

Title: PSI 2019/2020 - Standard Model and Beyond part 2 - Lecture 8

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Collection: PSI 2019/2020 - Standard Model and Beyond - Part 2

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SM+B #26

Last time: Todorov + Dubois Violette

$$\begin{array}{c}
 \text{Aut}(\underline{H_3(\mathbb{O})}) = F_4 \begin{array}{l} \nearrow \text{Spin}(9) \rightarrow \underline{(\mathbb{O})} \leftarrow H_2(\mathbb{O}) \\ \searrow \underline{\underline{\mathfrak{su}(3) \times \mathfrak{sh}(3)}} \end{array} \\
 \downarrow \\
 \text{Isom}(\underline{\mathbb{O}P^2}) \\
 \downarrow \\
 \underline{\underline{\mathbb{Z}_3}} \rightarrow 1, n, e, \dots, n_7 e_7 \\
 \uparrow \\
 \mathbb{O} = a1 + b\eta \rightarrow \mathbb{O} \quad \uparrow \quad = \hat{n} \cdot \vec{e} = q \rightarrow l
 \end{array}$$

intersection of $\text{Spin}(9) \cap \underline{\underline{\mathfrak{su}(3) \times \mathfrak{sh}(3)}}$ in F_4

1. The first step in the process of creating a business plan is to conduct a thorough market research. This involves identifying the target market, understanding their needs and preferences, and analyzing the competitive landscape. Market research can be conducted through various methods, including surveys, interviews, and focus groups.

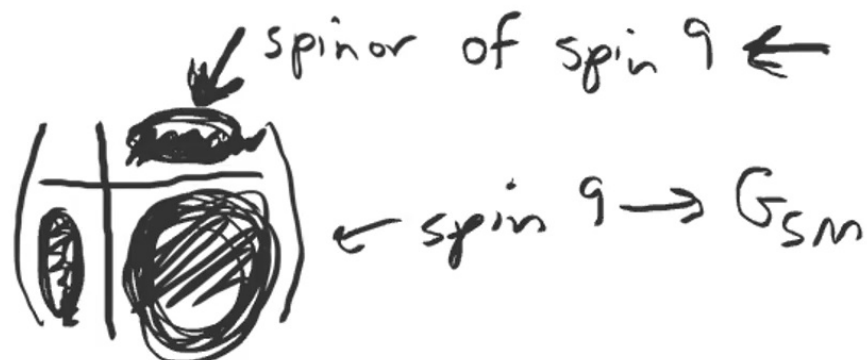
Pirsa: 20040016

$$\begin{array}{ccccccc}
 \left(\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right)_{3 \times 3} & \xrightarrow{\quad} & \underline{\underline{\mathbb{CP}^2}} & \xrightarrow{\quad} & \underline{\underline{M_{9,1}}} & \xrightarrow{\quad} & \underline{\underline{M_{3,1}}} \xleftarrow{\quad} \left(\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right)_{\mathbb{Z}_6} \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 H_2(\mathbb{C}) & & & & & & \frac{(SU(3) \times SU(2) \times U(1))}{\mathbb{Z}_6}
 \end{array}$$

$$H_2(\mathbb{C}) \ni X = \begin{pmatrix} x_0 + x_9 & x \\ \bar{x} & x_0 - x_9 \end{pmatrix} \xleftarrow{\quad} \underline{\underline{(x_0, \dots, x_9)}}$$

$$\det(X) = \underline{\underline{x_0^2 - x_9^2 - x_1^2 - x_2^2 - \dots - x_8^2}} \quad M_{9,1}$$

$$H_2(\mathbb{C}) \ni Y = \begin{pmatrix} y_0 + y_3 & y \\ \bar{y} & y_0 - y_3 \end{pmatrix} \Rightarrow \det(Y) = \underline{\underline{y_0^2 - y_1^2 - \dots - y_3^2}} \quad M_{3,1}$$



Spin 9, F_4

↓

$g \rightarrow R(g) \neq \underline{\underline{UR(g)h^T}}$

	$SU(3)$	$Sp(2)$	$U(1)$
q_L	3	2	$1/6$
u_R	3	1	$2/3$
d_R	3	1	$-1/3$
l_L	1	2	$-1/2$
ν_R	1	1	0
e_R	1	1	-1

$(3, 2, \frac{1}{6}) \oplus (\bar{3}, 1, -\frac{2}{3}) \oplus (\bar{3}, 1, \frac{1}{3})$
 $\oplus \dots$
 $(\bar{3}, 2, -\frac{1}{6}) \oplus (3, 1, \frac{2}{3}) \oplus (3, 1, -\frac{1}{3})$

$(3, 2, \frac{1}{6}) \oplus (\bar{3}, 2, -\frac{1}{6}) \oplus \dots$
 $q_L \leftarrow q_L' \rightarrow q_L'^c$
 $(3, 2, +\frac{1}{6})$

$m \bar{q}_L'^c q_L$

Complex reps:

$$\rightarrow \text{---} \text{---} \text{---} \text{---} \text{---} A_n - su(n+1)$$

$$\times \text{---} \text{---} \text{---} \text{---} \text{---} B_n - so(2n+1)$$

$$\times \text{---} \text{---} \text{---} \text{---} \text{---} C_n - sq(n)$$

$$\rightarrow \text{---} \text{---} \text{---} \text{---} \text{---} D_n - so(2n)$$

$$su(n) \ (n \geq 3) \leftarrow su(5)$$

$$so(2n) \ (n \text{ odd}) \leftarrow so(10)$$

$$E_6 \leftarrow$$

$$\times g_2 \text{---}$$

$$\times f_4 \text{---}$$

$$\rightarrow e_6 \text{---}$$

$$\times e_7 \text{---}$$

$$\times e_8 \text{---}$$

$$su(4) \times su(2) \times su(2)$$

$$E_8 \rightarrow E_7 \rightarrow E_6 \rightarrow so(10)$$

$$\rightarrow su(5)$$

$$\rightarrow su(3) \times su(2)$$

Todoov + DV

$$\mathbb{CP}^2 \rightarrow (\mathbb{R} \otimes \mathbb{C})P^2, (\mathbb{C} \otimes \mathbb{C})P^2, (\mathbb{H} \otimes \mathbb{C})P^2, (\mathbb{O} \otimes \mathbb{C})P^2$$



F_4

$\nearrow \underline{\underline{E_6}}$

E_7

E_8

$$L_3(\mathbb{C}, \mathbb{C}) = \text{Der}(\mathbb{C}) \oplus \text{Der}(\mathbb{C}) \oplus A'_3(\mathbb{C} \otimes \mathbb{C}) = 78 \quad (E_6)$$



$$= \text{Tr}_i(\mathbb{C}) \oplus \text{Tr}_i(\mathbb{C}) \oplus (\mathbb{C} \times \mathbb{C}) \oplus (\mathbb{C} \otimes \mathbb{C}) \oplus (\mathbb{C} \otimes \mathbb{C})$$

$$(\mathbb{R} \otimes \mathbb{C})P^2 \xrightarrow{\text{fix a point}} \underline{\underline{\text{spin}(9)}} \begin{pmatrix} \mathbb{C} \\ \mathbb{C} \end{pmatrix} \leftarrow$$

$$(\mathbb{C} \otimes \mathbb{C})P^2 \xrightarrow{\text{fix a point}} \underline{\underline{\underline{U(2) \times \text{spin}(10)}}} \begin{pmatrix} \mathbb{C} \otimes \mathbb{C} \\ \mathbb{C} \otimes \mathbb{C} \end{pmatrix} \leftarrow$$

$$L_3(\mathbb{C}, \mathbb{D}) = \text{Tr}_L(\mathbb{C}) \oplus \text{Tr}_L(\mathbb{D}) \oplus (\mathbb{C} \otimes \mathbb{D}) \oplus (\mathbb{C} \otimes \mathbb{D}) \oplus (\mathbb{C} \otimes \mathbb{D})$$

$u(1) \times \text{spin}(10)$

$$E_6 = \text{so}(2) \oplus \text{so}(2) \oplus \text{so}(8) \oplus (\mathbb{C} \otimes \mathbb{D}) \oplus (\mathbb{C} \otimes \mathbb{D}) \oplus (\mathbb{C} \otimes \mathbb{D})$$

$$u(1) \oplus \text{spin}(10) = \left(\begin{array}{c|c} \text{so}(8)^\vee & -1 \\ \hline \mathbb{C} \otimes \mathbb{D} & \text{so}(2)^\vee \end{array} \right)$$

$$\left(\begin{array}{c} \mathbb{C} \otimes \mathbb{D} \\ \mathbb{C} \otimes \mathbb{D} \end{array} \right)$$

$$\underline{\underline{\text{so}(8)}} \rightarrow \left(\begin{array}{c} g_1^V \\ g_2^{S^+} \\ g_3^{S^-} \end{array} \right)$$

$$(\mathbb{C} \otimes \mathbb{D}) \rightarrow \text{spin}(10) \times u(1)$$

$$\text{Klein} \rightarrow G/H$$

$$M_{3,1} = \mathbb{P}^2 / \text{so}(3,1) \leftarrow (\mathbb{C} \otimes \mathbb{D}) \mathbb{P}^2 / (\text{spin}(10) \times u(1))$$

$$L_3(\mathbb{C}, \mathbb{O}) = \text{Tr}_L(\mathbb{C}) \oplus \text{Tr}_L(\mathbb{O}) \oplus (\mathbb{C} \otimes \mathbb{O}) \oplus (\mathbb{C} \otimes \mathbb{O}) \oplus (\mathbb{C} \otimes \mathbb{O})$$

$u(1) \times \text{spin}(10)$

$$E_6 = \text{so}(2) \oplus \text{so}(2) \oplus \text{so}(8) \oplus (\mathbb{C} \otimes \mathbb{O}) \oplus (\mathbb{C} \otimes \mathbb{O}) \oplus (\mathbb{C} \otimes \mathbb{O})$$

$$u(1) \oplus \text{spin}(10) = \left(\begin{array}{c|c} \text{so}(8)^\leftarrow & -1 \\ \hline \mathbb{C} \otimes \mathbb{O} & \text{so}(2)^\leftarrow \end{array} \right)$$

$$\left(\begin{array}{c} \mathbb{C} \otimes \mathbb{O} \\ \mathbb{C} \otimes \mathbb{O} \end{array} \right)$$

$$\underline{\underline{\text{so}(8)}} \rightarrow \left(\begin{array}{c} g_1^V \\ g_2^{S^+} \\ g_3^{S^-} \end{array} \right)$$

$$s_{10} = \left(\begin{array}{c} \mathbb{C} \otimes \mathbb{O} \\ \mathbb{C} \otimes \mathbb{O} \end{array} \right)$$

$$(\mathbb{C} \otimes \mathbb{O}) \rightarrow \text{spin}(10) \times u(1)$$

$$\text{Klein} \rightarrow G/H$$

$$M_{3,1} = \mathbb{P} / \text{so}(3,1)^\leftarrow$$

$$(\mathbb{C} \otimes \mathbb{O}) \mathbb{P}^2 = E_6 / \text{spin}(10) \times u(1)$$