Title: PSI 2019/2020 - Cosmology Part 2 - Lecture 2

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Collection: PSI 2019/2020 - Cosmology Part 2

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Gauge-invariant variables:

$$\overline{\Phi}_B = \overline{\Phi} + \partial_t\left[a(a\dot{E} - B)\right]$$

$$\Psi_B = -\Psi + aH\left[B - a\dot{E}\right]$$

* Newtonian Gauge $(E = B = 0)$ $\overline{\Phi}_B = \overline{\Phi}$, $\Psi_B = -\Psi$

Cosmological PT

S_i ↓ $\quad \partial_{(i} F_{j)}$ ↓ $\quad V_{Bi} = S_i - \dot{F}_i$

$$ds^2 = -(1+2\Phi)dt^2 - 2a\partial_i B\, dt\, dx^i + a^2\left[(1+2\Psi)\delta_{ij} - 2\partial_i\partial_j E\right]dx^i dx^j$$

$$\tilde{X}^\mu \to X^\mu + \xi^\mu \qquad (\xi^0, \delta^{ij}\partial_j\xi)$$

$$\xi^i = S^i + \partial^i\xi + \varepsilon_{ijk}\partial_j\xi_k$$

gauge-trans. Scalars ↑ $\quad$ gauge-trans. vectors ↑

$$\tilde{\Phi} = \Phi - \xi^0$$
$$\tilde{B} = B - \frac{\xi^0}{a} + a\dot{\xi}$$
$$\tilde{\Psi} = \Psi - H\xi^0$$
$$\tilde{E} = E + \xi$$

choice of ξ^0, ξ → Gauge (any 2 metric functions to zero)

* New

$\hat{E} = E + \xi$

* Matter Sector

Perfect fluids – dark matter

$$T^{\mu}{}_{\nu} = (\rho + P) u^{\mu} u_{\nu} + P \delta^{\mu}{}_{\nu}$$

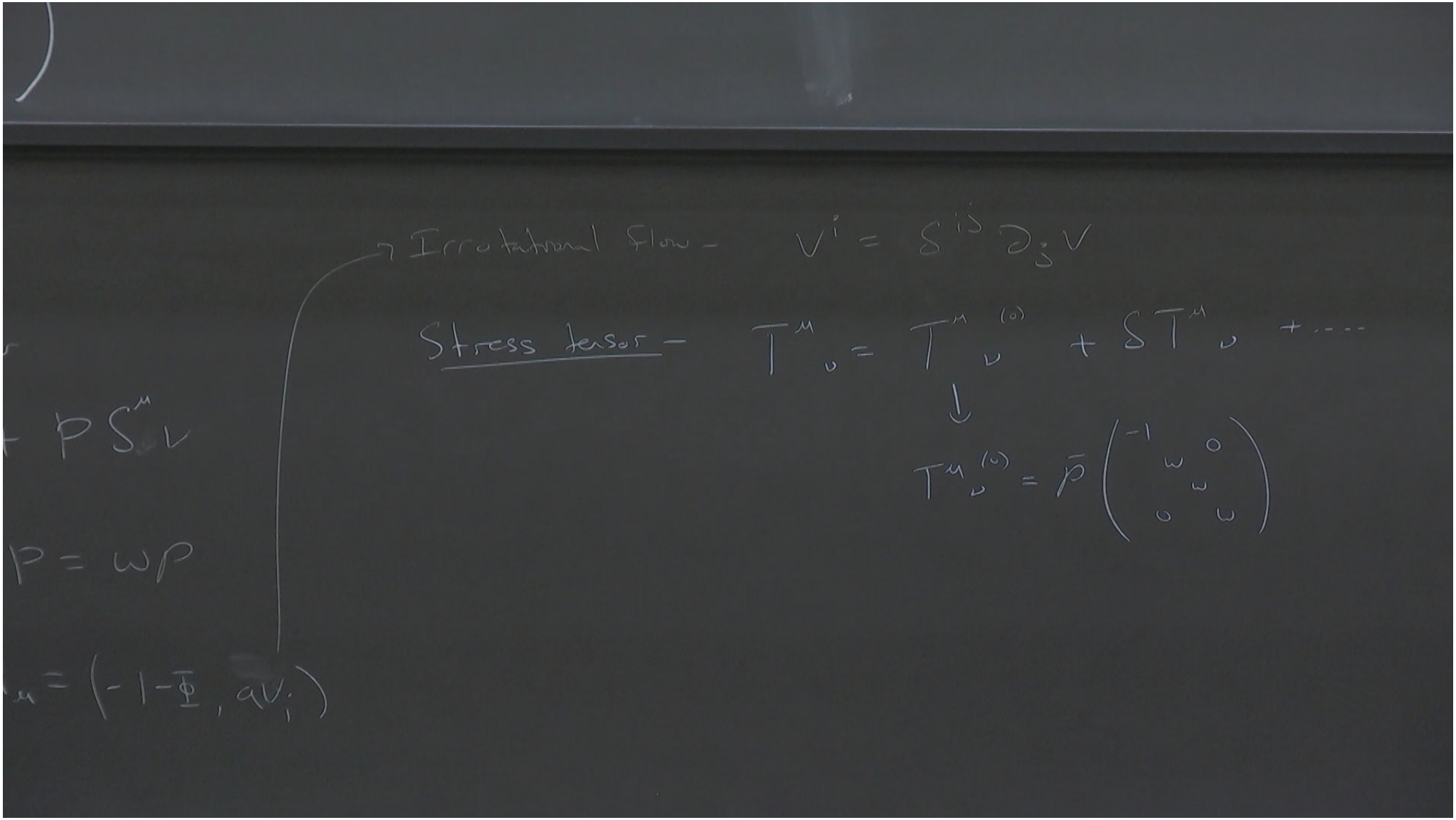
$-1 = g_{\mu\nu} u^{\mu} u^{\nu}$

$\rho = \bar{\rho}(1+\delta)$, $P = w\rho$

$u^{\mu} = (1-\Phi, v^{i}/a)$, $u_{\mu} = (-1-\Phi, a v_{i})$

→ Irrotation

Stress t


Irrotational flow - $V^i = \delta^{ij} \partial_j V$
Stress tensor - $T^\mu{}_\nu = T^\mu{}_\nu{}^{(0)} + \delta T^\mu{}_\nu + \cdots$
$T^\mu{}_\nu{}^{(0)} = \bar{p} \begin{pmatrix} -1 & & & 0 \\ & w & & \\ & & w & \\ 0 & & & w \end{pmatrix}$
$+ p S^\mu{}_\nu$
$p = w\rho$
$u = (-1 - \Phi, a V_i)$

$\delta^{ij}\partial_j V$

$$T^\mu{}_\nu = {}^{(0)}T^\mu{}_\nu + \delta T^\mu{}_\nu + \dots$$

$$\downarrow$$

$$T^\mu{}_\nu{}^{(0)} = \bar{\rho}\begin{pmatrix} -1 & & 0 \\ & w & \\ 0 & & w \end{pmatrix}$$

$$\delta T^0{}_0 = -\left(\bar{\rho}\delta + w\bar{\rho}\delta\right)\left(1-\Psi\right)\left(-1+\Psi\right) + w\bar{\rho}\delta$$

$$= -\bar{\rho}\delta$$

$$\delta T^0{}_i = \bar{\rho}\, a(1+w)(\partial_i V - \partial_i B)$$

$$\delta T^i{}_0 = -\bar{\rho}(1+w)\partial_i V / a$$

$$\delta T^i{}_j = \delta w \bar{\rho}\, \delta^i{}_j$$

* Can transform coordinates:

$$\delta\tilde{T}_{\mu\nu} = \delta T_{\mu\nu} - \left(\partial_\gamma T^{(0)}_{\mu\nu}\right)\xi^\gamma - \left(\partial_\nu \xi^b\right) T^{(0)}_{\mu b} - \left(\partial_\mu \xi^a\right) T^{(0)}_{a\nu}$$

$$\Rightarrow \quad \tilde{\delta} = \delta - \xi^0 \frac{\partial_t \bar{\rho}}{\bar{\rho}}$$

$$\tilde{v} = v + a\dot{\xi}$$

$-(\partial_\mu \xi^a)\, T^{(0)}_{a\nu}$

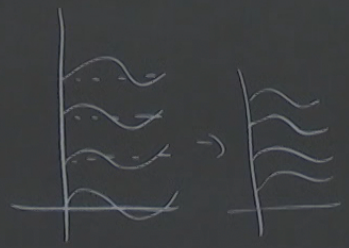
* Gauge-invariant variables:

$$\delta_B = \delta - a\frac{\partial_+ \bar{\rho}}{\bar{\rho}}(B - a\dot{E})$$

$$V_B = V - a\dot{E}$$

} In Newtonian Gauge $\delta_B = \delta$, $V_B = V$

* Constant-density gauge



$$\xi^0 = \frac{\bar{\rho}}{\partial_+ \bar{\rho}}\delta \rightarrow \tilde{\delta} = 0 \Rightarrow \tilde{\psi} = \psi - H\xi^0$$

$$\zeta = \psi - \frac{H}{\partial_+ \bar{\rho}}\bar{\rho}\delta$$

"Curvature perturbation"

$${}^{(3)}R = +\frac{\nabla^2 g}{4a^2}$$

* Can specify a gauge where $\delta T^0{}_i = 0 = \bar{\rho} a(1+w) \partial_i (V - B) \rightarrow V = B$

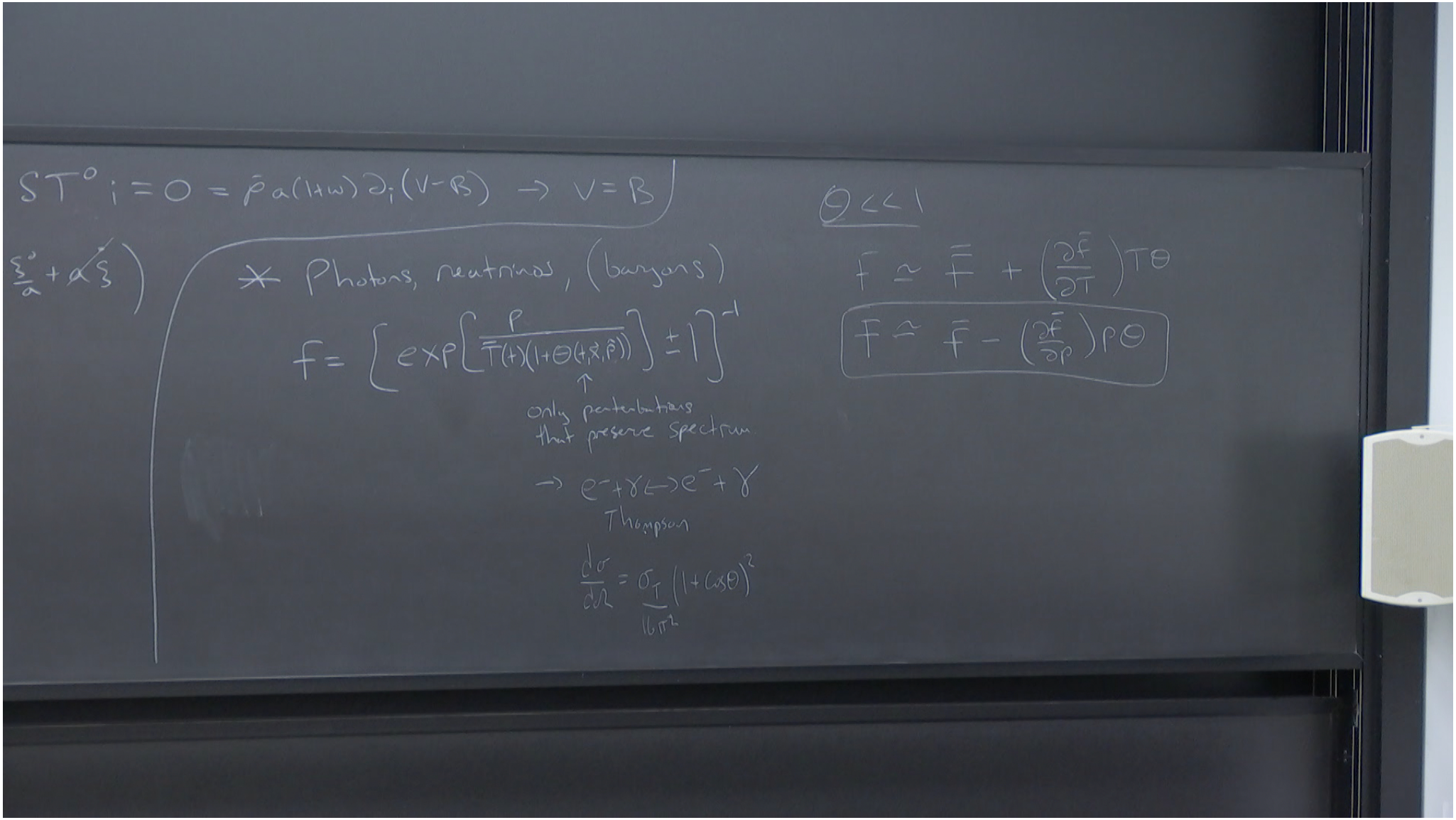
$$0 = \tilde{V} - \tilde{B} = V + \cancel{\alpha \xi} - \left(B - \frac{\xi^0}{a} + \cancel{\alpha \xi} \right)$$

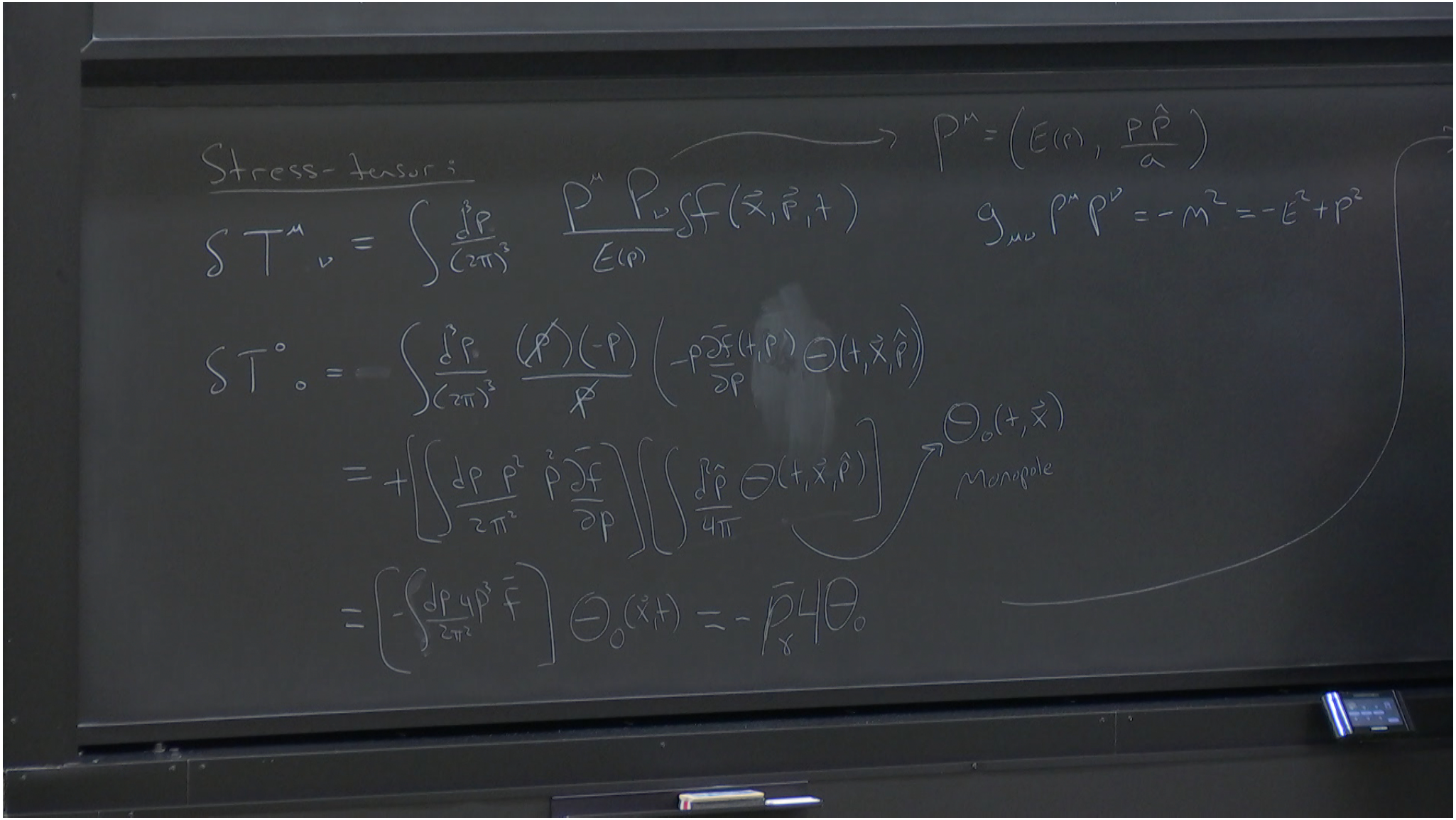
$$0 = V - B + \frac{\xi^0}{a}$$

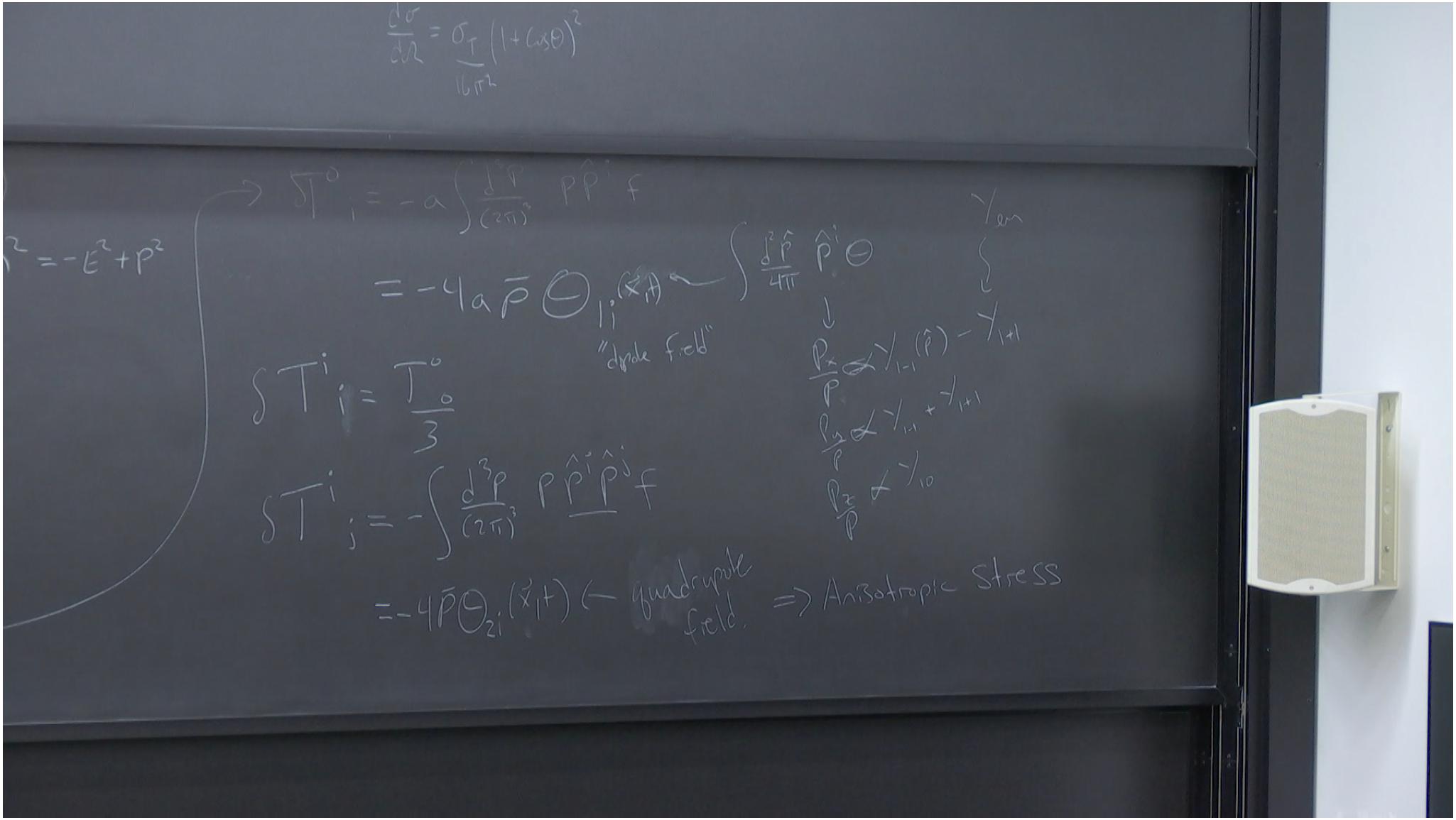
$$\Rightarrow \xi^0 = -a(V - B)$$

In this gauge: $\tilde{\psi} = \psi - H \xi^0$

"Comoving curvature perturbation" $\mathcal{R} = \psi + aH(V - B)$


$\delta T^0{}_i = 0 = \bar{\rho} a (1+w) \partial_i (v - B) \rightarrow v = B$
$\theta \ll 1$
$\bar{f} + \left(\frac{\partial \bar{f}}{\partial T}\right) T\theta$
$\bar{f} - \left(\frac{\partial \bar{f}}{\partial p}\right) p\theta$
* Photons, neutrinos, (baryons)
$f = \left[\exp\left[\frac{p}{\bar{T}(t)(1+\Theta(t,\vec{x},\hat{p}))} \right] \pm 1 \right]^{-1}$
only perturbations that preserve spectrum
$\rightarrow e^- + \gamma \leftrightarrow e^- + \gamma$
Thompson
$\frac{d\sigma}{d\Omega} = \frac{\sigma_T}{16\pi^2} (1 + \cos\theta)^2$


Stress-tensor:
$\delta T^{\mu}{}_{\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{P^\mu P_\nu}{E(p)} f f(\vec{x},\vec{p},t)$
$P^\mu = (E(p), \frac{p\hat{p}}{a})$
$g_{\mu\nu} P^\mu P^\nu = -m^2 = -E^2 + p^2$
$\delta T^{0}{}_{0} = \int \frac{d^3p}{(2\pi)^3} \frac{(p)(-p)}{p} \left(-p \frac{\partial \bar{f}(t,p)}{\partial p} \Theta(t,\vec{x},\hat{p})\right)$
$= +\left[\int \frac{dp\, p^2}{2\pi^2} \hat{p} \frac{\partial \bar{f}}{\partial p}\right]\left[\int \frac{d^2\hat{p}}{4\pi} \Theta(t,\vec{x},\hat{p})\right]$
$\Theta_0(t,\vec{x})$
Monopole
$= \left[-\int \frac{dp\, 4p^3}{2\pi^2} \bar{f}\right] \Theta_0(\vec{x},t) = -\bar{\rho}_\gamma 4\Theta_0$


$\frac{d\sigma}{d\Omega} = \frac{\sigma_T}{16\pi^2}(1+\cos\theta)^2$
$\delta T^0{}_i = -a\int \frac{d^3p}{(2\pi)^3} p\hat{p}^i f$
$= -4a\bar{\rho}\Theta_{1i}(\vec{x},t) \sim \int \frac{d^2\hat{p}}{4\pi}\hat{p}^i\Theta$
"dipole field"
$\delta T^i{}_i = \frac{T^0_0}{3}$
$\delta T^i{}_j = -\int \frac{d^3p}{(2\pi)^3} p\hat{p}^i\hat{p}^j f$
$= -4\bar{\rho}\Theta_{2ij}(\vec{x},t)$ ← quadrupole field ⇒ Anisotropic Stress
$\frac{P_x}{P} \propto Y_{1-1}(\hat{p}) - Y_{11}$
$\frac{P_y}{P} \propto Y_{1-1} + Y_{11}$
$\frac{P_z}{P} \propto Y_{10}$

* Equations of Motion:

$$\delta G_{\mu\nu} = 8\pi G_N \delta T_{\mu\nu}$$
$$\nabla_\mu \delta T^\mu{}_\nu = 0$$

} focus here.

$$\frac{df}{dt} = C[f]$$

* Trick: Solve for everything in Newtonian gauge, use

$$\Phi = \Phi_B, \quad \psi = \psi_B, \quad \delta = \delta_B, \quad v = v_B$$

$= \left[-\int \frac{dp\, q p^3}{2\pi^2} \bar{f} \right] \Theta_0(\vec{x},t) = -\bar{\rho}_\gamma 4\Theta_0$

$= -4\bar{\rho}\Theta_{2i}(\vec{x}$

$\Rightarrow ds^2 = -(1+2\Phi)dt^2 + (1+2\Psi)d\vec{x}^2$

* Equations of Motion:

$$\left.\begin{aligned} \delta G_{\mu\nu} &= 8\pi G_N \delta T_{\mu\nu} \\ \nabla_\mu \delta T^\mu{}_\nu &= 0 \end{aligned}\right\} \text{forms here.}$$

$$\frac{df}{dt} = C[f]$$

Einstein eqns

00 Component

$$\delta G_{00} = 8\pi G_N \delta T_{00}$$

$$3H\dot{\Psi} - \frac{\nabla^2\Psi}{a^2} - 3H^2\Phi = 4\pi G_N\left[\bar{\rho}\delta + 4\bar{\rho}_\gamma\Theta_0 + \cdots\right]$$

* Trick: Solve for everything in Newtonian gauge, use

$\Phi = \Phi_B$, $\Psi = \Psi_B$, $\delta = \delta_B$, $v = v_B$

$$\Rightarrow ds^2 = -(1+2\Phi)dt^2 + (1+2\Psi)dx^2$$

motion:

$8\pi G_N \delta T_{\mu\nu}$ } focus here.

$= 0$

$C[f]$

everything in gauge, use $\Psi = \Psi_B$, $\delta = \delta_B$, $v = v_B$

Einstein eqns

00 Component

$$\delta G_{00} = 8\pi G_N \delta T_{00}$$

$$3H\dot{\Psi} - \frac{\nabla^2\Psi}{a^2} - 3H^2\Phi = 4\pi G_N \left[\bar{\rho}\delta + 4\rho_\gamma \Theta_0 + \cdots\right]$$

$\Phi = \Phi_B$, $\psi = \psi_B$, $\delta = \delta_B$, $v = v_B$

$i - j$ $(i \neq j)$

$$\delta G_{ij} = 8\pi G_N \, \delta T_{ij}$$

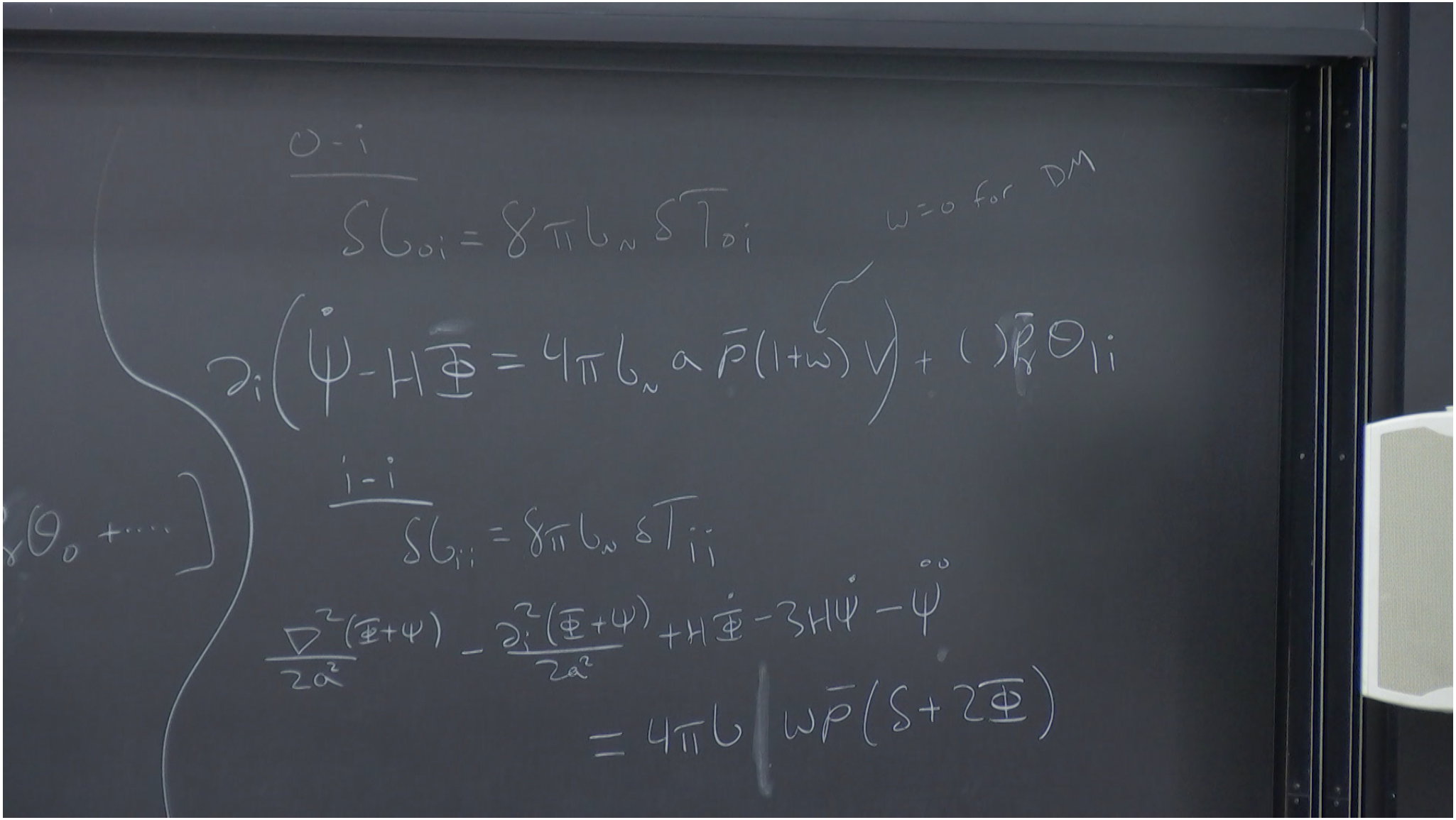
$$-\partial_i \partial_j (\Phi + \psi) \cong 0$$

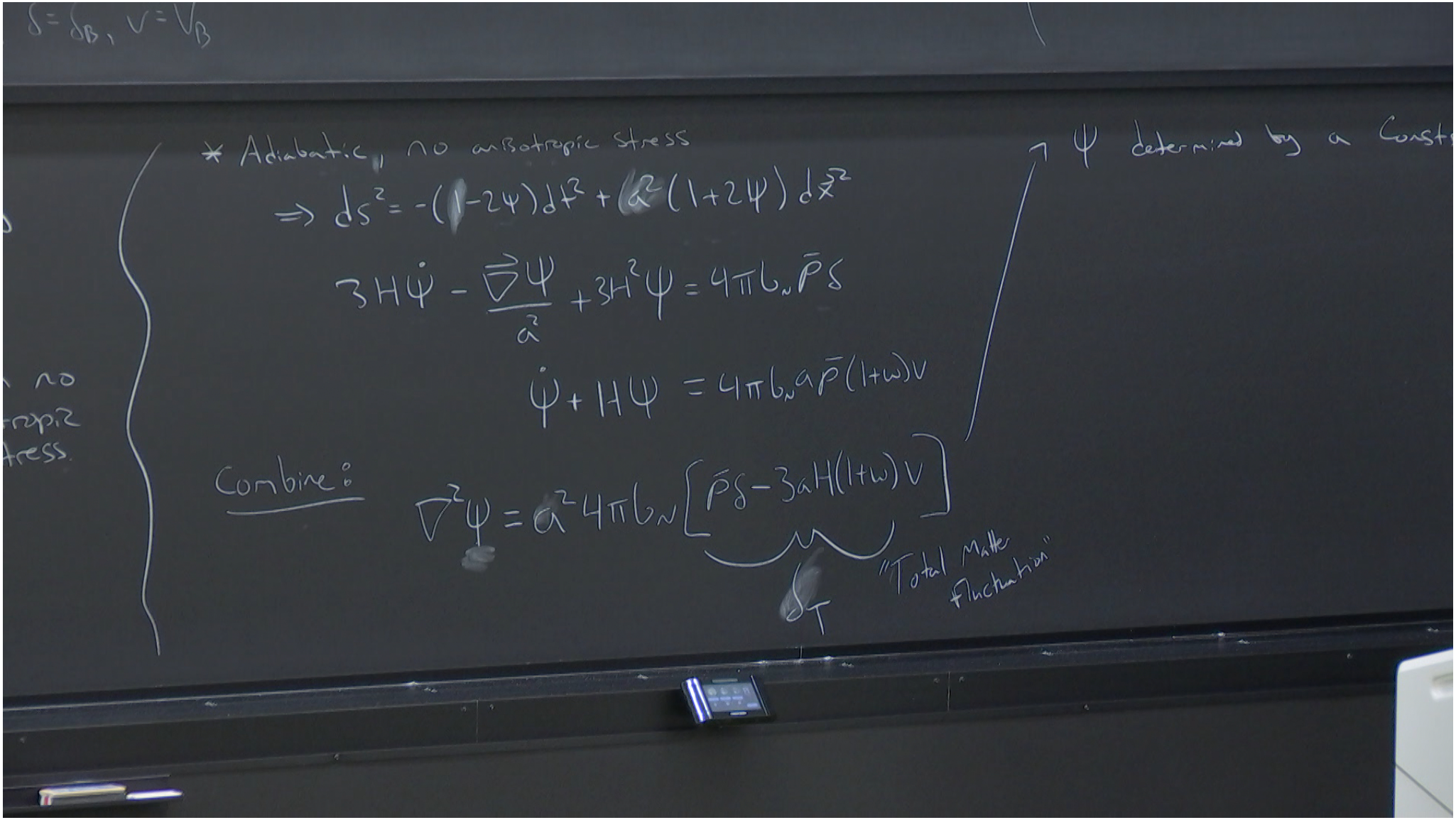
$$\Rightarrow \Phi + \psi = 0$$ when no anisotropic stress.

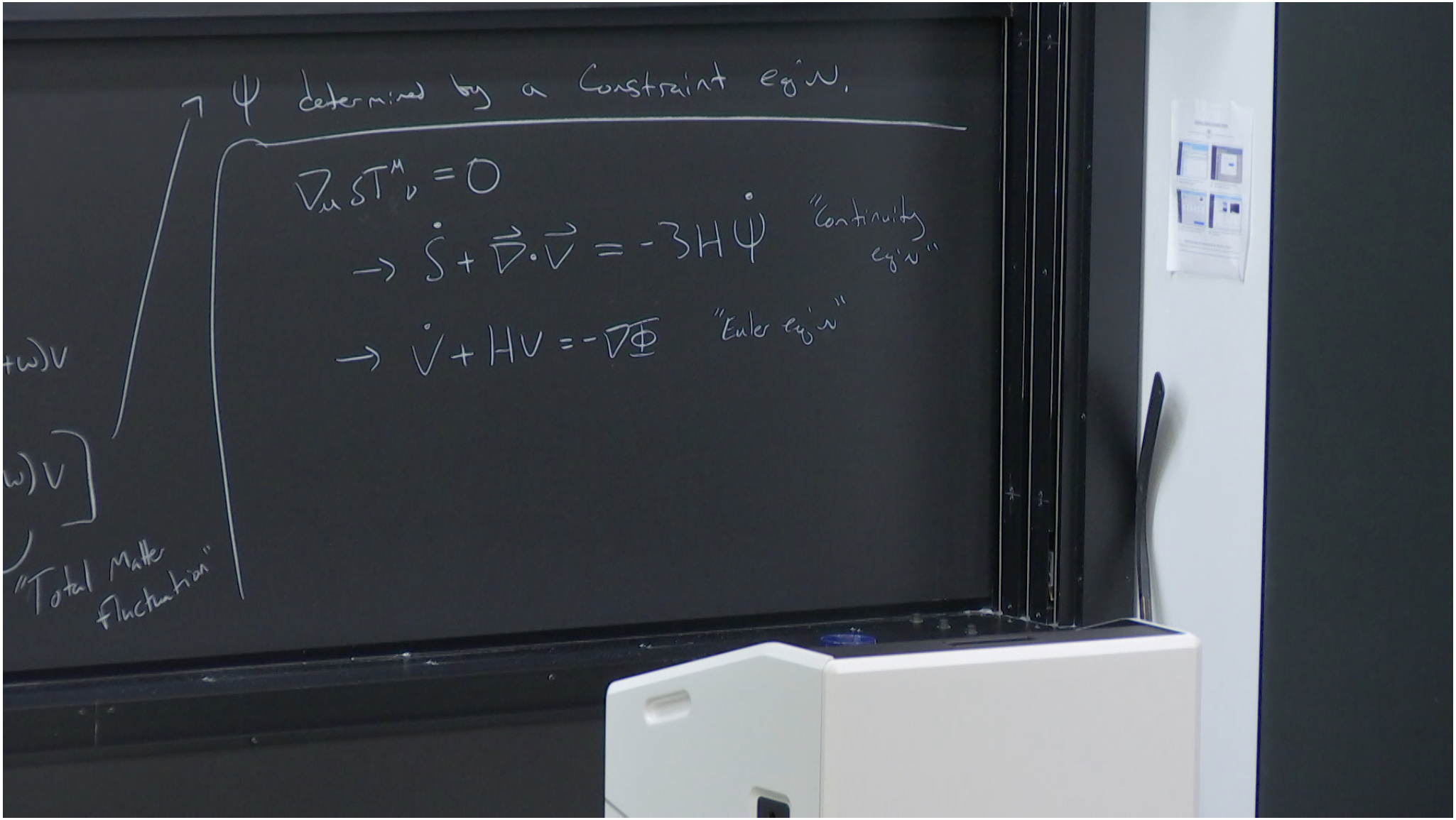
$V_B = V$

curvature perturbation"

$+ \frac{\nabla^2 \mathcal{R}}{4a^2}$


0 - i
$\delta G_{0i} = 8\pi G_N \delta T_{0i}$
$w=0$ for DM
$\partial_i\left(\dot{\Psi} - H\bar{\Phi} = 4\pi G_N a \bar{\rho}(1+w) V\right) + (\,)\bar{\rho}_\gamma \Theta_{1i}$
$\gamma\Theta_0 + \cdots$
i - i
$\delta G_{ii} = 8\pi G_N \delta T_{ii}$
$\frac{\nabla^2(\Phi+\Psi)}{2a^2} - \frac{\partial_i^2(\Phi+\Psi)}{2a^2} + H\dot{\Phi} - 3H\dot{\Psi} - \ddot{\Psi}$
$= 4\pi G\, w\bar{\rho}(\delta + 2\Phi)$


δ = δ_B, v = v_B
* Adiabatic, no anisotropic stress
⇒ ds² = -(1-2Ψ)dt² + a²(1+2Ψ)dx²
3HΨ̇ - ∇Ψ/a² + 3H²Ψ = 4πG_N ρ̄ δ
Ψ̇ + HΨ = 4πG_N a ρ̄ (1+w) v
Combine:
∇²Ψ = a² 4πG_N [ρ̄δ - 3aH(1+w)v]
δ_T
"Total Matter fluctuation"
Ψ determined by a Const


ψ determined by a Constraint eqⁿ.
$\nabla_\mu \delta T^\mu_\nu = 0$
$\rightarrow \dot{\delta} + \vec{\nabla}\cdot\vec{v} = -3H\dot{\psi}$ "Continuity eqⁿ"
$\rightarrow \dot{V} + HV = -\nabla\Phi$ "Euler eqⁿ"
+w)V
w)V]
"Total Matter fluctuation"

gauge, use

$\psi = \psi_B$, $\delta = \delta_B$, $v = v_B$

δT_{ij}

when no anisotropic stress

* Adiabatic, no anisotropic stress

$$\Rightarrow ds^2 = -((1-2\psi)dt^2 + a^2(1+2\psi)d\vec{x}^2$$

$$3H\dot{\psi} - \frac{\vec{\nabla}\psi}{a^2} + 3H^2\psi = 4\pi G_N \bar{\rho}\delta$$

$$\dot{\psi} + H\psi = 4\pi G_N a \bar{\rho}(1+w)v$$

Combine:

$$\nabla^2\psi = a^2 4\pi G_N [\bar{\rho}\delta - 3aH(1+w)v]$$

$\nabla^2 \to -k^2$

δ_T "Total Matter fluctuation"

ψ det

∇_u