Title: PSI 2019/2020 - Cosmology Part 2 - Lecture 1

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Collection: PSI 2019/2020 - Cosmology Part 2

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* Part II: Cosmological Perturbation Theory - FRW + Small fluctuations.

Background

FRW: $ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$

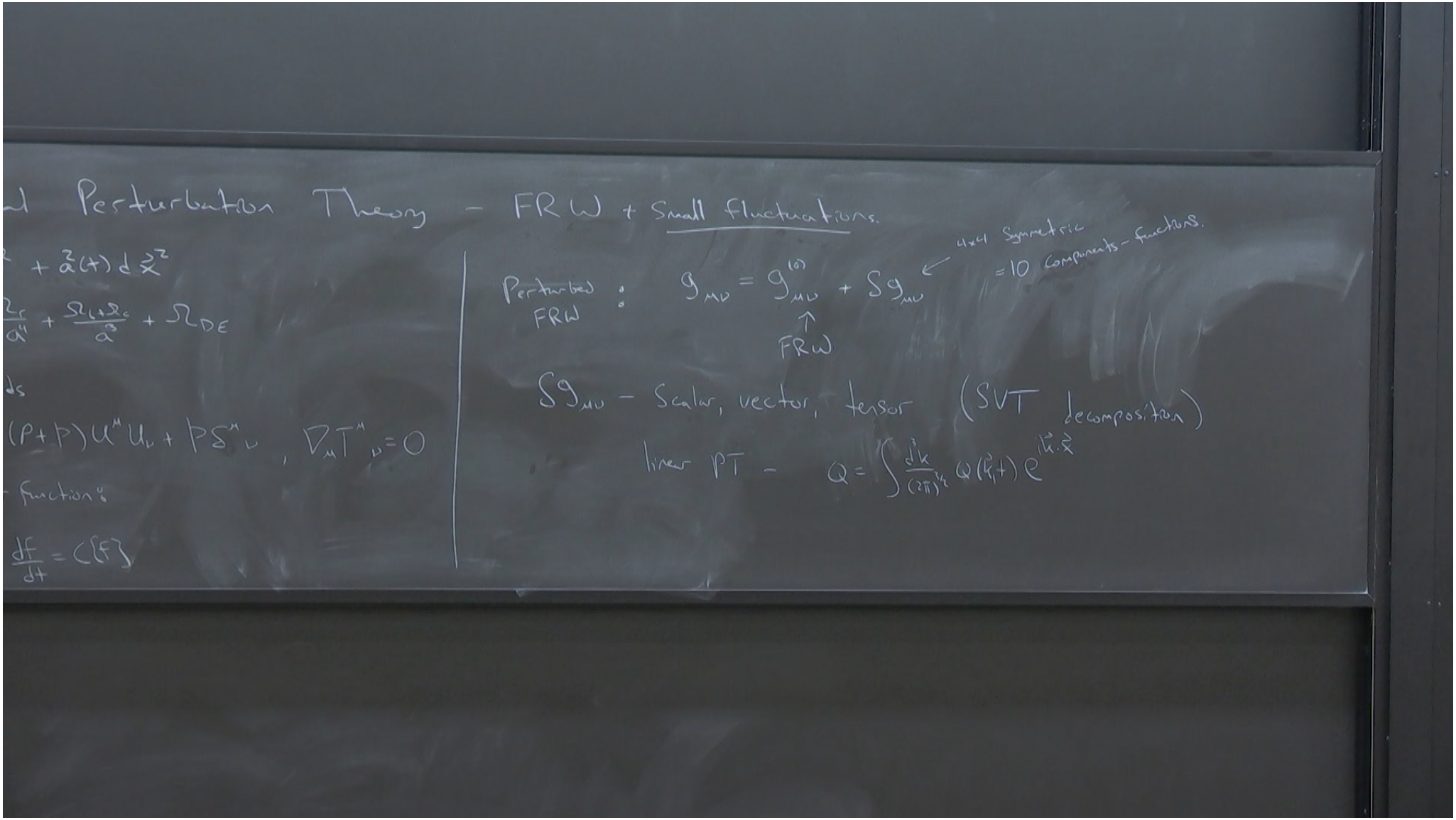
$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_b + \Omega_c}{a^3} + \Omega_{DE}$$

Matter: Perfect Fluids

$$T^\mu{}_\nu = (\rho + p) u^\mu u_\nu + p \delta^\mu{}_\nu \,, \quad \nabla_\mu T^\mu{}_\nu = 0$$

Distribution function:

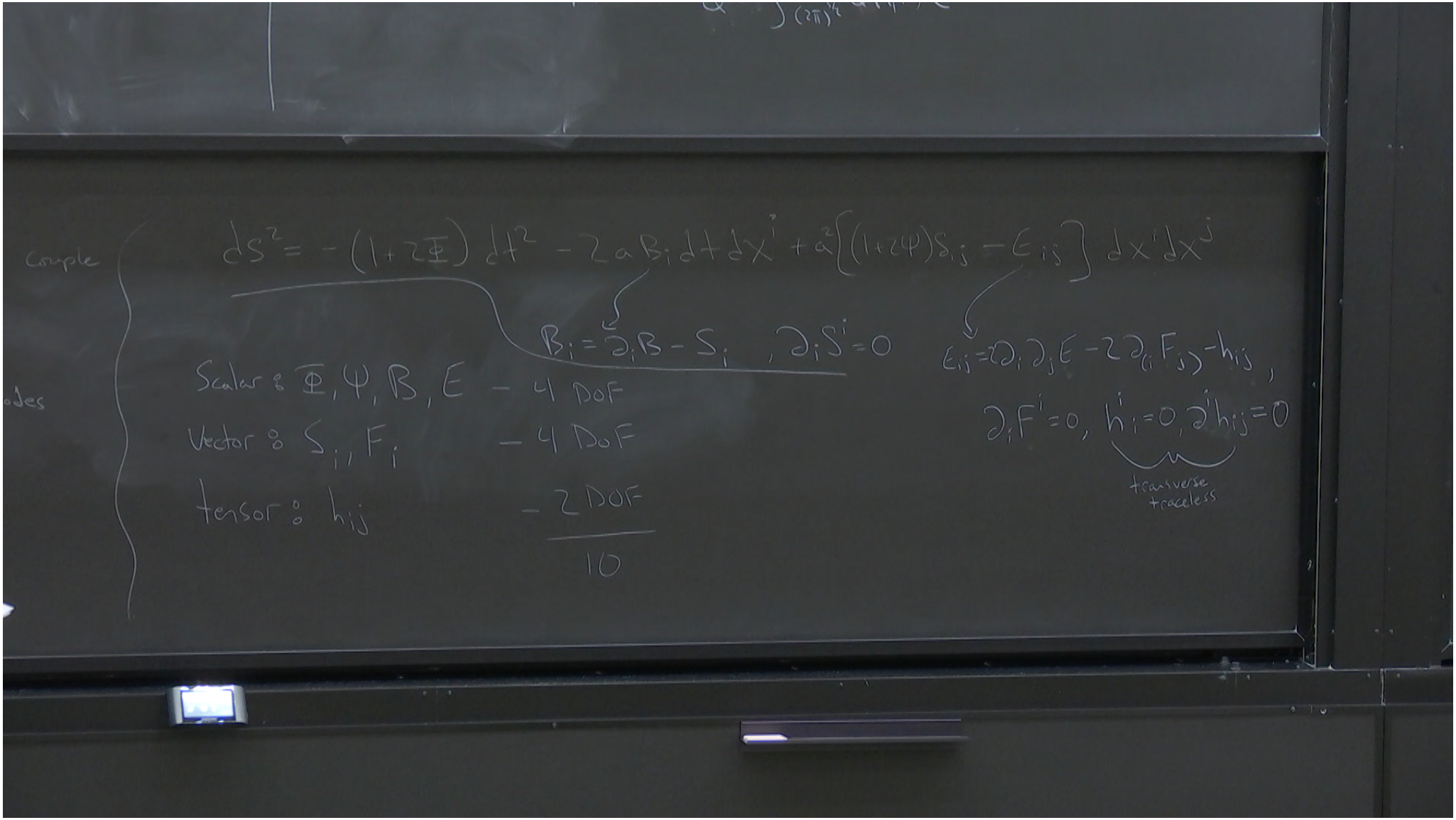
$$\frac{df}{dt} = C[f]$$

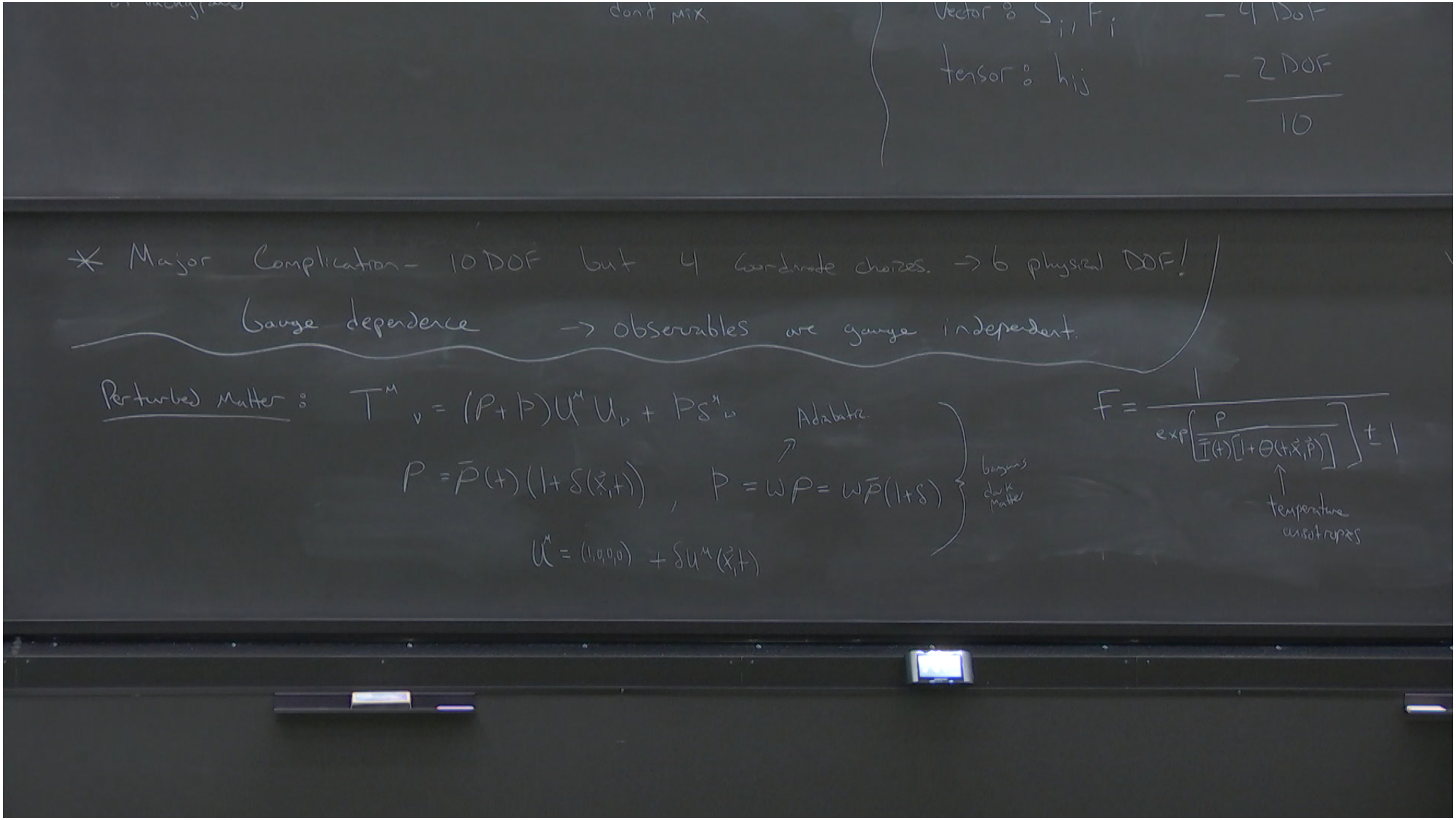

Perturbation Theory – FRW + Small fluctuations.
$+ \vec{a}^2(t)\, d\vec{x}^2$
$\frac{\Omega_r}{a^4} + \frac{\Omega_{(+)}\Omega_c}{a^3} + \Omega_{DE}$
ds
$(\rho+p)U^\mu U_\nu + p\delta^\mu{}_\nu \,,\; \nabla_\mu T^\mu{}_\nu = 0$
function
$\frac{df}{dt} = C[f]$
Perturbed FRW :
$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$
FRW
4×4 Symmetric
=10 components – functions.
$\delta g_{\mu\nu}$ – Scalar, vector, tensor (SVT decomposition)
linear PT – $Q = \int \frac{d^3k}{(2\pi)^{3/2}} Q(\vec{k},t)\, e^{i\vec{k}\cdot\vec{x}}$

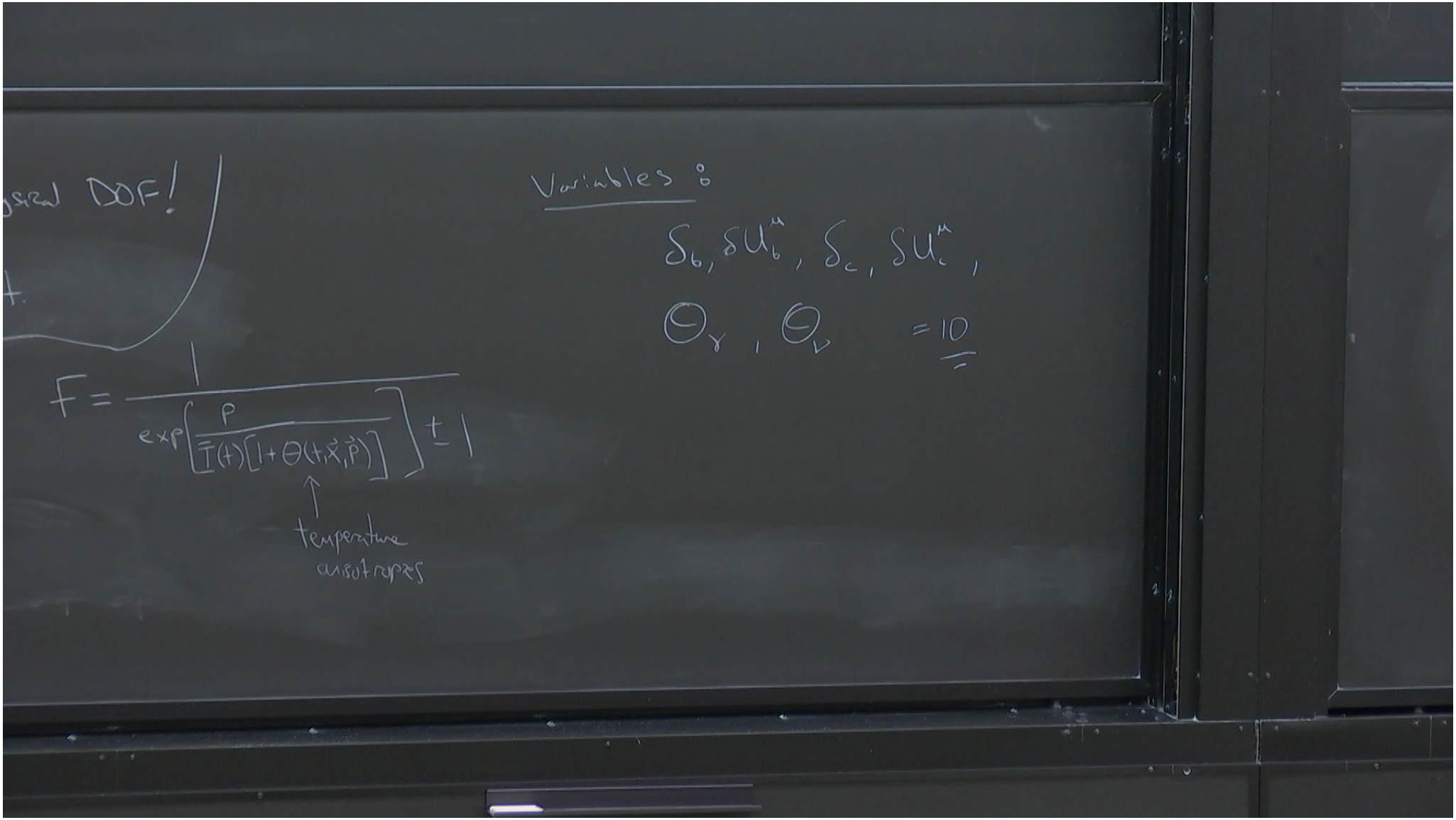
$\frac{df}{dt} = C\{f\}$

translation invariance of background (homogeneity) → $Q(\vec{k}, t)$ don't couple

rotational invariance of background (isotropy) → Scalar, vector, tensor modes don't mix


Couple
$ds^2 = -(1+2\Phi)\,dt^2 - 2aB_i\,dt\,dx^i + a^2\left[(1+2\Psi)\delta_{ij} - E_{ij}\right]dx^i dx^j$
$B_i = \partial_i B - S_i \quad , \partial_i S^i = 0$
$E_{ij} = 2\partial_i\partial_j E - 2\partial_{(i}F_{j)} - h_{ij}$,
$\partial_i F^i = 0, \; h^i{}_i = 0, \; \partial^i h_{ij} = 0$
transverse
traceless
Scalar : Φ, Ψ, B, E — 4 DoF
Vector : S_i, F_i — 4 DoF
tensor : h_{ij} — 2 DoF
10


don't mix
vector: S_i, F_i — 4 DoF
tensor: h_{ij} — 2 DoF
10
* Major Complication - 10 DOF but 4 coordinate choices. → 6 physical DOF!
Gauge dependence → observables are gauge independent.
Perturbed Matter: $T^{\mu}{}_{\nu} = (\rho + P)U^{\mu}U_{\nu} + P\delta^{\mu}{}_{\nu}$
Adiabatic.
$\rho = \bar{\rho}(t)(1+\delta(\vec{x},t))$, $P = w\rho = w\bar{\rho}(1+\delta)$
baryons dark matter
$U^{\mu} = (1,0,0,0) + \delta u^{\mu}(\vec{x},t)$
$f = \frac{1}{\exp\left[\frac{p}{\bar{T}(t)[1+\Theta(t,\vec{x},\hat{p})]}\right] \pm 1}$
temperature anisotropies


sical DOF!
t.
Variables :
$\delta_b, \delta u_b^\mu, \delta_c, \delta u_c^\mu,$
$\Theta_\gamma, \Theta_\nu$ = 10
$f = \frac{1}{\exp\left[\frac{p}{\bar{T}(t)\left[1+\Theta(t,\vec{x},\vec{p})\right]}\right] \pm 1}$
temperature anisotropies

$u^\mu = (1,0,0,0) + \delta u^\mu(\vec{x},t)$

Dynamics

Inflation $\rightarrow$ $\delta G_{\mu\nu} = 8\pi G_N \, \delta T_{\mu\nu}$

$\nabla_\mu \delta T^\mu{}_\nu = 0$

$\frac{df}{dt} = C[f]$

$\rightarrow$ growth of structure
CMB anisotropies
BAO
⋮

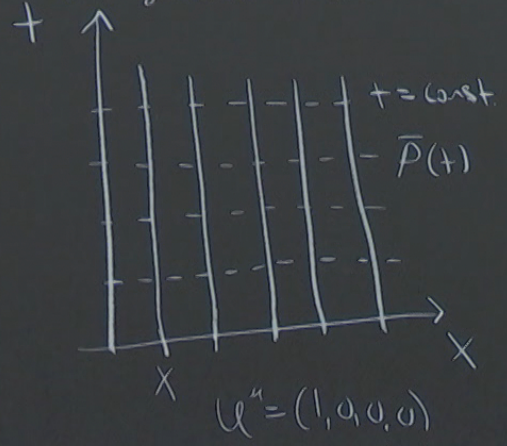
anisotropies

structure

ps

* Gauge dependence

homogeneous FRW

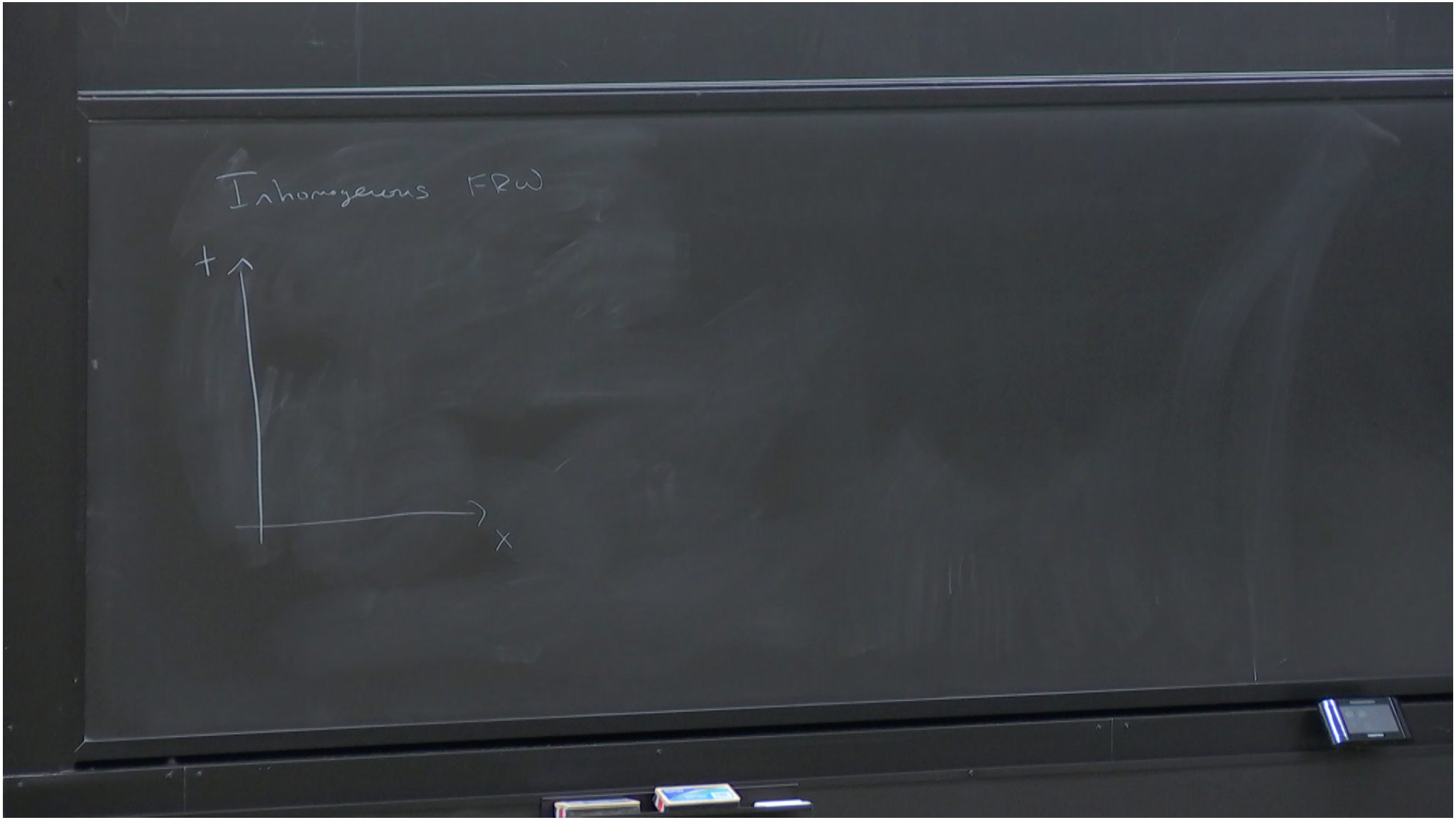


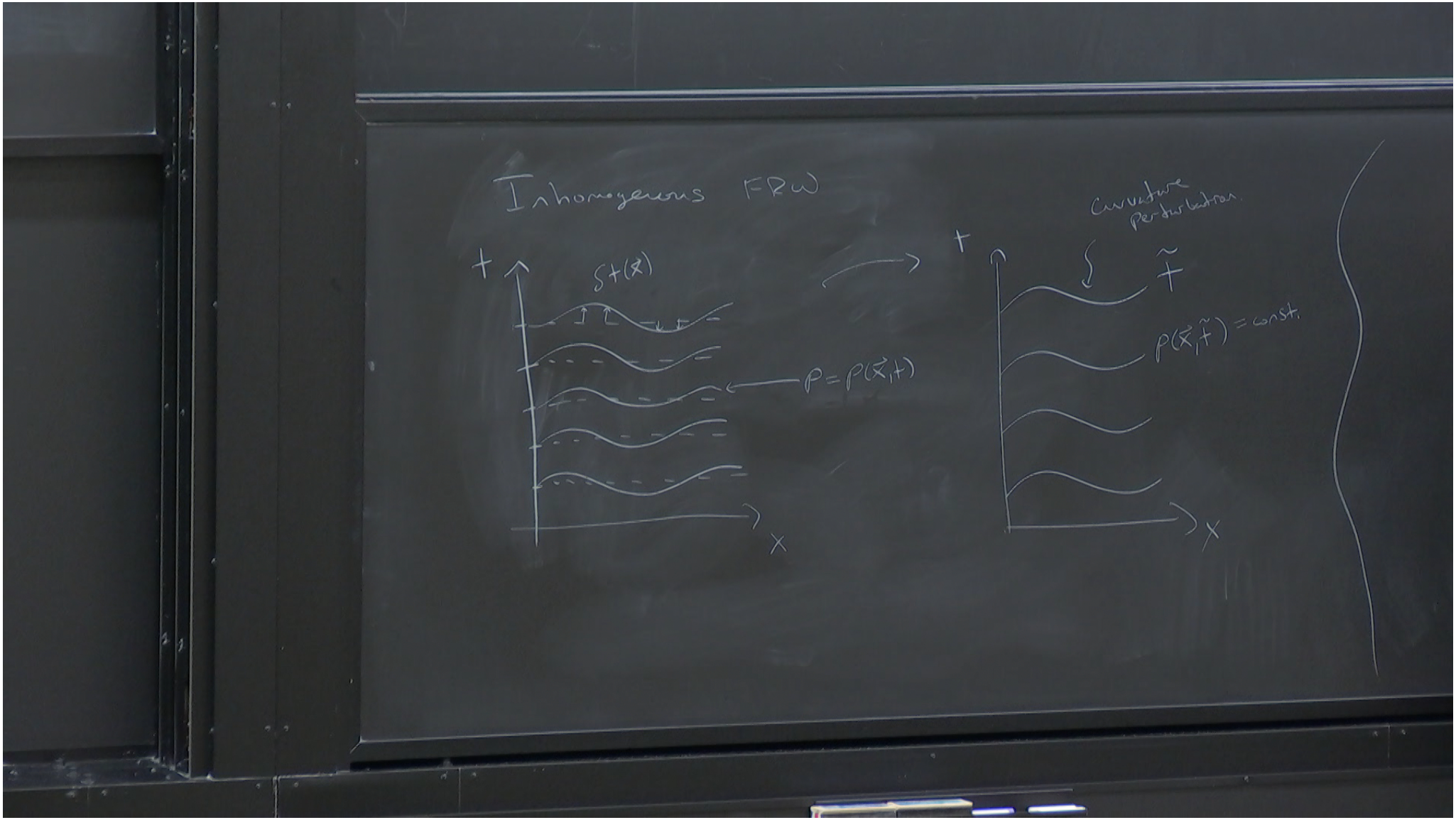
$u^\mu = (1,0,0,0)$

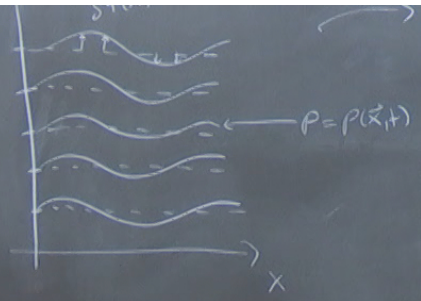
$ds^2 = -dt^2 + \vec{a}(t) d\vec{x}^2$

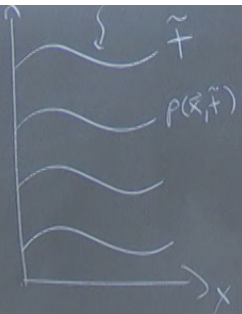
proper time for each particle

label an ensemble of particles


Inhomogeneous FRW
t
x


Inhomogeneous FRW
t
δt(x)
ρ = ρ(x,t)
x
t
Curvature Perturbation
t̃
ρ(x,t̃) = const.
x





$$= \left(\delta^{a}_{\mu} + \partial_{\mu}\xi^{a}\right)\left(\delta^{b}_{\nu} + \partial_{\nu}\xi^{b}\right)\left(g^{(0)}_{ab}(x^{\gamma}) + \delta\tilde{g}_{ab}(x^{\gamma}) + (\partial_{\gamma} g$$

$$g^{(0)}_{\mu\nu}(x^{\gamma}) + \delta g_{\mu\nu}(x^{\gamma}) = g^{(0)}_{\mu\nu}(x^{\gamma}) + \partial_{\nu}\xi^{b} g^{(0)}_{\mu b} + \partial_{\mu}\xi^{a} g^{(0)}_{a\nu} + \delta\tilde{g}_{\mu\nu}(x^{\gamma}) + \partial_{\gamma} g^{(0)}_{\mu\nu}$$

$$\Rightarrow \boxed{\delta\tilde{g}_{\mu\nu} = \delta g_{\mu\nu} - \left(\partial_{\gamma} g^{(0)}_{\mu\nu}\right)\xi^{\gamma} - \partial_{\nu}\xi^{b} g^{(0)}_{\mu b} - \partial_{\mu}\xi^{a} g^{(0)}_{a\nu}}$$

all a function of x^{γ}

Focusing on Scalars

$$ds^2 = -(1+2\Phi)\,dt^2 - 2a\partial_i B\,dt\,dx^i + a^2\left[(1+2\Psi)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i dx^j \quad , \quad \xi^{\mu} = \begin{pmatrix} \xi^0 \\ \delta^{ij}\partial_j \xi \end{pmatrix}$$

$$\tilde{\Phi} = \Phi - \dot{\xi}^0$$

$$\tilde{B} = B - \frac{\xi^0}{a} + a\dot{\xi}$$

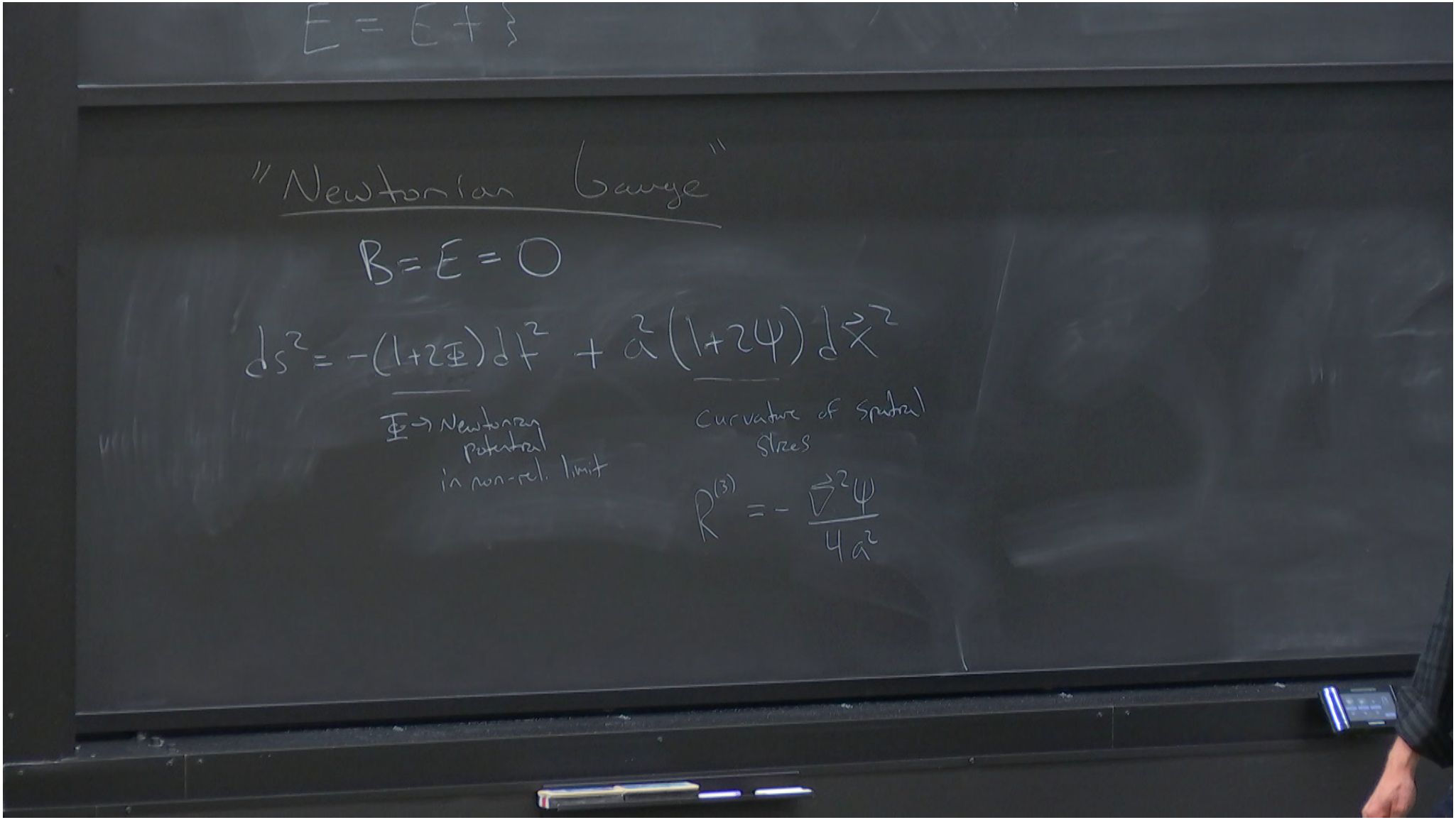
$$\tilde{\Psi} = \Psi - H\xi^0$$

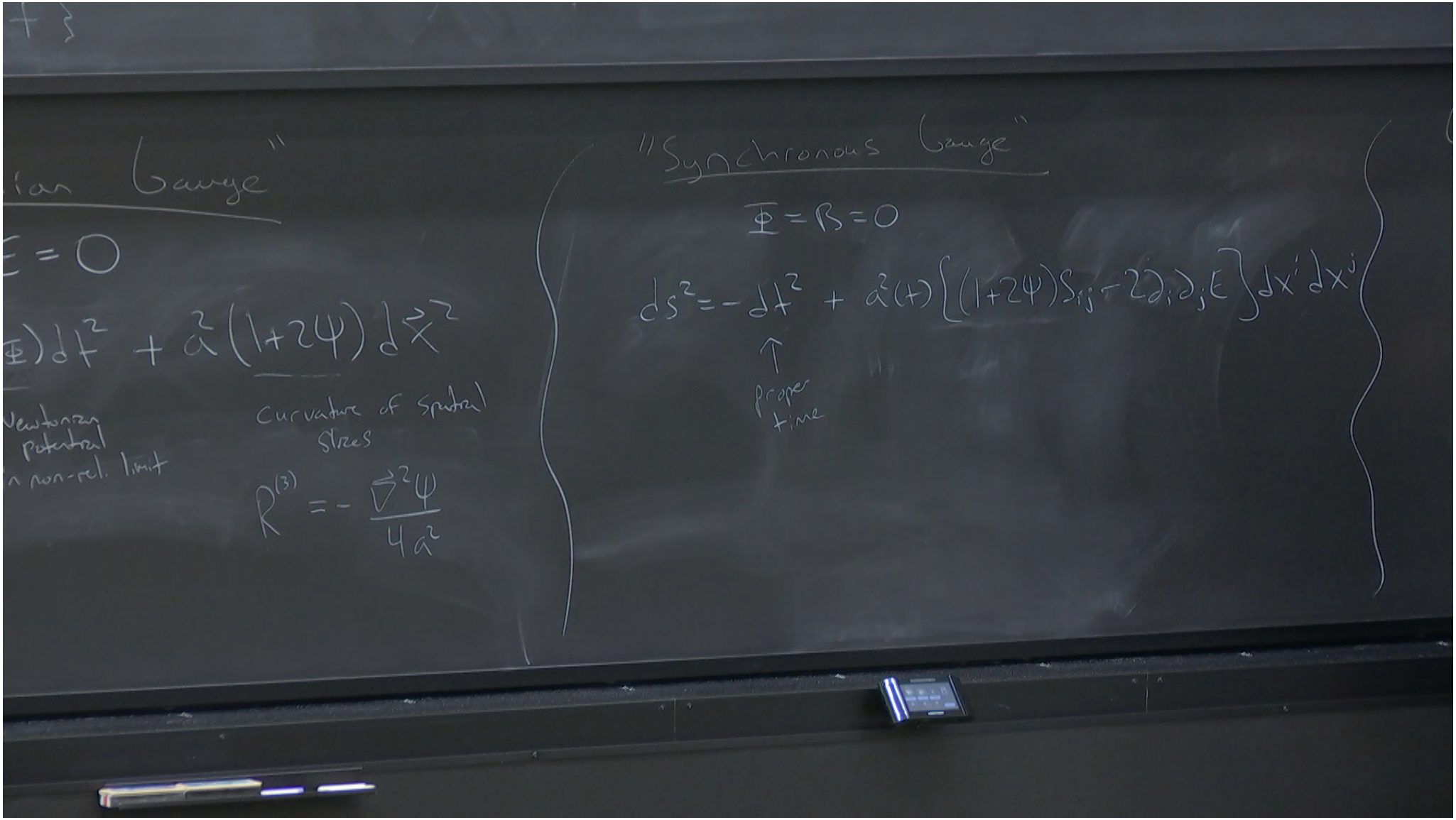
$$\tilde{E} = E + \xi$$

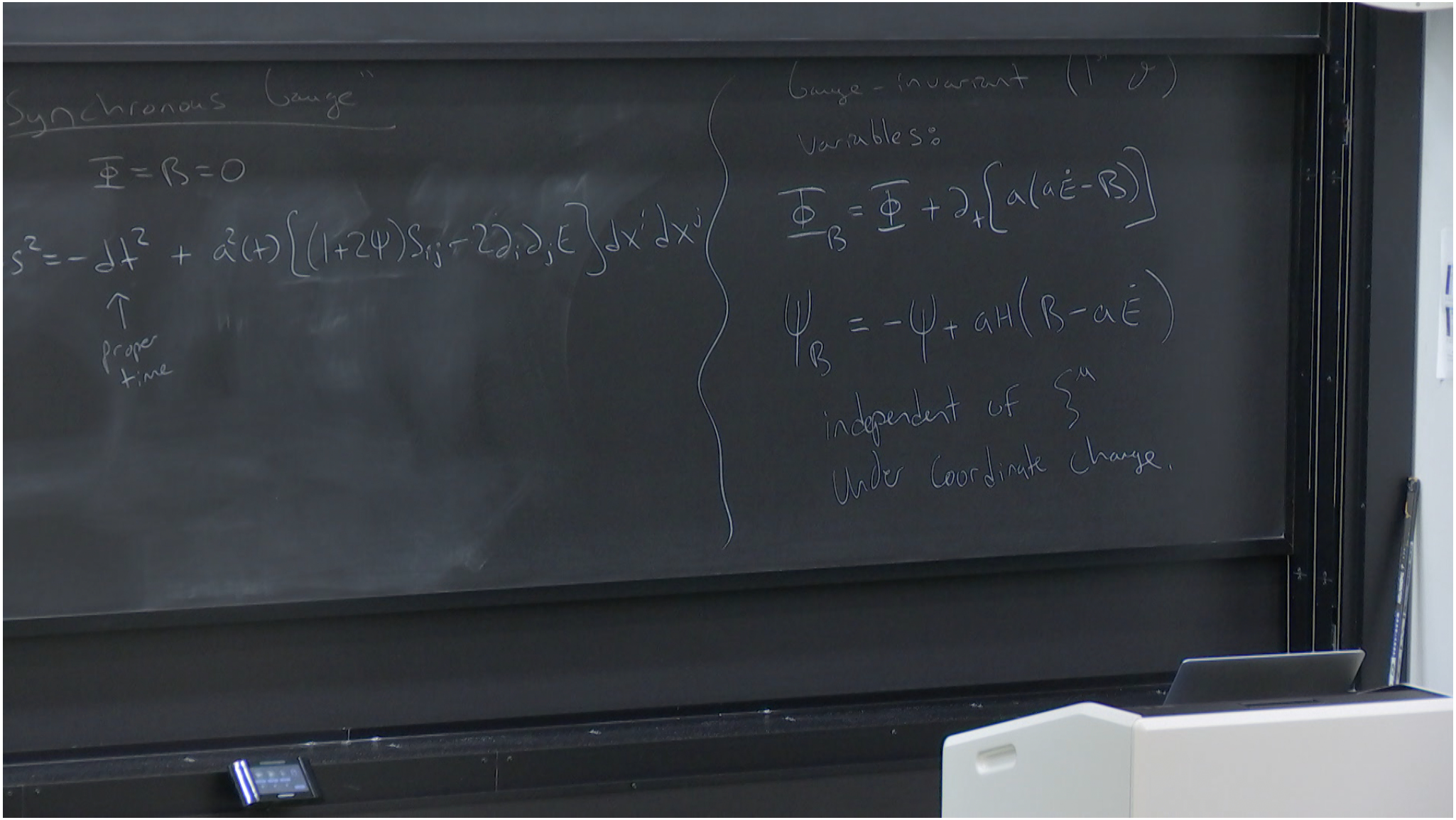
4 scalar functions

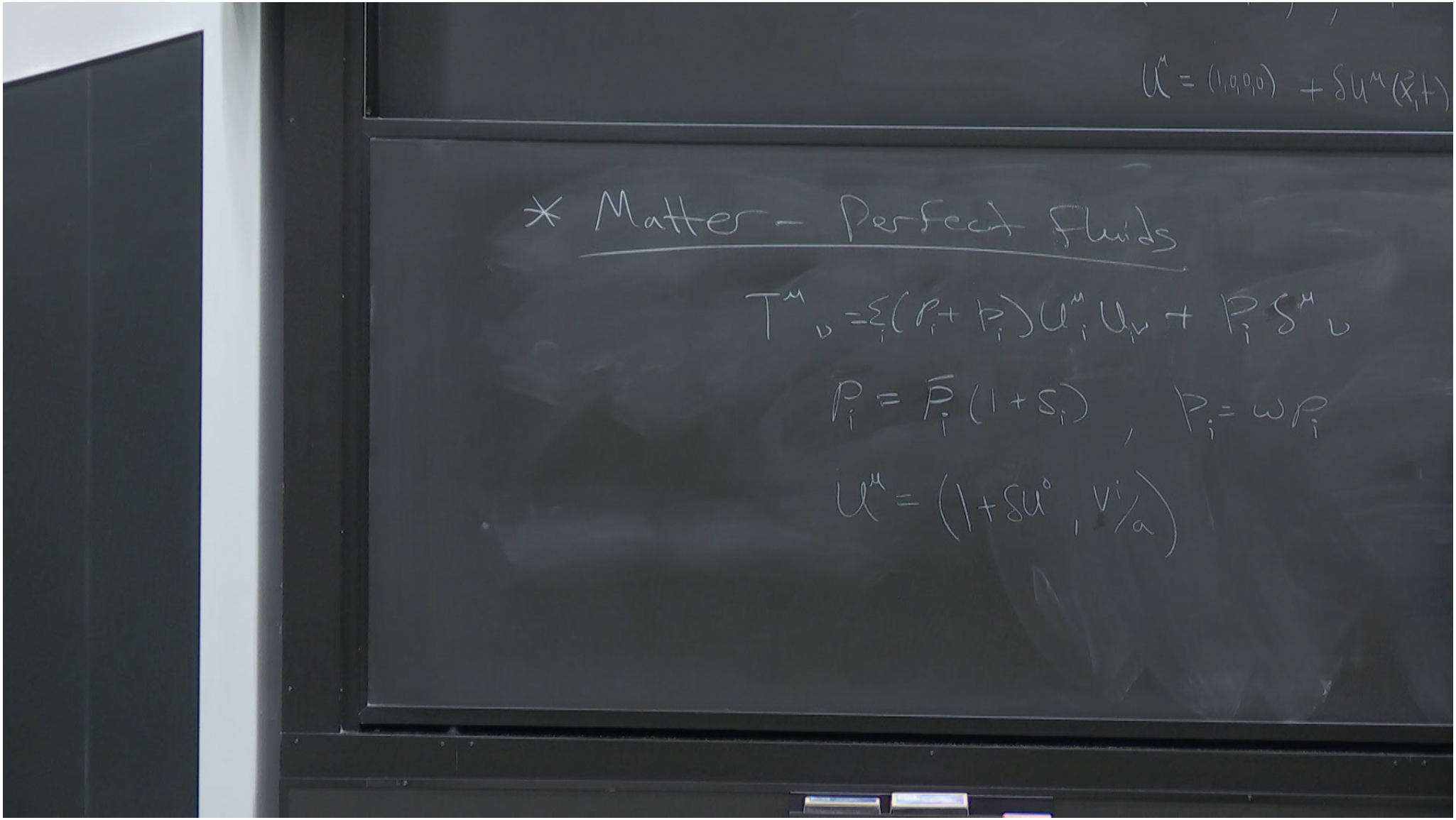
2 free choices for $\xi^0(\vec{x},t)$, $\xi(\vec{x},t)$

→ Eliminate any 2 Scalar metric functions via coordinate choice!


E = E + }
"Newtonian Gauge"
B = E = 0
$ds^2 = -(1+2\Phi)dt^2 + a^2(1+2\Psi)d\vec{x}^2$
Φ → Newtonian potential in non-rel. limit
Curvature of spatial slices
$R^{(3)} = -\frac{\vec{\nabla}^2\Psi}{4a^2}$


"Synchronous Gauge"
Φ = B = 0
ds² = −dt² + a²(t)[(1+2Ψ)δ_ij − 2∂_i∂_j E] dx^i dx^j
proper time
Curvature of Spatial Slices
R^(3) = − ∇²Ψ / 4a²


Synchronous Gauge
Φ = B = 0
ds² = −dt² + a²(t)[(1+2ψ)δ_ij − 2∂_i∂_jE]dx^i dx^j
proper time
Gauge-invariant (1st order) Variables:
Φ_B = Φ + ∂_t[a(aĖ − B)]
ψ_B = −ψ + aH(B − aĖ)
independent of ξ^μ
Under coordinate change.


$u^\mu = (1,0,0,0) + \delta u^\mu(\vec{x},t)$
* Matter - Perfect Fluids
$T^\mu{}_\nu = \sum_i (\rho_i + P_i) u^\mu_i u_{i\nu} + P_i \delta^\mu{}_\nu$
$\rho_i = \bar{\rho}_i (1+\delta_i)$, $P_i = w \rho_i$
$u^\mu = (1+\delta u^0, v^i/a)$

$= wP = wP(1+\delta)$ dark matter

temperature anisotropies

Use normalization: $-1 = g_{\mu\nu} u^\mu u^\nu$

$$\Rightarrow -1 = -(1+2\Phi)(1+\delta u^0)^2 + a^2\left(\frac{v^i}{a}\right)^2$$

$$-1 = -1 - 2\Phi - 2\delta u^0$$

$$\Rightarrow \delta u^0 = -\Phi$$

$$\rightarrow u^\mu = \left(1-\Phi, \frac{v^i}{a}\right), \quad u_\mu = \left(-1-\Phi, \frac{v^i}{a}\right)$$

* For scalars — irrotational flow

$$\vec{v} = \vec{\nabla} v$$

* Sub into $T^\mu{}_\nu$

$$T^\mu{}_\nu = T^{\mu\,(0)}_{\nu} + \delta T^\mu{}_\nu$$

$$\delta T^0{}_0 = (\rho + P) u^0 u_0 + P = -\delta\bar{\rho}$$

$$\delta T^0{}_i = \bar{\rho} a (1+w)(v^i - \partial_i B)$$

$$\delta T^i{}_0 = -\bar{\rho}(1+w) v^i / a$$

$$\delta T^i{}_j = \delta w \bar{\rho}\, \delta^i{}_j$$