Title: Rebooting Canonical Quantum gravity

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Series: Quantum Gravity

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Abstract: In this talk, we present a new outlook on canonical quantum gravity and its coupling to matter.

We will show how this fresh perspective combines the critical elements of holography, loop quantum gravity, and relative locality. I will first focus on the consequences of cutting open a portion of space and show that new symmetry charges and new degrees of freedom reveal themselves.
I will explain the nature of this boundary symmetry algebra in metric gravity and then first-order gravity. We will see that a rich structure appears that explains from the continuum perspective the non-commutativity of geometric flux, the quantization of the area spectra, the nature of the simplicity constraints but also reveals the dual momentum observables and finally allow to reconcile the elements of canonical gravity with Lorentz invariance.
I will discuss the issue of quantization as a challenge of finding a representation of the boundary algebra and will give clues about where we are in this process.

Rebooting Canonical gravity

Laurent Freidel PI.

PI 2020

Based on:

- arXiv:1507.02573 with Perez
- arXiv:1611.03668 with Donnelly
- arXiv:1601.04744 with Perez and Pranzetti
- arXiv:1806.03161 with Pranzetti
- arXiv:1810.09364 with Livine
- arXiv:1811.0436 with Girelli and Shoshany
- arXiv:1910.05642 with Pranzetti and Livine
- arXiv: I,II,III to appear with Pranzetti and Geiller
- arXiv: I,II to appear with Donnelly, Moosavian and Speranza

Quantum-Gravity

- A good time to revisit the issue of quantum gravity from first principles
- Many different approaches, lots of successes and failures. It is a good time to regroup and learn from these and understand what they have in common.
- First and foremost, The most important task we could do is to predict new phenomena that will eventually be observed. For me this means thinking about new phenomena and new observables.

Quantum-Gravity

- What have learned from
  Loop Gravity, Holography, String theory.
- What are key successes and commonalities?

Nonlocality: A fundamental scale, a fundamental tension, a fundamental quantum number N.
A discrete spectra of area or entropy.
A profound unification of matter and geometry for ST.

Questions

- What are the central questions, besides discovering new observables prediction and phenomena?

- Q1: How do we reconcile the presence of a fundamental scale with the relativity principle ?
- Q2: What are the fundamental degrees of freedom? What is the geometrical entropy counting?
- Q3: Can we think of quantum gravity as a fundamental unification of matter and geometry?

QG: matter = quantum geometrical defect.
ST: geometry is defined by interaction of extended probes.
Unifying matter and geometry is unifying the small and the large. This why it is hard.

Discretization and Symmetries

To define a quantum field theory system we need to introduce a cutoff, ie discretize it, and then learn how to remove it without breaking symmetries.

Take as an example Yang-Mills.

In the presence of a cutoff the gauge group as to be deformed: Maximal momenta gauge transformations do not form a group.

One resolution is to put the system on the lattice.

In order to preserve a lattice version of gauge symmetry.

There is a catch: the E field have to become non-commutative.

Continuum $[E(x)^a_I, E(y)^b_J] = 0, \quad [E^a_I(x), A^J_b(y)] = \delta^a_b \delta^J_I \delta^{(3)}(x,y)$

Discrete $\longrightarrow$ $[E^I_e, E^J_{e'}] = \delta_{ee'} C^{IJ}{}_K E^K_e$ Why?

Discretization and Symmetries

The non-commutativity of the electric fields
is necessary to have a lattice version of gauge invariance

$$[E_e^I, E_{e'}^J] = \delta_{ee'} C^{IJ}{}_K E_e^K$$

$$\mathrm{d}_A E^I = 0 \quad \rightarrow \quad \sum_{e \supset v} E_e^I = 0.$$

Can we understand this from the continuum?
Can we apply it to gravity ?

Which component of the metric should be taken as
commutative or not when discretized,
in order for symmetries to be preserve in the process ?
What are these discrete symmetries ?
Do they have a continuum analog

Loop gravity: Results

- Loop gravity/Spin foam focuses on the BF formulation of gravity where

$*B_{IJ} = \frac{1}{2}\epsilon_{IJKL}B^{KL}$

$$S = \int B^{IJ} \wedge F_{IJ}(\omega), \qquad \underbrace{B^{IJ} = *(e \wedge e)^{IJ} + \gamma^{-1}(e^I \wedge e^J)}_{\text{simplicity}}.$$

- LQG discretization assign to each edge of a network generators generators B_e^{IJ} which forms a Lorentz algebra

- The simplicity constraint is then written as a second class constraint Boost = γ Rotation

Immirzi parameter

$$B_e^{0i} = \frac{\gamma}{2}\varepsilon^i{}_{jk}B_e^{jk}.$$

- The Casimir of the Rotational part of B is the Area square.

$$A_e = \gamma\sqrt{j(j+1)}$$

Loop gravity: Puzzles

- The main puzzle face by people outside LQG when looking at it is the interplay there is between discretization and quantization.
- Is the discrete area spectra an artefact of the discretization?
- Or can it been shown to have its source in the continuum?
- Why the Immirzi parameter (a boundary term) plays such central role? Does it mean that everything commute in in metric gravity ? $\gamma = 0$
- The reason for the non-commutation of B is the implementation of the discrete Lorentz Gauss law but simplicity constraints implies a symmetry breaking

$$\mathrm{SL}(2,\mathbb{C}) \to \mathrm{SU}(2)$$

Alexandrov, Noui, Geiller, Rovelli, Speziale,...

- Is the discrete area in tension with Lorentz invariance?

Looking inside space

- How can we get a deeper understanding of what space is made of, of what are its fundamental degrees of freedom?
- We could admire it from far away, marvel at the perfect geometry of some of its most symmetric solution: the stationary black-hole and the mathematical beauty and regularity of asymptotic infinity like most modern physicists or we could do what any 5 year old would do when trying to understand a new puzzle…

Looking inside space

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- We could admire it from far away, marvel at the perfect geometry of some of its most symmetric solution: the stationary black-hole and the mathematical beauty and regularity of asymptotic infinity like most modern physicists or we could break it open and admire its granularity
- In QFT spacetime is a holistic unbreakable object, the source of all entanglement. It is not thinkable to cut it open.

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- In QG we discover new degrees of freedom that appear at the edge of space when we cut it open: A new set of boundary symmetries and conjugated edge modes.

Gauge theory and boundaries

- In gauge theory the main source of entanglement comes from the imposition of the constraints. In Yang-Mills for any slice Σ and in vacuum we have

$$\mathrm{d}_A E^I \overset{\Sigma}{=} 0 \qquad\qquad E \overset{\Sigma}{=} *F$$

- Due to the Bianchi identity any violation j of the Gauss law behave as electric charges and satisfy $\mathrm{d}_A j = 0$

- The canonical generator of gauge transformation $\delta_\alpha A = \mathrm{d}_A \alpha$ is the canonical charge

$$G_\alpha = \int_\Sigma \mathrm{d}_A \alpha \wedge E$$

Gauge theory and boundaries II

- In presence of boundaries and after imposing constraints th(canonical charge becomes

$$G_\alpha \approx \int_{\partial\Sigma} \alpha^I E_I$$

- It satisfy the higher loop algebra $[G_\alpha, G_\beta] = G_{[\alpha,\beta]}$
- E^I is the boundary charge aspect which satisfy the higher loop algebra

$$[E^I(x), E^J(y)] = C^{IJ}{}_K E^K(x)\delta^{(2)}(x-y)$$

- The non-commutativity is already present in the continuum. It is not an artefact of discretization. It is a consequence of gauge invariance.

Gauge theory and boundaries III

- Not only have we charges associated with constraints that that generates bulk gauge transformations, we also have charge associated with topological constraints

$$\mathrm{d}_A F_I = 0$$

- We have electric an magnetic charge aspects

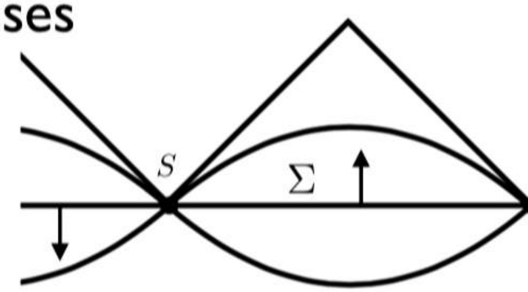
$$G_\alpha = \int_{\partial\Sigma} \alpha^I E_I \qquad \tilde{G}_\alpha = \int_S \alpha^I F_I$$

- The full boundary symmetry group is a centrally extended higher loop group $G^S \times \tilde{G}^S$ Pranzetti 2019
- Central extension being the hall-mark of duality

Metric Gravity

Donnelly 2016

- We can now do the same analysis for metric gravity. The bulk symmetry algebra is diffeomorphism,
- We chose a time foliation that possesses entangling spheres: places where time stop flowing.



Local holography: The hamiltonian generators respecting S are boundary operators

$$L = \frac{1}{2}\sqrt{|g|}R(g) \longrightarrow \quad H_\xi = \int_S \epsilon^{\perp}_b{}^a \nabla_a \xi^b.$$

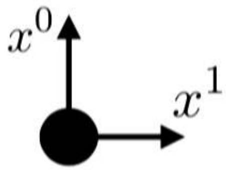
Komar, Wald, ...

$$\epsilon^{\perp}_a{}^b = \sqrt{q}(n_a s^b - s_a n^b)$$

unit normal bivector

Metric Gravity

Donnelly 2016



$$ds^2 = h_{ij}\mathrm{d}x^i\mathrm{d}x^j + q_{\alpha\beta}(\mathrm{d}\sigma^\alpha + N_i^\alpha \mathrm{d}x^i)(\mathrm{d}\sigma^\beta + N_i^\beta \mathrm{d}x^i)$$

normal metric — tangential metric — Normal lapse

Local holography: The bdy symmetry algebra is

$$\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S$$

The charge aspects are

$$H_i{}^j = \frac{\sqrt{q}}{\sqrt{|h|}} h_{ik}\epsilon^{kj}$$

$$K_0 = \tfrac{1}{2}(H_0{}^1 - H_1{}^0), \qquad K_1 = H_0{}^0 = -H_1{}^1, \qquad K_2 = \tfrac{1}{2}(H_0{}^1 + H_1{}^0)$$

They form a local SL(2,R) algebra

$$[K_i(\sigma), K_j(\sigma')] = \epsilon_{ij}{}^k K_k \delta^{(2)}(\sigma,\sigma')$$

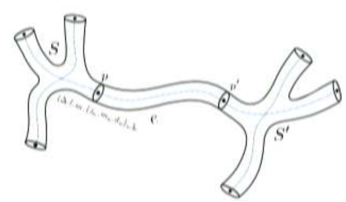
With Casimir

$$\sqrt{q} = K_1^2 + K_2^2 - K_0^2.$$

Continuous series

From bdy algebra to tensor network

- To quantize geometry we need to find representations of the boundary symmetry algebra
- States associated with bulk regions are representation state of the boundary symmetry group.
- Space can be partitioned into finite 3D regions bounded by surfaces.
- Gluing of two subregions are obtained by fusion of the algebraic data along boundary spheres (links).
- Coarse graining is obtained by truncating the boundary symmetry algebra into a finite dimensional algebra



Smolin, Rovelli, Crane, ...

Toward the full bdy symmetry

- In order to quantize gravity we need to answer the question: What is the full bdy symmetry group of gravity ?
- Different components of the symmetry groups are revealed by different gravity formulation.
- In the BF formulation of gravity

$$S = \int B^{IJ} \wedge F_{IJ}(\omega), \qquad \underbrace{B^{IJ} = *(e \wedge e)^{IJ} + \gamma^{-1}(e^I \wedge e^J)}_{\text{simplicity}}.$$

- We have new boundary charges generating an $\mathrm{SL}(2,\mathbb{C})^S$

$$[B_{IJ}(\sigma), B_{KL}(\sigma')] = (B_{IL}\eta_{JK} + \cdots)\delta^{(2)}(\sigma, \sigma')$$

- What is the role of simplicity?

Bulk simplicity

Pranzetti Geiller 2020

- To analyse simplicity we pick an internal unit normal vector and we decompose the B field in Boost and spin components

$$E_I := B_{IJ} n^J, \qquad S_I := *B_{IJ} n^J$$

$n^2 = -1$

- The pull-back of the simplicity constraint becomes $e^I n_I \overset{\Sigma}{=} 0$

$$E_I \overset{\Sigma}{=} \gamma S_I$$

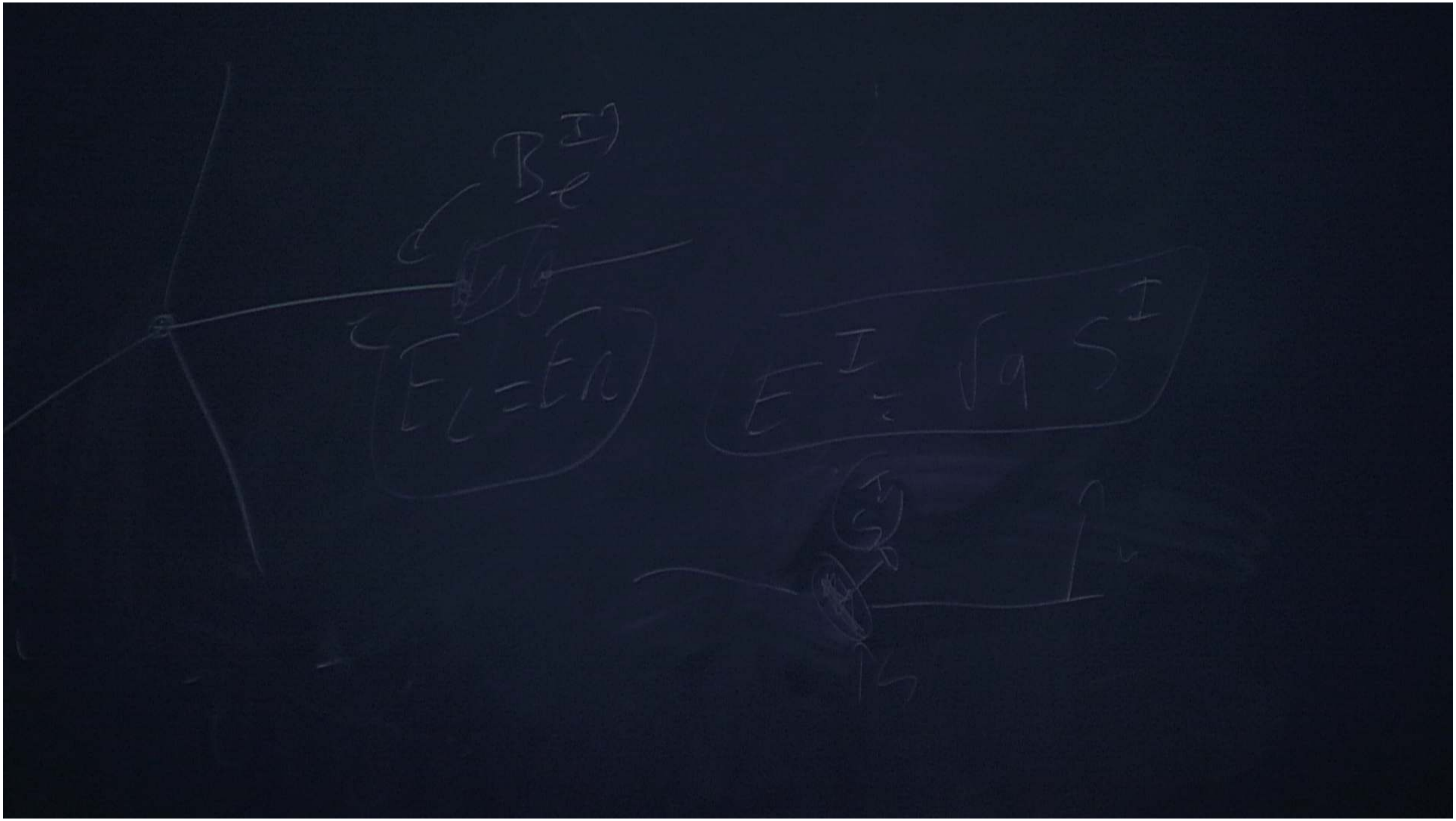
Vectorial area element

- The BF symplectic potential controls the commutations

$$\Theta_{\text{BF}} = \int_\Sigma B_{IJ} \delta\omega^{IJ}$$

- We can also decompose the connection using

$$\mathrm{d}_\omega n^I := K^I$$


$E_L = E_R$
$E^I_i = \sqrt{q}\, S^I$

Bulk simplicity

Pranzetti Geiller 2020

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Bulk simplicity

- After the decomposition the BF symplectic potential reads

$$\Theta_{\mathsf{BF}} = \int_\Sigma (K^I \delta E_I + \pi_I \delta n^I) + \Theta_{\mathsf{BF}}(\partial\Sigma)$$

- Bulk simplicity $\quad \pi_I = 0$

- And $\quad K^I \delta E_I = \underbrace{P^{ab}\delta\tilde{g}_{ab}}_{\text{canonical GR}} + \underbrace{2\delta(\sqrt{\tilde{g}}K)}_{\text{Gibbons-Hawking}}$

Boundary simplicity Pranzetti Geiller 2020

- The boundary BF symplectic potential reads

$$\Theta_{\mathsf{BF}}(S) = \int_S \left(E_I \delta n^I + \frac{1}{2\gamma} e_I \wedge \delta e^I \right)$$

- It is the symplectic potential of a local Poincare algebra with:
  - momenta $p_I = \gamma n_I$
  - Pauli-Lubanski vector $W_I = \gamma S_I \quad = \frac{1}{2}\epsilon_{IJKL}(e^J \wedge e^K) n^L$
  - Angular momenta $L_{IJ} = \frac{E_I}{\gamma} p_J - \frac{E_J}{\gamma} p_I$
  - Total angular momenta $B_{IJ} = E_I n_J - E_J n_I + \frac{1}{\gamma} e_I \wedge e_J$

$$[W_I, W_J] = \epsilon_{IJKL} W^K p^L \qquad [W_I, p_J] = 0$$

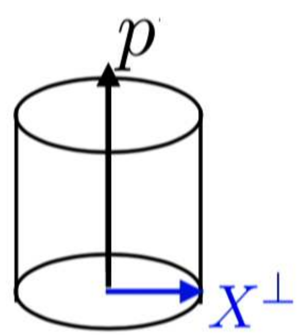
$$[E_I, E_J] = B_{IJ}$$

Boundary simplicity

- This geometrical Poincare algebra has Casimirs

$$p^2 = \gamma^2 \qquad W^2 = \det(q) = \gamma^2 j(j+1)$$

⟶ Discreetness of area element



- Simplicity $C_I = E_I - W_I = 0$
- Geometrically $X_I^{\perp} = S_I$

- The usual gravity limit corresponds to a massless limit

Boundary simplicity

- This geometrical Poincare algebra has Casimirs

$$p^2 = \gamma^2$$

$$W^2 = \det(q) = \gamma^2 j(j+1)$$

$\longrightarrow$ Discreetness of the area element

- Simplicity constraint is preserved by the Lorentz generators

Corresponds to a symmetry breaking

$$\mathrm{SL}(2,\mathbb{C}) \ltimes \mathbb{R}^4 \to \mathrm{SL}(2,\mathbb{C})$$

Which preserves Lorentz symmetry unlike $\mathrm{SL}(2,\mathbb{C}) \to \mathrm{SU}(2)$

Boundary simplicity

- The total angular moment is not the only Dirac observables commuting with the simplicity constraints

The tangential metric is also a good Dirac observable

It satisfy a local $\mathrm{SL}(2,\mathbb{R})$ algebra

$$[q_{ab}, q_{cd}] = -\gamma\left(q_{ac}\epsilon_{bd} + q_{bc}\epsilon_{ad} + q_{ad}\epsilon_{bc} + q_{bd}\epsilon_{ac}\right)$$

Its representation belong to the discrete series.
With Casimir $\det(q) = \gamma^2 j(j+1)$

Question Summary

- What are the discrete symmetries controlling non-commutativity ? Do they have a continuum analog?

  Yes they do. It is the boundary symmetry algebra.

- What are the fundamental degrees of freedom of space?

We have proposed that these are representation states of the boundary symmetry symmetries. That space can be understood algebraically as the fusion of these states.

- Which component of the metric should be taken as commutative or not when discretized ?

Boundary symmetries			
$\mathrm{diff}(S)$	$\mathfrak{sl}(2,\mathbb{R})_{\perp}$	$\mathfrak{sl}(2,\mathbb{R})_{\parallel}$	$\mathfrak{sl}(2,\mathbb{C})$
normal connection	normal metric	tangential metric	tangential bivector field

Question Summary

- Is the discrete area in tension with Lorentz invariance?

No. No more that the presence of discrete spin is in tension with Lorentz symmetry. The discrete area element appears both as a Casimir of the tangential SL(2,R) and as the spin of the elemental Poincare algebra. The simplicity constraint realizes the symmetry braking pattern Poincare $\rightarrow$ Lorentz, not Lorentz $\rightarrow$ Rotation.

- What role is played by the Immirzi parameter ?

A role similar to the one played by the θ parameter in Yang-mills. It labels different representation of the boundary symmetry group. It appears as the mass of the elemental Poincare.

More Questions

- Can we understand matter as a geometrical defect ?
- What is the relation between the material Poincare algebra and the elemental one ?
- Do we have the full boundary symmetry group ?
- Can we achieve a quantization of the boundary symmetry algebra without resorting to discretization ?
- How do we go from entangling spheres to evolving boundaries ?