Title: PSI 2019/2020 - Quantum Gravity Part 1 - Lecture 6

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Collection: PSI 2019/2020 - Quantum Gravity Part 1

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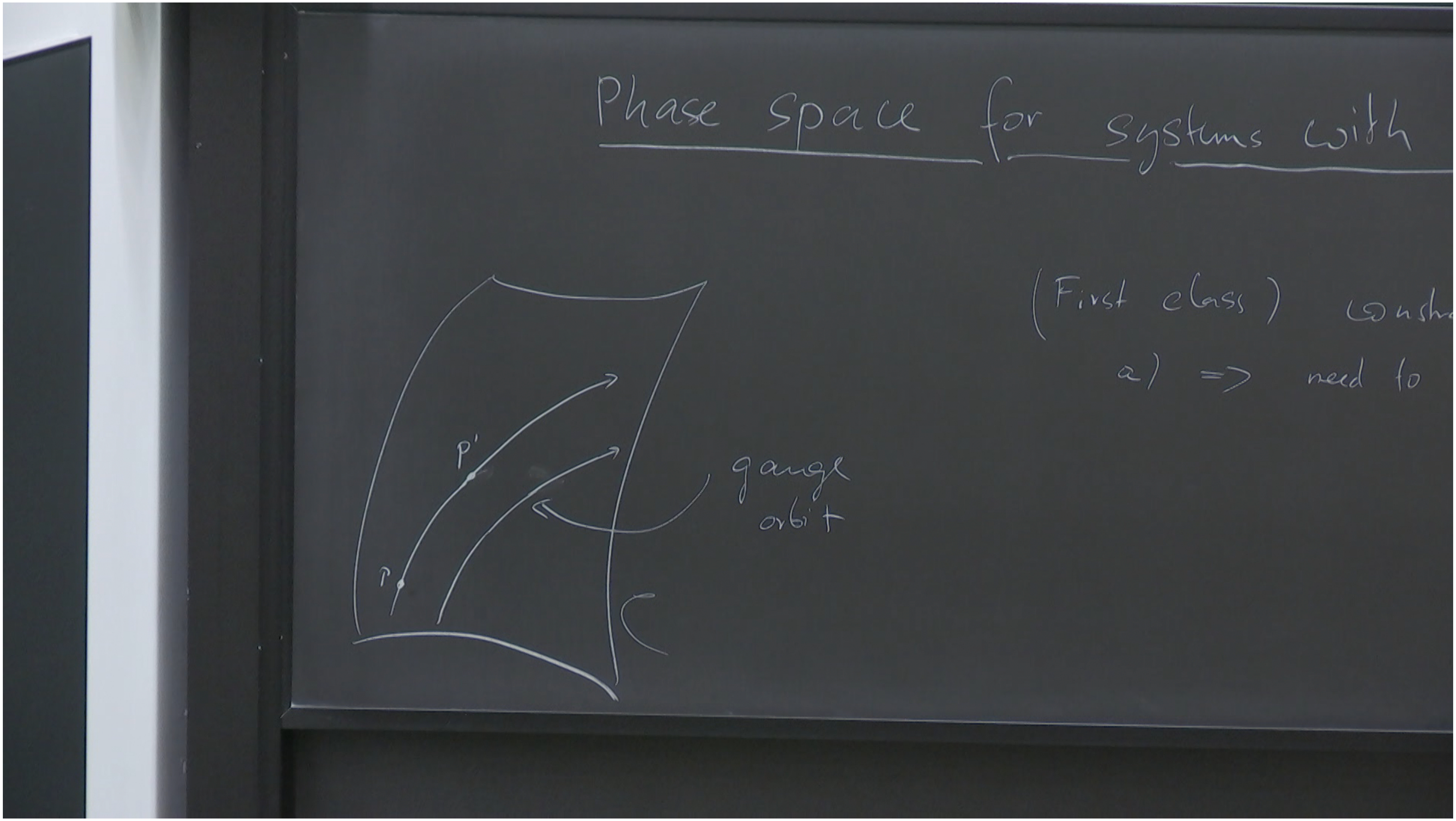

Phase space for systems with
(First class)
a) => need to
p'
p
gauge orbit
C

For systems with (first class) constraints

$$\{C_I, C_J\} = f_{IJ}{}^K C_K$$

(First class) constraint

a) => need to reduce to the constraint hypersurface

b) => gauge symmetry: gauge orbits along hypersurface constraints

$f_{IJ}{}^{K} C_K$

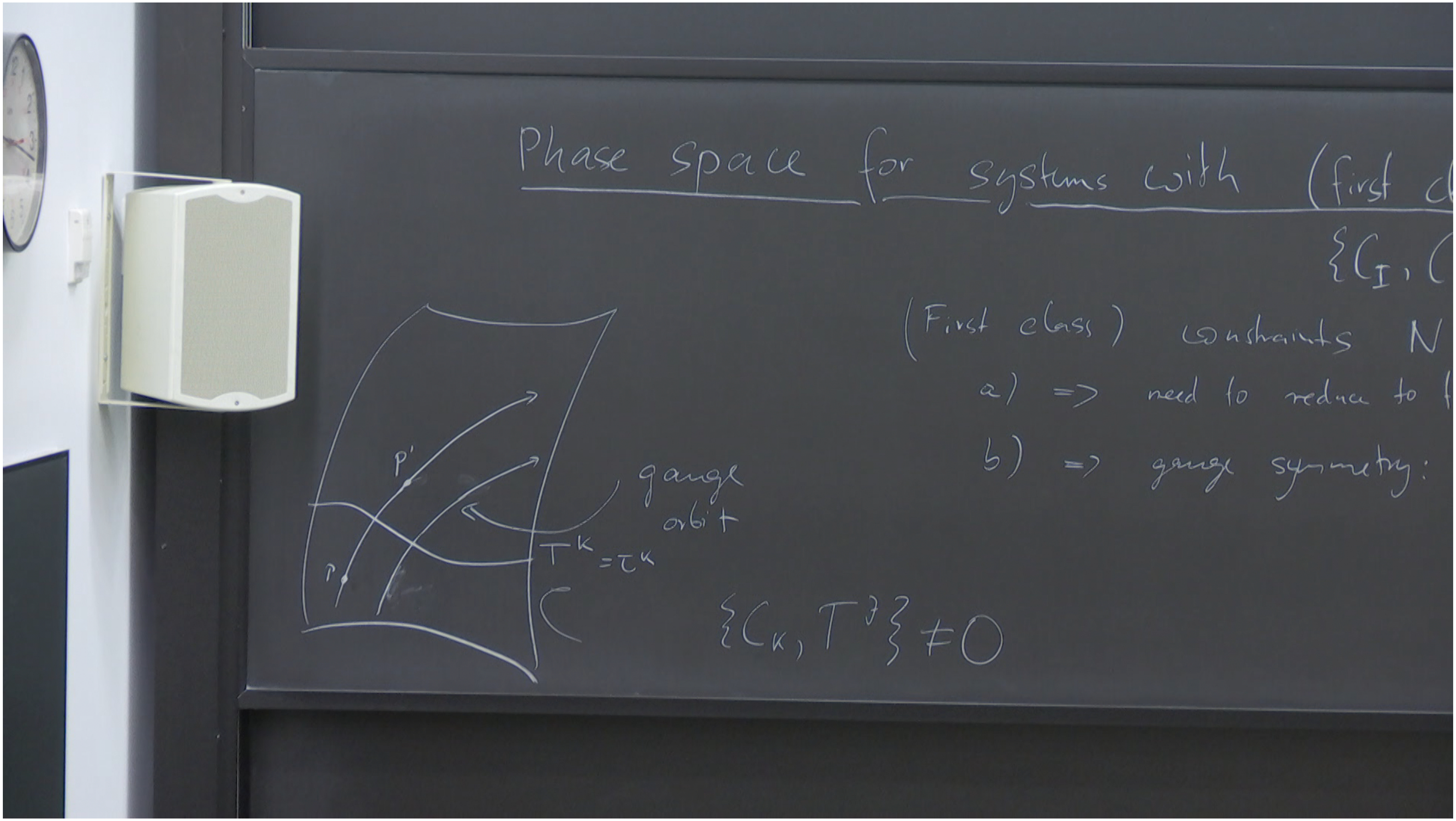
$2n$ dim phase space

nstraint hypersurface $(2n - N)$ dim

ge orbits along hypersurface N dim orbit. Need to quotient by gauge action.

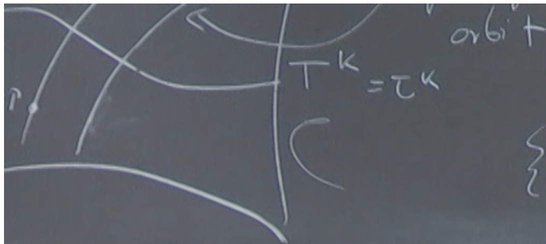
(Reduced) phase space. $\Rightarrow (2n - 2N)$ dim

Physical


Phase space for systems with (first cl
{C_I, C
(First class) constraints N
a) => need to reduce to f
b) => gauge symmetry:
p'
p
gauge orbit
T^K = τ^K
C
{C_K, T^J} ≠ 0

$$\{f, C_j\} \neq 0$$

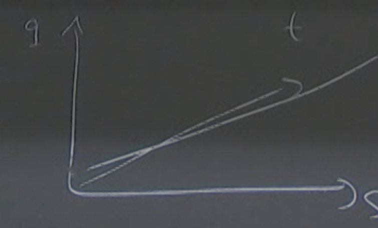
Physical / Dirac } Observables $\mathcal{O}$: $\{\mathcal{O}, C_j\} \simeq 0$



$\{C_k, T^j\} \neq 0$

(Reduced) phase space,
Physical

7 Quantization of Gauge System



7.1 Example: The parametrized particle

Usual action: $S = \int L(q, \dot{q})\, dt$ $\qquad L = \frac{m}{2}\dot{q}^2 - V(q)$

Configurations: $(t(s), q(s))$ $\qquad$ s: evolution parameter

$$\cong$$

$$(t(f(s)), q(f(s)))$$

$$S_p = \int L\left(q, \frac{q'}{t'}\right) t' \, ds$$

$$q' = \frac{dq}{ds} = \frac{dq(t(s))}{dt} \frac{dt(s)}{ds} = \dot{q} \, t'$$

$$dt = t' \, ds$$

- invariant under reparametrizations.

$$s \longmapsto \tilde{s} = f(s)$$

Canonical Analysis

$$L_p = L\left(q, \frac{q'}{t'}\right) t' \quad , \quad (q, t)$$

Hamiltonian for L

$$p_t = \frac{\partial L_p}{\partial t'} = L - \frac{\partial L}{\partial \dot{q}} \frac{q'}{(t')} = -(p_q \dot{q} - L) = -h(p_p, q)$$

$$p_q = \frac{\partial L_p}{\partial q'} = \frac{\partial L}{\partial \dot{q}} \frac{1}{t'} t' = m \frac{q'}{t'}$$

$p_t = -h(p_q, q) \Rightarrow$ Constraint $\quad C = p_t + h(p_q, q)$

$p_q = m\, q'/t'$

$$H = p_t t' + p_q \underbrace{q'}_{\dot{q} t'} - \underbrace{L t'}_{L_p} = \underbrace{t'}_{N} \big(\underbrace{p_t + h(p_q, q)}_{C} \big)$$

$$\underbrace{p_q q' - L t'}_{t'(\underbrace{p_q \dot{q} - L}_{h})}$$

$$t' = \{ t , NC \} = N$$

$$q' = \{ q , NC \} = N \frac{\partial h(p_q , q)}{\partial p_q}$$

"Time" evolution is a gauge transformation

- Physical observables $\{F, C\} \simeq 0$

- Relational observables: $F_f(\tau)$: value of f

parameter

phase space fct

· Time evolution

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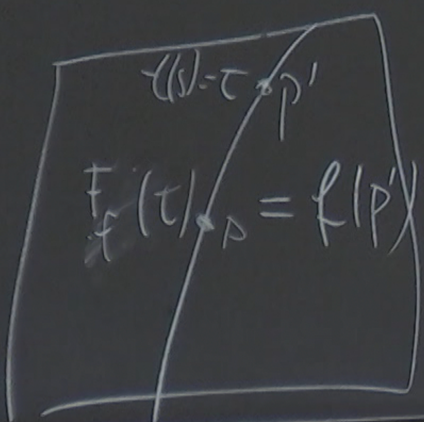


$F_q(\tau) =$

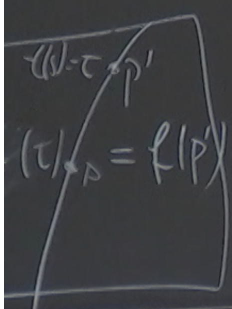
"Time" evolution is a gauge transformation

• Physical observables $\quad \{F, C\} \simeq 0$

• Relational observables: $\quad F_f(\tau)$: value of f for s where $t(s) = \tau$

parameter

phase space fct



(free particle) $\rightarrow F_q(\tau) = q + \frac{p_q}{m}(\tau - t) \qquad \{F_q(\tau), C\}$

$F_{p_q}(\tau) = p_q$

$$F_t(\tau) = \tau$$

$$F_{p_t}(\tau) = -h(p_q, q)$$

$$= -\frac{p_q^2}{m}$$

s where $t(s) = \tau$

$$\{F_q(\tau), C\}$$