Title: Geometry and 5d N=1 QFTs

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Series: Quantum Fields and Strings

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Abstract: I will explain that a geometric theory built upon the theory of complex surfaces can be used to understand wide variety of phenomena in five-dimensional supersymmetric theories, which includes the following:

Classification of 5d superconformal field theories (SCFTs).
Enhanced flavor symmetries of 5d SCFTs.
5d N=1 gauge theory descriptions of 5d and 6d SCFTs.
Dualities between 5d N=1 gauge theories.
T-dualities between 6d N=(1,0) little string theories.

This relationship between geometry and 5d theories is based on M-theory and F-theory compactifications.

M-theory on CY3 (intersection theory)

$\Big\downarrow$ Application to 5d $\mathcal{N}=1$ QFTs

1. Classification of 5d SCFTs
2. 5d $\mathcal{N}=1$ gauge theories $\underset{\text{mass deformation}}{\overset{\text{UV complete}}{\rightleftarrows}}$ 5d SCFTs / 6d SCFTs on S^1 with twist

e.g. $SU(2) + nF \longrightarrow$ 5d SCFT $\quad 0 \le n \le 7$

$SU(2) + 8F \longrightarrow$ 6d SCFT on S^1 (E-string)

$SU(2) + \geq 9F \longrightarrow$ No SCFT UV completion

3. Enhanced flavor symmetries of 5d SCFTs

e.g. $SU(2) + (0 \le n \le 7) F$ Classically $SO(2n) \longrightarrow E_{n+1}$

4. Dualities: $SU(3)_{-5+\frac{n}{2}} + nF$ $\searrow$ 5d SCFT $(0 \le n \le 9)$

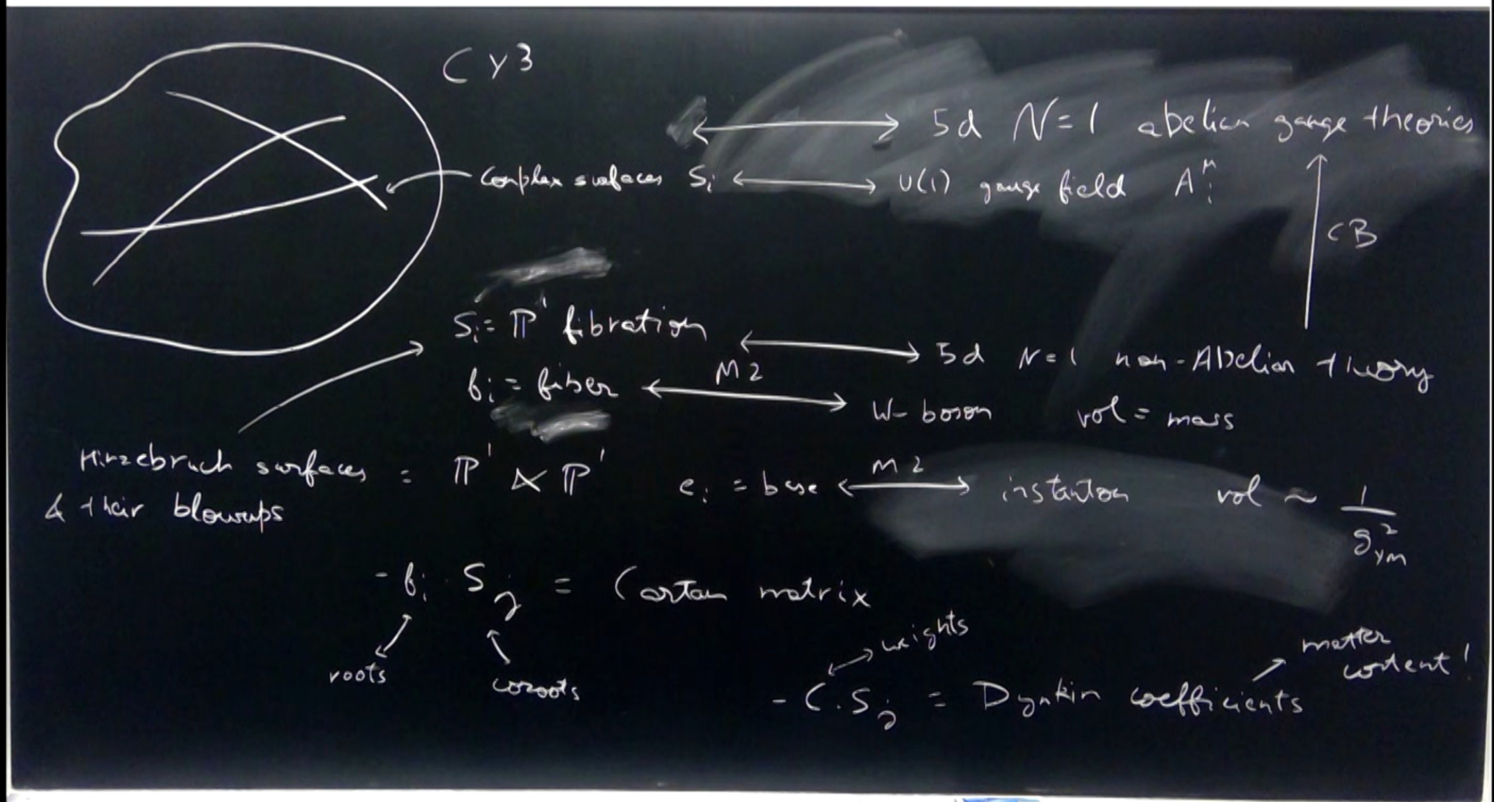
$Sp(2) + nF$ $\nearrow$ 6d SCFTs' $(n=10)$

$\downarrow$ TB

6d $sp(1)$ gauge theory

5. T-dualities b/w 6d $\mathcal{N}=(1,0)$ LSTs

e.g. $(2,0)$ LSTs $\overset{T}{\longleftrightarrow}$ $(1,1)$ LSTs


CY3
5d N=1 abelian gauge theories
Complex surfaces S_i ⟷ U(1) gauge field A^μ_i
CB
S_i = P^1 fibration ⟷ 5d N=1 non-Abelian theory
f_i = fiber ⟷ W-boson vol = mass
M2
Hirzebruch surfaces = P^1 ⋉ P^1
& their blowups
e_i = base ⟷ instanton vol ~ 1/g^2_YM
M2
- f_i · S_j = Cartan matrix
roots
coroots
weights
- C · S_j = Dynkin coefficients
matter content!

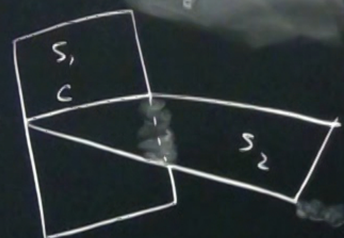
Intersection theory: $e_i^2=-n$ $\quad f_i^2=0$ $\quad e_i\cdot f_i=+1 \Rightarrow S_i=F_n$

Blowups $\quad x_a^2=-1$ $\quad x_a\cdot x_b=0$ $\quad x_\alpha\cdot(\alpha e+\beta f)=0 \Rightarrow S_i=F_n^b$

$C=\alpha e+\beta f-\gamma_a x_a$ $\qquad g(e)=g(f)=g(x_a)=0$

or $\alpha x_b-\beta_a x_a$

Gluing:



$C|_{S_1}=C_1$ $\qquad C|_{S_2}=C_2$ $\qquad g_{C_1}=g_{C_2}=g$

CY: $\boxed{C_1^2+C_2^2=2g-2}$

$f_i\cdot S_i=K_i\cdot f_i$

$F_2 \overset{e\qquad e}{\text{———}} F_0 \longrightarrow -f_i\cdot S_j=\begin{pmatrix}2&-1\\-1&2\end{pmatrix} \Rightarrow su(3)_1$ pure gauge theory $\qquad =2g-2-f_i^2$

$-2+0=-2$ $\qquad\qquad =0-2-0=-2$

CS level $\longleftrightarrow$ prepotential $\longleftrightarrow$ $S_i \cdot S_j \cdot S_k \longrightarrow$ reduces to intersection theory of complex surfaces

$F_0 \;\underline{\;2e+f \qquad e\;}\; F_6 \Rightarrow -f_i \cdot S_j = \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix} \Rightarrow \circ\Longrightarrow\circ \Rightarrow$ pure $sp(2)$ theory

$(2e+f)^2 = +4$

$F_0 = \mathbb{P}^1 \times \mathbb{P}^1 \qquad e \leftrightarrow f$ (isomorphism)

$F_0 \;\underline{\;e+2f \qquad e\;}\; F_6 \Rightarrow su(3)_{-5}$ pure

$$\boxed{sp(2) = su(3)_{-5}}$$

Add matter: $F_0 \;\underline{\;2e+f \qquad e\;}\; F_6^n = F_0 \;\underline{\;e+2f \qquad e\;}\; F_6^n$

$-x \cdot S_i = (0,1)$

= Highest wt. of F

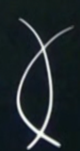
$$\boxed{sp(2) + nF = su(3)_{-5+\frac{n}{2}} + nF}$$

Dualities: ① Apply isomorphisms ② Flop ① ② ① .....

$$A_{ij} \longrightarrow A'_{ij} \qquad W \longrightarrow W'$$

$$F_0 \xrightarrow{\ 2e+f\ } F_6^{10} = F_0 \xrightarrow{\ 2e+f \qquad e+2f-\sum x_i\ } F_0^{10} \qquad (e+2f-\sum x_i)^2 = +4-10 = -6$$

$$= F_0 \xrightarrow{\ 2e+f \qquad 2e+f-\sum x_i\ } F_0^{10} \Rightarrow -f_i \cdot S_j = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

Fibers $f_i \longrightarrow$  = Kodaira I_2 fiber   affine $su(2)^{(1)}$ = o══o

$= T^2 \Rightarrow$ CY3 is elliptically fibered

Grauert $\longrightarrow$ Not shrinkable! $\Rightarrow$ Not 5d SCFT!

$CY3 \longrightarrow B$ $\quad$ B shrinkable $\Longrightarrow$ 6d SCFT on S^1_R

$$\frac{1}{R} \sim \mathrm{vol}\left(T^2\right)$$

Affine gauge algebra $\longleftrightarrow$ 6d gauge theory on S^1

$$\big\uparrow \mathrm{TB}$$

6d SCFTs

$\boxed{sp(2)+10F = \text{6d } sp(1) \text{ SCFT (carries 10F of } sp(1))}$

$\boxed{= su(3)_0 + 10F}$

1. Finite $\longleftrightarrow$ Finite = Duality of 5d gauge theories
2. Finite $\longleftrightarrow$ Affine = 5d gauge theory descends from 6d gauge theory on S^1
3. Affine $\longleftrightarrow$ Affine = T-duality of 6d g.t. on S^1 $\xleftarrow{\mathrm{TB}}$ 6d LSTs

Shrinkability: Can be formulated in terms of positivity cond^n on intersection theory

1. **Bottom-up**: Manually search for all CY3. (very complex problem)
2. **Top-down**: Use F-theory classification of 6d SCFTs! (easy)

$\downarrow$

{6d gauge algebras + matter}

$\downarrow$ on S^1

Parent CY3s $\longleftrightarrow$ convert into EFCY3 with affine gauge algebra (sitting at the edge of shrinkability)

$\downarrow$ Blowdown $\longleftrightarrow$ destroys T^2 fibration

Descendant CY3s $\longleftrightarrow$ 5d SCFTs

$\Downarrow$

Shrinkability

∴ Huge class of 5d SCFTs <u>classified</u> as descendants of 6d SCFTs

Is this classification complete? $\longrightarrow$ No!

e.g. consider $\mathbb{F}_4 + 3F \longrightarrow$ satisfies shrinkability $\longleftrightarrow$ 5d SCFT

Decoupling $su(2)_{inst}$ — Blowup $\downarrow$ $\uparrow$ Blowdown?

$\mathbb{F}_4 + 3F + R \longrightarrow$ not shrinkable!

However

$$6d \;\; \mathbb{F}_4^{(1)} + 3F \;=\; su(2)_{inst} - (\mathbb{F}_4 + 3F)$$

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1. Blowdown = Removing a curve from CY3 = Integrating out a BPS particle
2. Decoupling = " " surface " " = " " " " string

Conjectured classification:

$$\{\text{5d SCFTs}\} \xleftarrow[\text{BPS particles \& strings}]{\text{Integrating out}} \{\text{6d SCFTs on } S^1\}$$

$$\{\text{Shrinkable CY3}\} \xleftarrow[\text{surfaces}]{\text{Removing curves \&}} \{\text{EF CY3 with shrinkable } B\}$$

Evidence: {5d g.t. w/ simple G satisfying some necessary criteria} [JKVZ '17]

1. Known to be shrinkable $\longrightarrow$ inside above classification
2. " " non-" $\longrightarrow$ not " " " (trivial)
3. Unknown
   (can be counted on fingers)

① $5d \xleftarrow[\text{particles only}]{\text{BPS}} 6d$ (general case)

② $5d \xleftarrow[\text{strings + particles}]{\text{BPS}} 6d$ (exceptional cases)

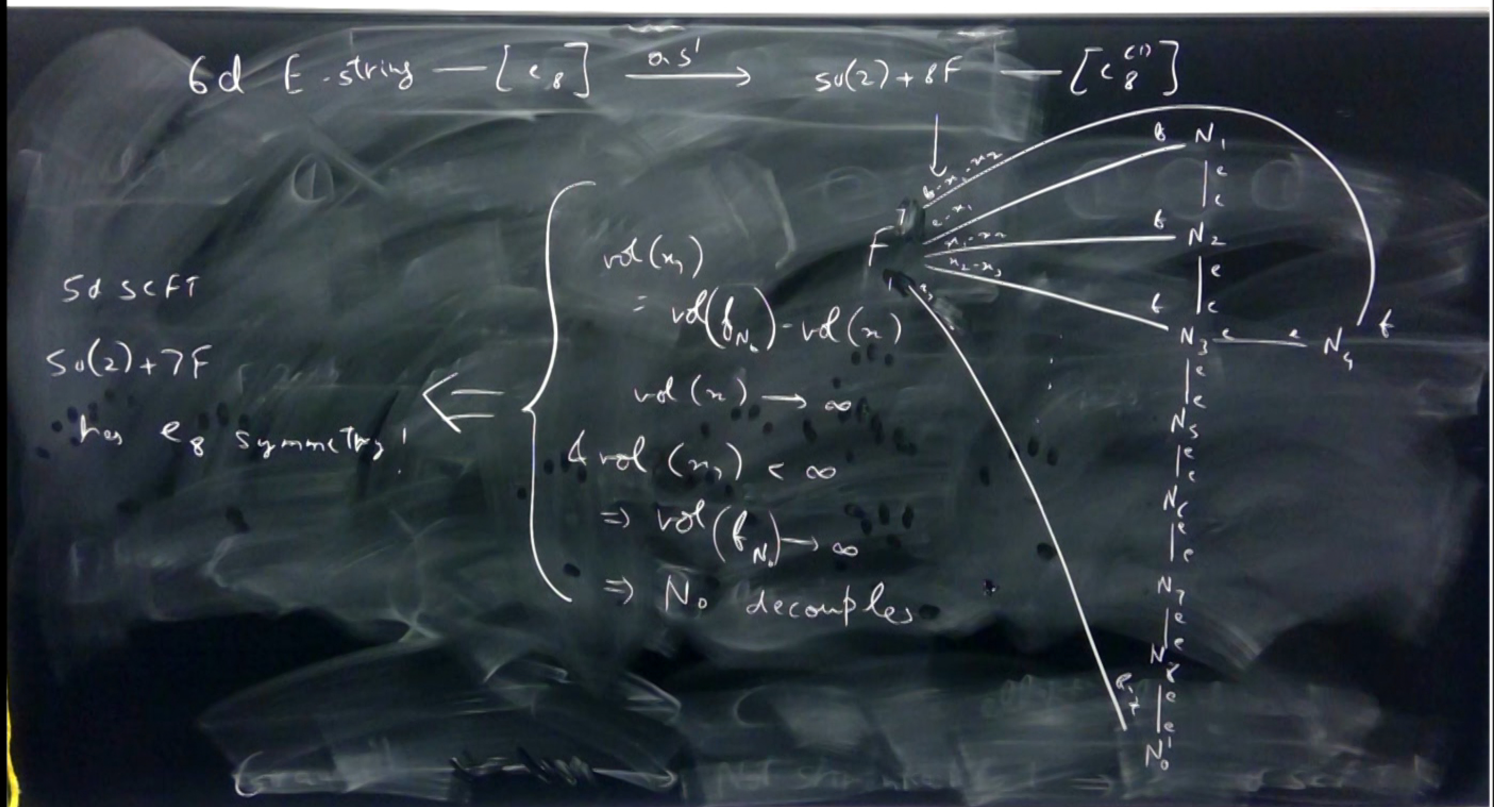
<u>Enhanced flavor symmetries</u>: Works only for ①

6d quiver SCFT $\quad \mathfrak{g}_1 - \mathfrak{g}_2 \xrightarrow{\text{on } S^1} \mathfrak{g}_1^{(1)} - \mathfrak{g}_2^{(1)} \longrightarrow \{F_{\mathfrak{g}_1}\} - \{F_{\mathfrak{g}_2}\}$

$\downarrow$ expanding all $\mathfrak{g}_2$ to ∞ vol

$\mathfrak{g}_1 - [\mathfrak{g}_2] \xrightarrow{\text{on } S^1} \mathfrak{g}_1^{(1)} - [\mathfrak{g}_2^{(1)}] \longrightarrow \{F_{\mathfrak{g}_1}\} - \{N_{\mathfrak{g}_2}\}$

$N_{\mathfrak{g}_2}$ are $\mathbb{P}^1$-fibered <u>non-compact</u> surfaces


6d E-string — [e8] —a.s'→ su(2)+8F — [e8(1)]
5d SCFT
su(2)+7F
has e8 symmetry!
vol(x1) = vol(fN0) - vol(x)
vol(x) → ∞
& vol(x1) < ∞
⇒ vol(fN0) → ∞
⇒ N0 decouples
N1
N2
N3
N4
N5
N6
N7
N8
N0'

More details:

1. Detailed study of CY3 associated to 6d SCFTs on S^1 [1809.01650, 1811.10616, 1909.11666]

2. $5d \xleftarrow[\text{only}]{\text{BPS particles}} 6d$ classification detailed calculations of flops, blowdowns [1909.05635]

3. $5d \xleftarrow{\text{BPS strings}} 6d$ [1912.00025]

4. Dualities b/w 5d g.t. [1909.05250]

5. 5d & 6d SCFT UV completions of JKVZ theories [soon w/ Gabi Zafrir]

6. T-duality of LSTs & enhanced symmetries [soon]