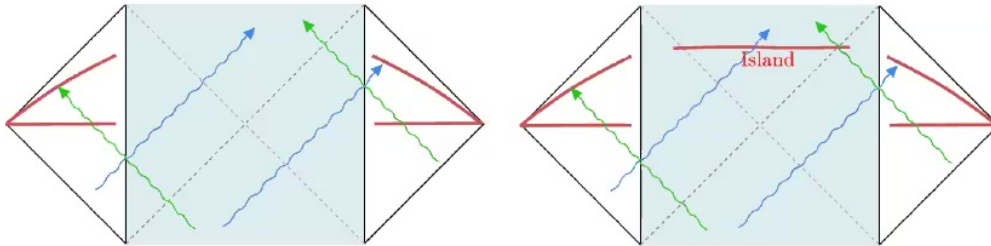
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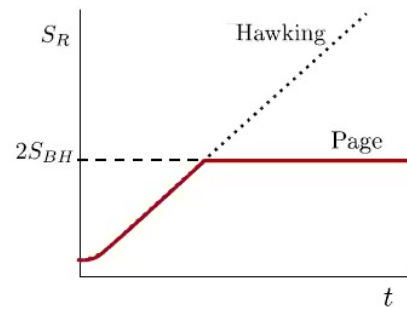
$n \rightarrow 1$ limit

Replica wormholes leave imprint as $n \rightarrow 1$, reproducing “island” rule

$$S_R = \min_I \text{ext}_I [S_{\text{eff}}(R \cup I) + S_{\text{grav}}(I)]$$



and Page curve



Deep implication: entanglement wedge reconstruction implies that the operator algebra of the island is contained in the exterior of the black hole.



Local argument for QES

Consider general dilaton gravity

$$I = \int_{\Sigma_2} (\phi R + U(\phi)) + I_{CFT}$$

Take metric $ds^2 = e^{2\rho} dz d\bar{z}$, place defect at $z = 0$ and work locally near here:

$$\rho \rightarrow \rho + (n-1)\delta\rho, \quad \phi \rightarrow \phi + (n-1)\delta\phi, \quad T_{\mu\nu}^{flat} \rightarrow T_{\mu\nu}^{flat} + (n-1)\delta T_{\mu\nu}^{flat}$$

Smoothness on replica manifold $w = z^{1/n}$ implies $\delta\rho \sim -\frac{1}{2} \log(z\bar{z})$.

Conformal ward identity gives

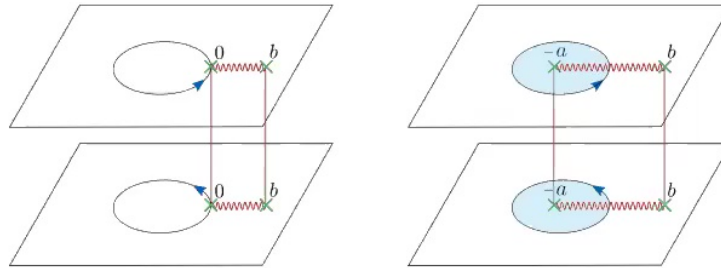
$$2\pi\delta T_{zz}^{flat} = \frac{c/12}{z^2} - \frac{\partial S_{CFT}^{flat}}{z} + \text{reg}, \quad \text{barred}$$

Expanding dilaton in series respecting replica symmetry around $z = 0$ and solving singular terms of constraint equations gives

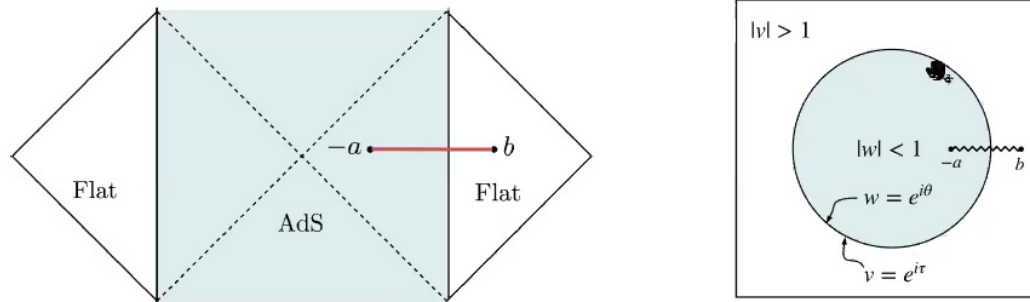
$$\partial(S_{BH} + S_{CFT}) = \bar{\partial}(S_{BH} + S_{CFT}) = 0.$$



Single interval



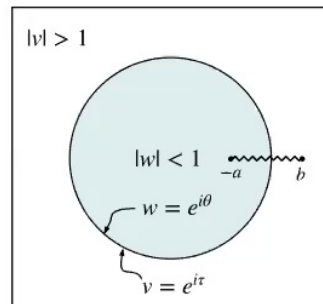
Want to calculate entanglement entropy in region $[0, b]$. As an ansatz consider extending branch point to $-a$ (free parameter!). Additional branch points are topologically suppressed. So our configuration is



“ $-a$ ” twist point sources a stress tensor for metric EOM, or Schwarzian EOM.



Single interval



Branch points at $w = A = e^{-a}$, $v = B = e^b$. Uniformize disk interior with

$$\tilde{w} = \left(\frac{w - A}{1 - Aw} \right)^{1/n}.$$

Place standard hyperbolic metric in \tilde{w} -disk, $ds^2 = \frac{4|d\tilde{w}|^2}{(1-|\tilde{w}|^2)^2}$. $\tilde{w} = e^{i\tilde{\theta}}$ at boundary of disk, so Schwarzian EOM becomes standard:

$$\frac{\phi_\tau}{2\pi} \partial_\tau \{e^{i\tilde{\theta}}, \tau\} = i(T_{yy}(i\tau) - T_{\bar{y}\bar{y}}(-i\tau)), \quad v = e^y.$$

Equation in w plane written using Schwarzian composition identity:

$$\{e^{i\tilde{\theta}}, \tau\} = \{e^{i\theta}, \tau\} - \{e^{i\tilde{\theta}}, e^{i\theta}\} \left(\frac{\partial e^{i\theta}}{\partial \tau} \right)^2.$$

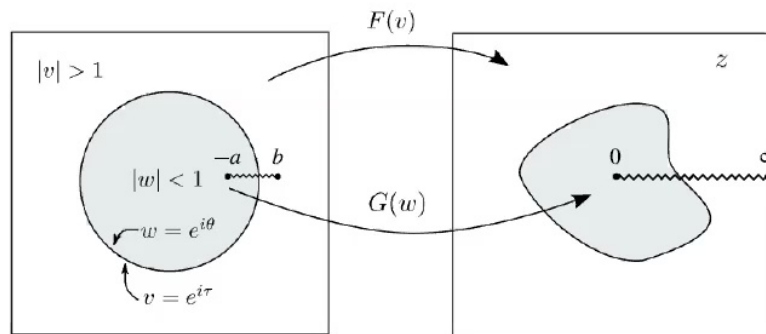


Conformal welding

So we have the EOM in our original coordinates,

$$\frac{\phi_r}{2\pi} \partial_\tau \left[\{e^{i\theta}, \tau\} - \frac{1}{2} \left(1 - \frac{1}{n^2} \right) \frac{(1 - A^2)^2 (\partial_\tau \theta)^2}{|1 - Ae^{i\theta}|^4} \right] = i(T_{yy}(i\tau) - T_{\bar{y}\bar{y}}(-i\tau)).$$

Stress tensor on RHS found by uniformization map a la Cardy-Calabrese.
But need a single coordinate across interior/exterior.



Conformal welding: given gluing $\theta(\tau)$, find $F(v)$ and $G(w)$ holomorphic in their respective disks, agree on boundary of disks. Uniformizing the z plane ($\tilde{z} = z^{1/n}$) gives $T_{\tilde{z}\tilde{z}} = 0 \rightarrow T_{zz} \rightarrow T_{vv} \rightarrow T_{yy}$.



Equation of motion

$$\frac{24\pi\phi_r}{c\beta} \partial_\tau \left[\{e^{i\theta(\tau)}, \tau\} + \frac{1}{2} \left(1 - \frac{1}{n^2}\right) R(\theta(\tau)) \right] = ie^{2i\tau} \left[-\frac{1}{2} \left(1 - \frac{1}{n^2}\right) \frac{F'(e^{i\tau})^2}{F(e^{i\tau})^2} - \{F, e^{i\tau}\} \right] + cc$$

This is *not* a local equation for $\theta(\tau)$, due to F . Simple in two limits: $n \rightarrow 1$ and $\beta \rightarrow 0$. Welding drops out as $\beta \rightarrow 0$, perturbative as $n \rightarrow 1$.

As $n \rightarrow 1$, solution imposes a constraint on a :

$$\frac{c}{6\phi_r} \frac{\sinh \frac{a-b}{2}}{\sinh \frac{a+b}{2}} = \frac{1}{\sinh a}.$$

This is precisely what you get from $\partial_a S_{gen} = 0$, aka the island rule.

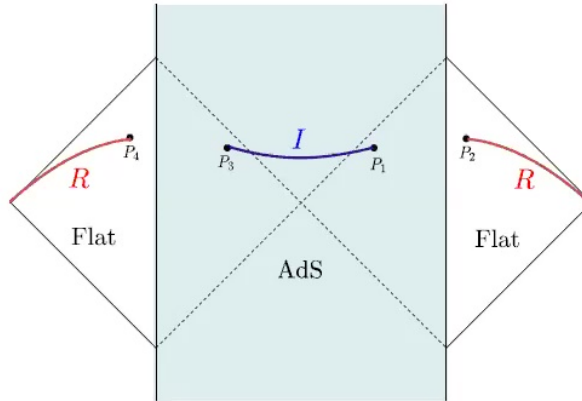
Plugging the solution back into the action at leading order in $n - 1$ gives us the entropy:

$$S([0, b]) = \min_a \left(\underbrace{S_0 + \frac{\phi(-a)}{4G} + S_{CFT}([-a, b])}_{S_{gen}([-a, b])} \right)$$



Two intervals

Growth of wormhole at late times means $S[\overline{R \cup I}]$ should factorize.



Encoding the interior

We have

$$S[\rho_R] = S[\tilde{\rho}_{RI}] + \frac{A}{4G}$$

for ρ_R the exact density matrix and $\tilde{\rho}_{RI}$ the one obtained from a semiclassical calculation. Like in AdS/CFT, this implies

$$S_{rel}(\rho_R|\sigma_R) = S_{rel}(\tilde{\rho}_{RI}|\tilde{\sigma}_{RI})$$

through argument of JLMS [Jafferis, Lewkowycz, Maldacena, Suh].



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Equality of relative entropies is basis of entanglement wedge reconstruction:

$$\mathcal{O}_I|i\rangle = \mathcal{O}_R|i\rangle$$

for $|i\rangle \in \mathcal{H}_{code}$ (space of perturbative excitations on top of state with given island).



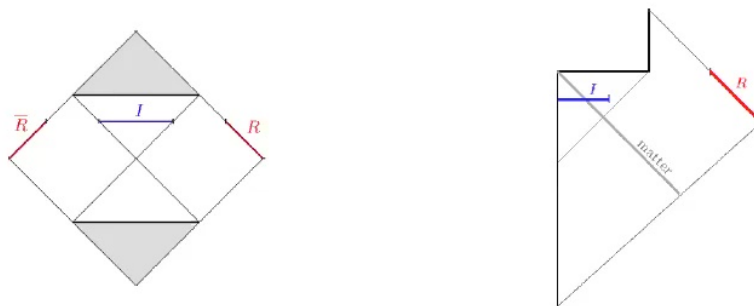
Extension to flat space

\mathcal{I}^+ seems like a natural stand-in for the non-dynamical region.

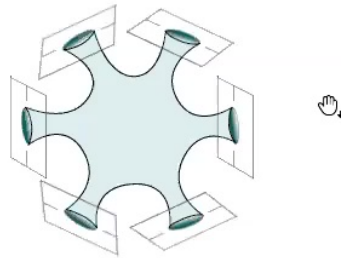
Dilaton-gravity models with asymptotically flat boundary conditions studied in the 80s, including CGHS/RST. Models were determined to destroy information.

A modification of the model which includes replica wormholes gives results consistent with unitarity [Hartman, Shaghoulian, Strominger].

Suggests an encoding of the black hole interior on \mathcal{I}^+



Outlook



- ▶ Replica wormholes are a replica non-diagonal (but symmetry-preserving!) contribution to the entropy in gravitational systems.
- ▶ Mechanism is general: replica wormholes, perhaps as off-shell configurations, should exist in any number of dimensions for generic theories of gravity.
- ▶ Microscopic picture? + many more...



Extension to flat space

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