

Title: TBA

Speakers: Bianca Dittrich

Series: Quantum Gravity

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Recording...



Quantum Geometry

vs

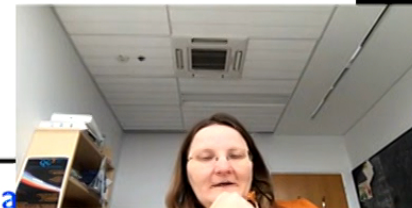
Quantum Gravity

Bianca Dittrich

with

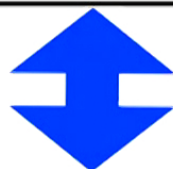
**Seth Asante,
Hal Haggard.**

PI-Cyberspace March 19, 2020



Quantum
Geometry

Discrete (locally independent) area
Resulting from symplectic geometry
of (extended) phase space for geometries.



Quantum
Gravity

Dynamics.

Input: Discrete (locally independent) area spectra.

What dynamics can we expect from, e.g. a path integral?
Is this dynamics consistent with GR dynamics?



Discrete area spectra: a robust feature

(Kinematical level, Euclidean signature)



- LQG: From a rigorous (continuum) quantization of phase space of geometries⁺.
Via extended phase spaces (gauge theory, higher gauge theory):
Only way so far to deal with triangle inequalities. But issues with reduction.
- But also heuristically from Regge calculus: $\{A_l(l), \theta_l\} = 1$.
Suggest an (asymptotically) equidistant spectrum for the areas.
But possibly not 'locally independent'.

↙ angle
- (weak) holographic principle
- (black hole) entropy counting
- Ryu Takayanagi conjecture: Areas as more fundamental variables?
- boundary observable algebras

Area spectra:

$$A_l = \gamma \sqrt{j(j+1)} \ , \quad j = 0, \frac{1}{2}, \dots$$

$$A_l = (\gamma)(n + \frac{1}{2}) \ , \quad n = 0, 1, \dots$$





Minimal input: Discrete (locally independent) area spectra.

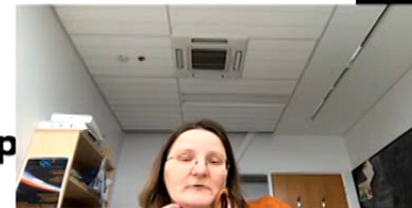
What dynamics can we expect from, e.g. a path integral?
Is this dynamics consistent with GR dynamics?

To be able to say something about the dynamics:
Build simple models (as simple as possible).
Concentrate on key issue.



Frank opinion slide (No discussion!)

Why are spin foam amplitudes so    comp



At least five different models.

No agreement on flatness problem after 10 years (even for simplest example: three 4-simplex configuration).

- Historically: Hoped to be an exact quantization of quantum gravity.
- However: Models with local amplitudes break diffeomorphism symmetry, and are triangulation dependent. Do not impose the Hamiltonian and diffeomorphism constraints. Without this, no criteria to resolve the many discretization ambiguities. No agreement (on whether or) how to do sum over triangulation. (You should not.) Do not deliver on key criteria for a background independent theory of quantum gravity.
- Rather: Spin foams can serve as starting point for a dynamics constructed via renormalization flow. Flow from IR (known) to UV (unknown): which UV amplitudes are consistent with IR dynamics? There is no washing out of UV physics.

[BD 2012, 2014, ...
+ Bahr, Cunningham,
Delcamp,
Martin-Benito
Steinhaus, ...]

For coarse graining and everything else:

Need a model **you can calculate and reason with. Otherwise it is a waste of time.**

Make sure it delivers on IR dynamics.

Many details will be fixed by requirement of (coarse graining) consistent amplitudes.

Frank opinion slide (No discussion!)

Why are spin foam amplitudes so  **comp**



At least five different models.

No agreement on flatness problem after 10 years (even for simplest example: three 4-simplex configuration).

Necessary (?) part of the complication:

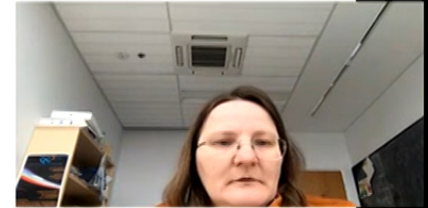
Quantum geometry techniques arising from (higher) gauge reformulations of gravity:
to be able to quantize phase spaces with a very complicated topology.

[Ashtekar, Rovelli, Smolin, Isham, Lewandowski, ... BD, Freidel, Krasnov, Livine, Speziale, Ryan, ...]
[Quantum Geometry via BFCG: Asante, BD, Girelli, Riello, Tsimiklis 2019]

Remark: Here we happen to choose (unconsciously) locality over the non-local restrictions imposed by geometricity.



Overview



1. Semi-classical limit and classical actions for discrete GR
2. Quantization: imposing discrete area spectrum and GR dynamics (as good as LQG can)
3. Features: spin foam model with resolution of the flatness problem (as good as LQG can)
4. Two different choices for quantization: phenomenological consequences



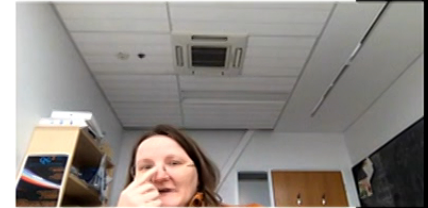
Semi-classical limit of spin foams

All models involve intricate recoupling symbols and give for the one-simplex amplitude in the limit of large lengths:

$$\mathcal{A}_\sigma = \cos(\text{Regge action}) + \text{degenerate configurations}$$

[lots of works]

- but models differ in which variables are used: will be essential
- cosine: from sum over (tetrad) orientations
- degenerate configurations: from first order/ gauge reformulation possibly dominating, highly problematic



Semi-classical limit of spin foams

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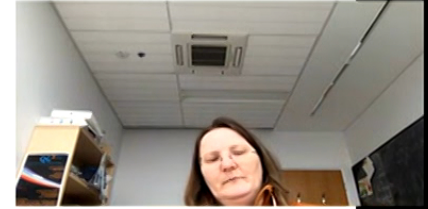
[lots of works]

- but models differ in which variables are used: will be essential
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- ~~degenerate configurations: from first order/ gauge reformulation possibly dominating highly problematic~~

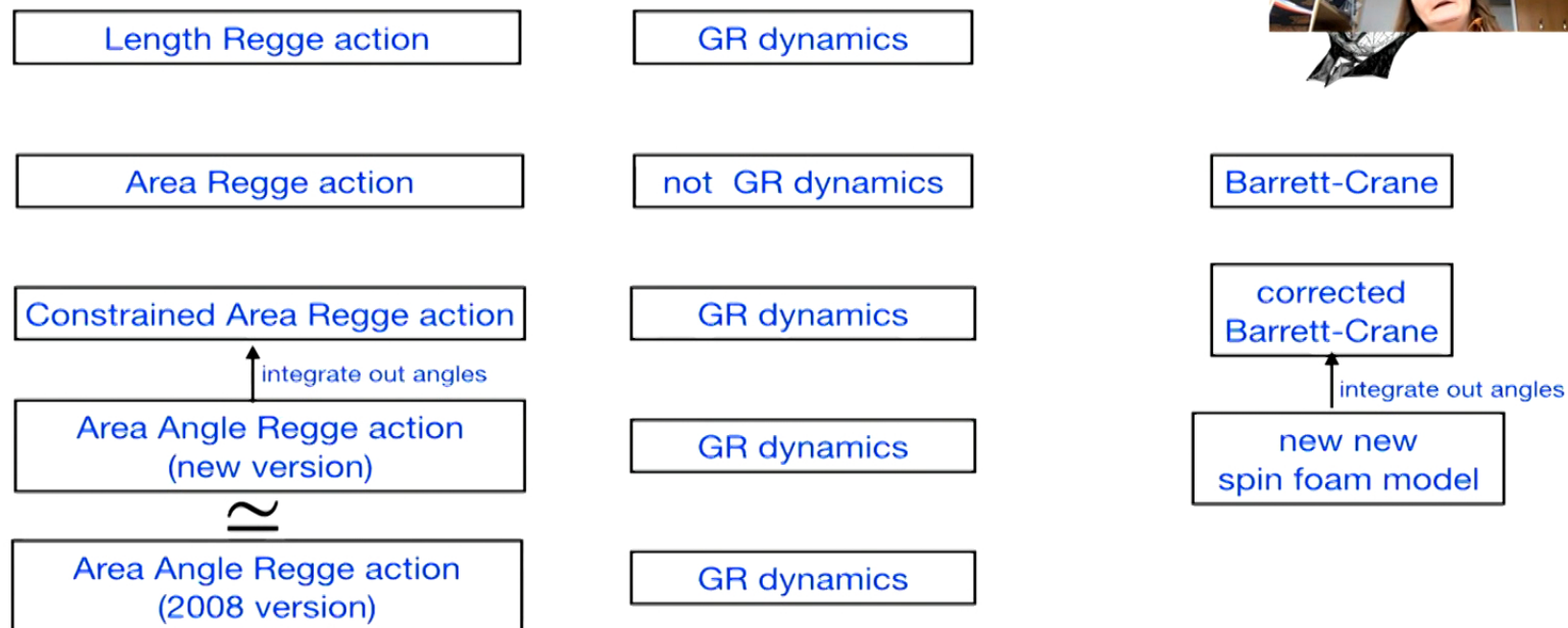
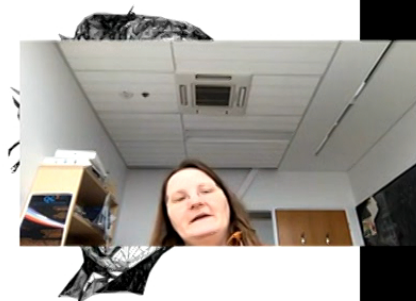
- To be able to reason about the model and to calculate something we work with:

$$\mathcal{A}_\sigma = \exp(i \text{Regge action})$$

(Could also use the cosine.)



Regge calculus: discretization of geometries



Note: Historic inconsistency in naming. Area Angle Regge actions are also with constraints.

Length Regge calculus — a discretization of GR

$$S_{\text{LR}} = \sum_{t \in \text{bdry}} \pi A_t(l) + \underbrace{\sum_{t \in \text{bulk}} 2\pi A_t(l)}_{S_t(l)} - \sum_{\sigma} \underbrace{\sum_{t \supset \sigma} A_t(l) \theta_t^{\sigma}(l)}_{S_{\sigma}(l)}$$

dihedral angle



Equations of motion (Einstein equations):

$$\sum_{t \supset e} \frac{\partial A_t(l)}{\partial l_e} \epsilon_t(l) = 0$$

Deficit angle = curvature:

$$\epsilon_t(l) = 2\pi - \sum_{\sigma \supset t} \theta_t^{\sigma}(l)$$

Allow for solutions
with curvature.

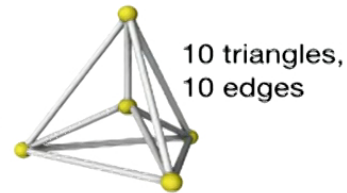
But we want to impose discrete area spectrum: need area variables.

Area Regge calculus — imposing flatness, featur

A 4-simplex has 10 triangles and 10 edges.

We can invert lengths for areas: $L_e^\sigma(a_t)$.

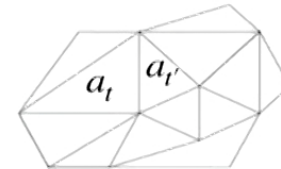
(Modulo discrete ambiguities, ignored for this talk.)



Can write action contributions as functions of areas:

$$S_t(a_t) = 2\pi a_t$$

$$S_\sigma(a_t) = S_\sigma(L_e^\sigma(a_t))$$



$$S_{\text{AR}}(a_t) = \sum_{t \in \text{bulk}} S_t(a_t) + \frac{1}{2} \sum_{t \in \text{bdry}} S_t(a_t) - \sum_{\sigma} S_\sigma(a_t) \sim \sum_{t \in \text{blk}} a_t \epsilon_t(a_t)$$

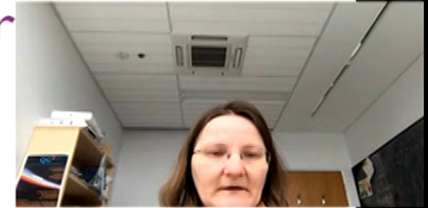
Equation of motion:

$$\epsilon_t(a_t) = 0$$

Solutions are flat.
Obviously not the dynamics of GR.

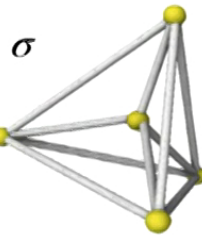
[Barrett, Roček Williams 1997,
Asante, BD, Haggard 2018]

Quantization of Area-Regge calculus (most likely) given by Barrett Crane model.
We will see how to 'repair' it!

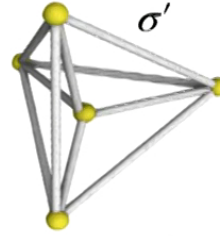


Constrained Area Regge calculus

10 triangles,
10 edges



σ

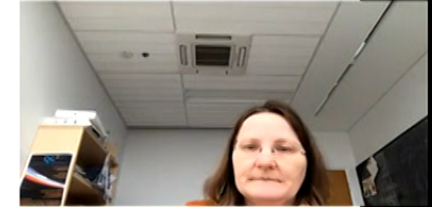


σ'

10 triangles,
10 edges

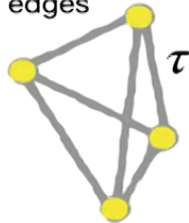


16 triangles \rightarrow 16 area variables
14 edges \rightarrow 14 length variables



Additional degrees
of freedom!

4 triangles,
6 edges



τ

When we glue we match for the
shared tetrahedron:

-6 length variables: determines geometry of tetrahedron

-but only 4 area variables: underdetermines geometry of tetrahedron

Need to match two further geometric quantities:

Two 3D dihedral angles (at non-opposite edges)

Other choices are possible.

Gluing Constraints:

$$\Phi_{e_1}^{\tau, \sigma}(a_t) = \Phi_{e_1}^{\tau, \sigma'}(a_t)$$

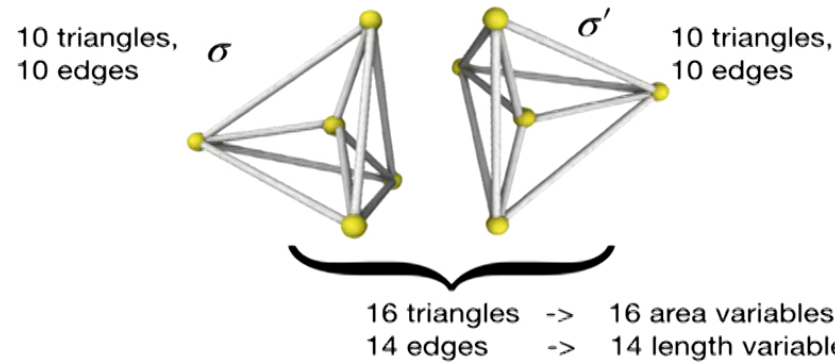
$$\Phi_{e_2}^{\tau, \sigma}(a_t) = \Phi_{e_2}^{\tau, \sigma'}(a_t)$$

10 areas from σ

10 areas from σ'

For the two-simplex configuration:
Reduce from 16 to 14 independent variables.

Constrained Area Regge calculus



$$S_{\text{CAR}}(a_t, \lambda_{\tau}^{e_i}) = S_{\text{AR}}(a_t) + \underbrace{\sum_{\tau=\sigma \cap \sigma'} \sum_{e_i \in \tau} \lambda_{\tau}^{e_i} \left(\Phi_{e_i}^{\tau, \sigma}(a_t) - \Phi_{e_i}^{\tau, \sigma'}(a_t) \right)}_{\text{NOT additive over 4-simplices. Depends on all 16 areas of the two 4-simplices.}}$$

NOT additive over 4-simplices.
Depends on all 16 areas of the two 4-simplices.

Equations of motion: equivalent to length Regge calculus.

BUT: Apart from keeping areas fixed on boundary,

need to also fix 3D dihedral angles: functions of areas (one step) away from the 3D boundary.

NO strict 3D boundary/ canonical formulation: need to know areas of all the 4-simplices glued to boundary.

(Constrained) Area Angle Regge calculus V.I

Let us add as variables two 3D dihedral angles $(\phi_{e_1}^\tau, \phi_{e_2}^\tau)$ for each tetrahedron τ .
(We can choose to take all six angles, but would have to impose closure/ Gauss constraints.)

Adding more variables we have to add more constraints:

For each pair $\tau \subset \sigma$:

$$\phi_{e_1}^\tau = \Phi_{e_1}^{\tau, \sigma}(a_t)$$

$$\phi_{e_2}^\tau = \Phi_{e_2}^{\tau, \sigma}(a_t)$$

10 areas from σ

Impose gluing constraints: $\Phi_{e_1}^{\tau, \sigma}(a_t) = \Phi_{e_1}^{\tau, \sigma'}(a_t)$

$$\Phi_{e_2}^{\tau, \sigma}(a_t) = \Phi_{e_2}^{\tau, \sigma'}(a_t)$$

Action (leading to length Regge EOM):

Additive over 4-simplices.

$$S_{AAR1}(a_t, \phi_{e_i}^\tau, \lambda_{\tau, \sigma}^{e_i}) = S_{AR}(a_t) + \sum_{\sigma} \sum_{\tau \subset \sigma} \lambda_{\tau, \sigma}^{e_i} (\Phi_{e_i}^{\tau, \sigma}(a_t) - \phi_{e_i}^\tau)$$

Admits
boundary/ canonical
formulation.

→
solve for 3D angles

$$S_{CAR}(a_t, \lambda_{\tau}^{e_i})$$

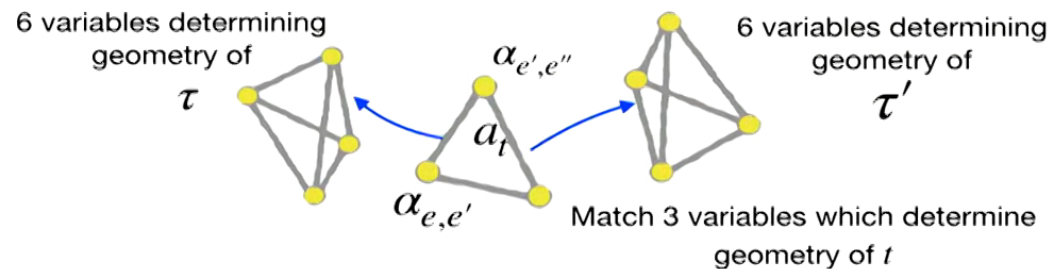
Reproduce Constrained Area Regge action.



(Constrained) Area Angle Regge calculus V.2 (b)

[BD, Speziale 2008] The same as before but rather use a different (more local) form
For each triangle $t \subset \tau \cap \tau'$ with $\tau, \tau' \subset \sigma$ we need two constraints:

$$\text{For two 2D angles in } t: \quad \alpha_{e,e'}^{\tau}(a_t, \phi_{e_i}^{\tau}) = \alpha_{e,e'}^{\tau'}(a_t, \phi_{e_i}^{\tau'})$$



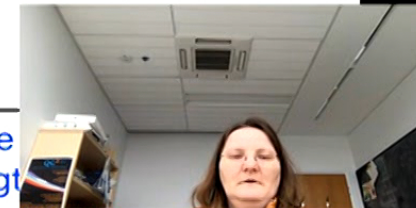
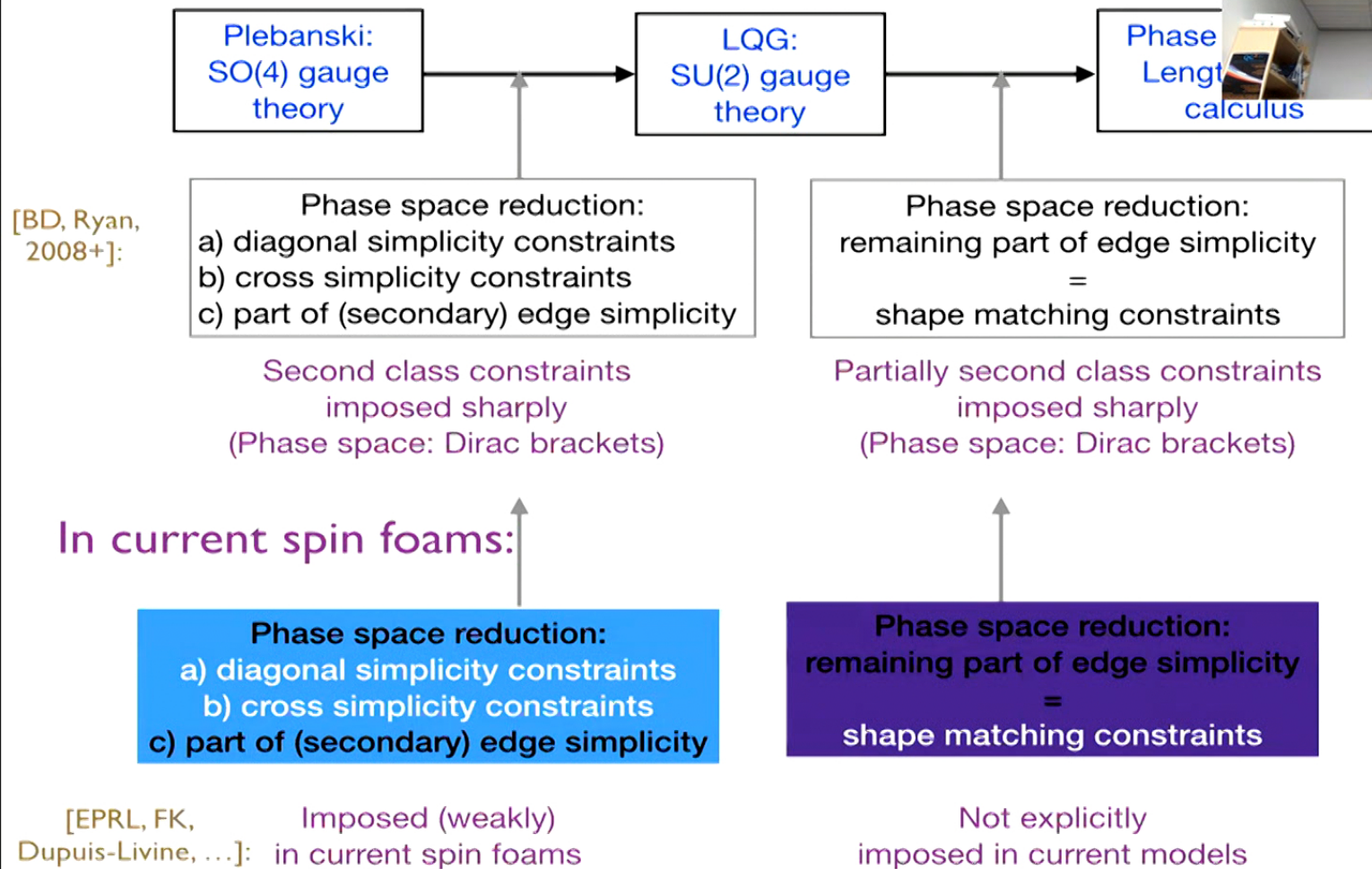
Constraints are equivalent to previous set.
(Reduce 20 to 10 variables on a 4-simplex.)

This form of the constraints can be restricted to 3D hypersurface: Shape matching constraints.
Arise from (part of the) secondary simplicity constraints.
Need to be imposed on LQG to obtain Length Regge.

[BD, Ryan 2009-12]

See [BD, Speziale 2008] for the full action: equivalent to length Regge.

Orientation for the spin foam fan:



Path integral



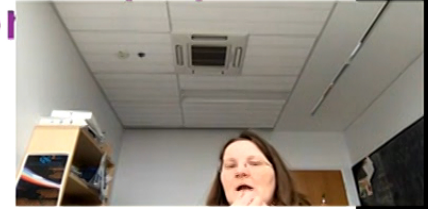
To impose discrete area spectrum:

Use areas as variables.

Simplest choice: Constrained Area Regge Formulation



Path integral for Constrained Area Regge For



$$Z = \sum_{a_t \in \mathbf{S}_a} \mu(a_t) \prod_t \mathcal{A}_t \prod_{\sigma} \mathcal{A}_{\sigma} \prod_{\tau} G_{\tau=\sigma \cap \sigma'}^0(\{a_t\}_{t \in \sigma \cup \sigma'})$$

area spectrum \uparrow \mathbf{S}_a Triangle weight $\mathcal{A}_t = \exp(\frac{i}{\hbar} 2\pi a_t)$ Simplex amplitude $\mathcal{A}_{\sigma} = \exp(\frac{i}{\hbar} S_{\sigma}(a_t))$ Couples two 4-simplices

$$G_{\tau=\sigma \cap \sigma'}^0(\{a_t\}_{t \in \sigma \cup \sigma'}) = \begin{cases} 1 & \text{if constraints are satisfied.} \\ 0 & \text{if not.} \end{cases}$$

$$\begin{aligned} \Phi_{e_1}^{\tau, \sigma}(a_t) &= \Phi_{e_1}^{\tau, \sigma'}(a_t) \\ \Phi_{e_2}^{\tau, \sigma}(a_t) &= \Phi_{e_2}^{\tau, \sigma'}(a_t) \end{aligned}$$

Model does not work.

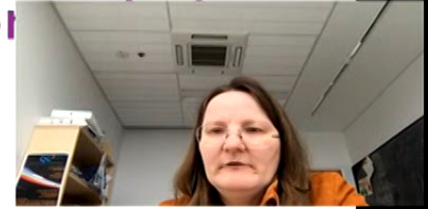
Follows also from constraining (ala spin foams) higher gauge (KBF) model for quantum flat space time.

[Mikovic, Vojinovic]

[Korepanov, Baratin, Freidel, Asante, BD, Girelli, Riello, Tsimiklis;]

Path integral for Constrained Area Regge For

Model does not work.



$$G_{\tau=\sigma\cap\sigma'}^0(\{a_t\}_{t\in\sigma\cup\sigma'}) = \begin{cases} 1 & \text{if constraints are satisfied.} \\ 0 & \text{if not.} \end{cases}$$

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$$\Phi_{e_1}^{\tau,\sigma}(a_t) = \Phi_{e_1}^{\tau,\sigma'}(a_t)$$

$$\Phi_{e_2}^{\tau,\sigma}(a_t) = \Phi_{e_2}^{\tau,\sigma'}(a_t)$$

For discrete (approximately equidistant) area values this gives diophantine like equations.

Almost never satisfied. Leads to huge reduction in density of states.

Solution: Weaken the constraints. But by how much?

Need to find compromise between: Sufficiently many states and correct dynamics.

LQG phase space or Kapovich-Millson phase space:

3D dihedral angles in a tetrahedron do not (Poisson) commute.

Constraints cannot be implemented sharply.

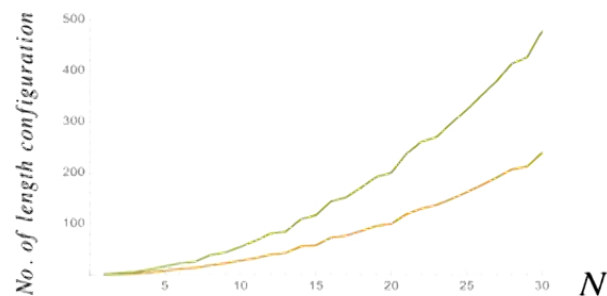
Conjecture:

Following LQG phase space (quantization) we obtain the “correct” density of states.

Can be done “fast and simple” or by making use of quantum geometry/ LQG techniques.

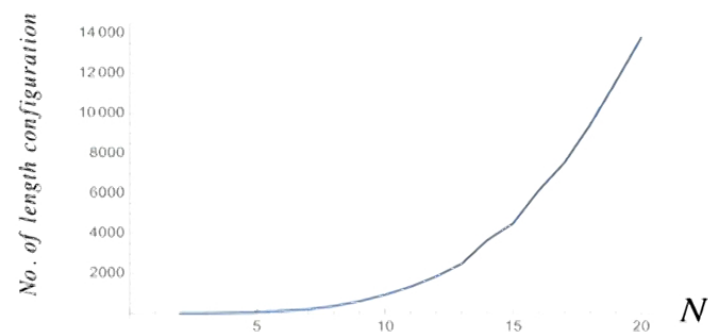
Density of states

One (symmetry reduced)
4-simplices with P lengths
parameters



Number of configurations with
all lengths between N and $(N + 1)$:
 $\sim N^P$

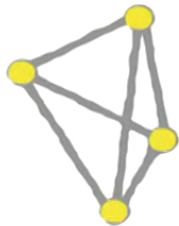
Two (symmetry reduced)
4-simplices with P lengths
parameters, with shape matching
weakly imposed



Number of configurations with
all lengths between N and $(N + 1)$:
 $\sim N^P$



Phase space for a tetrahedron / Intertwiner s

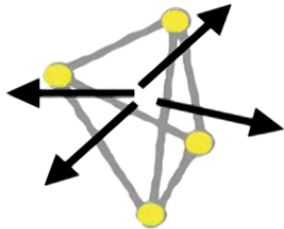


Classical:

- fix areas of a tetrahedron
- remaining degree of freedom: two 3D dihedral angles
- Kapovich-Millson: symplectic structure on this configuration
- results in two-dim. phase space with Poisson brackets

$$\{\phi_1, \phi_2\} = (\gamma) \frac{\sin \alpha}{a_t}$$

γ cancels out if
 γ -dependent spectrum
is used

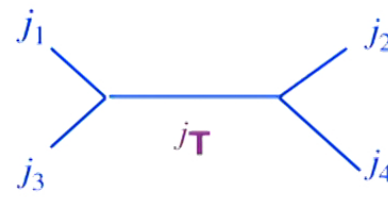
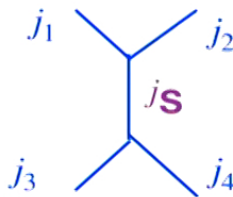


Quantum:

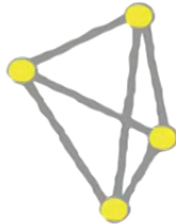
- four angular momentum vectors with global rotation symmetry imposed

$$\text{Inv} \mathcal{H}(j) = V_{j_1} \otimes V_{j_2} \otimes V_{j_3} \otimes V_{j_4} / \text{SU}(2)$$

- one quantum number: recoupling spin — encodes dihedral angle(s)



Phase space for a tetrahedron / Intertwiner s

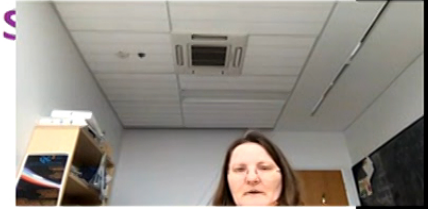


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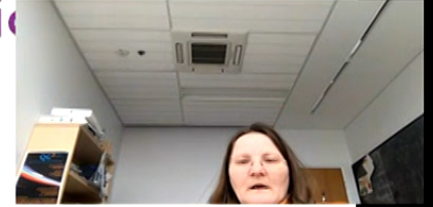


Tetrahedral coherent states / coherent intertwiners:

- states peaked on phase space point (Φ_1, Φ_2)
- resolution of unity provides measure
- different choices for parameterization and variables

[Perelomov \rightarrow Livine-Speziale, Bonzom-Livine, Freidel-Hnybida, ...]

Path integral for Area Angle Regge Formulation



Need to implement constraints:

$$\phi_{e_1}^\tau = \Phi_{e_1}^{\tau,\sigma}(a_t)$$

$$\phi_{e_2}^\tau = \Phi_{e_2}^{\tau,\sigma}(a_t)$$

Coherent states implement the constraints as good as possible (with minimal uncertainty):

$$\mathcal{K}_{\Phi_1, \Phi_2} = \mathcal{K}((\phi_{e_1}^\tau, \phi_{e_1}^\tau); (\Phi_{e_1}^\tau, \Phi_{e_1}^\tau))$$

argument of
wave function

wave function peaked
around this phase space point

New simplex amplitude:

$$\mathcal{A}'_\sigma = \exp\left(\frac{i}{h} S_\sigma(a_t)\right) \prod_{\tau \subset \sigma} \mathcal{K}((\phi_{e_1}^\tau, \phi_{e_1}^\tau); (\Phi_{e_1}^{\tau,\sigma}(a_t), \Phi_{e_1}^{\tau,\sigma}(a_t)))$$

Coherent states determine
integration measure for angles:

$$\prod_{\tau} d\mu_{\text{coh}}^\tau(\phi_{e_1}^\tau, \phi_{e_2}^\tau)$$

$$Z = \sum_{a_t \in \mathbf{S}_a} \mu(a_t) \int \prod_{\tau} d\mu^\tau(\phi_{e_1}^\tau, \phi_{e_2}^\tau) \prod_t \mathcal{A}_t \prod_{\sigma} \mathcal{A}'_{\sigma}$$

Coherent state transform: Express coherent states as wave-functions of e.g. S-channel intertwiner → 15j symbol.

The new new spin foam models

$$Z = \sum_{a_t \in \mathbf{S}_a} \mu(a_t) \int \prod_{\tau} d\mu^{\tau}(\phi_{e_1}^{\tau}, \phi_{e_2}^{\tau}) \prod_t \mathcal{A}_t \prod_{\sigma} \mathcal{A}'_{\sigma}$$

$$\mathcal{A}'_{\sigma} = \exp\left(\frac{i}{\hbar} S_{\sigma}(a_t)\right) \prod_{\tau \subset \sigma} \mathcal{K}((\phi_{e_1}^{\tau}, \phi_{e_2}^{\tau}); (\Phi_{e_1}^{\tau}, \Phi_{e_2}^{\tau}))$$



Integrate out 3D dihedral angles.

$$Z = \sum_{a_t \in \mathbf{S}_a} \mu(a_t) \prod_t \mathcal{A}_t \prod_{\sigma} \mathcal{A}_{\sigma} \prod_{\tau} G_{\tau=\sigma \cap \sigma'}^{\text{fuzzy}}(\{a_t\}_{t \in \sigma \cup \sigma'})$$

Corrected
Barrett-Crane model.

$$\mathcal{G}_{\tau=\sigma \cap \sigma'}^{\text{fuzzy}} = \langle \mathcal{K}_{\Phi_{e_1}^{\tau, \sigma}, \Phi_{e_2}^{\tau, \sigma}} | \mathcal{K}_{\Phi_{e_1}^{\tau, \sigma'}, \Phi_{e_2}^{\tau, \sigma'}} \rangle$$

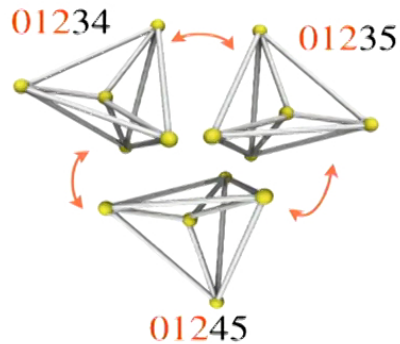
Peaked on constraints.

Heuristic versions: replace constraint amplitudes by Theta-functions on uncertainty intervals.



Joined on LQG
Hilbert space.

Flatness issue test: 3-3 Pachner move config



Three 4-simplices glued together.

No bulk edge → No summation in length Regge calculus.

One bulk triangle → In Area Regge calculus summation over triangle area, forcing flatness.

Hope:

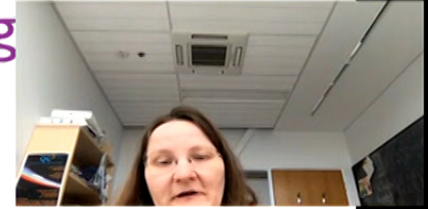
Our (weakened) constraints on Area Regge calculus allow only a sum over a number of order 1 area values.

A simple argument shows: (Supported by numerical study)
The number of configuration one sums over scales with $\sqrt{\text{Area}}$

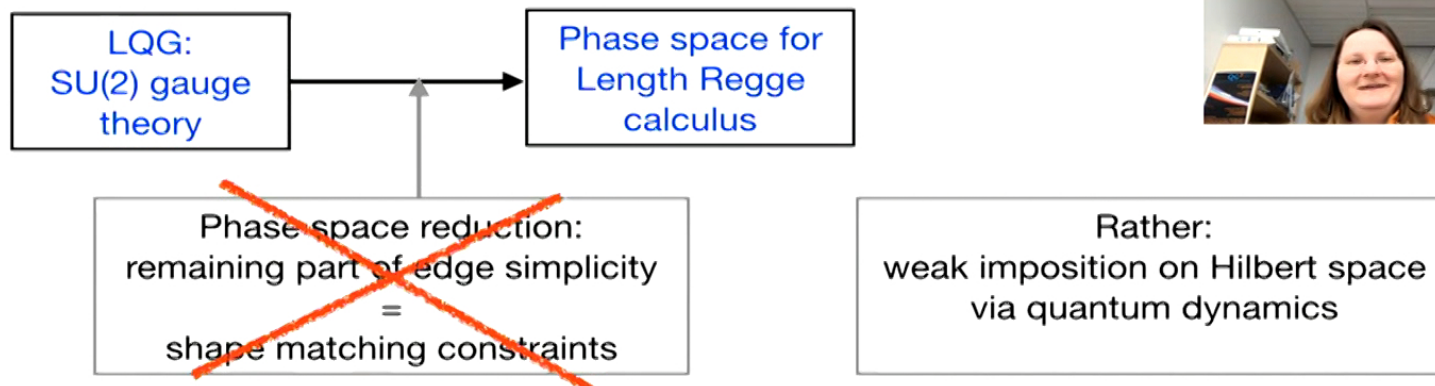
$$\delta\phi \sim \frac{1}{\sqrt{\text{Area}}} \quad \text{and} \quad \frac{\partial \text{Area}}{\partial \phi} \sim \text{Area}$$

$$\Rightarrow \delta \text{Area} \sim \sqrt{\text{Area}}$$

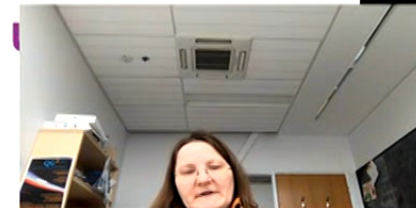
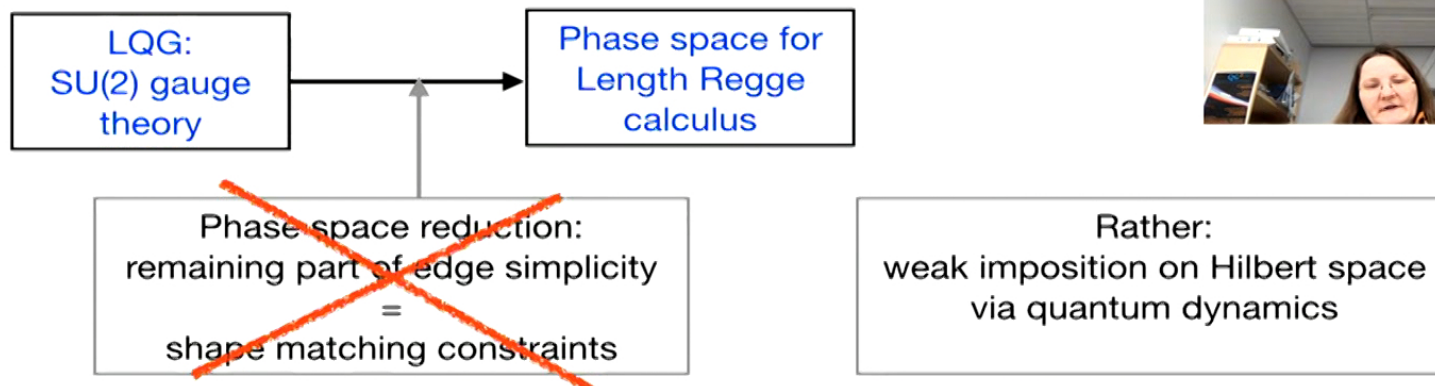
Constraining, but less than we initially hoped for.
(Quantum) Corrections to the dynamics: torsion fluctuations average over curvature.



Phenomenology and Falsifiability (starring at ...)



Phenomenology and Falsifiability (starring at ...)



We allow certain torsion degrees of freedom to (vacuum) fluctuate.
Really different from ADM/ metric quantization.

It is really torsion: shape matching imposes $d_\omega d_\omega e = 0$.

[Asante, BD, Girelli,
Riello, Tsimiklis 2019]

Forced by demanding discrete (locally independent) area spectrum.

Bound to have phenomenological consequences.

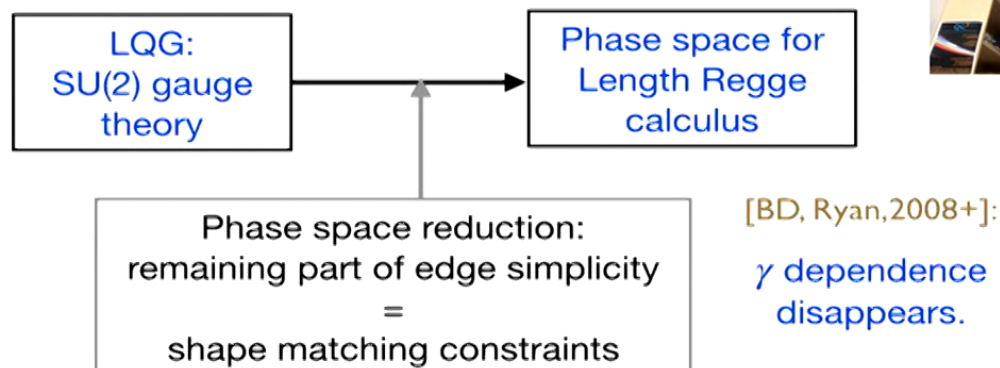
γ -dependence due to not imposing shape matching.

Key question: How do the torsion fluctuations behave under coarse graining?

Effective continuum description?

Krasnov: Family of models with choice of potential for simplicity constraints.

There is an alternative (ignored so far):



That is a proper/ rigorous quantization of length Regge phase space.

Challenge: triangle inequalities usually resolved by going to gauge (or higher gauge) description.

Reduction leads to **non-local** Dirac brackets resulting from shape matching constraints.

Most detailed analysis so far: [BD, Ryan, 2008+]:

Even on boundary of 4-simplex:

Discrete (locally independent) area spectrum \rightarrow highly interdependent length.

Would BH entropy counting work out?

Areas are not independent anymore.

Arises also for LQG with (weak) imposition of shape matching.



Summary and Outlook

Input: Discrete (locally independent) area spectra.



Path integral which imposes GR dynamics as close as possible.
(As allowed by LQG Hilbert space/ quantum tetrahedron.)



- Can use simpler or more involved versions of the model
Theta functions vs coherent states. $\text{Exp}(i \text{ Regge action})$ vs recoupling symbols.
[Address one problem at a time.](#)
- Key questions:
 - How do torsion fluctuations behave under renormalization flow?
 - Diffeomorphism symmetry violated by curvature and torsion.
 - Understand quantum corrections for the dynamics resulting from torsion fluctuations.
 - Continuum description?
- Alternative quantization: rigorous quantization of lengths Regge phase space.
- Difference in density of states? Black hole entropy?

[Bahr, BD 09+
Asante, BD, Haggard 18]