Title: Geometry and 5d N=1 QFTs

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Series: Quantum Fields and Strings

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Abstract: I will explain that a geometric theory built upon the theory of complex surfaces can be used to understand wide variety of phenomena in five-dimensional supersymmetric theories, which includes the following:

Classification of 5d superconformal field theories (SCFTs).
Enhanced flavor symmetries of 5d SCFTs.
5d N=1 gauge theory descriptions of 5d and 6d SCFTs.
Dualities between 5d N=1 gauge theories.
T-dualities between 6d N=(1,0) little string theories.

This relationship between geometry and 5d theories is based on M-theory and F-theory compactifications.

M-theory on CY_3 (intersection theory)

↓ Application to 5d $\mathcal{N}=1$ QFTs

1. Classification of 5d SCFTs
2. 5d $\mathcal{N}=1$ gauge theories $\underset{\text{mass deformation}}{\overset{\text{UV complete}}{\rightleftarrows}}$ 5d SCFTs / 6d SCFTs on S^1 with twist

e.g. $SU(2) + nF \longrightarrow$ 5d SCFT $\quad 0 \le n \le 7$

$SU(2) + 8F \longrightarrow$ 6d SCFT on S^1 (E-string)

$SU(2) + \geq 9F \longrightarrow$ No SCFT UV completion

3. Enhanced flavor symmetries of 5d SCFTs

e.g. $SU(2) + (0 \le n \le 7)F$ — Classically $SO(2n) \longrightarrow E_{n+1}$

4. Dualities: $SU(3)_{-5+\frac{n}{2}} + nF$ and $Sp(2) + nF$ $\longrightarrow$ 5d SCFT $(0 \le n \le 9)$

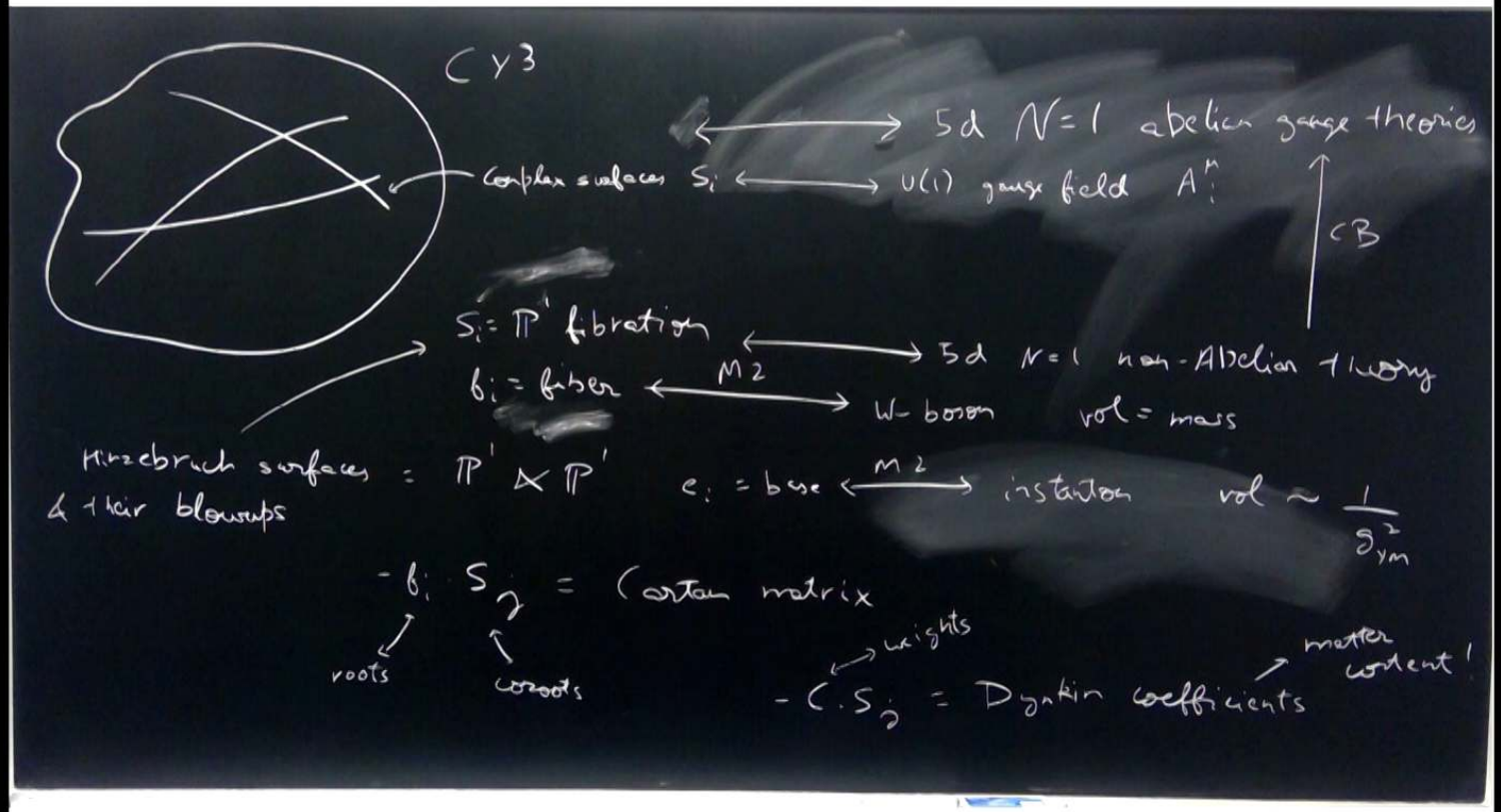
6d SCFT on S^1 $(n = 10)$

$\downarrow$ TB

6d $Sp(1)$ gauge theory

5. T-dualities b/w 6d $\mathcal{N} = (1,0)$ LSTs

e.g. $(2,0)$ LSTs $\xleftrightarrow{T}$ $(1,1)$ LSTs


CY3
5d N=1 abelian gauge theories
complex surfaces S_i ⟷ U(1) gauge field A^μ_i
CB
$S_i = \mathbb{P}^1$ fibration
5d N=1 non-Abelian theory
f_i = fiber ⟷ M2 ⟷ W-boson
vol = mass
Hirzebruch surfaces = $\mathbb{P}^1 \times \mathbb{P}^1$ & their blowups
e_i = base ⟷ M2 ⟷ instanton
vol ~ $\frac{1}{g^2_{ym}}$
$-f_i \cdot S_j$ = Cartan matrix
roots
coroots
weights
matter content
$-C \cdot S_j$ = Dynkin coefficients

Intersection theory: $e_i^2 = -n$ $f_1^2 = 0$ $e_i \cdot f_i = +1 \Rightarrow S_i = F_n$

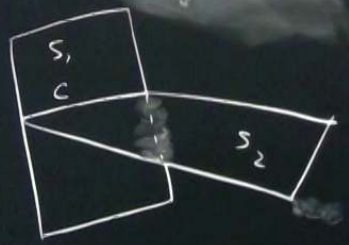
Blowups $x_a^2 = -1$ $x_a \cdot x_b = 0$ $x_\alpha \cdot (\alpha_i e + \beta_i f) = 0 \Rightarrow S_i = F_n^b$

$C = \alpha e + \beta f - \gamma_a x_a$

or $\alpha x_\beta - \beta_a x_a$

$g(e) = g(f) = g(x_a) = 0$

Glueing:



$C|_{S_1} = C_1$ $C|_{S_2} = C_2$ $g_{C_1} = g_{C_2} = g$

CY: $\boxed{C_1^2 + C_2^2 = 2g - 2}$

$f_i \cdot S_i = K_i \cdot f_i$

$= 2g - 2 - f_i^2$

$= 0 - 2 - 0 = -2$

$F_2 \xrightarrow{e} F_0$ $\longrightarrow f_i \cdot S_j = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \Rightarrow SU(3)_1$ pure gauge theory

$-2 + 0 = -2$

CS level $\longleftrightarrow$ prepotential $\longleftrightarrow$ $S_i\, S_j\, S_k$ $\longrightarrow$ reduces to intersection theory of complex surfaces

$F_0 \overset{2e+f}{\text{---}} e\, F_6 \Rightarrow -f_i\ S_{ij} = \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix} \Rightarrow$ $\Rightarrow$ pure $sp(2)$ theory

$(2e+f)^2 = +4$

$F_0 = \mathbb{P}^1 \times \mathbb{P}^1 \qquad e \longleftrightarrow f \qquad$ (isomorphism)

$F_0 \overset{e+2f}{\text{---}} e\, F_1 \Rightarrow SU(3)_{-5}$ pure

$\boxed{Sp(2) = SU(3)_{-5}}$

Add matter $\quad F_0 \overset{2e+f}{\text{---}} e\, F_6^n = F_0 \overset{e+2f}{\text{---}} e\, F_1^n$

$-x\ S_i = (0,1)$ = Highest wt. of F

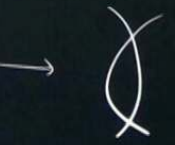
$$sp(2) + nF = su(3)_{-5+\frac{n}{2}} + nF$$

Dualities: (1) Apply isomorphisms (2) Flop (1) (2) (1) ....

$$A_{ij} \longrightarrow A'_{ij} \qquad W \longrightarrow W'$$

$$F_0^{2c+b} \circ F_c^{10} = F_0^{2c+b} \; {}^{c+b-\Sigma x_i}\!\!\diagdown\!\!\diagup\; F_0^{10} \qquad (c+2b-\Sigma x_i)^2 = +4-10 = -6$$

$$= F_0^{2c+b} \; {}^{2c+b-\Sigma x_i}\!\!\diagdown\!\!\diagup\; F_0^{10} \Rightarrow -f_i \cdot S_j = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

Fibers $f_i \longrightarrow$  = Kodaira I_2 fiber affine $su(2)^{(1)}$ =

$= T^2 \Rightarrow CY3$ is elliptically fibered

Gravity $\longrightarrow$ Not shrinkable! $\Rightarrow$ Not 5d scFT!

$CY_3 \to B$ B shrinkable $\Longrightarrow$ 6d SCFT on S^1_R

$\frac{1}{R} \sim \mathrm{vol}\left(T^2\right)$

Affine gauge algebra $\longleftrightarrow$ 6d gauge theory on S^1

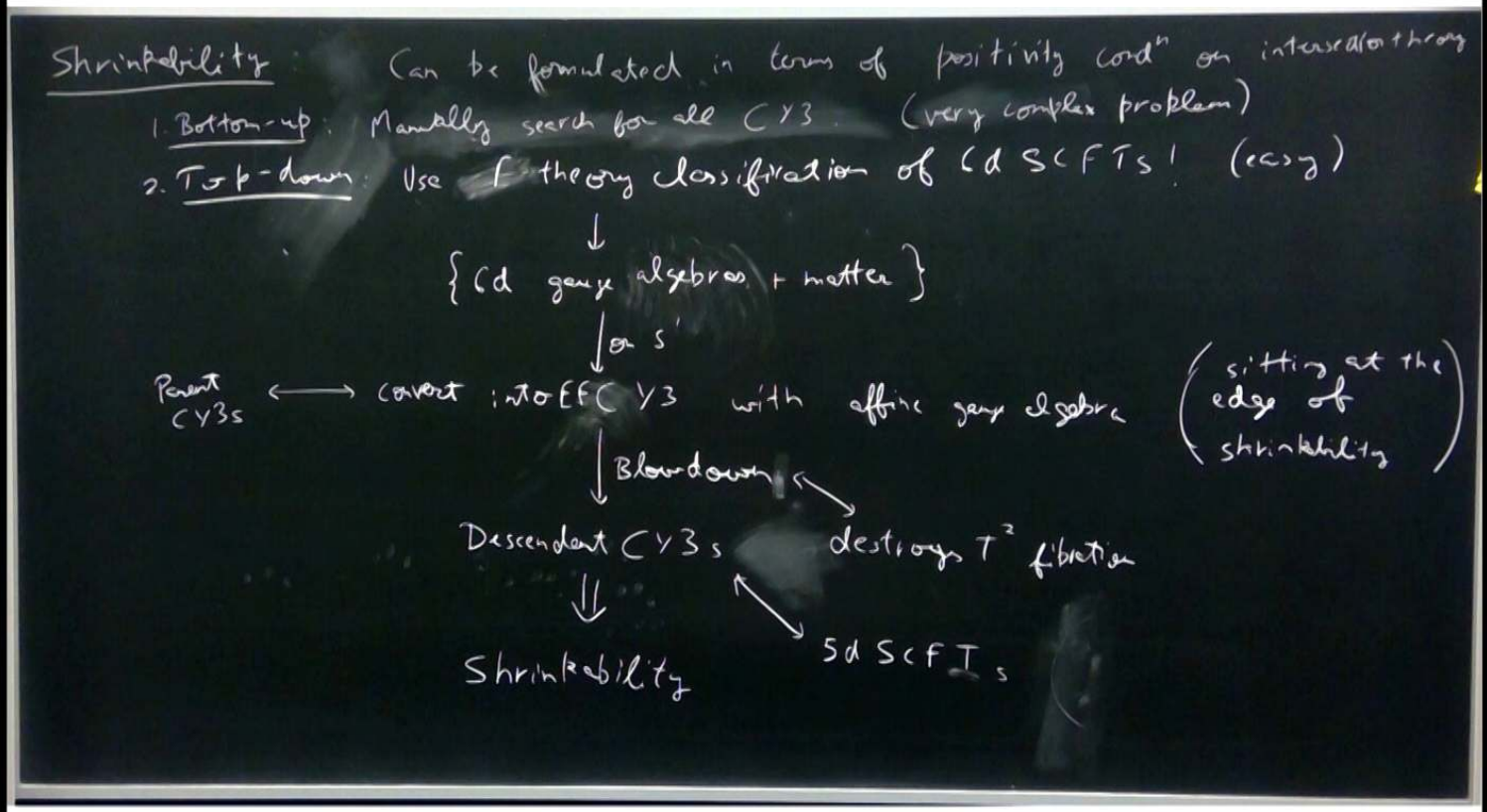
$\uparrow$ TB

6d SCFTs

$\therefore$ $Sp(2) + 10F$ = 6d $sp(1)$ SCFT (carries $10F$ of $sp(1)$)

$= su(3)_0 + 10F$

1. Finite $\longleftrightarrow$ Finite = Duality of 5d gauge theories
2. Finite $\longleftrightarrow$ Affine = 5d gauge theory descends from 6d gauge theory on S^1
3. Affine $\longleftrightarrow$ Affine = T-duality of 6d g.t. on S^1 $\xleftarrow{TB}$ 6d LSTs


Shrinkability: Can be formulated in terms of positivity cond^n on intersection theory
1. Bottom-up: Manually search for all CY3. (very complex problem)
2. Top-down: Use F-theory classification of 6d SCFTs! (easy)
{6d gauge algebras + matter}
on S'
Parent CY3s
convert into EFC Y3 with affine gauge algebra
(sitting at the edge of shrinkability)
Blowdown
Descendant CY3s
destroys T^2 fibration
5d SCFTs
Shrinkability

∴ Huge class of 5d SCFTs classified as descendants of 6d SCFTs

Is this classification complete? ⟶ No!

e.g. consider $f_4 + 3F$ ⟶ satisfies shrinkability ⟷ 5d SCFT

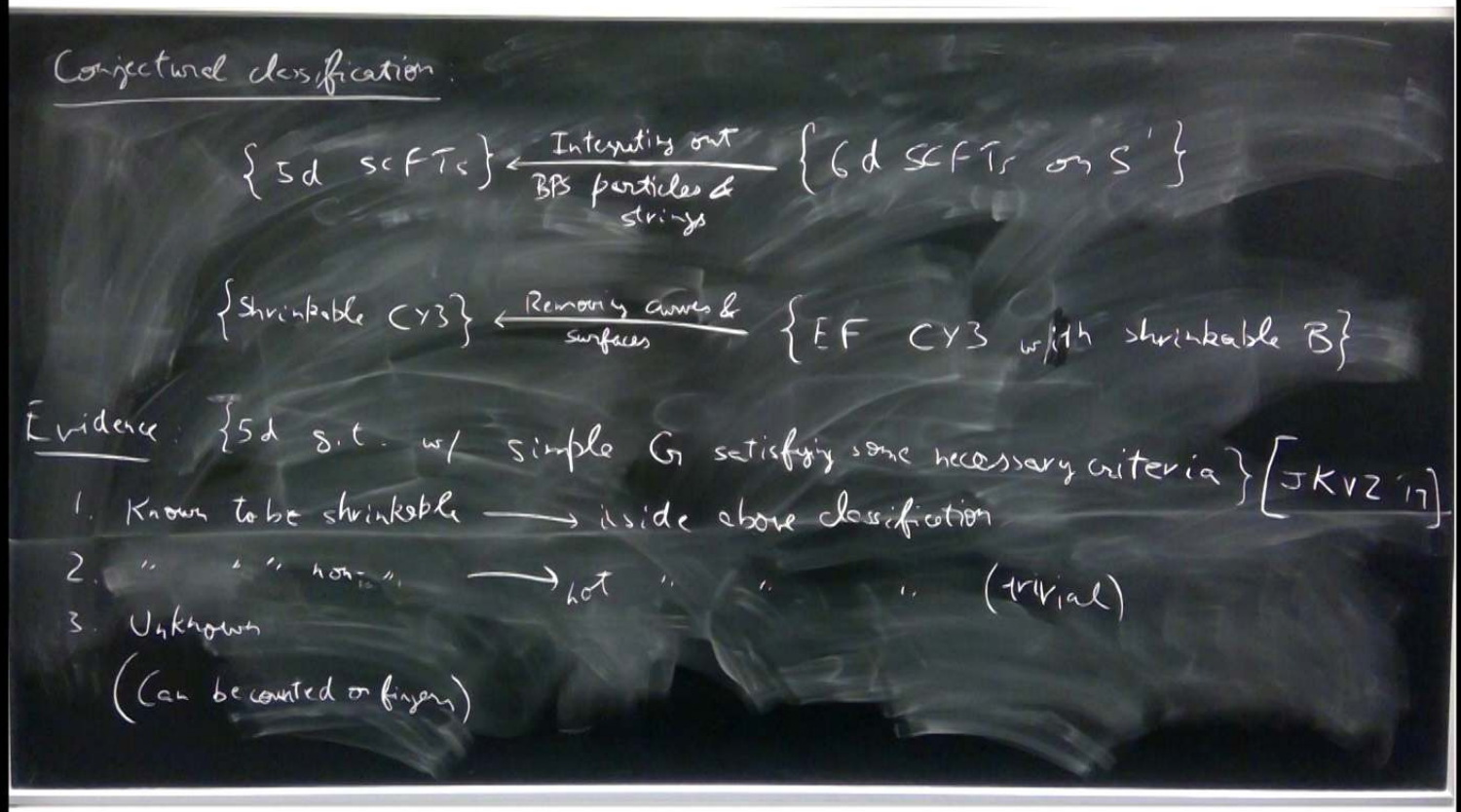
Blowup ↑ ↓ Blowdown?

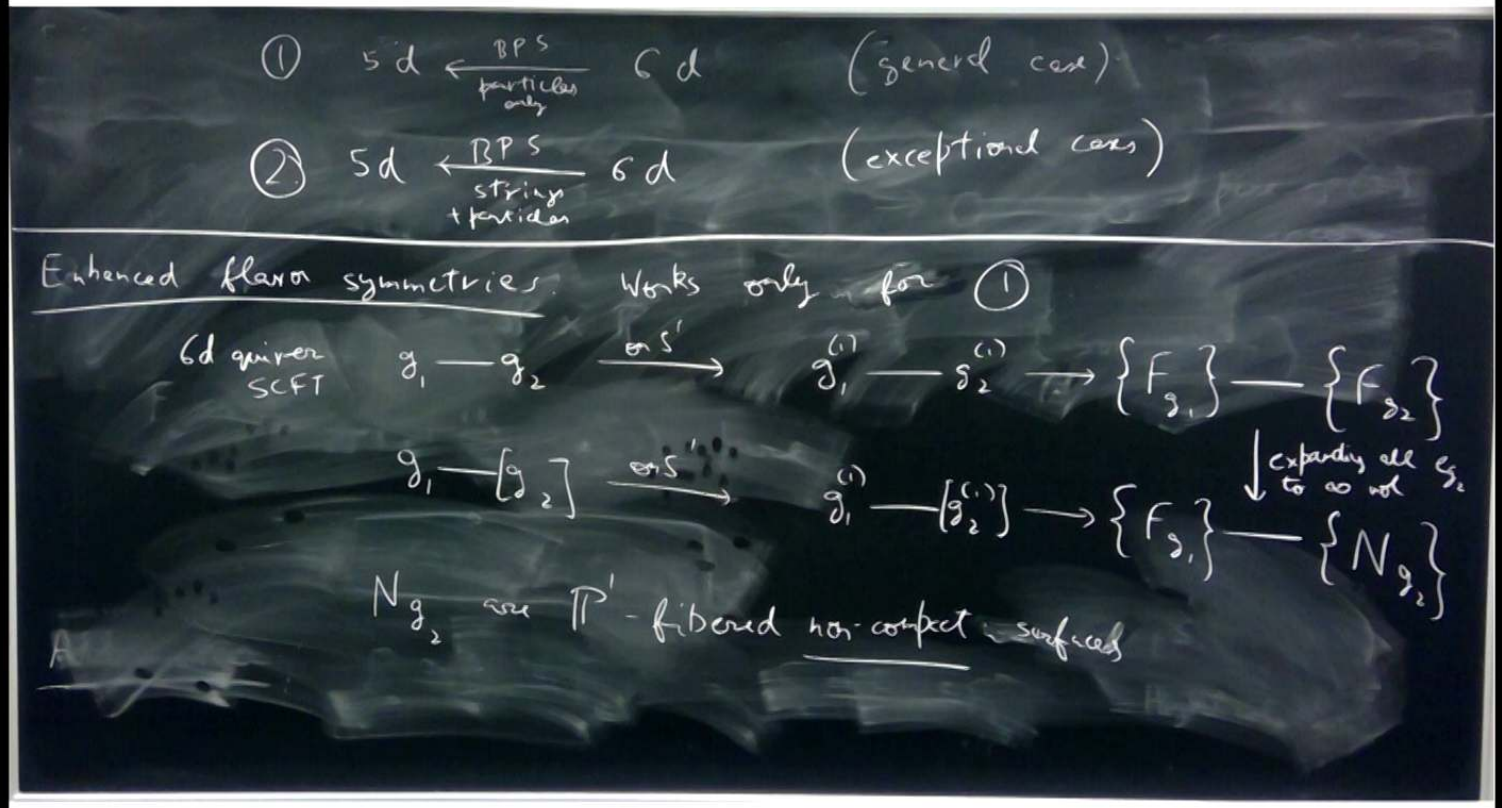
Decoupling $su(2)_{inst}$

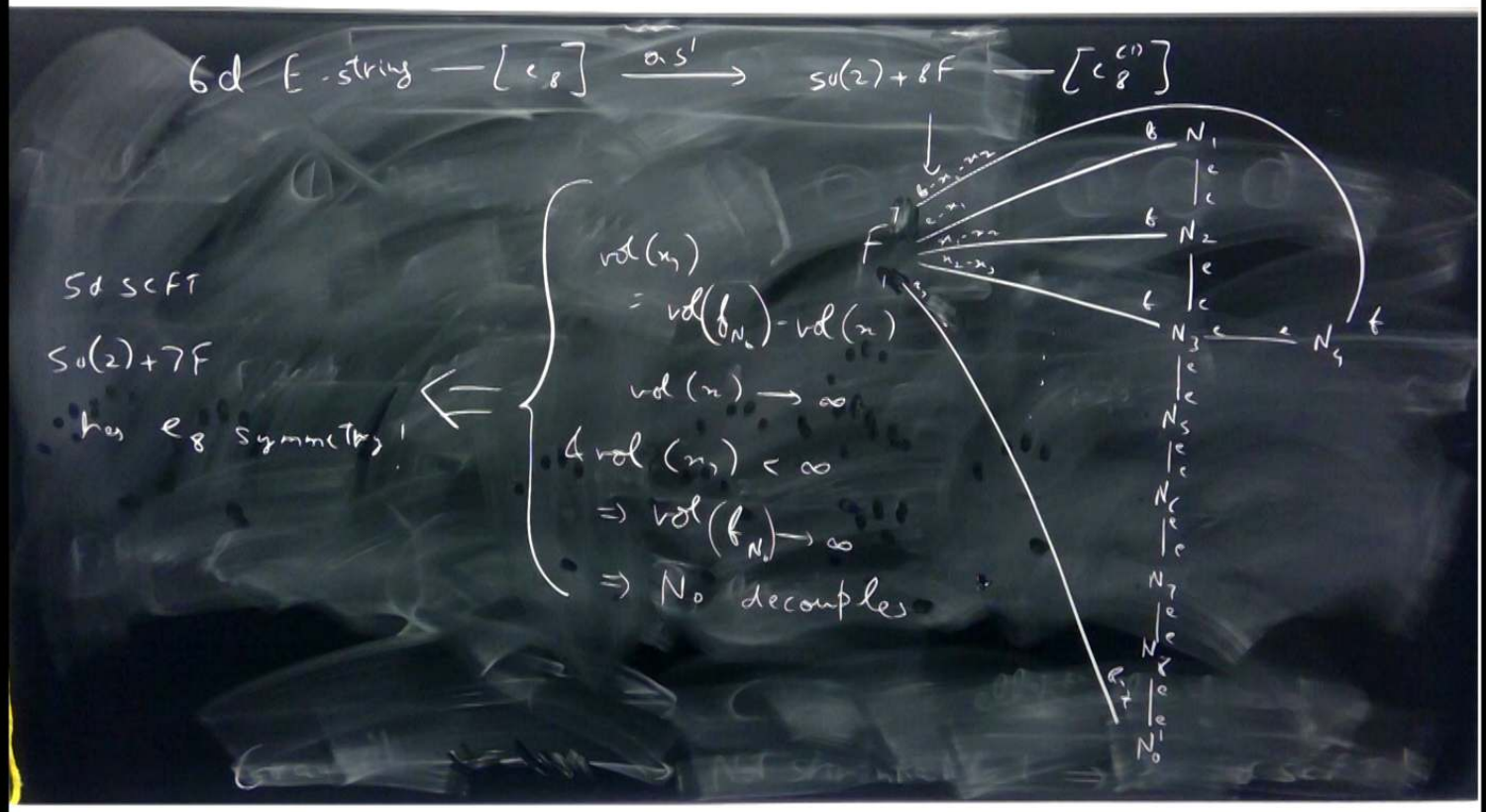
$f_4 + 3F + R$ ⟶ not shrinkable!

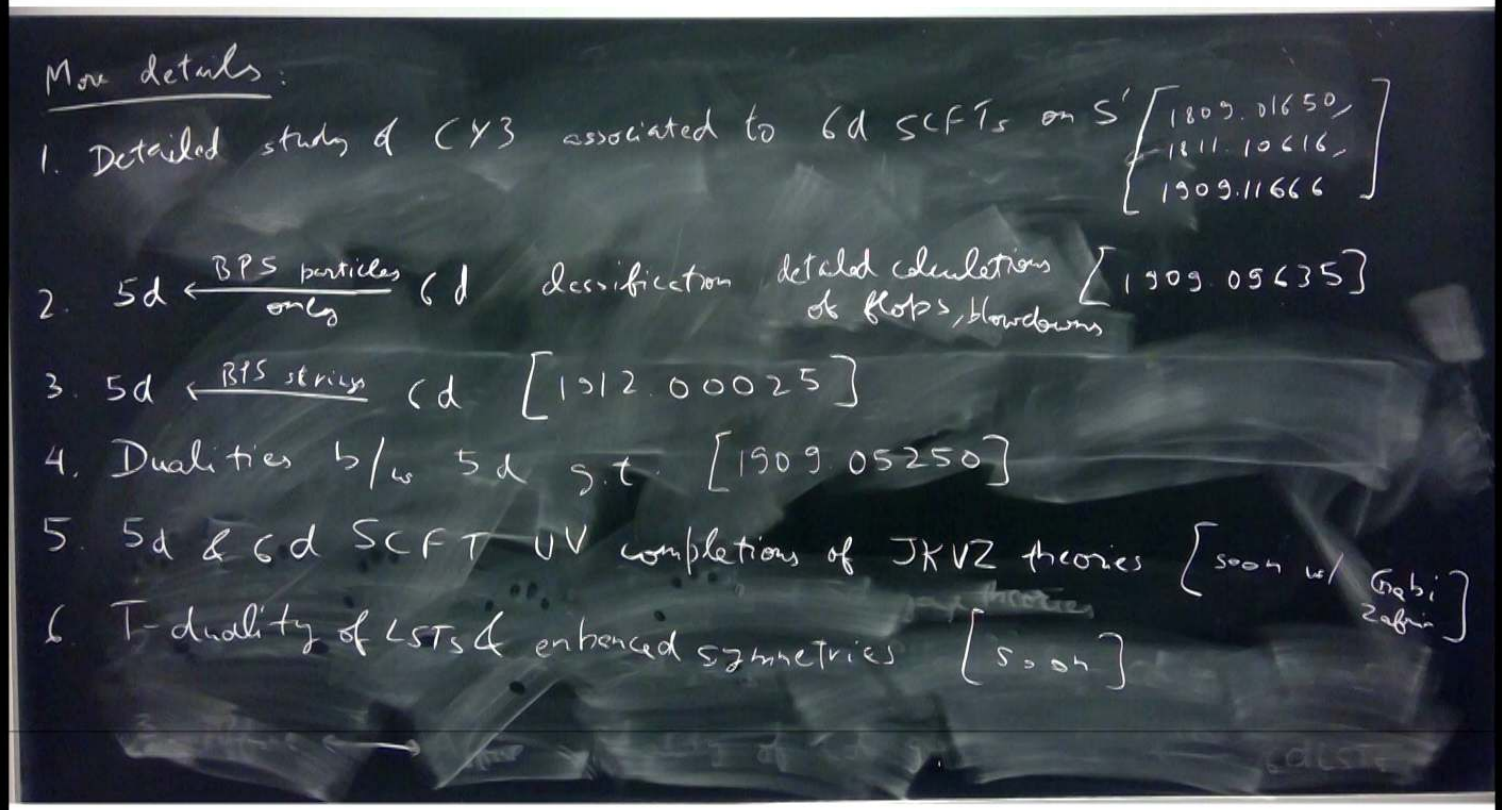
However 6d $f_4^{(1)} + 3F = su(2)_{inst} - (f_4 + 3F)$

1. Blowdown = Removing a curve from CY3 = Integrating out a BPS particle
2. Decoupling = " " surface " " = " " " " string


Conjectural classification:
{5d SCFTs} ← Integrating out BPS particles & strings — {6d SCFTs on S¹}
{shrinkable CY3} ← Removing curves & surfaces — {EF CY3 with shrinkable B}
Evidence: {5d s.t. w/ simple G satisfying some necessary criteria} [JKV2 '17]
1. Known to be shrinkable → inside above classification
2. " " "non-" → not " " " (trivial)
3. Unknown
((Can be counted on fingers))


① 5d ← BPS, particles only — 6d (general case)
② 5d ← BPS, string + particles — 6d (exceptional cases)
Enhanced flavor symmetries. Works only for ①
6d quiver SCFT
on S'
expanding all g_2 to ∞ vol
N_{g_2} are Π' - fibered non-compact surfaces


6d E-string — [e_8] —on S^1→ SU(2)+8F — [$e_8^{(1)}$]
5d SCFT
SU(2)+7F
has e_8 symmetry!
vol(x_1) = vol(f_{N_0}) − vol(x)
vol(x) → ∞
∫ vol(x_1) < ∞
⇒ vol(f_{N_0}) → ∞
⇒ N_0 decouples
8
F
1
N_1
N_2
N_3
N_5
N_6
N_7
N_8
N_0


More details:
1. Detailed study of CY3 associated to 6d SCFTs on S' [1809.01650, 1811.10616, 1909.11666]
2. 5d ← BPS particles only 6d descriptions detailed calculations of flops, blowdowns [1909.09635]
3. 5d ← BPS strings 6d [1912.00025]
4. Dualities b/w 5d s.t. [1909.05250]
5. 5d & 6d SCFT UV completions of JKVZ theories [soon w/ Gabi Zafrir]
6. T-duality of LSTs & enhanced symmetries [soon]