Title: Holomorphic-topological twists and TFT

Speakers: Dylan Butson

Series: Mathematical Physics

Date: March 12, 2020 - 11:00 AM

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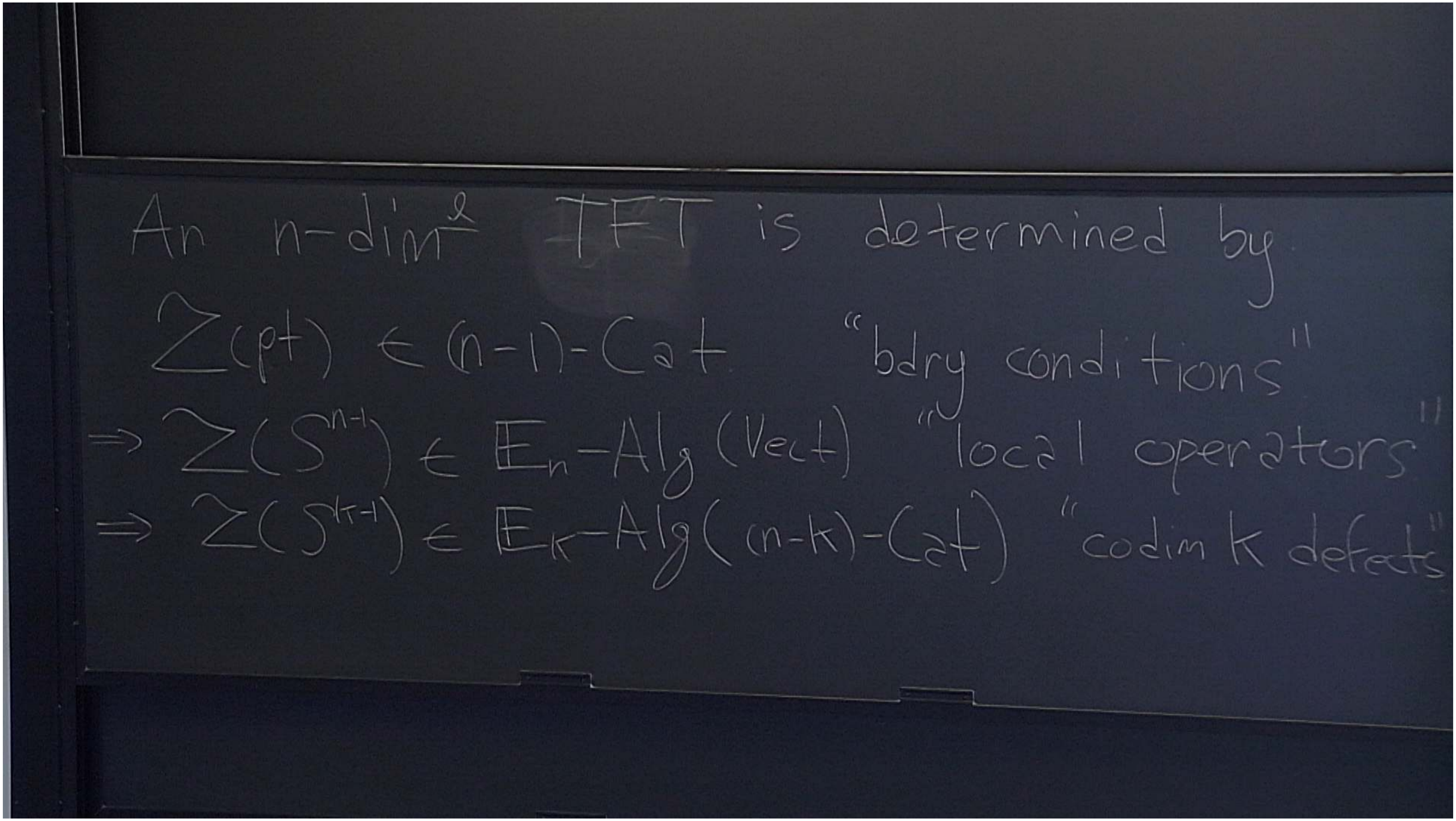
Abstract: I'll explain the TFT perspective on holomorphic-topological twists of 3d N=4 and 4d N=2 theories, and outline some connections between the topics discussed in Justin and Davide's previous lectures, and various ongoing work of Justin, Philsang, Kevin, Davide, Tudor, myself, etc.

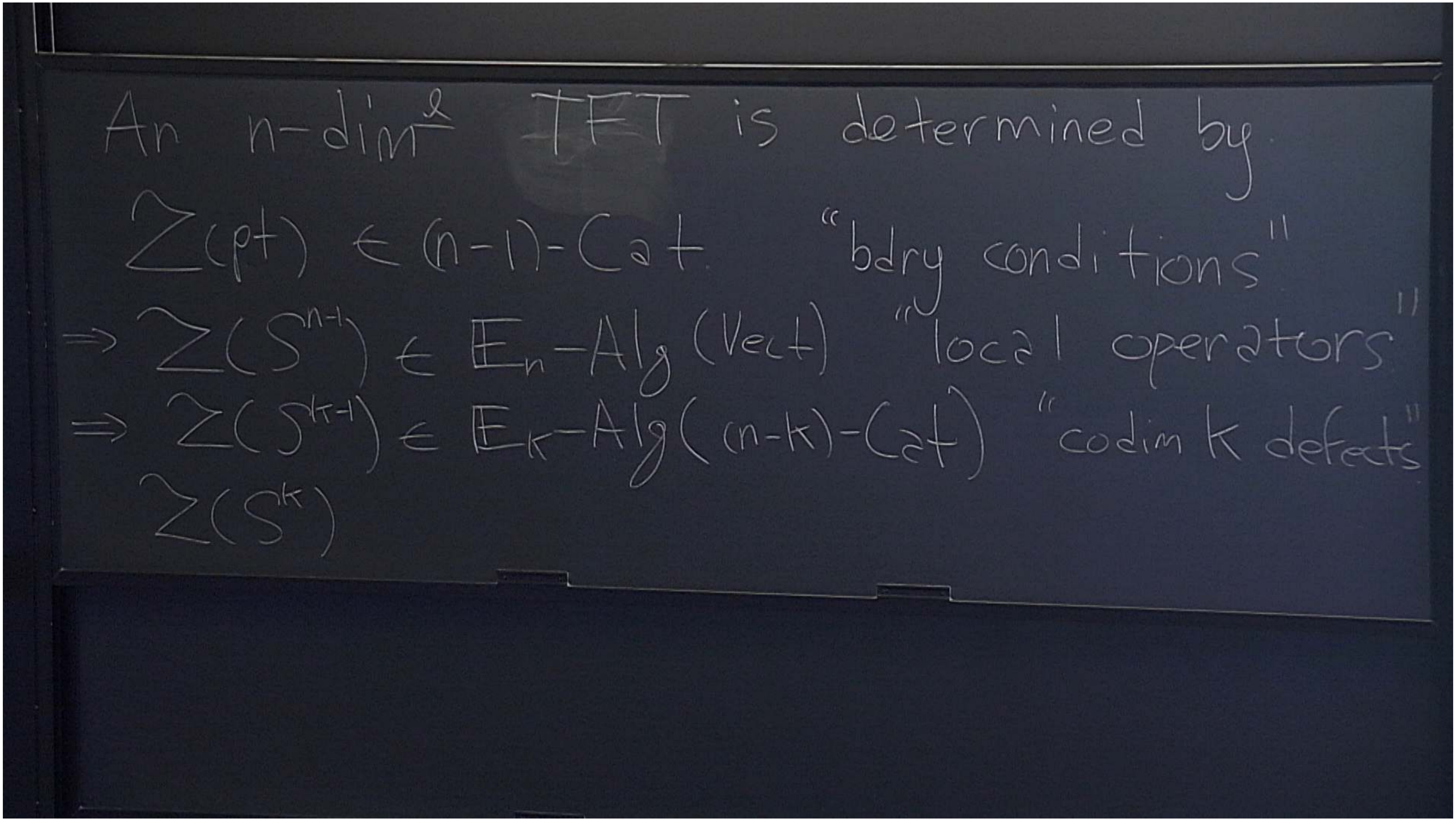


An n-dim$^{\text{l}}$ TFT is determined by

$Z(pt) \in (n-1)\text{-Cat}$ "bdry conditions"

$\Rightarrow Z(S^{n-1}) \in \mathbb{E}_n\text{-Alg}(\text{Vect})$ "local operators"

$\Rightarrow Z(S^{k-1}) \in \mathbb{E}_k\text{-Alg}((n-k)\text{-Cat})$ "codim k defects"



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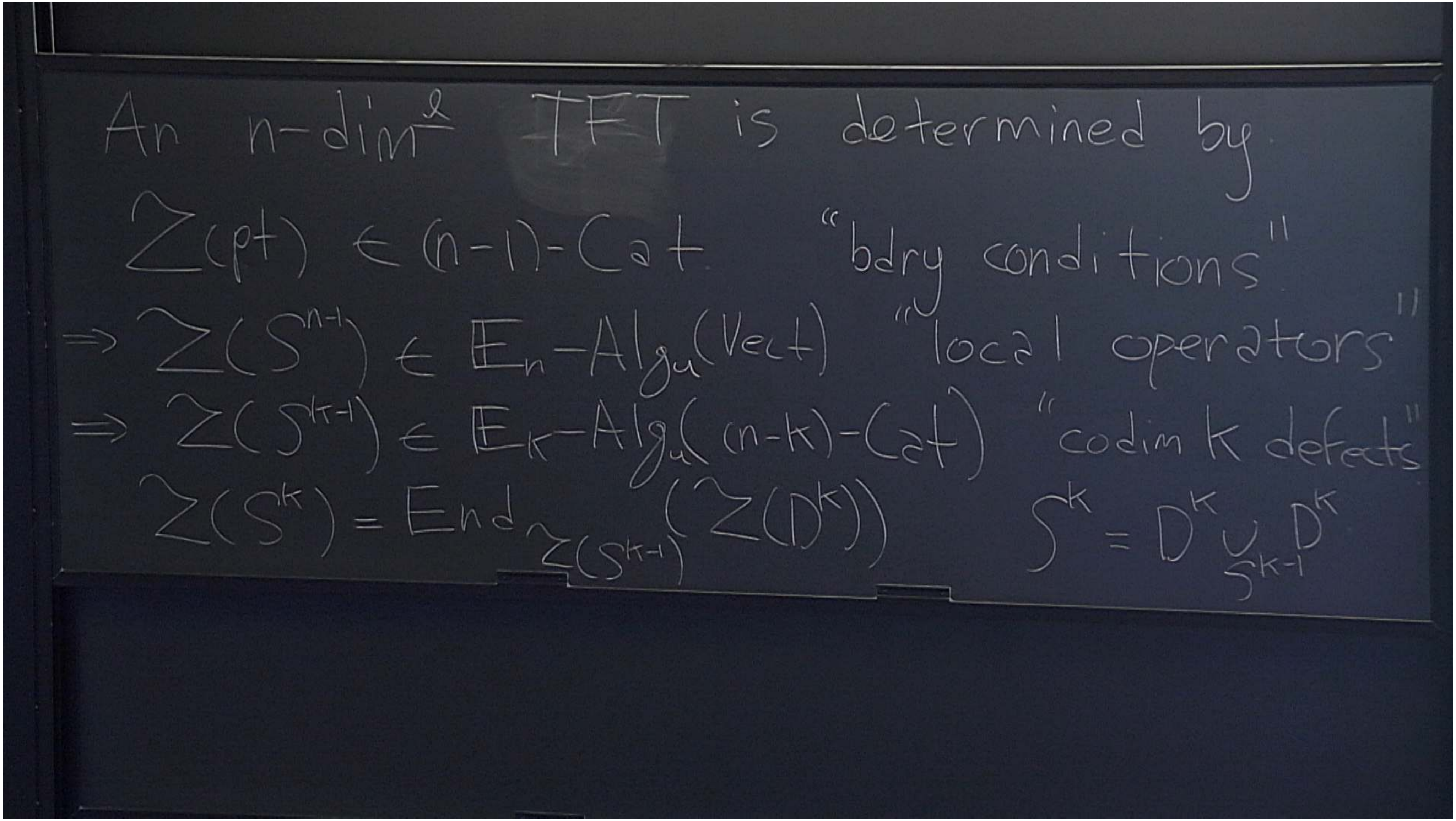
$Z(S^k)$



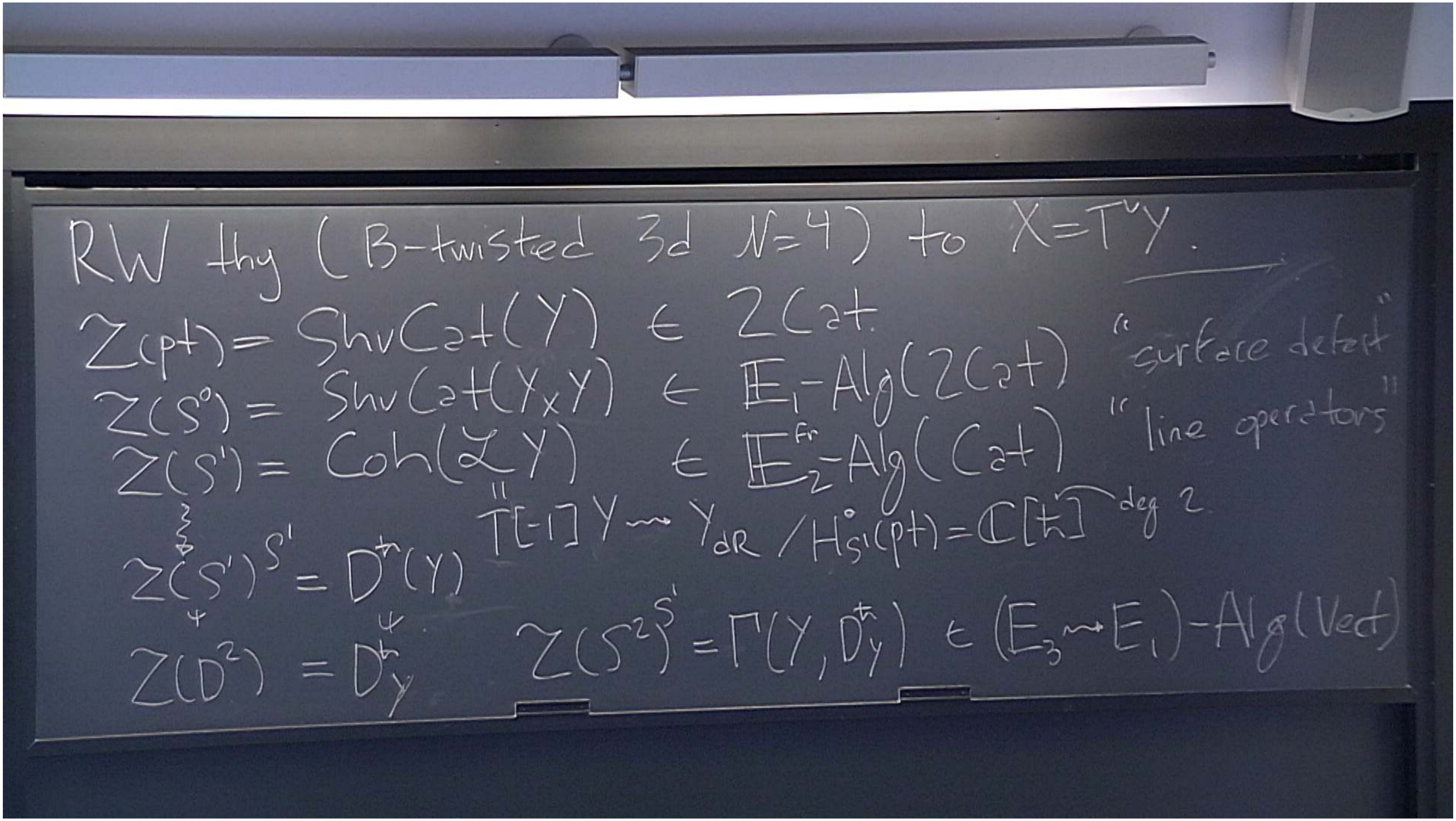
An n-dim$^{\ell}$ TFT is determined by

$\mathcal{Z}(pt) \in (n-1)\text{-Cat}$ "bdry conditions"

$\Rightarrow \mathcal{Z}(S^{n-1}) \in \mathbb{E}_n\text{-Alg}_u(\text{Vect})$ "local operators"

$\Rightarrow \mathcal{Z}(S^{k-1}) \in \mathbb{E}_k\text{-Alg}_u((n-k)\text{-Cat})$ "codim k defects"

$\mathcal{Z}(S^k) = \text{End}_{\mathcal{Z}(S^{k-1})}(\mathcal{Z}(D^k))$ $\quad S^k = D^k \cup_{S^{k-1}} D^k$



RW thy (B-twisted 3d $\mathcal{N}=4$) to $X = T^\vee Y$.

$Z(pt) = ShvCat(Y) \in 2Cat$ "surface defect"

$Z(S^0) = ShvCat(Y \times Y) \in \mathbb{E}_1\text{-}Alg(2Cat)$ "line operators"

$Z(S^1) = Coh(\mathcal{L}Y) \in \mathbb{E}_2^{fr}\text{-}Alg(Cat)$

$\quad\quad\quad\quad\quad\quad \| $

$\quad\quad\quad\quad\quad\quad T[-1]Y \rightsquigarrow Y_{dR} / H^\bullet_{S^1}(pt) = \mathbb{C}[\hbar]$ deg 2

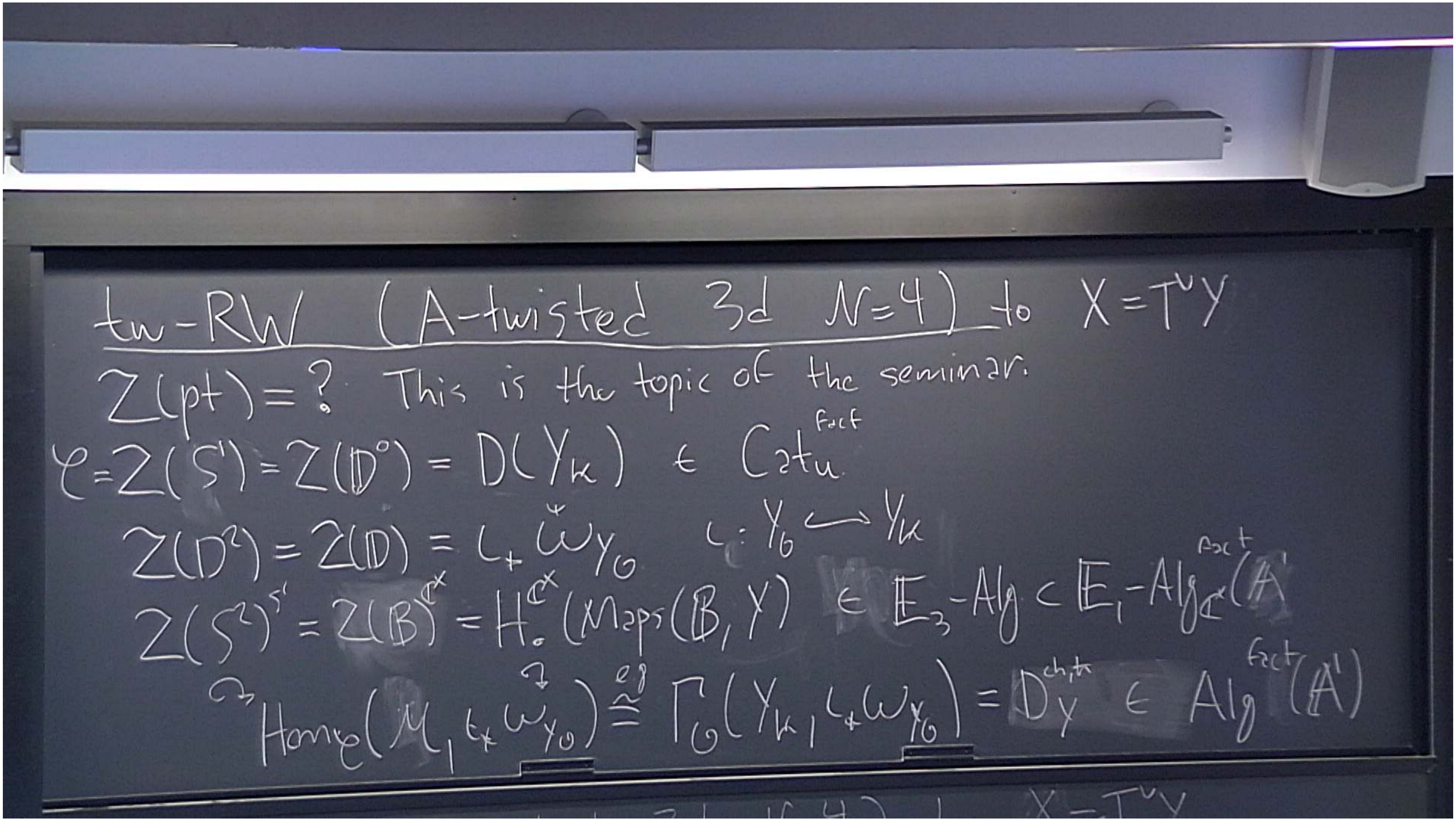
$\quad\rotatebox{-90}{$\rightsquigarrow$}$

$Z(S^1)^{S^1} = D^{\hbar}(Y)$

$\quad\psi \quad\quad\quad\quad \psi$

$Z(D^2) = D^{\hbar}_Y$

$Z(S^2)^{S^1} = \Gamma(Y, D^{\hbar}_Y) \in (E_3 \rightsquigarrow E_1)\text{-}Alg(Vect)$



tw-RW (A-twisted 3d $\mathcal{N}=4$) to $X=T^\vee Y$

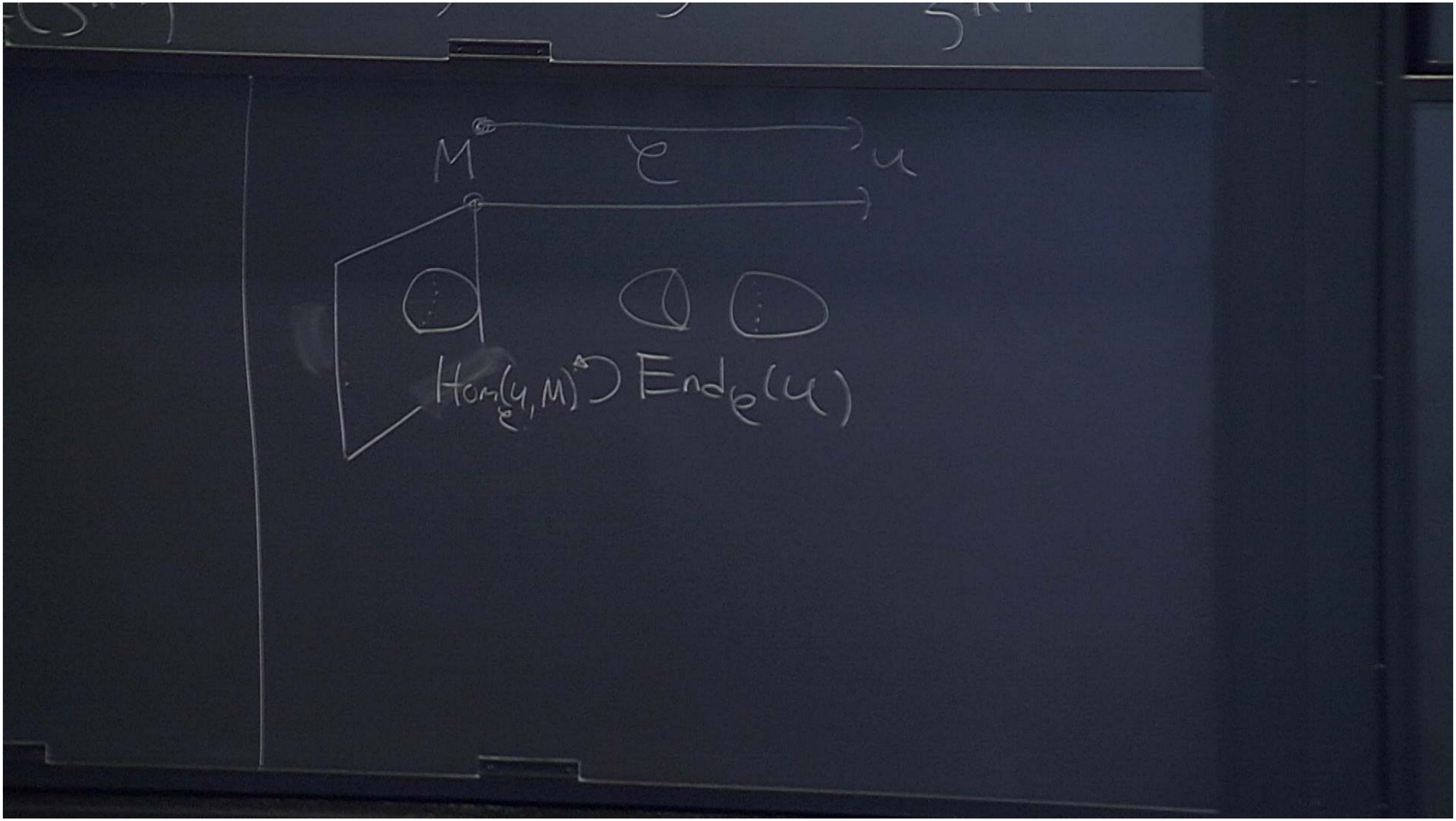
$Z(pt)=?$ This is the topic of the seminar.

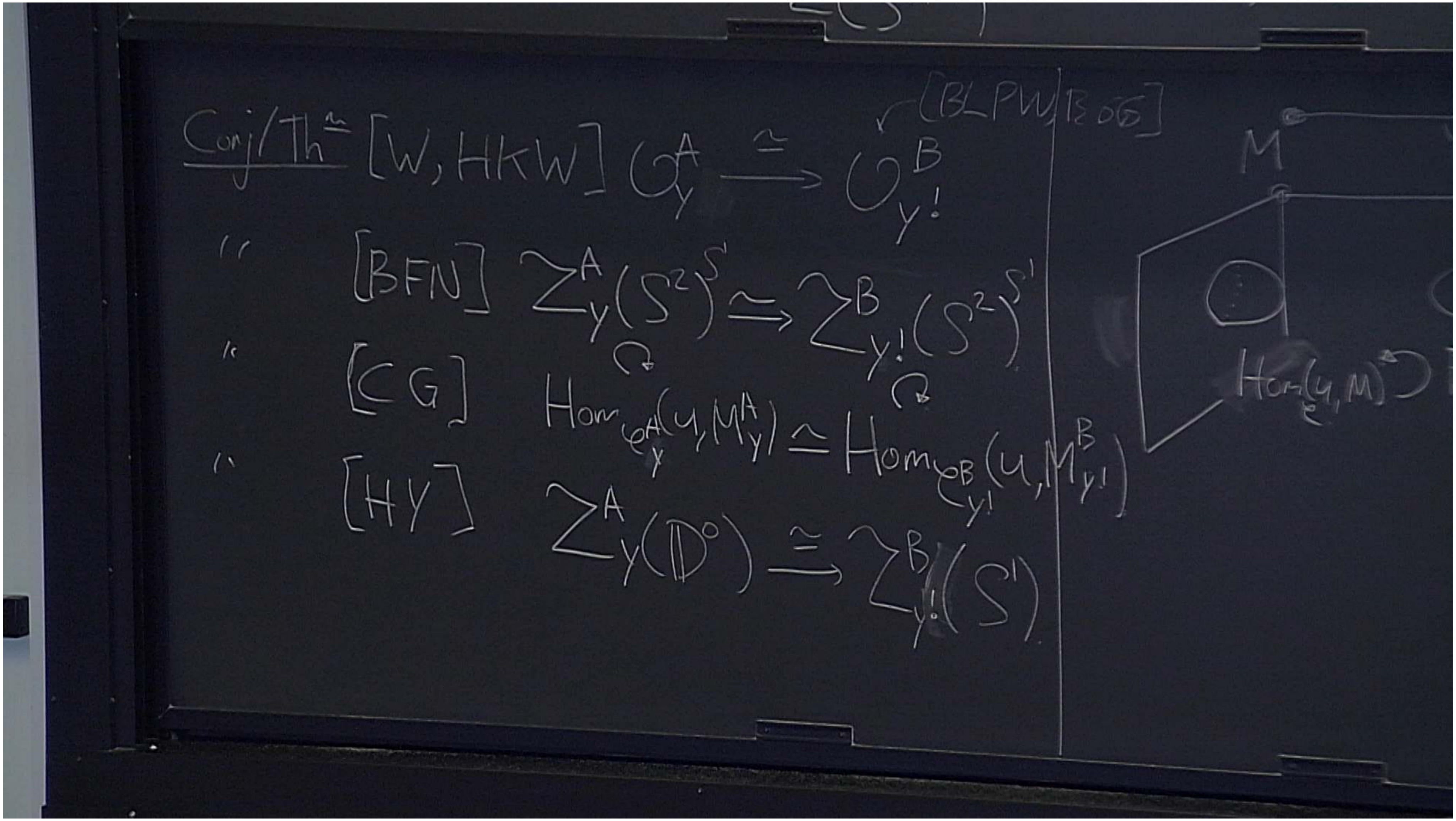
$\mathcal{C}=Z(S^1)=Z(\mathbb{D}^\circ)=D(Y_K) \in \mathrm{Cat}_u^{fact}$

$Z(D^2)=Z(\mathbb{D})=\iota_*\overset{\psi}{\omega}_{Y_O}$ $\quad \iota : Y_O \hookrightarrow Y_K$

$Z(S^2)^{S^1}=Z(\mathbb{B})^{\mathbb{C}^\times}=H_\bullet^{\mathbb{C}^\times}(\mathrm{Maps}(\mathbb{B},Y) \in \mathbb{E}_3\text{-}\mathrm{Alg} \subset \mathbb{E}_1\text{-}\mathrm{Alg}_{\mathbb{C}^\times}^{fact}(\mathbb{A}^1$

$\curvearrowright \mathrm{Hom}_{\mathcal{C}}(\mathcal{M}, \iota_*\overset{?}{\omega}_{Y_O}) \overset{e.g.}{\cong} \Gamma_O(Y_K, \iota_*\omega_{Y_O}) = \mathcal{D}_Y^{ch,\hbar} \in \mathrm{Alg}^{fact}(\mathbb{A}^1)$


M
$\mathcal{C}$
$\mathcal{U}$
$\mathrm{Hom}_{\mathcal{C}}(\mathcal{U}, M) \supset \mathrm{End}_{\mathcal{C}}(\mathcal{U})$

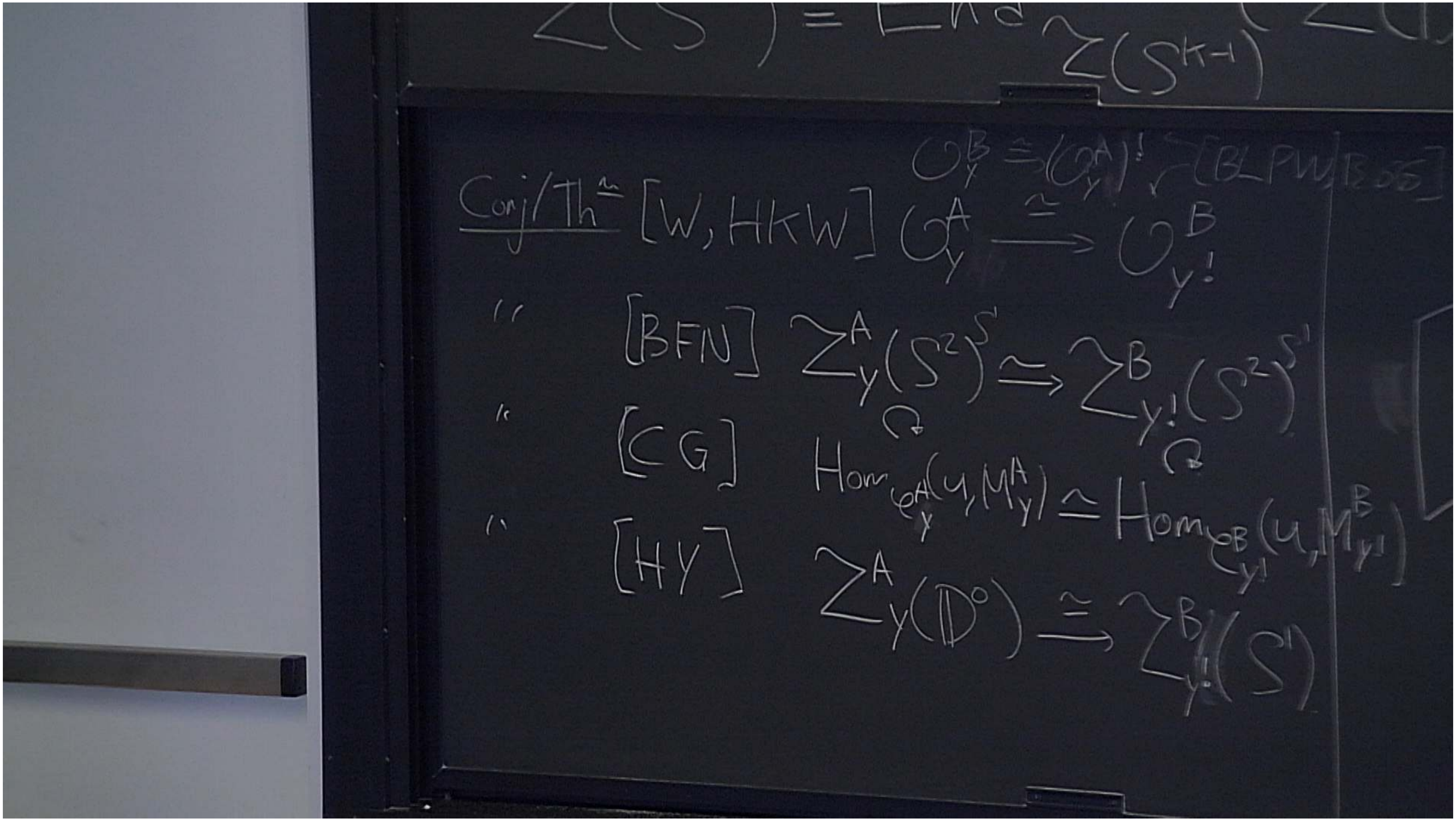


Conj/Th$^{\underline{m}}$ [W, HKW] $\mathcal{O}^A_y \xrightarrow{\simeq} \mathcal{O}^B_{y^!}$ ← [BLPW/BDG]

" [BFN] $\underset{\circlearrowright}{\Sigma^A_y(S^2)^{S^1}} \xrightarrow{\simeq} \underset{\circlearrowright}{\Sigma^B_{y^!}(S^2)^{S^1}}$

" [CG] $\mathrm{Hom}_{\mathcal{C}^A_y}(u, M^A_y) \simeq \mathrm{Hom}_{\mathcal{C}^B_{y^!}}(u, M^B_{y^!})$

" [HY] $\Sigma^A_y(\mathbb{D}^\circ) \xrightarrow{\simeq} \Sigma^B_{y^!}(S^1)$



$\mathcal{Z}(S\ ) = \mathrm{End}_{\mathcal{Z}(S^{k-1})}(\mathcal{Z}(1$

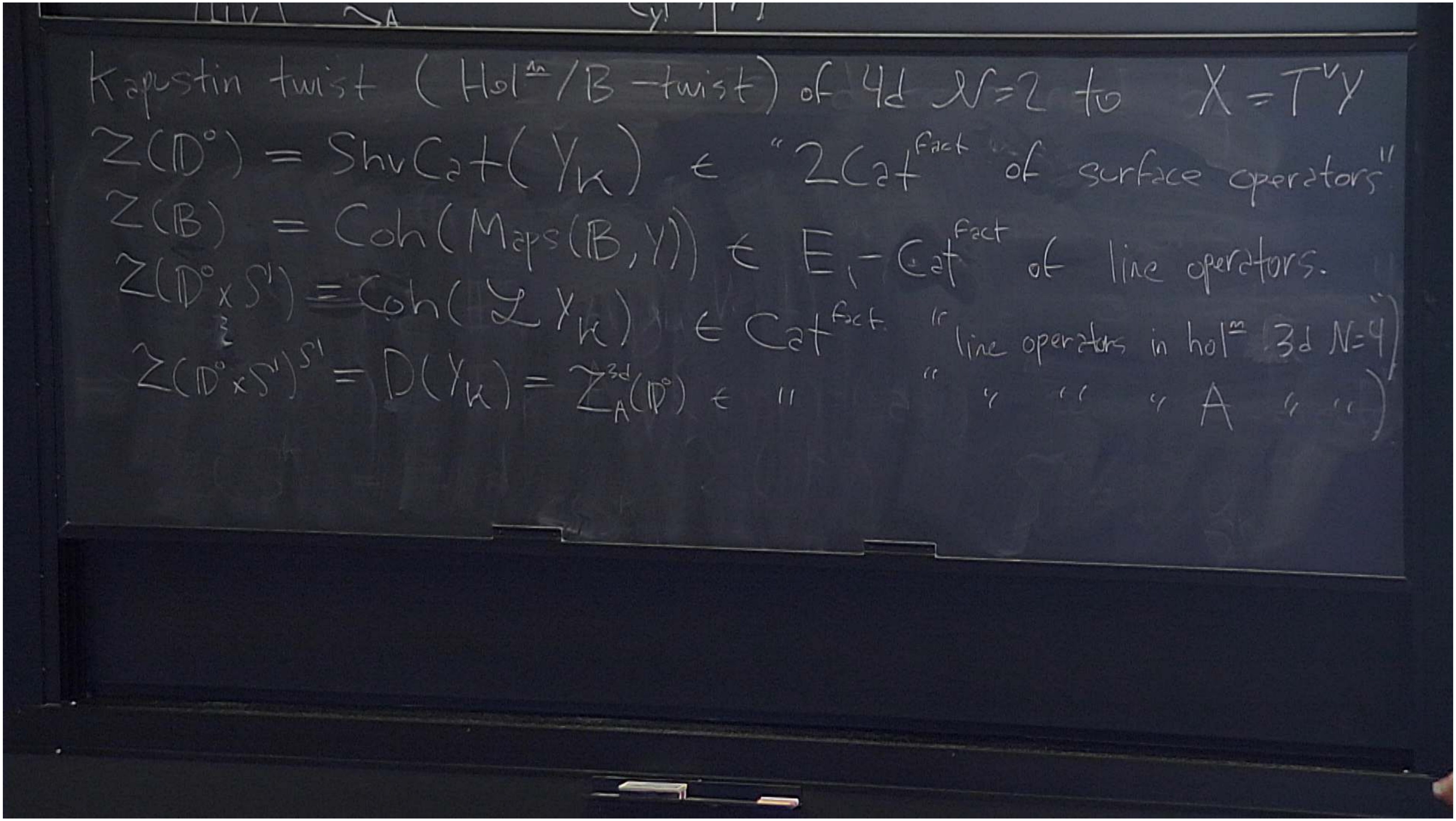
$\mathcal{O}^B_y \simeq (\mathcal{O}^A_y)^!$ ✓ [BLPW] [BDS]

Conj/Thm [W, HKW] $\mathcal{O}^A_y \xrightarrow{\simeq} \mathcal{O}^B_{y^!}$

" [BFN] $\underset{\circlearrowright}{\mathcal{Z}^A_y}(S^2)^{S^1} \xrightarrow{\simeq} \underset{\circlearrowright}{\mathcal{Z}^B_{y^!}}(S^2)^{S^1}$

" [CG] $\mathrm{Hom}_{\mathcal{C}^A_y}(\mathbb{1}, M^A_y) \simeq \mathrm{Hom}_{\mathcal{C}^B_{y^!}}(\mathbb{1}, M^B_{y^!})$

" [HY] $\mathcal{Z}^A_y(\mathbb{D}^o) \xrightarrow{\simeq} \mathcal{Z}^B_{y^!}(S^1)$



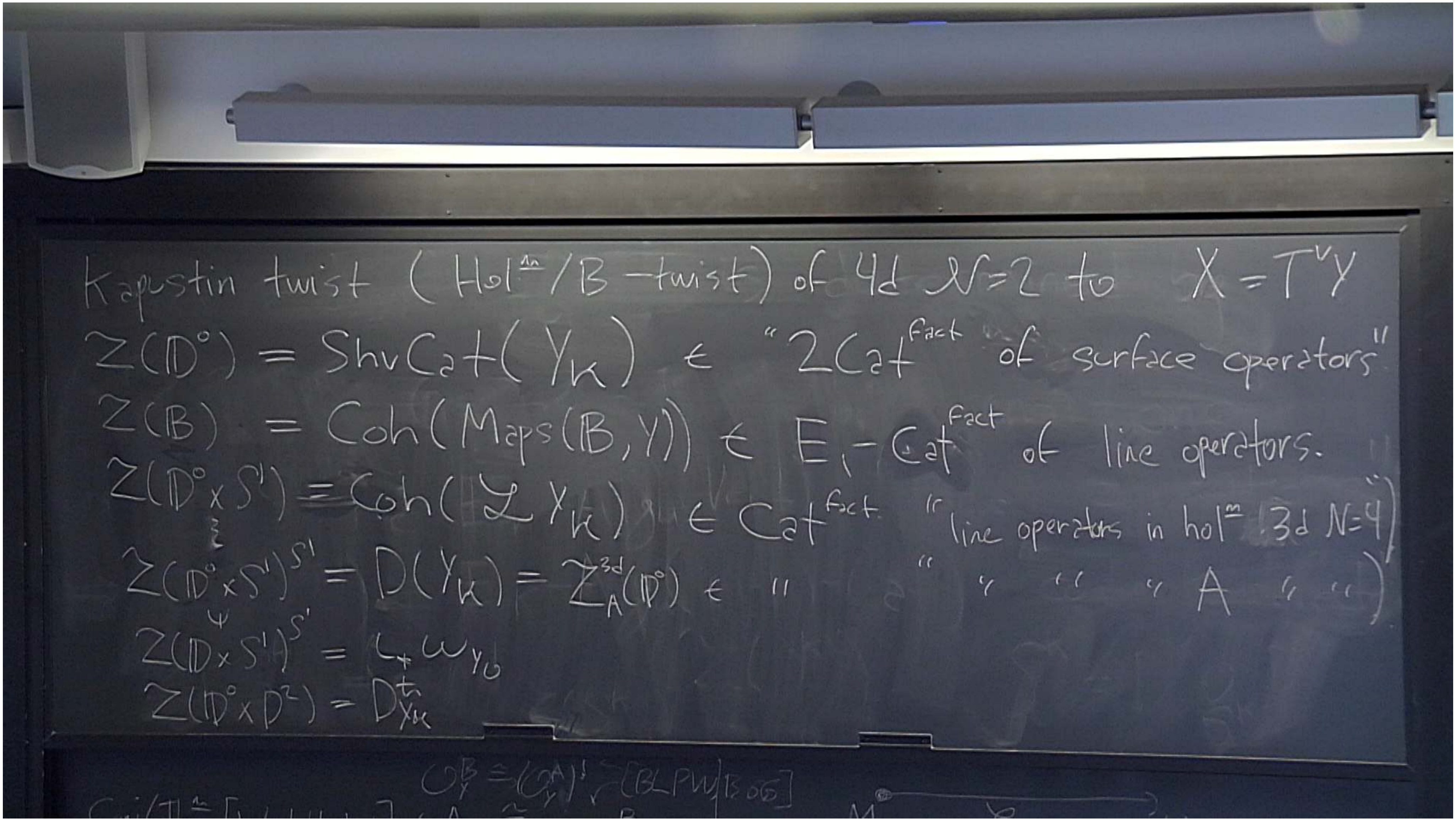
Kapustin twist (Hol$^{\text{an}}$/B-twist) of 4d $\mathcal{N}=2$ to $X = T^\vee Y$

$Z(\mathbb{D}^\circ) = \mathrm{ShvCat}(Y_\kappa) \in$ "2Cat$^{\text{fact}}$ of surface operators"

$Z(\mathbb{B}) = \mathrm{Coh}(\mathrm{Maps}(\mathbb{B}, Y)) \in E_1$-Cat$^{\text{fact}}$ of line operators.

$Z(\mathbb{D}^\circ \underset{\wr}{\times} S^1) = \mathrm{Coh}(\mathcal{L} Y_\kappa) \in \mathrm{Cat}^{\text{fact}}$ "line operators in hol$^{\text{m}}$ 3d $N=4$")

$Z(\mathbb{D}^\circ \times S^1)^{S^1} = D(Y_\kappa) = Z^{3d}_A(\mathbb{D}^\circ) \in$ " " " " A " ")



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$Z(\mathbb{D}^\circ \times S^1)^{S^1} = D(Y_\kappa) = Z_A^{3d}(\mathbb{D}^\circ) \in$ (" " " " A " ")

$Z(\mathbb{D} \times S^1)^{S^1} = \iota_* \omega_{Y_0}$

$Z(\mathbb{D}^\circ \times D^2) = D^{\mathrm{tr}}_{Y_\kappa}$

$\mathcal{O}_Y^B \simeq (\mathcal{O}_Y^A)^!$ [BLPW] [B.06]

$) = \mathrm{Sh}(\mathcal{L} Y_k) \in \mathrm{Cat}^{\mathrm{fact.}}$ "line operators in hol"

$S^1)^{S^1} = D(Y_k) = Z^{3d}_A(\mathbb{D}^0) \in$ " " " " " " A

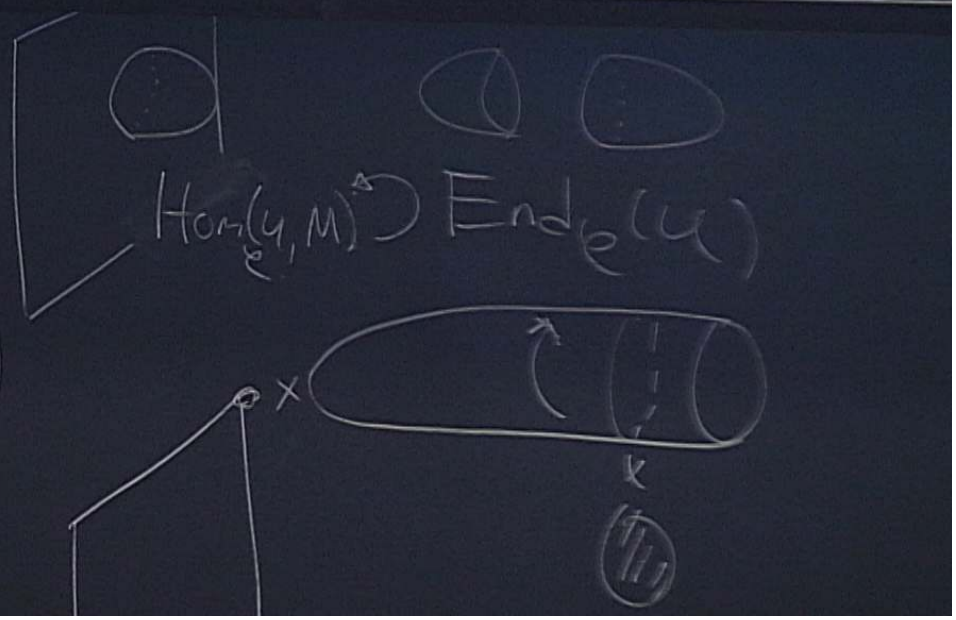
$S^1)^{S^1} = \iota_* \omega_{Y_0}$ $\qquad \mathbb{D} \times S^1 \underset{\mathbb{D}^0 \times S^1}{\cup} \mathbb{D}^0 \times D^2 \simeq S^3_{\text{"}hol^{\text{m}}/top^{\text{c}}\text{"}}$

$\times D^2) = D^{tr}_{Y_{tr}}$

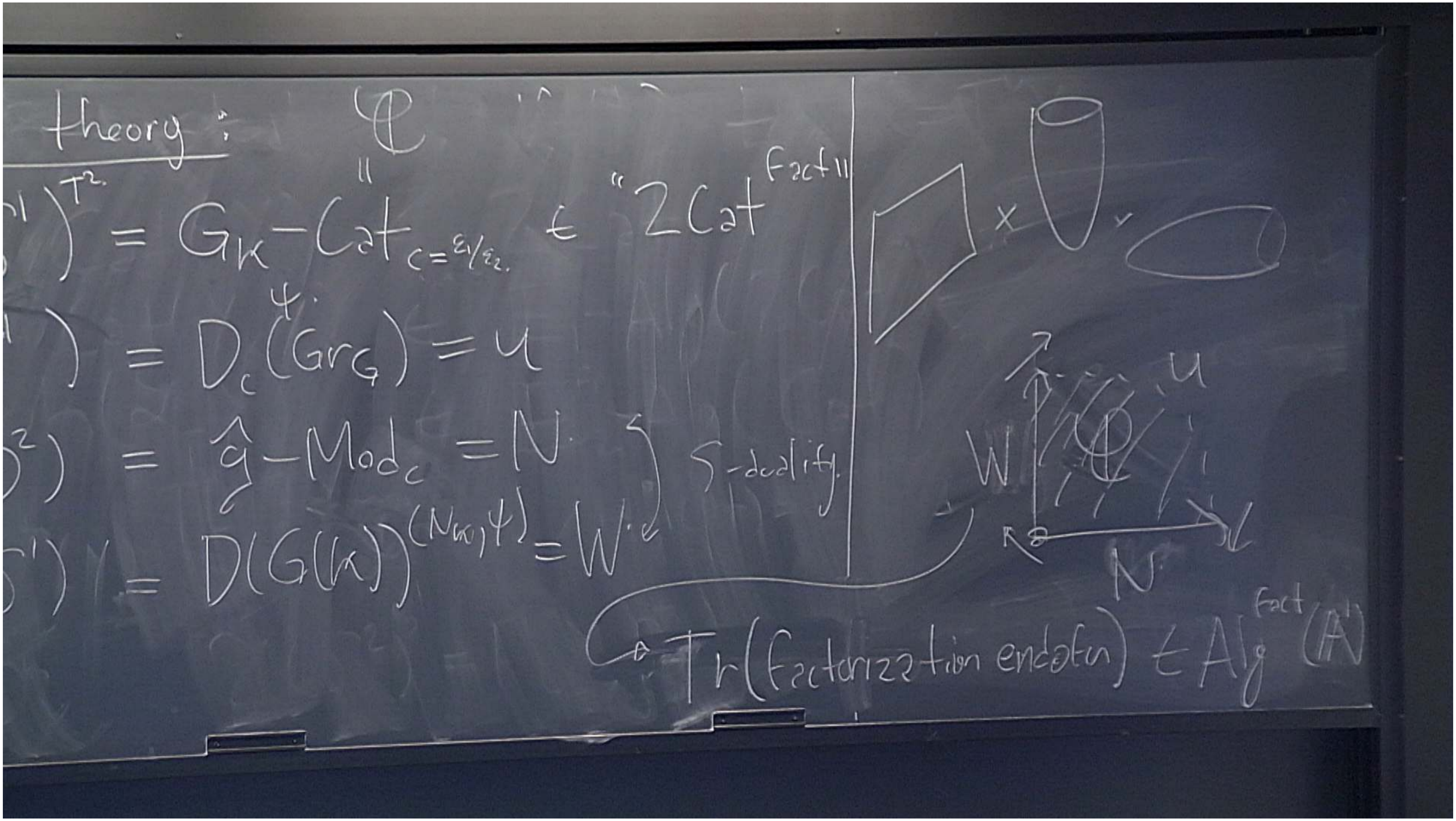
$\mathrm{BFN}]$ $\qquad \underset{\circlearrowright}{Z^A_y}(S^2)^{S^1} \xrightarrow{\simeq} \underset{\circlearrowright}{Z^B_{y^!}}(S^2)^{S^1}$

$G]$ $\qquad \mathrm{Hom}_{\mathcal{C}^A_y}(u, M^A_y) \simeq \mathrm{Hom}_{\mathcal{C}^B_{y^!}}(u, M^B_{y^!})$

$\mathrm{HY}]$ $\qquad Z^A_y(\mathbb{D}^0) \xrightarrow{\simeq} Z^B_{y^!}(S^1)$



$\mathrm{Hom}_{\mathcal{C}}(u, M) \supset \mathrm{End}_{\mathcal{C}}(u)$



theory :

$\mathcal{C}$

$\shortparallel$

$)^{T^2} = G_K\text{-}Cat_{c=\varepsilon_1/\varepsilon_2} \in$ "$2Cat^{Fact}$"

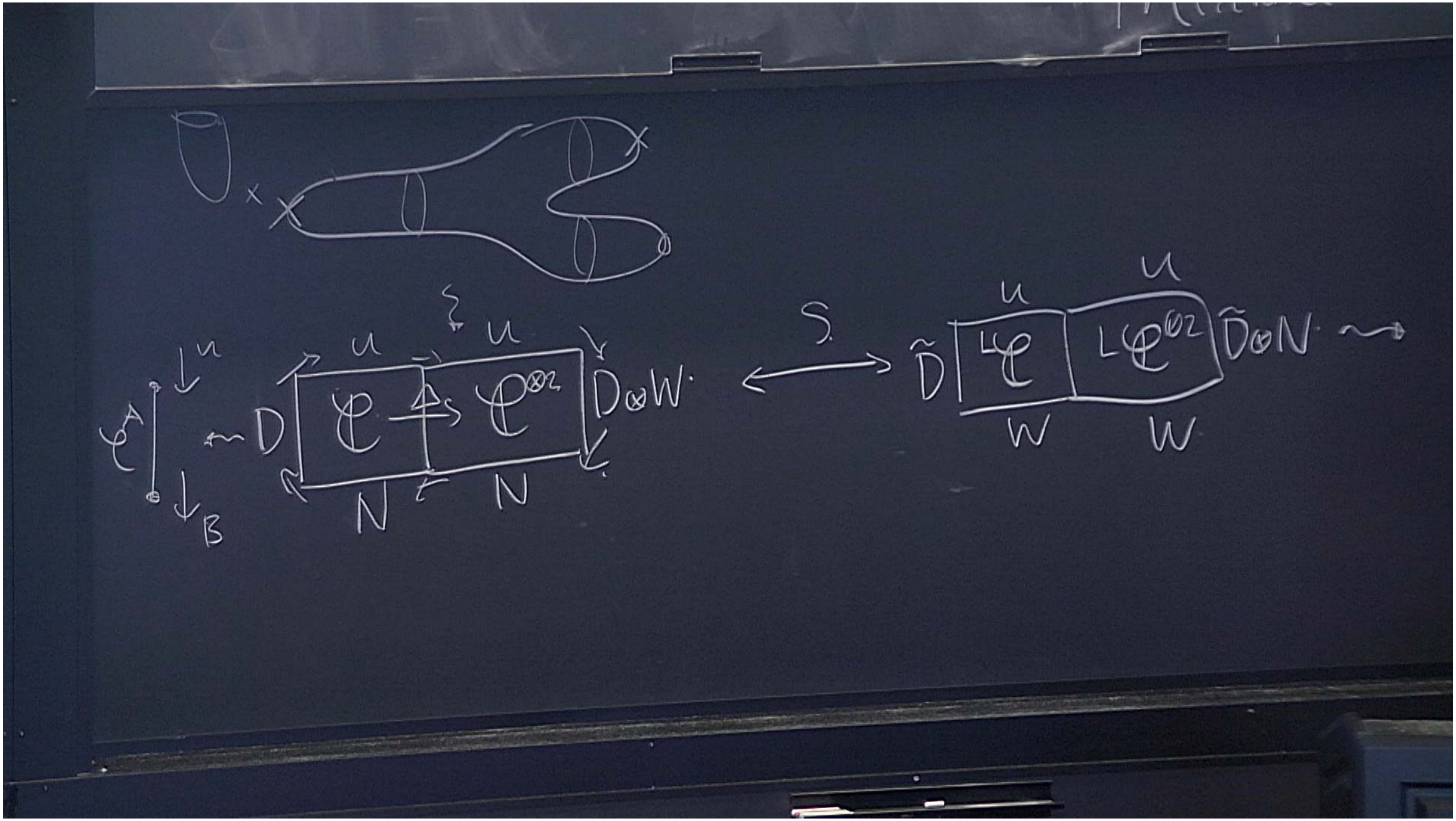
$) = D_c(\overset{\psi}{Gr_G}) = U$

$^2) = \hat{\mathfrak{g}}\text{-}Mod_c = N$

$^1) = D(G(\mathcal{K}))^{(N_{(K)},\psi)} = W$

S-duality

$\hookrightarrow Tr(\text{factorization endofun}) \in Alg^{fact}(\mathbb{A}^1)$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W