Title: ISCOs in AdS/CFT

Speakers: David Berenstein

Series: Quantum Fields and Strings

Date: April 07, 2020 - 2:30 PM

URL: http://pirsa.org/20040008

Abstract: I describe a novel way to produce states associated to geodesic motion for classical particles in the bulk of AdS that arise from particular operator insertions at the boundary

at a fixed time. When extended to black hole setups, one can understand how to map back the geometric information of the geodesics back to

the properties of these operators. In particular, the presence of stable circular orbits in global AdS are analyzed. The classical Innermost Stable Circular Orbit

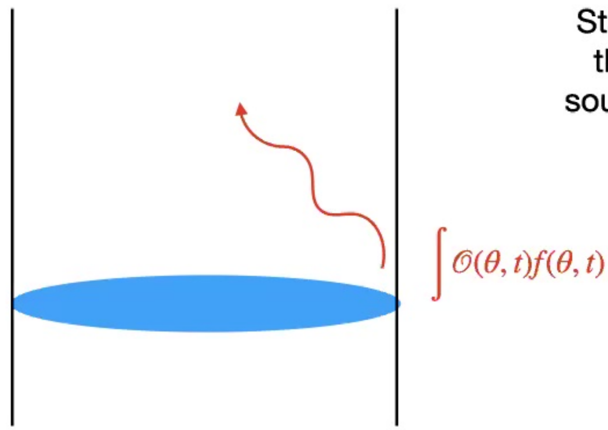
(ISCO) play an important role separating metastable state excitations from those that quickly fall in the horizon of black hole. In the dual CFT, this metastability effect must be a non-perturbative effect due to the curvature on the boundary.

ISCOs in AdS/CFT

David Berenstein
UCSB

Based on joint work with Joan Simon, arXiv:1910.10227
Work in progress with Ziyi (Frazier) Li and Joan Simon

Research supported by U.S. DEPARTMENT OF ENERGY | Office of Science



Standard AdS/CFT: we perturb the boundary and we have a source for waves that propagate into the interior

Part of the GKPW dictionary:
the source f is the boundary condition for the gravity theory.

HKLL prescription

We can also construct local fields in the bulk

Leading order

$$\phi_{bulk}(\theta, t, \rho) = \int K(\theta, t, \rho; \theta', t')\mathcal{O}(\theta', t')$$

Gets non-linear corrections to higher order (a power series in $\hbar$).
It also depends on the background metric in the bulk.

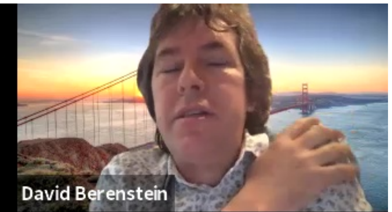
Problem

Operators can have dimension $\Delta > d$

They are irrelevant perturbations: badly behaved in the UV
Gravity side: infinite back-reaction.
Suggests HKLL and these insertions are only well defined in a perturbation series sense.

It is hard to understand how localized particles and energy can arise in the bulk:
one needs to interfere the incoming pulse
to generate localized excitations and in the process
kill the infinite back reaction to all orders.

What is the CFT dual to geodesics in the bulk?



This is really a question about preparation of states:
how do we make the CFT prepare initial conditions for geodesics in a way that does not suffer from infinities (non-perturbatively)?

If you’re asking this question, you might as well ask:
what geodesics are special in black hole backgrounds?

OR, WHAT IS THE DUAL TO ISCO’s in AdS/CFT?

ISCO: Innermost stable circular orbit

Rest of talk

- Properties of states created by operators at fixed time.
- Simple regularization and the physics of the cutoff.
- Saddle point computations and passing to a semiclassical limit in AdS.
- Adding black holes to the mix
- ISCO's and their dual physics
- Conclusion

Idea: Start with ground state in the CFT

Consider

$$\int d\Omega \mathcal{O}_\Delta(\theta, t)|0\rangle$$

This is a sum over descendants of the operator $\mathcal{O}$ **at a fixed time t**.
The integral over the angle makes it spherically symmetric: an s-wave

We can project on the individual states to obtain matrix elements.
One can use Euclidean two point functions to extract this information.

$$A_{\Delta+2k} \simeq \langle \Delta + 2k | \int d\Omega\, \mathcal{O}_\Delta(\theta, t) \rangle 0$$

$(\partial_\mu \partial^\mu)^k \mathcal{O}$

Complete list of spherically symmetric descendants (CFT in d>2)

We computed these: D.B, A. Miller arXiv:1406.4142

Result is finite

$$|A_{\Delta+2k}|^2 \propto \frac{\Gamma[k+\Delta]\Gamma[\Delta-\frac{d}{2}+k+1]}{k!\,(\Gamma[1+\Delta-\frac{d}{2}])^2\Gamma[k+\frac{d}{2}]}$$

The total state generated is not normalizable.

$$\sum_{k=0}^{\infty} |A_{2k+\Delta}|^2 = \infty$$

Using Stirling's approximation

$$|A_{\Delta+2k}|^2 \propto \exp((2\Delta - d)\log(k) + \mathcal{O}(1))$$

Observation: Amplitudes grow polynomially for all operator dimensions

Convergence requires

$$\sum_{k=1}^{\infty} k^{2\Delta-d} < \infty$$

$$2\Delta < d-1$$

Window where it works is too small.
A classical particle is a very massive particle that will have large dimension.

Any polynomial growth can be cut by an exponential decay:
we modify the amplitudes accordingly.

$$A_{\Delta+2k,\epsilon} \simeq \exp(-(2k+\Delta)\epsilon)\langle \Delta + 2k | \int d\Omega\, \mathcal{O}_\Delta(\theta, t) | 0 \rangle$$

Equivalent to conjugating by the Hamiltonian in Euclidean time
to regularize the operator insertions.

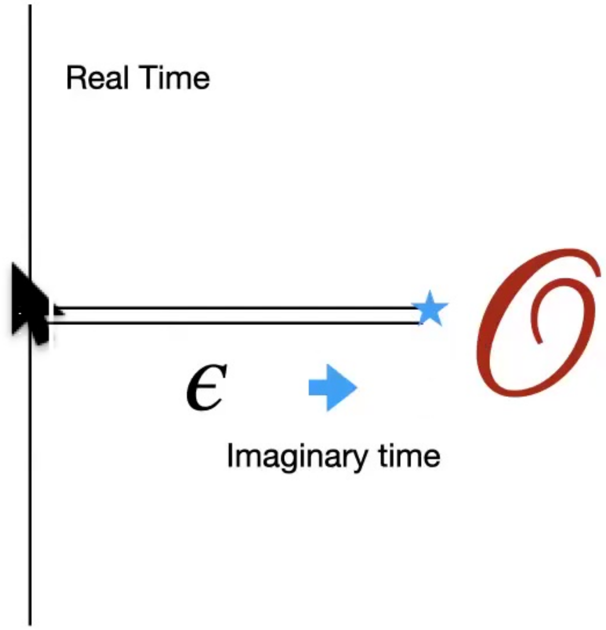
$$\mathcal{O}_{\Delta,\epsilon}(\theta, t) = \exp(-\epsilon H)\, \mathcal{O}_{\Delta}(\theta, t)\exp(+\epsilon H)$$

Takayanagi et al. have considered these also for local quenches
arXiv:1302.5703, arXiv:1605.02835, arXiv:1704.00053.

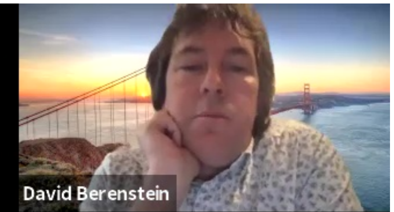
Upshot: the norm and the energy are now **finite**,
but depend on the regulator parameter ϵ.

Works for operators of arbitrarily high dimension.

Want to explore the physics of this regulator, especially for large Δ.



Flow of time



Saddle point: what is the most likely excited state?

Hamiltonian formalism: we computed amplitudes already.

Use Stirling's approximation

$$|A_{2k+\Delta,\epsilon}|^2 \simeq \exp\left[(2\Delta - d)\log(k) - 2\epsilon(2k + \Delta)\right]$$

Compute the saddle in the amplitude for k

$$k_{\max} = \frac{2\Delta - d}{4\epsilon}$$

$$E \simeq 2k_{max} + \Delta$$

Energy obviously goes to infinity when $\epsilon \to 0$

Variance

$$\partial_k^2 \log(|A_{\Delta+2k,\epsilon}|^2) = -\frac{2\Delta - d}{k^2} \simeq -\frac{1}{\sigma^2}$$

$$\frac{|\delta k|^2}{k_{\max}^2} \simeq \frac{1}{2\Delta - d}$$

It is small for large dimension operators (there is an $\hbar$ hidden in the expression: easier to see in AdS).
Can think of insertion at fixed classical energy.

Semiclassical in AdS

Relation between mass and dimension

$$m = \sqrt{\Delta(\Delta - d)} \simeq \Delta - d/2 + \mathcal{O}(1/\Delta)$$

Restoring units

$$m_{classical} = \hbar m R_{AdS}^{-1}$$

Classical m is equivalent to large dimension!
Fluctuations in energy are of order $\hbar$
It is appropriate to think about this process semi-classically in AdS.

Because s-wave, there is no angular momentum.
Classically, we need a radial geodesic
to describe the motion of the point particle, this has a simple action.

$$S = m \int d\tau \sqrt{\cosh^2 \rho - \dot{\rho}^2}$$

In global coordinates

$$ds^2 = -\cosh^2 \rho \, dt^2 + d\rho^2 + \sinh^2 \rho \, d\Omega^2_{d-1}$$

$$t = \tau$$ **Gauge**

Energy match

$$E = m\,\frac{\cosh^2\rho}{\sqrt{\cosh^2\rho - \dot{\rho}^2}} = m\cosh(\rho_*)$$

Defines a radial position from turning point of trajectory.

- All amplitudes computed in CFT are real
- This is consistent with a configuration with time reversal symmetry.
- The only classically time symmetric configuration is the turning point of radial trajectory (all velocities need to vanish).
- Upshot: the energy is injected at the turning point of radial geodesic.

The insertion at the radial position is independent of the dimension

$$\cosh \rho_\star = \frac{1}{\epsilon} + 1$$

The regulator has a geometric interpretation.
Same idea: Takayanagi et al.

Radial fluctuations

$$\delta\rho_\star \sinh \rho_\star = \frac{\delta E}{m} \simeq \frac{1}{m}\frac{E}{\sqrt{m}}$$

We get that the particle is very well localized on distances that are small compared to the AdS radius.

$$1 \gg \delta\rho_\star \simeq \frac{1}{\sqrt{m}} \gg \lambda_{\textbf{compton}} \simeq \frac{1}{m}$$

It can also be considered at rest at insertion point (much much larger than Compton wavelength)

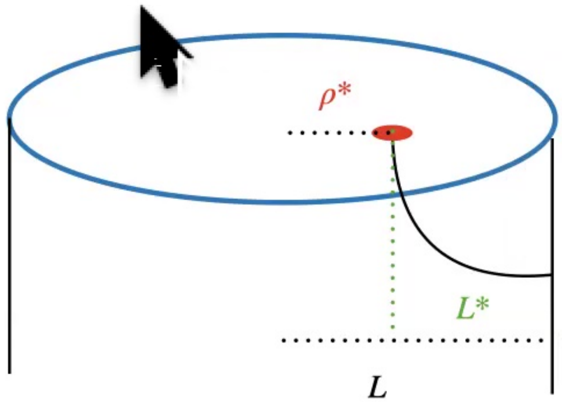
Side note

$$\delta x \simeq \frac{1}{\sqrt{m}}$$

Is the spatial uncertainty of point particle of mass m inserted at bottom of global AdS in the ground state.

Result involves imaginary time Hamiltonian evolution: go to path integral?

Tunneling!



Result

$$A_{2k_{max}+\Delta} \simeq \exp(m\rho_\star)$$

Why this is like tunneling

$$-\rho^* \sim L^* - L$$

Final answer is simple:

$$L_\star = -\log(\sinh \rho_\star) \simeq -\rho_\star \equiv \tilde{L}_\star$$

Takayanagi et al did similar calculations in Poincare AdS coordinates.

To add angular momentum

Remember that an operator insertion is

$$\mathcal{O}_\Delta(x) = \sum \frac{1}{[a]!} \partial^{[a]} \mathcal{O}_\Delta(0)$$

a generating series for all descendants.

We can project onto states of fixed angular momentum

Insert favorite spherical harmonics.

$$A_{\Delta+2k+\ell} \simeq \langle \Delta + 2k + \ell \,|\, \int d\Omega\, Y_\ell(\Omega)\mathcal{O}_\Delta(\Omega, t)\,|\,0\rangle$$

A direct computation (D.B., J.S.) leads to similar physics as before.

$$\left|A_{\Delta+2k+\ell}\right|^2 \propto \frac{\Gamma[\Delta + k + \ell]\Gamma[\Delta + k + 1 - d/2]}{k!\,\Gamma[\ell + k + d/2]\left(\Gamma[\Delta + 1 - d/2]\right)^2}$$

Fitzpatrick, Kaplan arXiv:1104.2597
Terashima: arXiv:1710.07298
Cotaescu: gr-qc/9903029

Use Stirling

$$\log |A_{\Delta+2k+\ell,\epsilon}|^2 \simeq (\Delta - \frac{d}{2})\log(k+\ell) + (\Delta - \frac{d}{2})\log\ k - 2\epsilon(\Delta + 2k + \ell)$$

We get a modified equation. We need to take scaling limit in angular momentum: it has to compete with k.

$$\frac{1}{k_{\mathbf{max}}+\ell} + \frac{1}{k_{\mathbf{max}}} \sim \frac{4\epsilon}{\Delta - d/2}.$$

AdS dual

Need to solve for geodesics with fixed angular momentum and energy.
Use normalized dimensionless constants

$\ell = J/m, e = E/m$

We get in standard global AdS coordinates

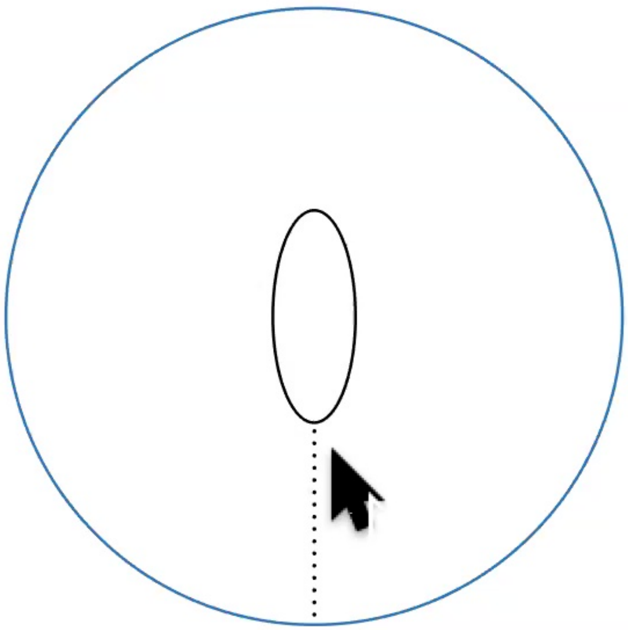
$$\dot{\rho}^2 = \frac{\cosh^4 \rho}{\hat{E}^2} - \cosh^2 \rho - \frac{\cosh^2 \rho}{\tanh^2 \rho} \frac{\ell^2}{e^2}$$

We now get two turning points for geodesic

$$2\cosh^2\rho_{\pm\star} = e^2 - \ell^2 + 1 \pm \sqrt{\left(e^2 - \ell^2 + 1\right)^2 - 4e^2}$$

Natural injection of energy is at aphelion of orbit in AdS.
(Continuity with vanishing angular momentum)

Similar uncertainty in radial direction



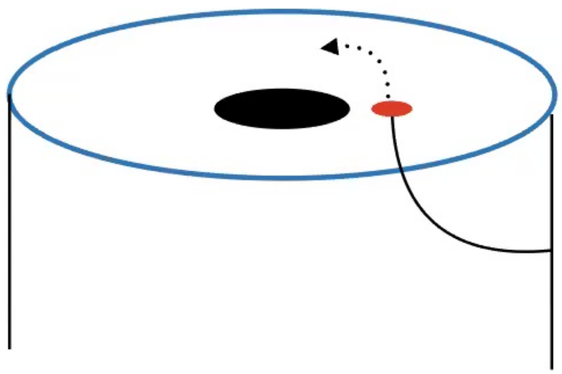
Timelike geodesic
with angular momentum.

Tunneling must happen to Aphelion: farthest point from origin in geodesic.

- The insertions are at an instant of time (Euclidean continuation)
- No integrations over time, in contrast to HKLL.
- Produce finite energy particle excitations at sub AdS scales.
- Non-perturbatively well defined (regulator is physical and geometric).
- Good for initial conditions: Gaussian localized.

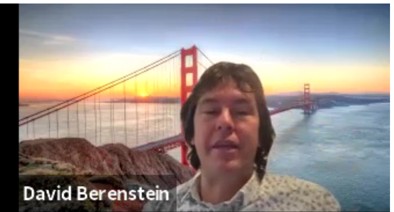
In the presence of a back hole

Do classical analysis first: find simplest circular Rotating geodesics in AdS



$$\int Y_{\ell,m}\mathcal{O}(-\epsilon)$$

Work in progress with Z. Li, J. Simon



Change coordinates for AdS_{d+1} Schwarzschild

$$ds^2 = -H(r)dt^2 + H^{-1}(r)dr^2 + r^2 d\Omega^2$$

$$H(r) = r^2 + 1 - \frac{2M}{r^{d-2}}$$

Classical analysis

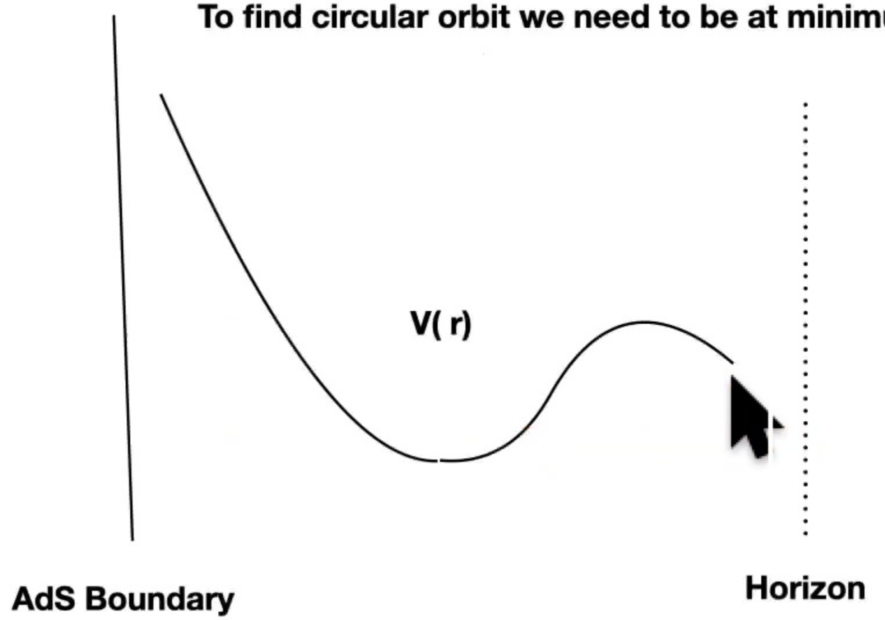
Impose angular momentum conservation + energy conservation in geodesic equations. We get the following constraint:

$$e^2 = \dot{r}^2 + 1 + \ell^2 + \frac{\ell^2}{r^2} + r^2 - \frac{2M\ell^2}{r^d} - \frac{2M}{r^{d-2}}$$

Of the form

$$e^2 = \dot{r}^2 + V(r)$$

To find circular orbit we need to be at minimum of V(r)



And the final answer for angular momentum (parametrized by r) is:

$$\ell^2 = r^4 \frac{(r^d + (d-2)M)}{r^d - dMr^2}$$

Circular orbits stop existing when denominator vanishes

$$r^d - dMr^2 = 0$$

This is the light-ring of the black hole: same as without AdS curvature.
These are unstable.

Stable when V”(r)>0

ISCO if V”(r)=0 (not pretty in general)

We get a scaling for the radius

$$r \sim M^{1/(d-2)}$$

MOST INTERESTING IN LARGE M LIMIT: high temperature in dual CFT

Check scaling of light ring and use in **r, l**

$$r^2 \sim \ell^2/r^2 \simeq M^{2/(d-2)}$$

$$M\ell^2 r^{-d} \simeq M^{4/(d-2)}$$

$$M r^{-(d-2)} \simeq M^0$$

Throw away terms in the potential that are the most subleading.

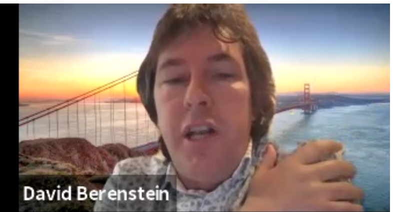
We can then find an analytical solution to the ISCO

$$r_{ISCO} = \left(\frac{d(d+2)M}{4}\right)^{1/(d-2)}$$

Notice, the horizon radius scales as $r_H \simeq M^{1/d}$

$$r_{ISCO} \gg r_H$$

These orbits are pushed very far away from the horizon.



Converting to temperature

$$e, \ell \simeq M^{2/(d-2)} \simeq T^{\frac{2d}{d-2}}$$

Also in **Festuccia, Liu: arXiv:0811.1033**,
study of semiclassical normal modes. (Hard to read in their paper)

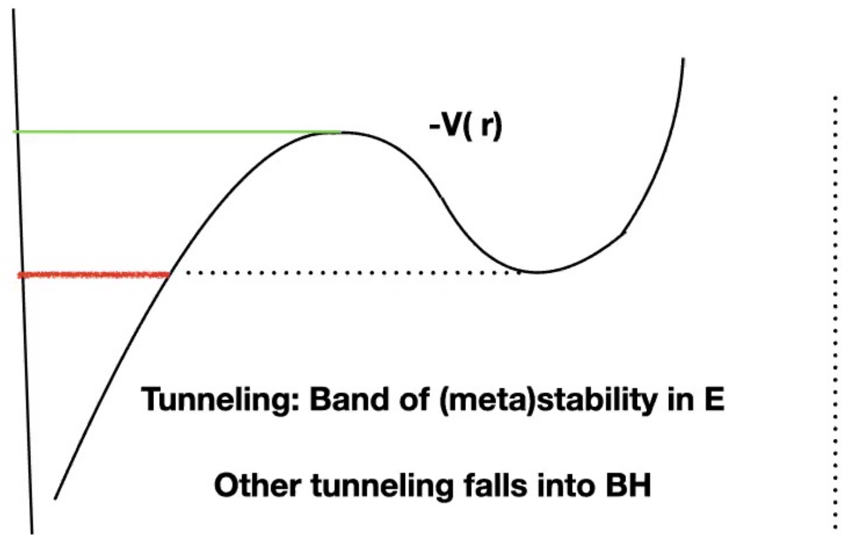
Thermal excitations scale differently:

$$e, \ell \sim T$$

Dual excitations to stable geodesics are way more energetic than thermal: one can say the have the wrong units in the boundary.

How to fix units

- We need another scale that is not T in the CFT
- This must be the curvature of the spatial boundary in CFT
- The effect is absent in BTZ (2+1): no stable geodesics.


-V(r)
Tunneling: Band of (meta)stability in E
Other tunneling falls into BH

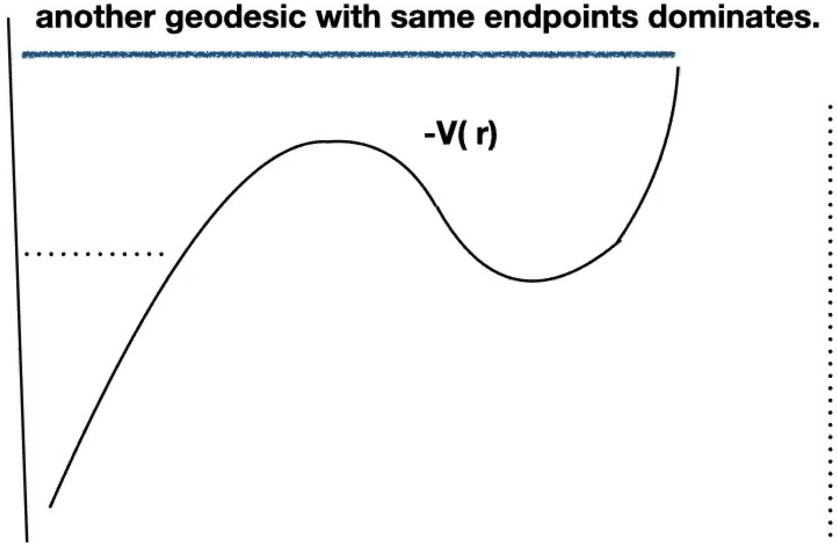

David Berenstein

$r(\ell)$ **Not single valued**

More to the point: more than one trajectory for fixed Euclidean time.

$e(\epsilon)$ **Not single valued either: at ISCO the Euclidean time of trajectory goes to infinity.**

Tunneling above potential hump not allowed: another geodesic with same endpoints dominates.



Predictions for CFT

- Band of metastable orbits
- Translates to a band of regularized operators that produce tunneling into the appropriate geodesics.
- The decay in these orbits is exponentially suppressed.
- Must be non-perturbative in CFT (the answer is not polynomial). The effect must be sourced by background curvature in CFT

$$E = \Delta e$$

$$J = \Delta \ell$$

Geodesic band depends on a Euclidean time ϵ,
but not on dimension of operator.

Conclusions

- Set of regularized operators that source particles in bulk geodesics directly.
- They tunnel particles into the geodesics. With or without angular momentum: can choose the geodesic.
- Their definition does not depend on knowing that we are in a black hole background (depends on Hamiltonian, not on state).
- Complementary to HKLL

- Interesting predictions for black hole backgrounds.
- Band of metastable operators
- There are also bands of “inner geodesics” that are inaccessible.

To do

- Next step: understand complex tunneling geodesics better.
- Rotating black holes
- Semiclassical quantization