Title: The trinity of relational quantum dynamics

Speakers: Philipp Hoehn

Series: Quantum Gravity

Date: March 12, 2020 - 2:30 PM

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Abstract: In order to solve the problem of time in quantum gravity, various approaches to a relational quantum dynamics have been proposed. In this talk, I will exploit quantum reduction maps to illustrate a previously unknown equivalence between three of the well-known ones: (1) relational observables in the clock-neutral picture of Dirac quantization, (2) Page and Woottersâ€™ (PW) SchrÃ¶dinger picture formalism, and (3) the relational Heisenberg picture obtained via symmetry reduction. Constituting three faces of the same dynamics, we call this equivalence the trinity. To establish the equivalence, a quantization procedure for relational Dirac observables is developed using covariant POVMs which encompass non-ideal clocks. The quantum reduction maps reveal this procedure as the quantum analog of the classical method of gauge-invariantly extending gauge-fixed quantities. The quantum reduction maps also allow one to extend a recent â€˜clock-neutralâ€™ approach to changing temporal reference frames, transforming relational observables and states between different clock choices, and demonstrate a clock dependent temporal nonlocality effect. Using the trinity, I will discuss how Kuchar's three fundamental criticisms against the PW formalism, namely that its conditional probabilities would (i) yield the wrong localization probabilities for relativistic particles, (ii) violate the constraints, and (iii) produce incorrect transition probabilities, can be resolved. Given the trinity, these resolutions also apply to approaches (1) and (3) and corroborate the PW formalism, if done correctly, as a viable approach to the problem of time. Time permitting, I will explain, however, why the slogan `time from entanglement' in the PW formalism is misleading.

The trinity of relational quantum dynamics

Philipp Höhn
University College London
and
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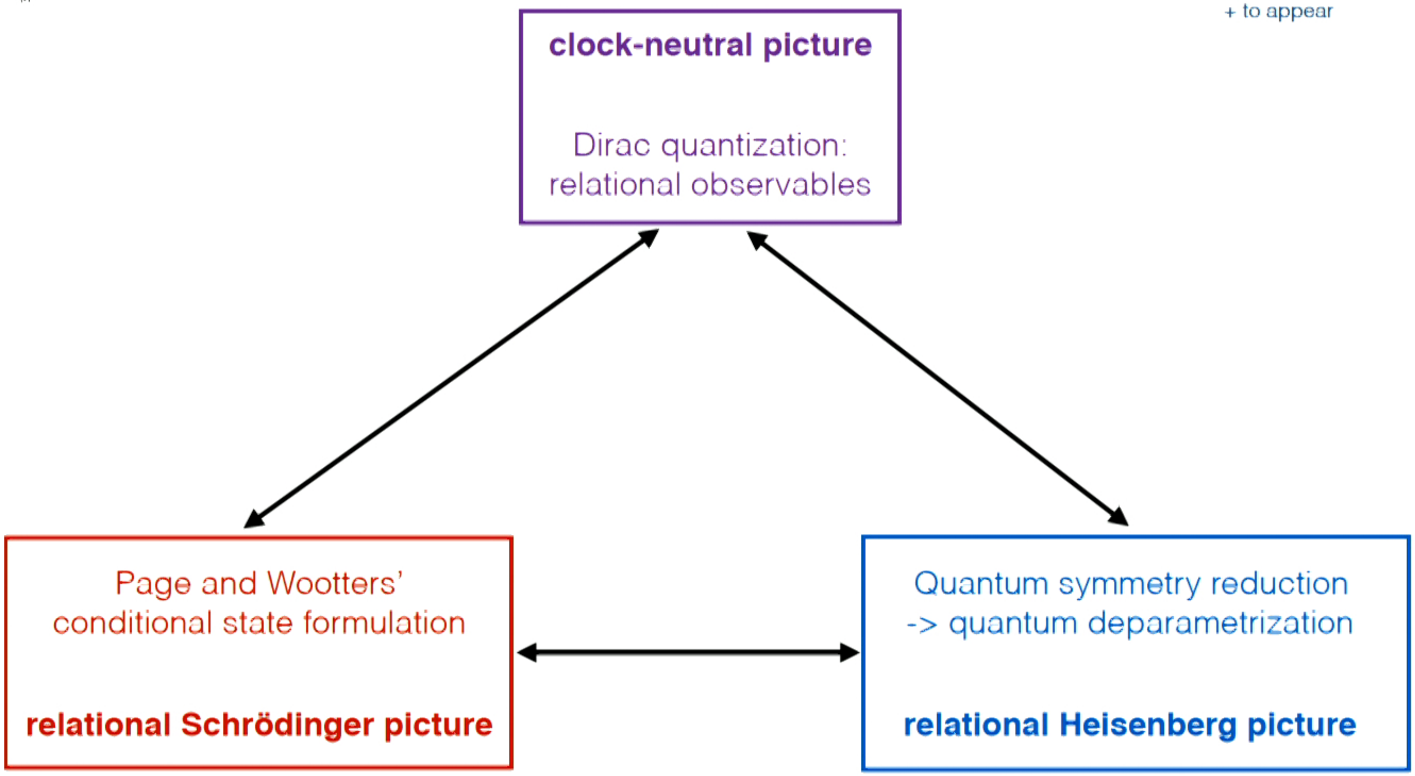
PI QG seminar
March 12 2020

work with Alex Smith (Dartmouth) and Max Lock (IQOQI Vienna)
1912.00033 + to appear

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033
+ to appear



General covariance

"All the laws of physics are the same in <u>all</u> reference frames."

↓

laws as tensor equations

↑

reference-frame-neutral

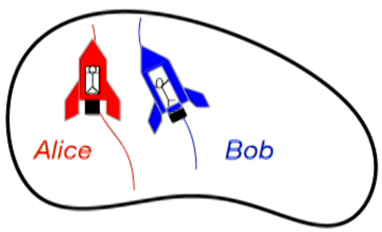
General covariance

“All the laws of physics are the same in <u>all</u> reference frames.”

<u>frames usually</u>

- idealized
- external

frame-neutral



spacetime

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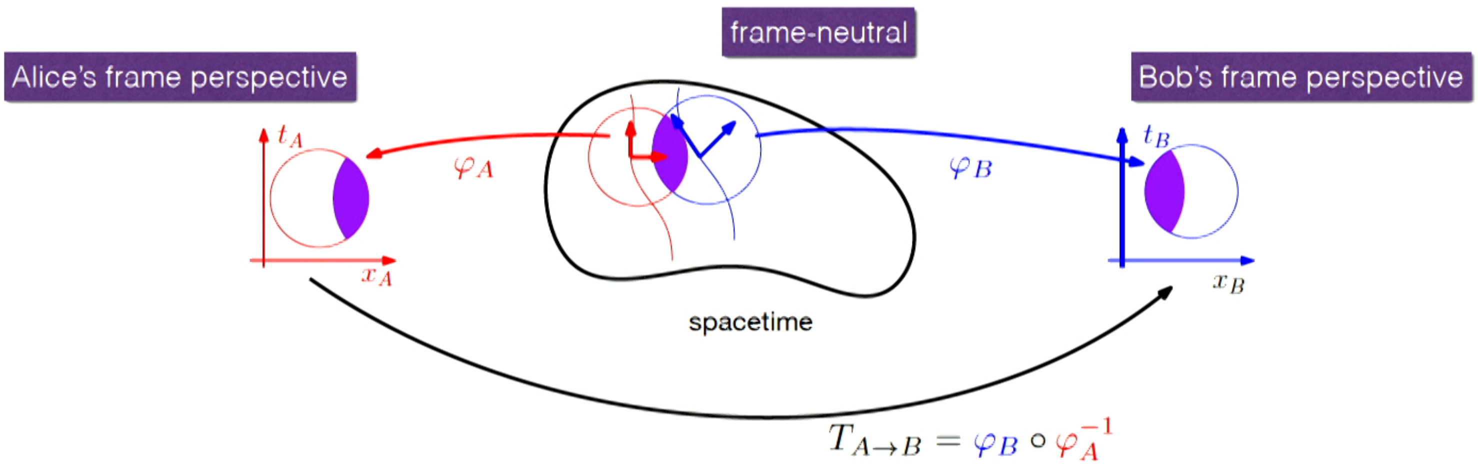
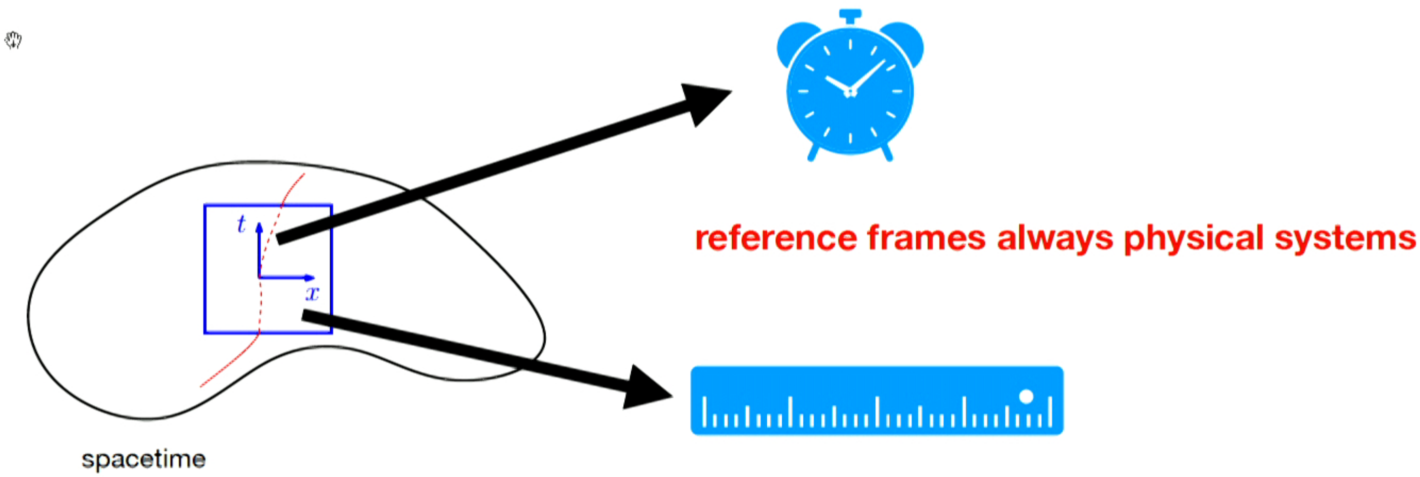


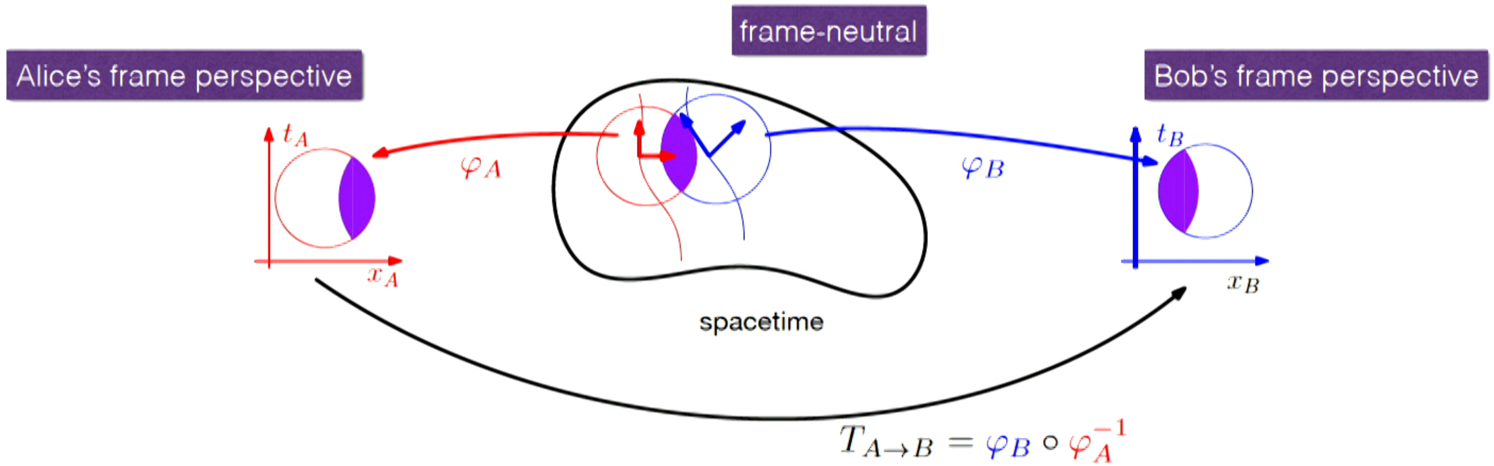
Quantum reference frames/systems



Universality of QT $\Rightarrow$ RF subject to QM itself

Towards quantum general covariance

"All the laws of physics are the same in all reference frames."



How to make sense of general covariance when frames are quantum?

$\Rightarrow$ especially in QG when imposing diffeo invariance

Bojowald, PH, Tsobanjan 2011; PH, Kubalova, Tsobanjan 2012; Palmer, Girelli, Bartlett 2014; PH 2017; Giacomini, Castro-Ruiz, Brukner 2019; Vanrietvelde, PH, Giacomini, Castro-Ruiz 2020;
+ various recent works: Belenchia, Brukner, Castro-Ruiz, Giacomini, PH, Locke, Smith, Vanrietvelde, Yang 2018-20;

Plan

1. Perspective-neutral approach to quantum general covariance
2. Trinity of relational quantum dynamics
3. Changing quantum clocks
4. Conclusions

Strategy

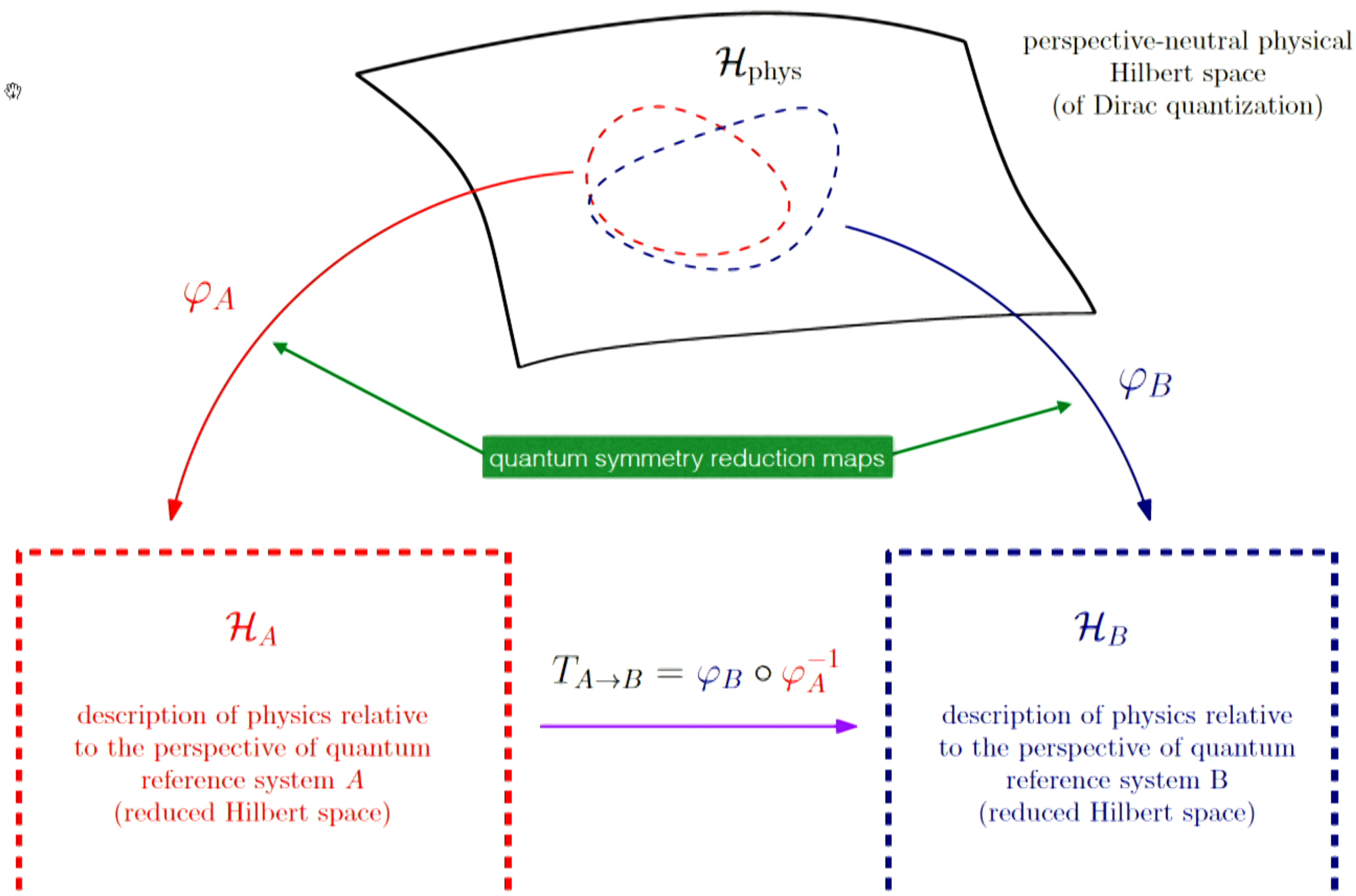
shift focus from passive to active diffeos

$\Rightarrow$ get away from coordinate description, focus on dynamical DoFs

better for QG, were coordinates a priori absent

key: symmetries $\Rightarrow$ redundancy

‘Quantum coordinate changes’:

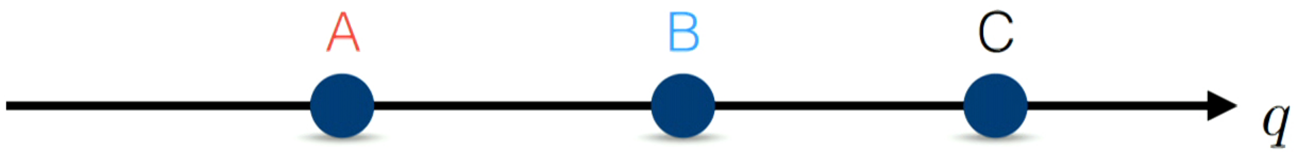


Warm-up:
Quantum symmetry reduction in 1D space

Illustration: spatial QRF changes in 1D

Require invariance under translations

Illustration: spatial QRF changes in 1D



Require invariance under translations

<u>classical:</u>

kinem. phase space $\mathcal{P}_{\text{kin}} = \mathbb{R}^2_A \times \mathbb{R}^2_B \times \mathbb{R}^2_C$

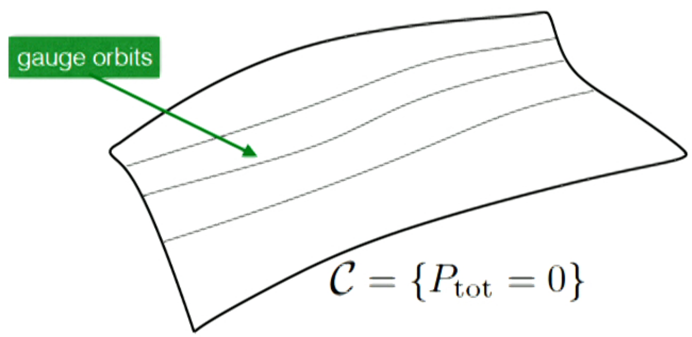
constraint $P_{\text{tot}} = p_A + p_B + p_C = 0$

gauge variables q_i

phys. observables

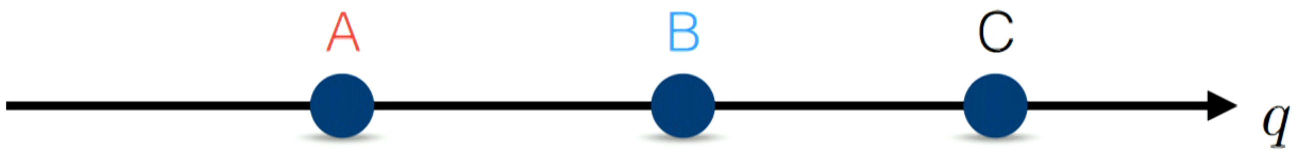
$q_B - q_A, q_C - q_A, q_B - q_C$

p_A, p_B, p_C



redundancy in gauge-inv. description
on constraint surface

Redundant description = perspective-neutral description*



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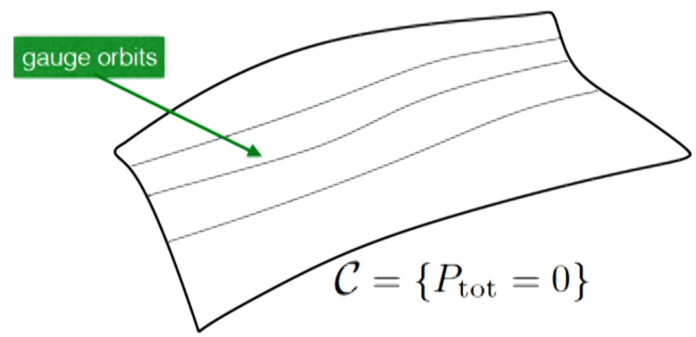

redundancy in gauge-inv. description on constraint surface

* global description of physics prior to having chosen a reference frame/system from whose perspective to describe remaining DoFs

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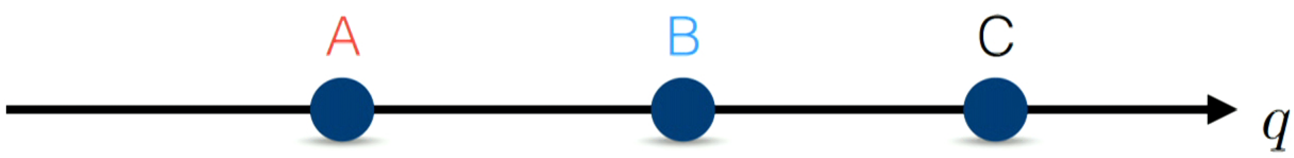

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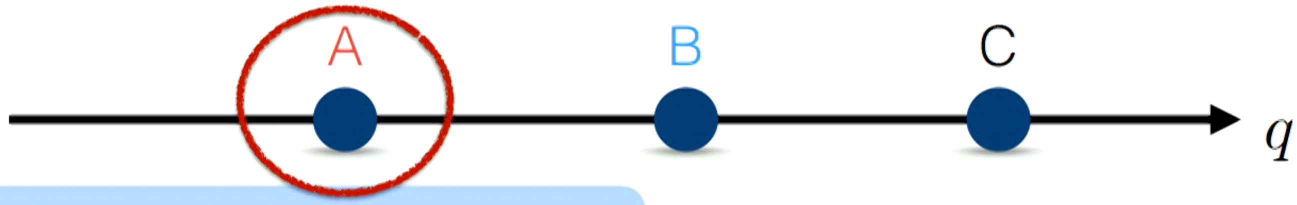
Reference system DoFs = redundant DoFs

$\Rightarrow$ avoid self-reference



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1. Choose perspective, e.g. A

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$$\mathcal{T}_A : (q_i, p_i)_{i=A,B,C} \mapsto (q_A, P_{\text{tot}})$$
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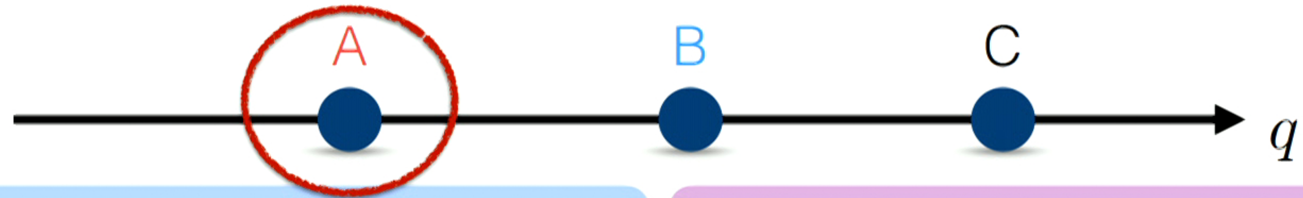

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red. phase space $\mathcal{P}_{BC|A} \simeq \mathbb{R}^2_B \times \mathbb{R}^2_C$

gauge-fixed description of gauge-inv. info: A's internal perspective

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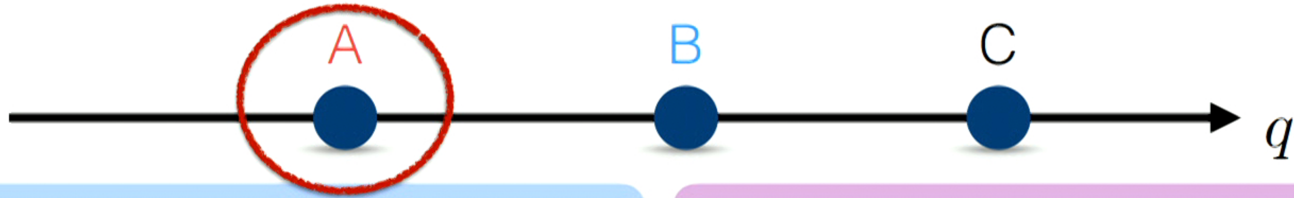

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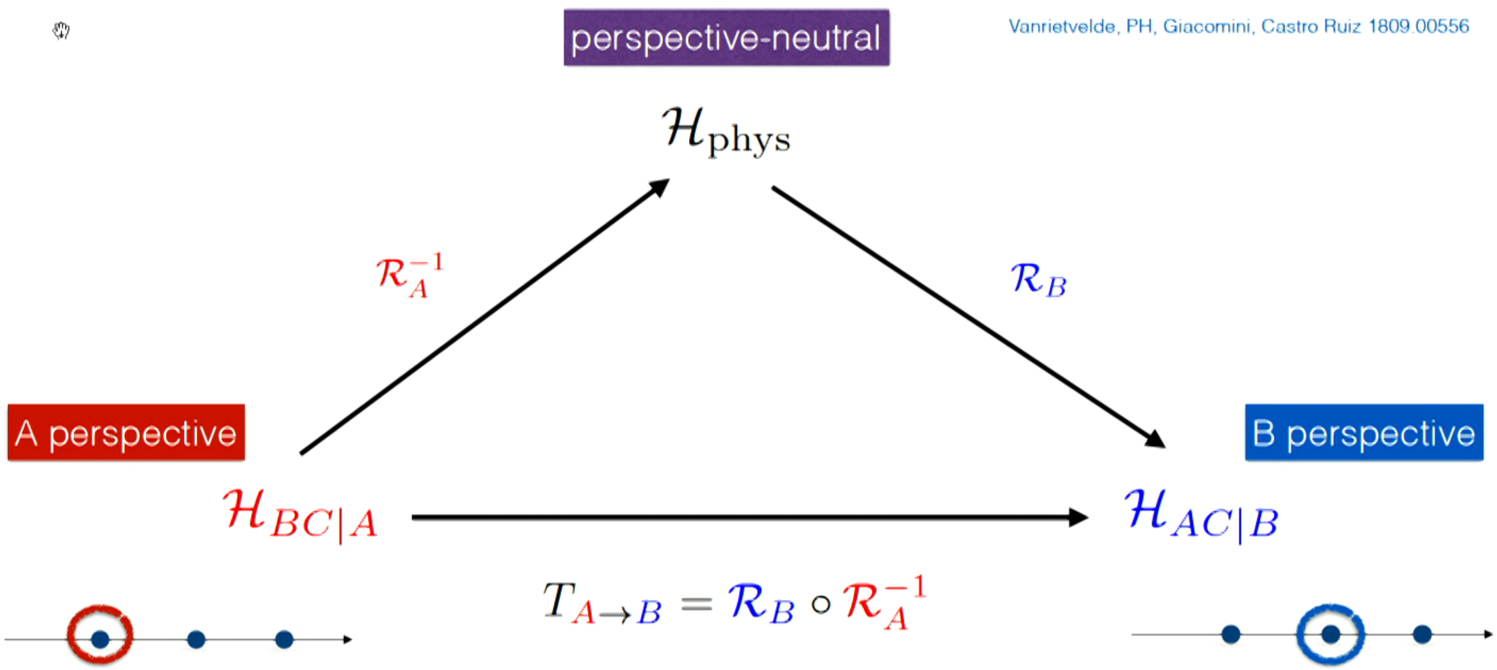
Change of perspective

Vanrietvelde, PH, Giacomini, Castro Ruiz 1809.00556



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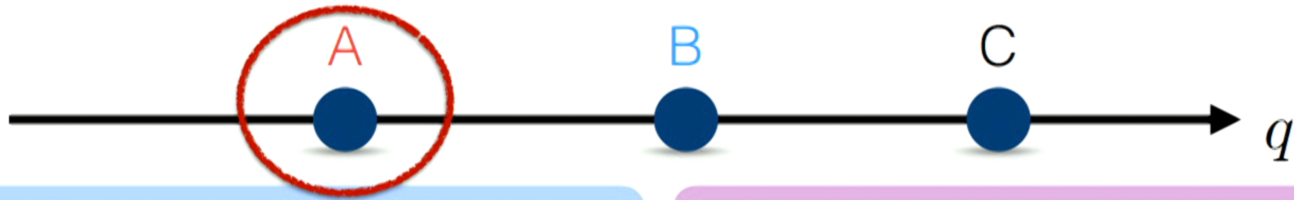

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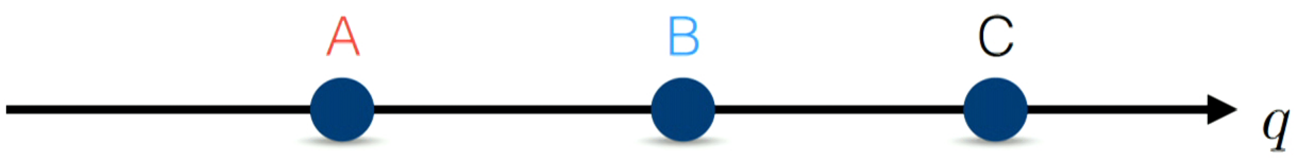

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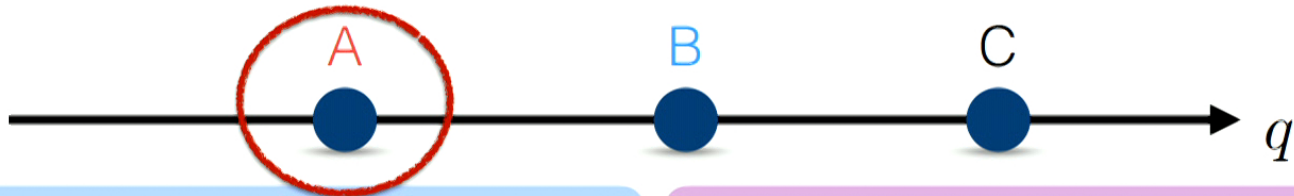

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$\leftarrow$ “$-\hat{p}_A$”

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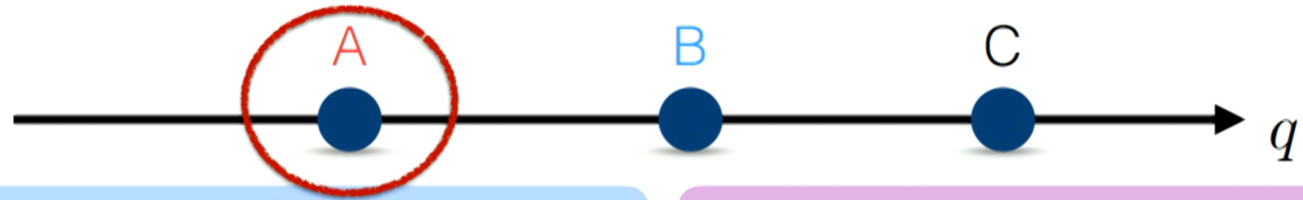

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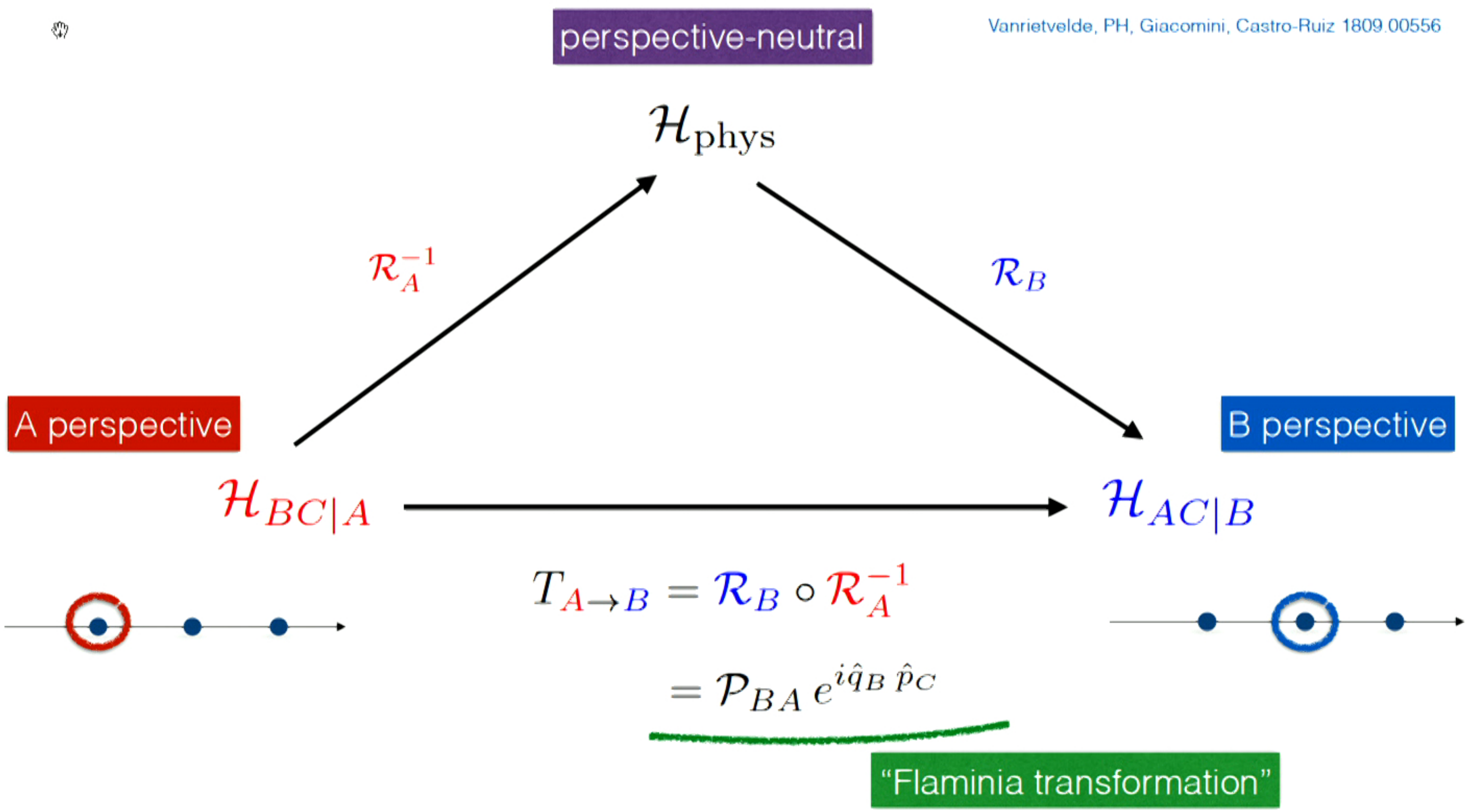
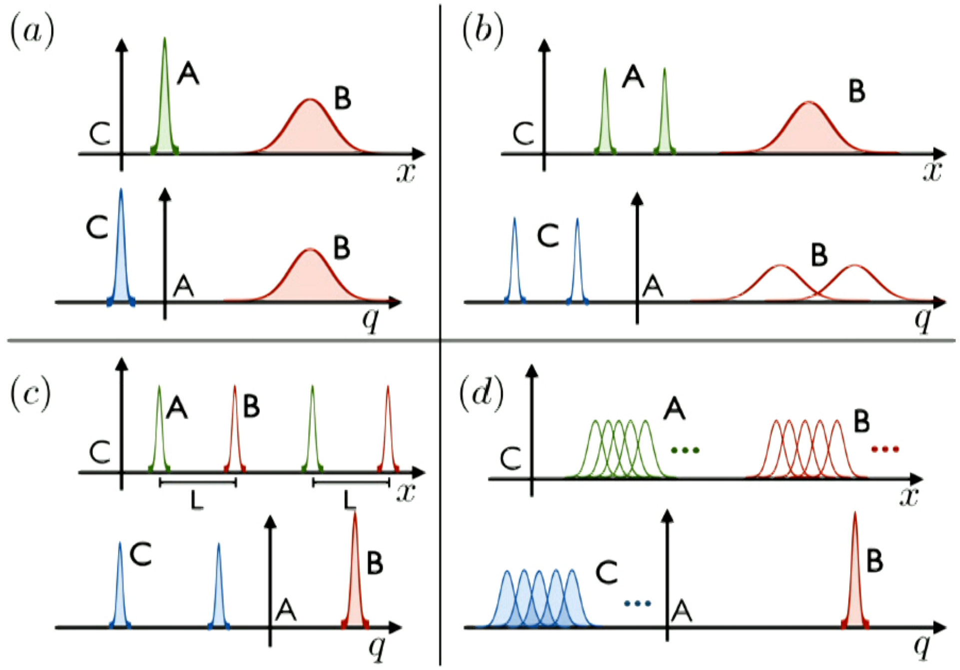
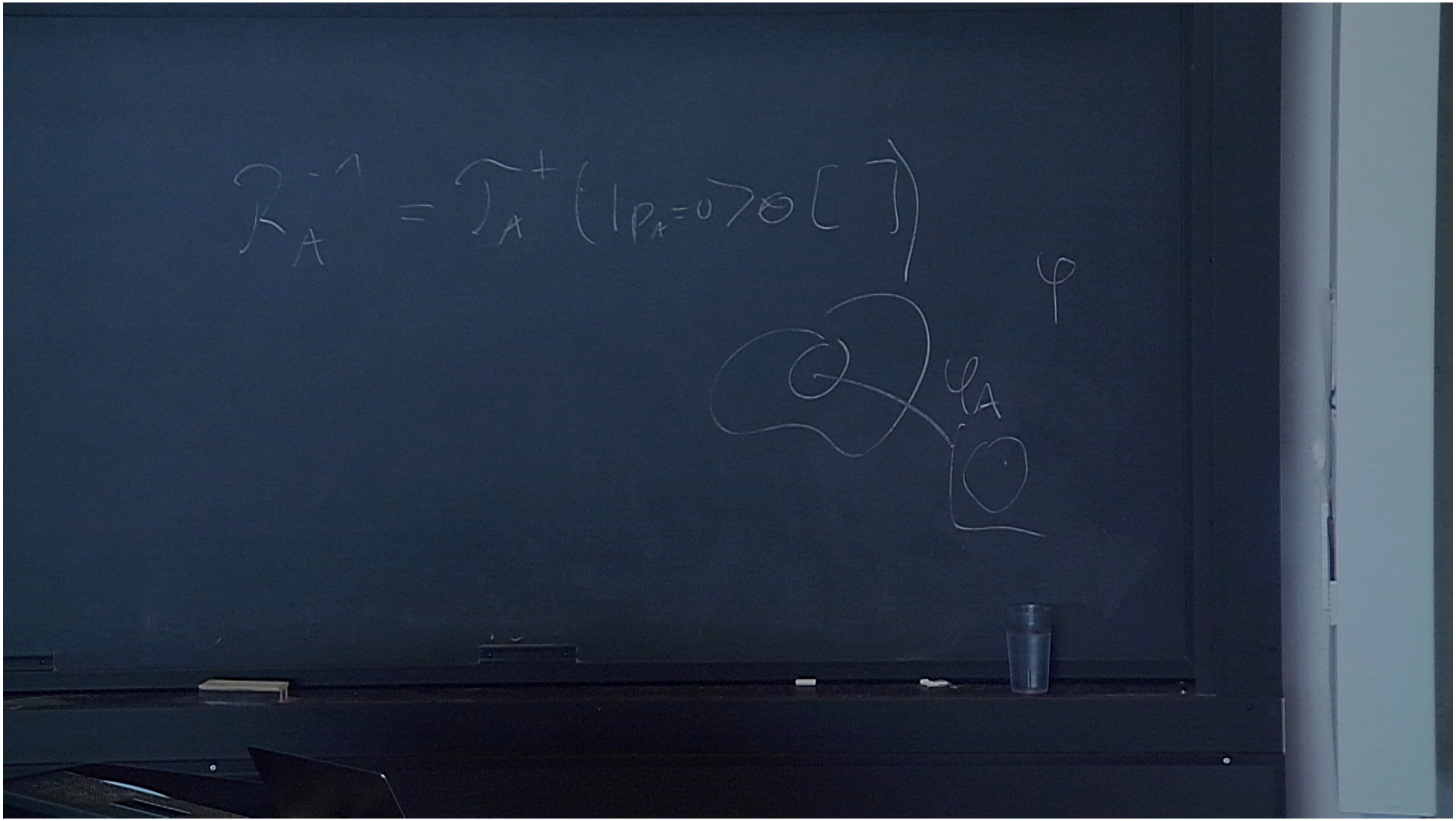


Giacomini, Castro-Ruiz, Brukner, *Nat Com* **10**, 494 (2019)

QRF dependence of correlations and superpositions



see Giacomini, Castro-Ruiz, Brukner *Nat. Comm.* **10**, 494 (2019)


$R_A^{-1} = T_A^{+} (|p_A=0\rangle \otimes [\ \ ])$
φ
φ_A

Perspective-neutral explanation

redundancy

$$|\psi_{\text{phys}}\rangle = \int \prod_{i=A,B,C} dp_i\, \psi_{\text{kin}}(\{p_i\})\, \delta(P_{\text{tot}})\, |p_A\rangle \otimes |p_B\rangle \otimes |p_C\rangle$$

"A disentangler" $\mathcal{T}_A$

Push phys. info into BC factor

$$|p_A = 0\rangle_A \otimes |\psi_{BC|A}\rangle$$

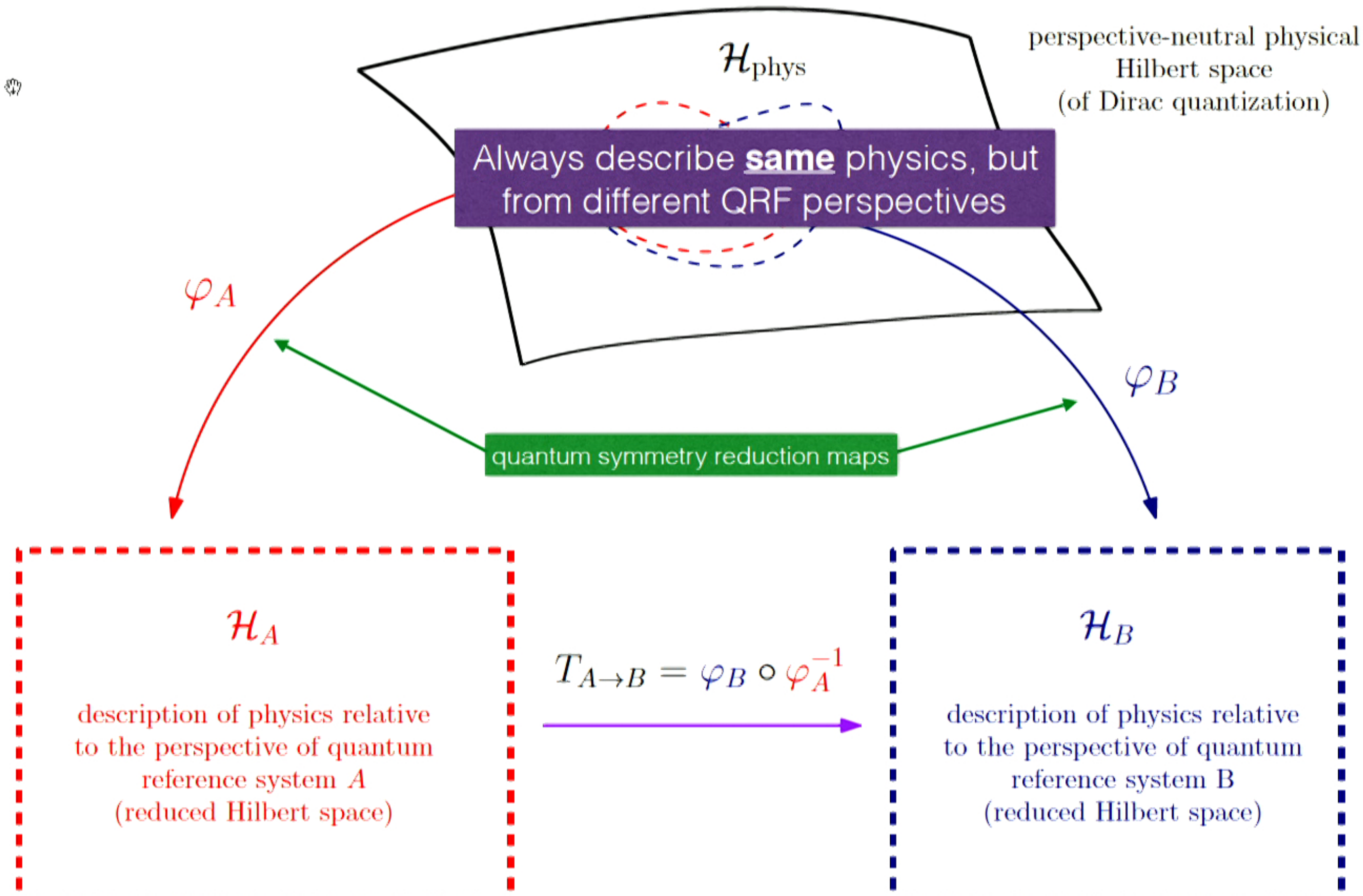
"B disentangler" $\mathcal{T}_B$

Push phys. info into AC factor

$$|p_B = 0\rangle_B \otimes |\psi_{AC|B}\rangle$$

‘Quantum coordinate changes’:

Vanrietvelde, PH, Giacomini, Castro-Ruiz, arXiv:1809.00556
Vanrietvelde, PH, Giacomini, arXiv:1809.05093
PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611



Hamiltonian constraints and clock-neutrality

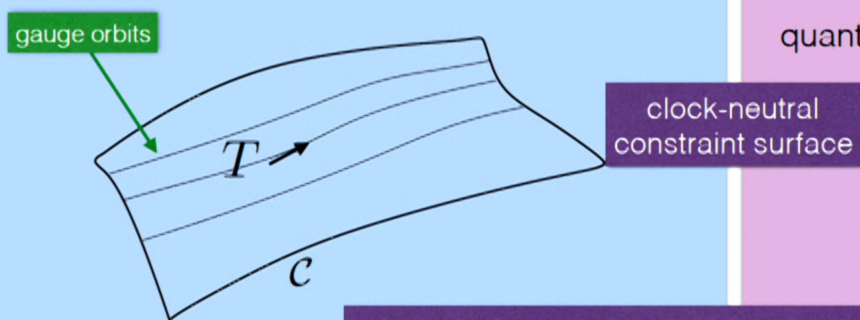
$$S = \int ds\, L(q^a, dq^a/ds) \qquad L(q^a, dq^a/ds) \mapsto L(q^a, dq^a/d\tilde{s})\frac{d\tilde{s}}{ds}$$

- diffeo & reparametrization inv. lead to Hamiltonian constraint

<u>classical:</u>

$$N(s)C_H(\{\text{phase space variables}\}) \approx 0$$

generator of dynamics <u>and</u> symmetry



<u>quantum:</u>

$$\hat{C}_H\,|\psi_{\text{phys}}\rangle = 0$$

problem of time, frozen formalism

quantum constraint surface $\mathcal{H}_{\text{phys}}$

"timeless" quantum theory

choose dynamical 'clock' (temporal reference system) and evolve other DoFs w.r.t. it

[DeWitt; Isham; Page & Wootters, Rovelli, Dittrich…]

Hamiltonian constraints and clock-neutrality

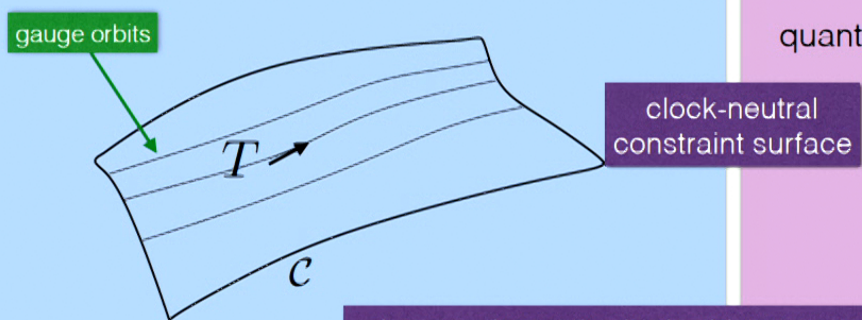
$$S = \int ds\, L(q^a, dq^a/ds) \qquad L(q^a, dq^a/ds) \mapsto L(q^a, dq^a/d\tilde{s})\frac{d\tilde{s}}{ds}$$

- diffeo & reparametrization inv. lead to Hamiltonian constraint

<u>classical:</u>

$$N(s)C_H(\{\text{phase space variables}\}) \approx 0$$

generator of dynamics <u>and</u> symmetry



<u>quantum:</u>

$$\hat{C}_H\,|\psi_{\text{phys}}\rangle = 0$$

problem of time, frozen formalism

quantum constraint surface $\mathcal{H}_{\text{phys}}$

~~"timeless" quantum theory~~

Clock-neutral

choose dynamical 'clock' (temporal reference system) and evolve other DoFs w.r.t. it

[DeWitt; Isham; Page & Wootters, Rovelli, Dittrich…]

Restriction

No interaction between clock and evolving DoFs

$$C_H = H_C + H_S$$

clock — system

H_C continuous spectrum (generates group $G \simeq \mathbb{R}$) and either

1. Non-degenerate, or
2. Doubly degenerate $H_C = \pm p_t^2$

H_S arbitrary

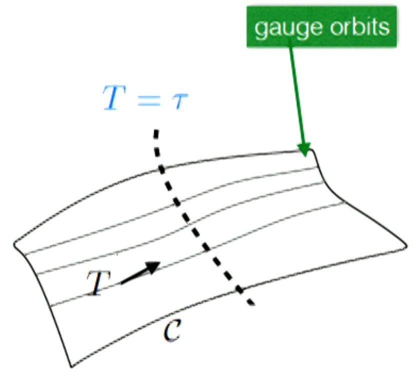
- Vacuum Bianchi models
- FRW + m=0 scalar field
- Relativistic particle
- Many non-relativistic models
- . . .

Classical relational dynamics

Choose time function T s.t.

$$\{T, H_C\} = 1$$

locally always possible



Relational Dirac observables: evolve system observable relative to time function

What is value of f_S when clock T reads τ?

$$F_{f_S,T}(\tau) = \alpha^s_{C_H} \cdot f_S \Big|_{\alpha^s_{C_H} \cdot T = \tau} = \sum_{n=0} \frac{(\tau - T)^n}{n!} \{f_S, H_S\}_n$$

Rovelli '90s, Dittrich '00s

solution to eom $\frac{df_S}{ds} = \{f_S, C_H\}$

gauge invariant $\{C_H, F_{f_S,T}(\tau)\} \approx 0$

gauge-inv. evol. rel. to T
=
"scanning with T=const surfaces through constraint surface"

Going beyond constraints linear in momenta

aim: quantize $\{T, H_C\} = 1$, and T^n

For relational observables…

$$F_{f_S,T}(\tau) = \sum_{n=0} \frac{(\tau - T)^n}{n!} \{f_S, H_S\}_n$$

… but also quantum symmetry reduction

$$\mathcal{T}_A := e^{i\hat{q}_A(\hat{p}_E + \hat{p}_F)}$$

"$-\hat{p}_A$"

conjugate to $\hat{P}_{\text{tot}}$

Covariant clock POVMs

POVM for clock readings: $E_T(\Delta t) = \int_{\Delta t \subset \mathbb{R}} E_T(dt) \geq 0, \quad E_T(\mathbb{R}) = \mathbf{1}$

covariance w.r.t. $\hat{H}_C$ $\qquad E_T(\Delta t + t) = U_C(t)\, E_T(\Delta t)\, U_C^\dagger(t)$

[Holevo, Busch, Milburn, Caves, Braunstein, ...
+ Smith for relational dynamics]

how? $\qquad E_T(dt) = \sum_\sigma |t,\sigma\rangle\langle t,\sigma| \qquad |t,\sigma\rangle = \int d\varepsilon\, e^{-i\varepsilon t}\, |\varepsilon,\sigma\rangle$

σ : degeneracy label for $\hat{H}_C$

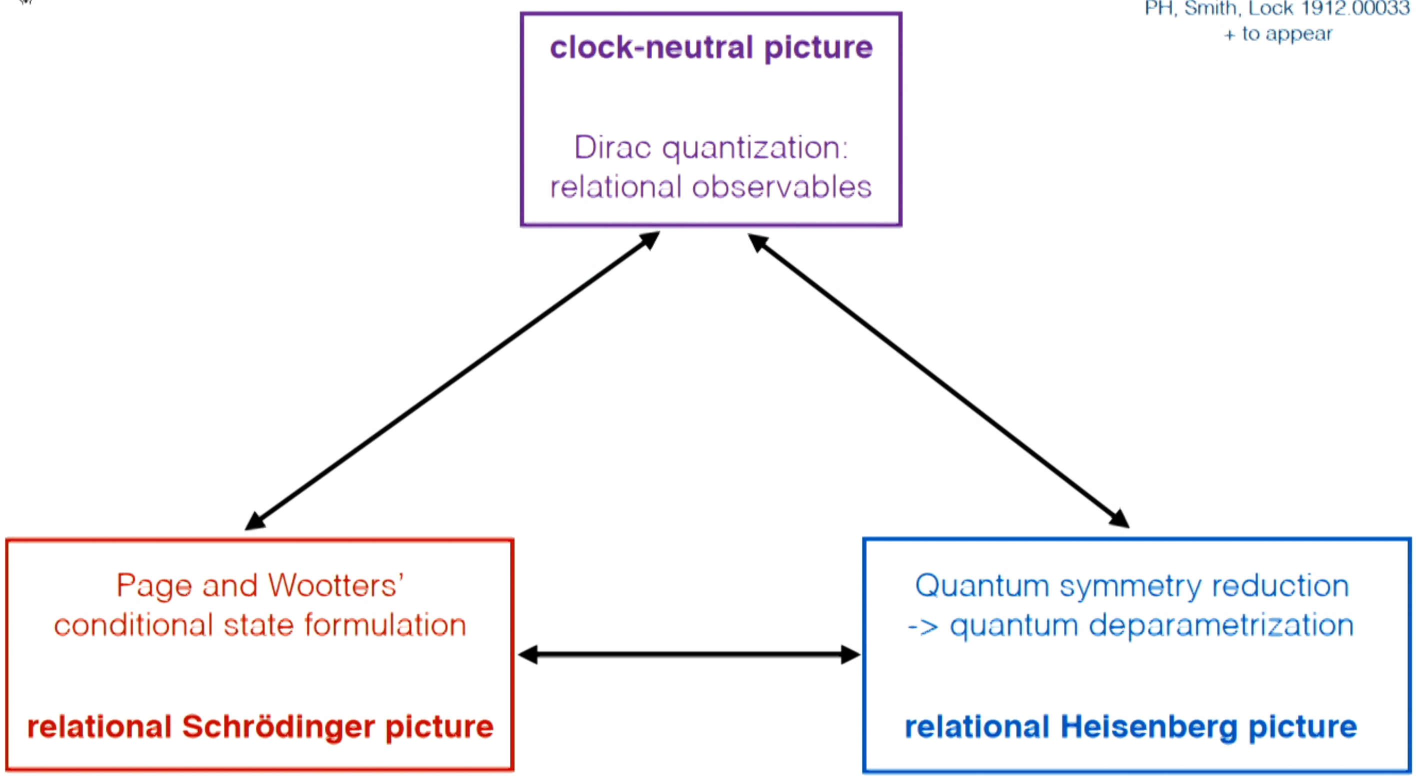
n-th moment operator $\qquad \hat{T}^{(n)} = \int_{\mathbb{R}} t^n\, E_T(dt)$ $\qquad$ not necessarily self-adjoint

generalization of canon. conjugacy $\qquad [\hat{T}^{(n)}, \hat{H}_C] = i\, n\, \hat{T}^{(n-1)}$

e.g. $\qquad \hat{H}_C = -\hat{p}_t^2 \qquad \Rightarrow \qquad \hat{T}^{(1)} = -\frac{1}{4}(\hat{t}\,\hat{p}_t^{-1} + \hat{p}_t^{-1}\,\hat{t})$

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033
+ to appear



I: Quantum relational Dirac observables

'clock-neutral' states

$$\hat{C}_H \, |\psi_{\rm phys}\rangle = (\hat{H}_C + \hat{H}_S) \, |\psi_{\rm phys}\rangle = 0$$

want to quantize relational observables

$$F_{f_S,T}(\tau) = \sum_{n=0} \frac{(\tau - T)^n}{n!} \{f_S, H_S\}_n$$

$\Rightarrow$ need quantization of T^n

covariant clock POVMs

I: Quantum relational Dirac observables

PH, Smith, Lock 1912.00033
+ to appear
[+ related work by Chataignier '19]

'clock-neutral' states

$$\hat{C}_H\,|\psi_{\rm phys}\rangle = (\hat{H}_C + \hat{H}_S)\,|\psi_{\rm phys}\rangle = 0$$

What is value of $\hat{f}_S$ when clock reads τ?

$$\hat{F}_{f_S,T}(\tau) := \int E_T(dt) \otimes \sum_{n=0} \frac{i^n}{n!}(t-\tau)^n\,[\hat{f}_S, \hat{H}_S]_n$$

$$= \sum_\sigma \int dt\, e^{-i\hat{C}_H\,t}\left(|\tau,\sigma\rangle\langle\tau,\sigma| \otimes \hat{f}_S\right) e^{i\hat{C}_H\,t}$$

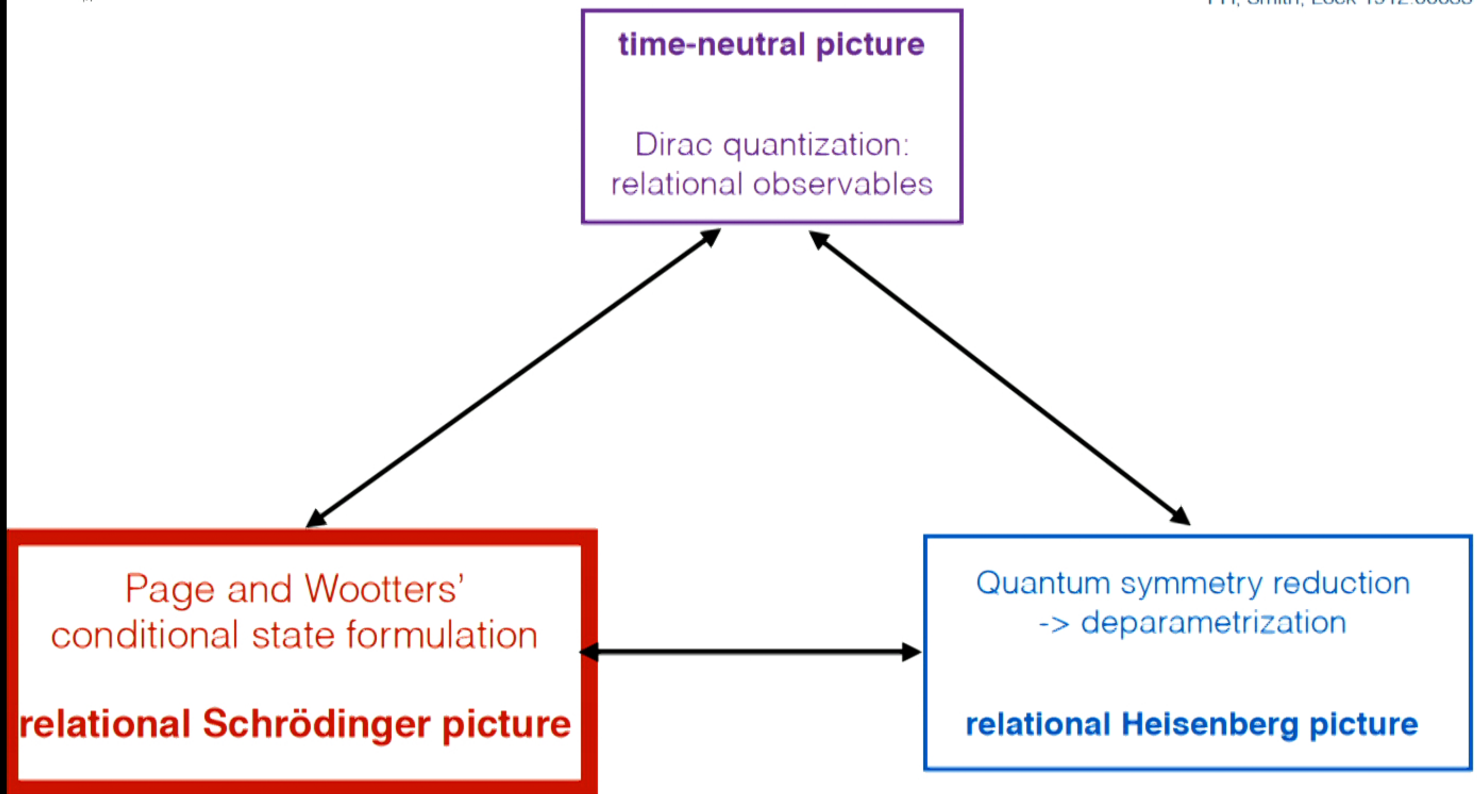
coherent group averaging or G-twirl

'projector' onto clock time τ

gauge-inv. $[\hat{F}_{f_S,T}, \hat{C}_H] = 0$

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033



II: Page-Wootters formalism

Extract dynamics from physical states through conditional probabilities

$$\hat{C}_H \, |\psi_{\rm phys}\rangle = (\hat{H}_C + \hat{H}_S) \, |\psi_{\rm phys}\rangle = 0$$

What is probability that $\hat{f}_S$ has outcome f_S given that clock reads τ?

$$P(f_S \text{ when } \tau) = \frac{\langle\psi_{\rm phys}|(|\tau\rangle\langle\tau| \otimes |f_S\rangle\langle f_S|)|\psi_{\rm phys}\rangle_{\rm kin}}{\langle\psi_{\rm phys}|(|\tau\rangle\langle\tau| \otimes I_S)|\psi_{\rm phys}\rangle_{\rm kin}}.$$

Page, Wootters '83

- conditional state of system when clock reads τ

$$|\psi_S^\sigma(\tau)\rangle := (\langle\tau,\sigma| \otimes I_S) \, |\psi_{\rm phys}\rangle$$

covariant

Projection onto classical gauge fixing (and degeneracy sector)

- solves **relational Schrödinger eq.**

Page, Wootters '80s;
PH, Smith, Locke 1912.00033 + to appear

$$i\partial_\tau |\psi_S^\sigma(\tau)\rangle = \hat{H}_S \, |\psi_S^\sigma(\tau)\rangle$$

Kuchar's 3 criticisms of PW formalism

Kuchar 1992

1. violation of constraint (ignore degeneracy sectors)

inner product on $\mathcal{H}_{\rm kin}$

$$\langle \psi_S(\tau) | \, \hat{f}_S \, | \psi_S(\tau) \rangle = \langle \psi_{\rm phys} | (| \tau \rangle \langle \tau | \otimes \hat{f}_S) | \psi_{\rm phys} \rangle_{\rm kin}$$

Does not commute with $\hat{C}_H$

$\Rightarrow$ throws $|\psi_{\rm phys}\rangle$ out of $\mathcal{H}_{\rm phys}$

2. wrong propagators for non-relativistic systems

$$P(q' \text{ when } \tau' | q \text{ when } \tau) = \frac{\langle \psi_{\rm phys} | (|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)(|\tau'\rangle\langle\tau'| \otimes |q'\rangle\langle q'|)(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|) | \psi_{\rm phys} \rangle_{\rm kin}}{\langle \psi_{\rm phys} | (|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|) | \psi_{\rm phys} \rangle_{\rm kin}}$$

$$= |\delta(\tau - \tau')\delta(q - q')|^2$$

3. wrong localization probability for Klein-Gordon systems

$$P(\vec{q} \text{ when } t) \sim |\psi(\vec{q}, t)|^2$$

sol. to KG eqn

conditioning w.r.t. Minkowski time (not covariant)

Kuchar's 3 criticisms of PW formalism

Kuchar 1992

1. violation of constraint (ignore degeneracy sectors)

in... product on $\mathcal{H}_{\text{kin}}$

$$\langle \psi_S(\tau)|\, \hat{f}_S \,|\psi_S(\tau)\rangle = \langle \psi_{\text{phys}}|(|\tau\rangle\langle\tau| \otimes \hat{f}_S)|\psi \ldots$$

Does not comm... C_H

$\Rightarrow$ thr... $\text{phys}\rangle$ out of $\mathcal{H}_{\text{phys}}$

2. wrong propagators for non... ...istic systems

$$P(q' \text{ when } \tau' | q \text{ when } \tau) = \frac{\langle \ldots \rangle\langle\tau| \otimes |q\rangle\langle q|)(|\tau'\rangle\langle\tau'| \otimes |q'\rangle\langle q'|)(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)|\psi_{\text{phys}}\rangle_{\text{kin}}}{\langle\psi_{\text{phys}}|(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)|\psi_{\text{phys}}\rangle_{\text{kin}}}$$

$$= |\delta(\tau - \tau')\delta(q - q')|^2$$

3. w...g localization probability for Klein-Gordon systems

$$P(\vec{q} \text{ when } t) \sim |\psi(\vec{q}, t)|^2$$

sol. to KG eqn

conditioning w.r.t. Minkowski time (not covariant)

effectively ended research on PW formalism for a while

Equivalence of PW and relational observables

PH, Smith, Lock 1912.00033;
+ to appear

conditioning $|\psi_S^\sigma(\tau)\rangle := (\langle\tau,\sigma|\otimes I_S)\,|\psi_{\text{phys}}\rangle$ defines reduction

$\mathcal{H}^\sigma_{\text{phys}}$ ← σ – sector of $\mathcal{H}_{\text{phys}}$

$$\mathcal{R}^\sigma_{\text{PW}}(\tau) = \langle\tau,\sigma|\otimes I_S \qquad (\mathcal{R}^\sigma_{\text{PW}}(\tau))^{-1} = \int dt\,|t,\sigma\rangle\otimes U_S(t-\tau)$$

$\mathcal{H}^{\text{phys}}_{S,\sigma}$ ← conditional (Schrödinger) states

1. equiv. of observables

$$(\mathcal{R}^\sigma_{\text{PW}}(\tau))^{-1}\,\hat{f}_S^{\text{phys}}\,\mathcal{R}^\sigma_{\text{PW}}(\tau) = \hat{F}^\sigma_{f_S,T}(\tau)$$

2. equiv. of expectation values

$$\langle\psi_S^\sigma(\tau)|\,\hat{f}_S^{\text{phys}}\,|\psi_S^\sigma(\tau)\rangle = \langle\psi_{\text{phys}}|\,\hat{F}^\sigma_{f_S,T}(\tau)\,|\psi_{\text{phys}}\rangle_{\text{phys}}$$

`gauge-fixed' — manifestly gauge-inv.

Equivalence of PW and relational observables

PH, Smith, Lock 1912.00033;
+ to appear

conditioning $|\psi_S^\sigma(\tau)\rangle := (\langle\tau,\sigma|\otimes I_S)\,|\psi_{\text{phys}}\rangle$ defines reduction

$\mathcal{H}^\sigma_{\text{phys}}$ ← σ – sector of $\mathcal{H}_{\text{phys}}$

$\mathcal{R}^\sigma_{\text{PW}}(\tau) = \langle\tau,\sigma|\otimes I_S$ (↓)

$(\mathcal{R}^\sigma_{\text{PW}}(\tau))^{-1} = \int dt\,|t,\sigma\rangle\otimes U_S(t-\tau)$ (↑)

$\mathcal{H}^{\text{phys}}_{S,\sigma}$ ← conditional (Schrödinger) states

1. equiv. of observables

$$(\mathcal{R}^\sigma_{\text{PW}}(\tau))^{-1}\,\hat{f}_S^{\text{phys}}\,\mathcal{R}^\sigma_{\text{PW}}(\tau) = \hat{F}^\sigma_{f_S,T}(\tau)$$

quantum analog of `gauge-inv. extension of gauge-fixed quantity'

2. equiv. of expectation values

$$\langle\psi_S^\sigma(\tau)|\,\hat{f}_S^{\text{phys}}\,|\psi_S^\sigma(\tau)\rangle = \langle\psi_{\text{phys}}|\,\hat{F}^\sigma_{f_S,T}(\tau)\,|\psi_{\text{phys}}\rangle_{\text{phys}}$$

PW is (quantum) gauge-fixed formulation of manifestly gauge-invar. theory on physical Hilbert space

Resolving Kuchar's 3 criticisms

PH, Smith, Lock 1912.00033;
+ to appear

1. ~~violation of constraint~~ (ignore degeneracy sectors)

$$\langle\psi_S(\tau)|\,\hat{f}_S\,|\psi_S(\tau)\rangle = \langle\psi_{\rm phys}|(|\tau\rangle\langle\tau|\otimes\hat{f}_S)|\psi_{\rm phys}\rangle_{\rm kin}$$

Does not commute with $\hat{C}_H$

$\Rightarrow$ throws $|\psi_{\rm phys}\rangle$ out of $\mathcal{H}_{\rm phys}$

No violation of constraints since

$$\langle\psi^\sigma_S(\tau)|\,\hat{f}^{\rm phys}_S\,|\psi^\sigma_S(\tau)\rangle = \langle\psi_{\rm phys}|\,\hat{F}^\sigma_{f_S,T}(\tau)\,|\psi_{\rm phys}\rangle_{\rm phys}$$

\`gauge-fixed' — manifestly gauge-inv.

Resolving Kuchar's 3 criticisms

2. wrong propagators for non-relativistic systems

$$P(q' \text{ when } \tau' | q \text{ when } \tau) = \frac{\langle\psi_{\rm phys}|(|\tau\rangle\langle\tau|\otimes|q\rangle\langle q|)(|\tau'\rangle\langle\tau'|\otimes|q'\rangle\langle q'|)(|\tau\rangle\langle\tau|\otimes|q\rangle\langle q|)|\psi_{\rm phys}\rangle_{\rm kin}}{\langle\psi_{\rm phys}|(|\tau\rangle\langle\tau|\otimes|q\rangle\langle q|)|\psi_{\rm phys}\rangle_{\rm kin}}$$

$$= |\delta(\tau-\tau')\delta(q-q')|^2$$

[different from Gambini et al '09; Giovanetti et al '15]

$$P(q' \text{ when } \tau' | q \text{ when } \tau) = \frac{\langle\psi_{\rm phys}|\hat{F}_{|q\rangle\langle q|,T}(\tau)\,\hat{F}_{|q'\rangle\langle q'|,T}(\tau')\,\hat{F}_{|q\rangle\langle q|,T}(\tau)|\psi_{\rm phys}\rangle_{\rm phys}}{\langle\psi_{\rm phys}|\hat{F}_{|q\rangle\langle q|,T}(\tau)|\psi_{\rm phys}\rangle_{\rm phys}}$$

$$= |\langle q'|U_S(\tau'-\tau)|q\rangle|^2$$

PH, Smith, Lock 1912.00033;

Resolving Kuchar's 3 criticisms

3. wrong localization probability for Klein-Gordon systems

$$P(\vec{q} \text{ when } t) \sim |\psi(\vec{q}, t)|^2$$

conditioning w.r.t. Minkowski time (not covariant)

sol. to KG eqn

Newton-Wigner localization probability for Klein-Gordon systems

$$P(\vec{q} \text{ when } \tau, \sigma) = |\psi_S^\sigma(\tau, \vec{q})|^2$$

conditioning w.r.t. covariant clock POVM

sol. to Schrödinger eqn

$\sim t/p_t$

PH, Smith, Lock to appear

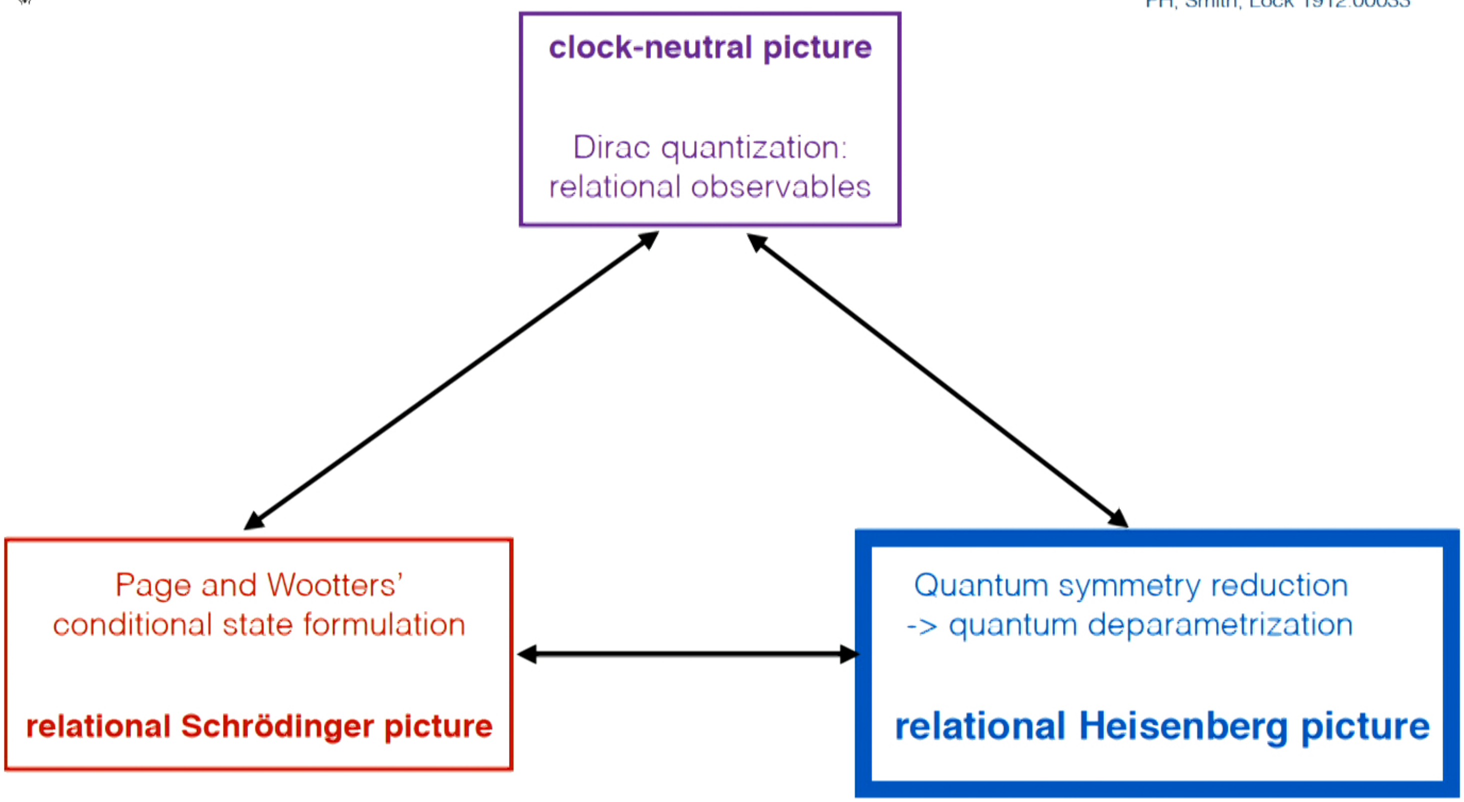
$$(\psi^\sigma_{\text{phys}}, \psi^\sigma_{\text{phys}})_{\text{KG}} \equiv \int d\vec{q}\, |\psi_S^\sigma(\tau, \vec{q})|^2$$

upshot:

PW just as viable an approach to relational dynamics as relational Dirac observables
+ implications also for interpretation of relational observables

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033



Quantum symmetry reduction

PH, Smith, Lock 1912.00033;
+ to appear

$|\psi_{\rm phys}\rangle \in \mathcal{H}_{\rm phys}$

analogous to particle example

constraint trivialization

$$\mathcal{T}_T = \int E_T(dt) \otimes e^{it(\hat{H}_S+\varepsilon)}$$

$$\sum_\sigma |\varepsilon,\sigma\rangle_C \otimes |\psi_S^\sigma\rangle$$

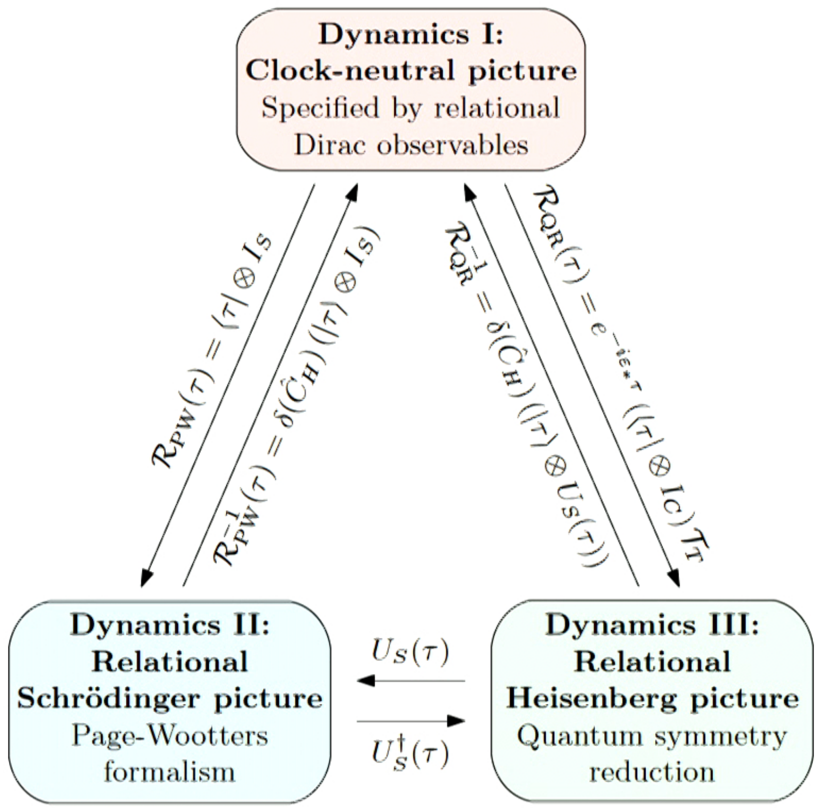
proj. onto class. gauge $\langle\tau,\sigma|$

$$|\psi_S^\sigma\rangle \in \mathcal{H}_{S,\sigma}^{\rm phys}$$

+ observables transform correctly $\Rightarrow$ **get relational Heisenberg picture**

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033;
+ to appear



Changing quantum clocks

Bojowald, PH, Tsobanjan, CQG 28, 035006 (2011)
Bojowald, PH, Tsobanjan, PRD 83,125023 (2011)
PH, Kubalova, Tsobanjan, PRD 86, 065014 (2012)

PH, Vanrietvelde 1810.04153
PH 1811.00611
PH, Smith, Lock 1912.00033 + to appear

Multiple choice problem

- many possible choices for relational clocks $\Rightarrow$ inequivalent quantum dynamics

e.g., 2 clocks variables T_1, T_2

$$T_1(T_2) \qquad \text{vs} \qquad T_2(T_1)$$

What if T_1, T_2 operators?

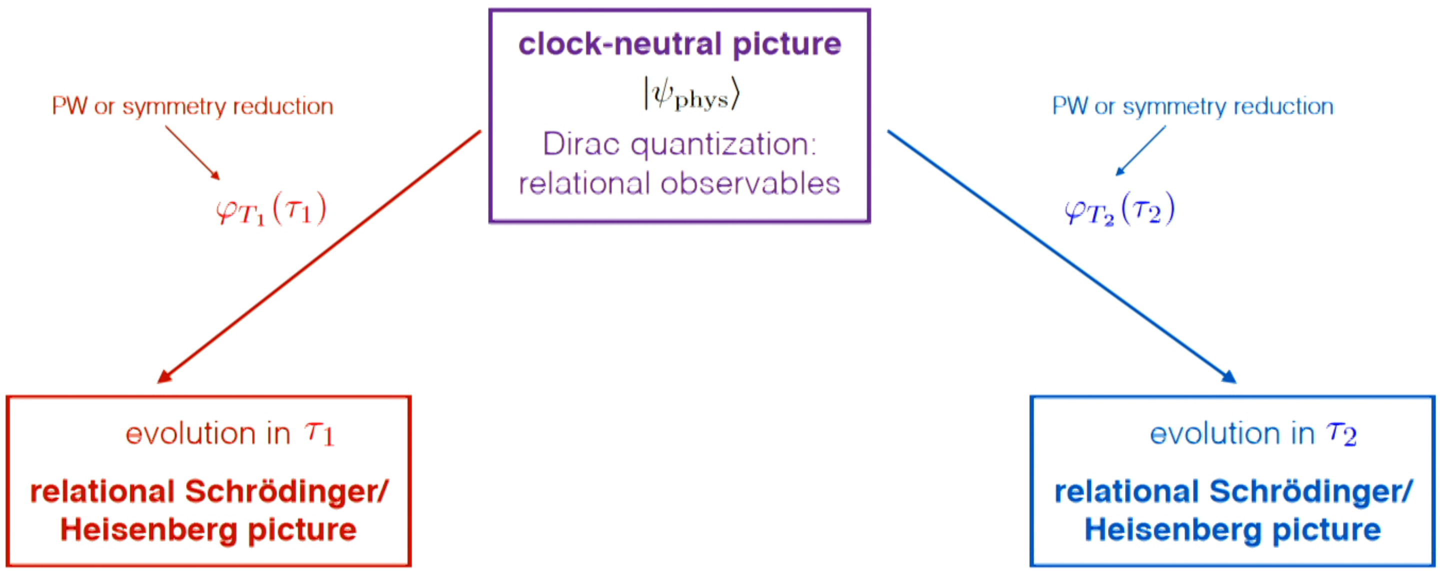
e.g. $T_1 = a, T_2 = \varphi$
in quantum cosmology

Kuchar (1992):
"The multiple choice problem is one of an embarrassment of riches: out of many inequivalent options, one does not know which one to select."

Isham (1993):
"Can these different quantum theories be seen to be part of an overall scheme that is covariant?… It seems most unlikely that a single Hilbert space can be used for all possible choices of an internal time function."

Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611
PH, Smith, Lock 1912.00033 + to appear



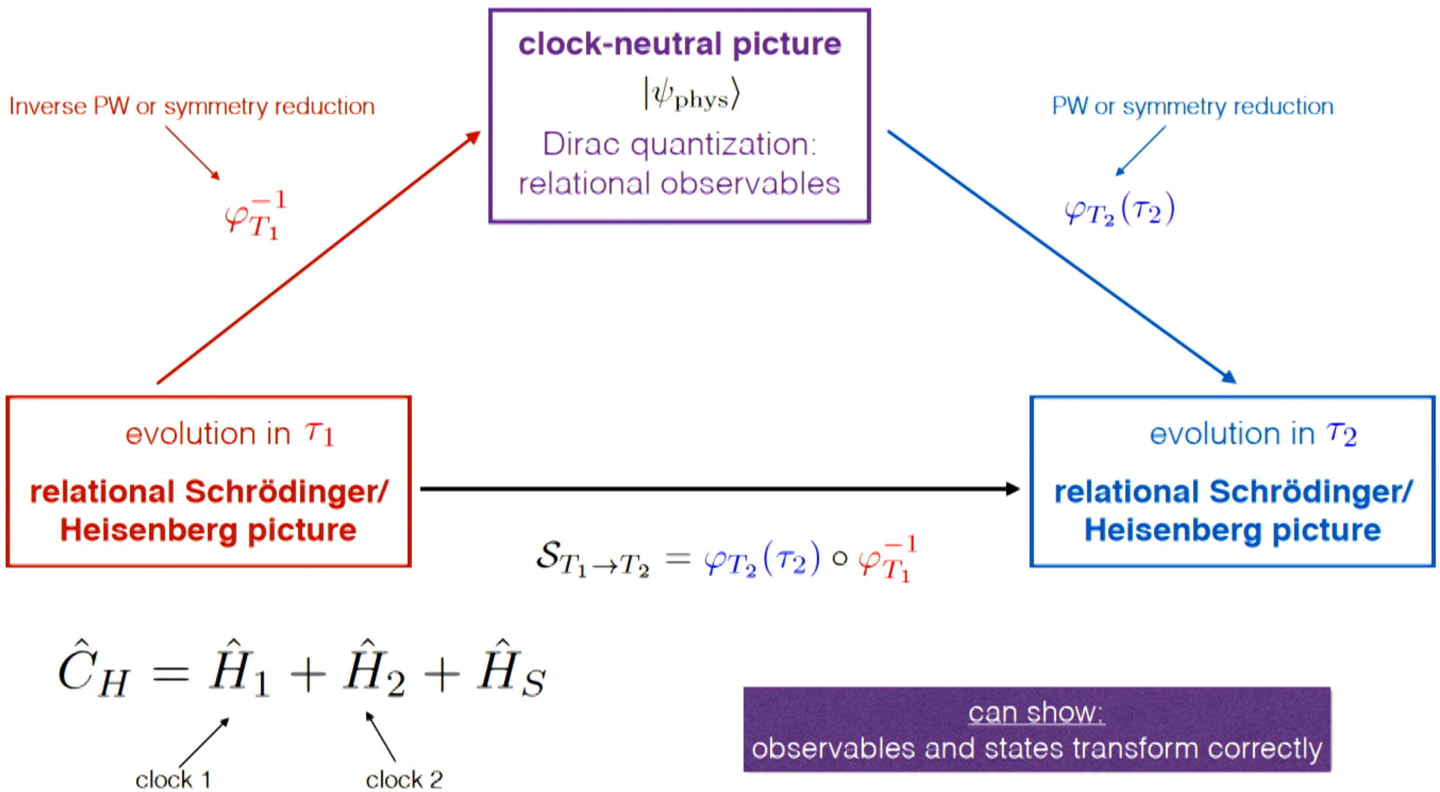
$$\hat{C}_H = \hat{H}_1 + \hat{H}_2 + \hat{H}_S$$

clock 1 ($\hat{H}_1$), clock 2 ($\hat{H}_2$)

encompasses various classes of non-relativistic and both special and general relativistic models

Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611
PH, Smith, Lock 1912.00033 + to appear



can show:
observables and states transform correctly

Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611
PH, Smith, Lock 1912.00033 + to appear

Always describe <u>same</u> dynamics, but relative to different clocks

Inver

relational observables

$\varphi_{T_1}^{-1}$

$\varphi_{T_2}(\tau_2)$

evolution in τ_1

relational Schrödinger/ Heisenberg picture

evolution in τ_2

relational Schrödinger/ Heisenberg picture

$$\mathcal{S}_{T_1 \to T_2} = \varphi_{T_2}(\tau_2) \circ \varphi_{T_1}^{-1}$$

$$\hat{C}_H = \hat{H}_1 + \hat{H}_2 + \hat{H}_S$$

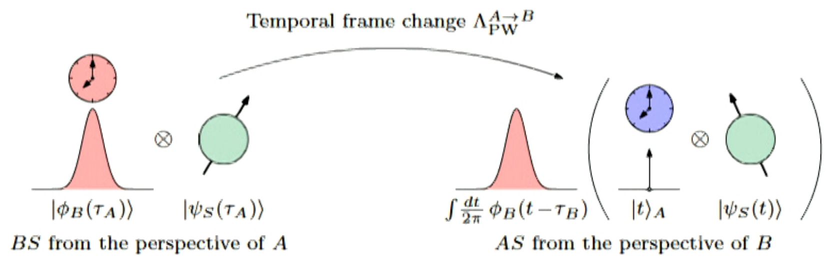
clock 1

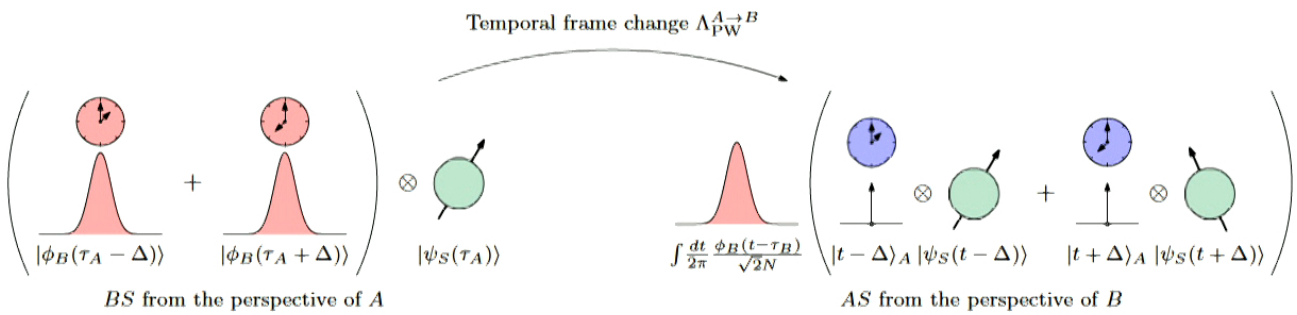
clock 2

can show:
observables and states transform correctly

Clock-dependent temporal (non-)locality

PH, Smith, Lock 1912.00033
Castro-Ruiz et al 1908.10165





explanation analogous to QRF dependent spatial correlations

superposition of time evolutions

Multiple choice problem

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$$T_1(T_2) \qquad \text{vs} \qquad T_2(T_1)$$

e.g. $T_1 = a, T_2 = \varphi$
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Multiple choice ~~problem~~ feature?

- many possible choices for relational clocks $\Rightarrow$ inequivalent quantum dynamics

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$$T_1(T_2) \qquad \text{vs} \qquad T_2(T_1)$$

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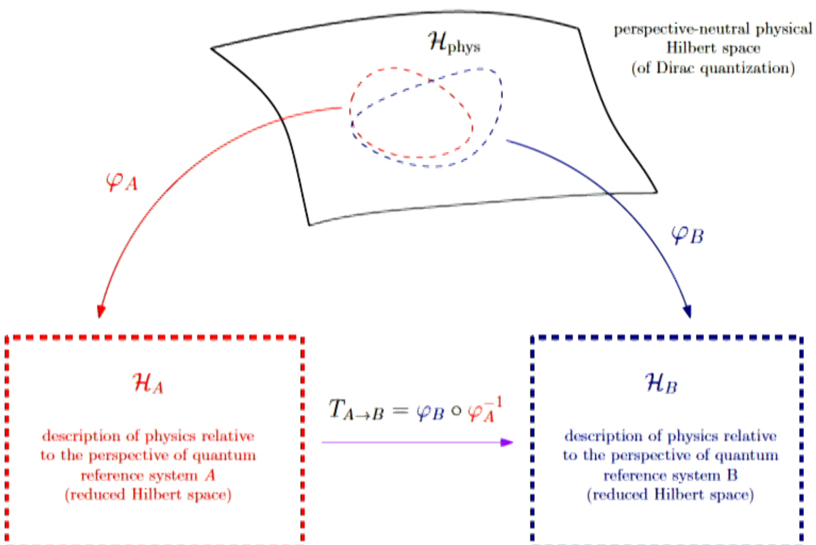
Conclusions



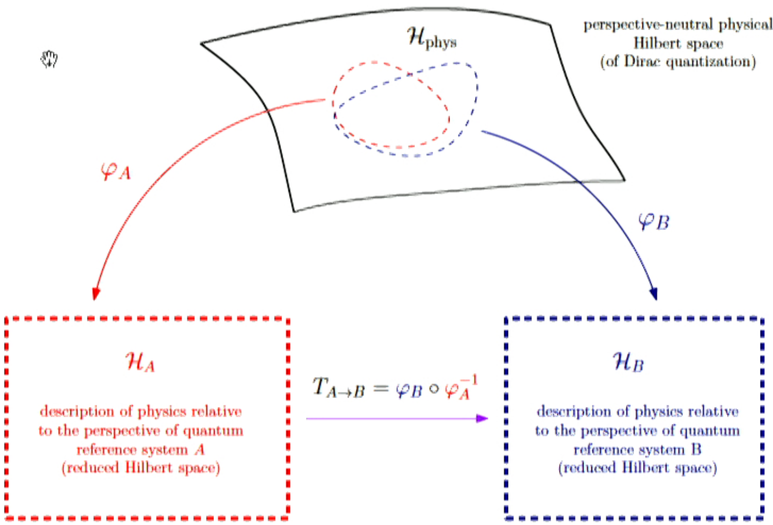
systematic changes of quantum reference system perspectives

- key: quantum reduction method
- both spatial and temporal

$\Rightarrow$

- structure for exploring **quantum version of general covariance**
- **new perspective on problem of time**
  (i) trinity: 3 faces of same rel. quantum dynamics
  (ii) multiple choice feature rather than problem

Outlook



Spatial QRFs:
Vanrietvelde, PH, Giacomini, Castro-Ruiz 1809.00556
Vanrietvelde, PH, Giacomini 1809.05093

Internal clocks:
PH, Vanrietvelde 1810.04153
PH 1811.00611

Trinity:
PH, Smith, Lock 1912.00033
+ relativistic version to appear

applications/extensions:

- Further abstraction (axiomatization) and QFT
- Measurement problem and Wigner's friend paradox
- Frame dependence of correlations in cosmology
- Import QRF machinery from QI into QG
- New method for constructing quantum rel. Dirac obs.