Title: PSI 2019/2020 - Quantum Gravity Part 1 - Lecture 4

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Symmetries of 3D Palatini action

$S = \frac{1}{2}\int e_{\alpha l} F^{l}_{\mu\nu} \tilde{\epsilon}^{\sigma\mu\nu} d^3x$

$\sim \int e \wedge F$

$\int (d_\omega N_i) \wedge F^i$
$= \int N_i \wedge \underbrace{d_\omega F^i}_{\text{Bianchi identity}} = 0$

- internal rotations:

$$\delta^R_\Lambda e^i_\mu = \epsilon^i_{k i} e^k_\mu \Lambda^i$$

$$\delta^R_\Lambda \omega^m_\mu = \partial_\mu \Lambda^m + \epsilon^m{}_{l i}\, \omega^l_\mu \Lambda^i$$

$$= D_\mu \Lambda^m$$

- diffeomorphism:

$$\delta^D_v e^i_\mu = \mathcal{L}_v e^i_\mu = v^\nu \partial_\nu e^i_\mu + e^i_\nu \partial_\mu v^\nu$$

$$\delta^D_v \omega^m_\mu = \mathcal{L}_v \omega^m_\mu = v^\nu \partial_\nu \omega^m_\mu + \omega^m_\nu \partial_\mu v^\nu$$

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action $\quad S = \frac{1}{2}\int e_{a} F^{l}_{\mu\nu} \tilde{\epsilon}^{\sigma\mu\nu} d^3x$

$\sim \int e \wedge F$

$\int (d_\omega N_j) \wedge F^j$
$= \int N_j \wedge \underbrace{d_\omega F^j}_{\text{Bianchi identity} \, = 0}$

$= \epsilon^j_{ki} e^k_\mu \Lambda^i$

$= \partial_\mu \Lambda^m + \epsilon^m{}_{li} \omega^l_\mu \Lambda^i$

$= D_\mu \Lambda^m$

$= \mathcal{L}_v e^j_\mu = v^\nu \partial_\nu e^j_\mu + e^j_\nu \partial_\mu v^\nu$

$= \mathcal{L}_v \omega^m_\mu = v^\nu \partial_\nu \omega^m_\mu + \omega^m_\nu \partial_\mu v^\nu$

Translational symmetry:

$\delta^T_N e^j_\mu = D_\mu N^j = \partial_\mu N^j + \epsilon^j{}_{mk} \omega^m_\mu N^k$

space time scalar, internal vector

$= (d_\omega N^j)$

$\delta^T_N \omega^j_\mu = 0$

Symmetries of 3D Palatini action $\quad S = \frac{1}{2}\int e_{\alpha}\, F_{\mu\nu}\, \epsilon \, d^3x$

$\sim \int e \wedge F$

- internal rotations:

$$\delta^R_\Lambda e^j_\mu = \epsilon^j_{k i}\, e^k_\mu \Lambda^i$$

$$\delta^R_\Lambda \omega^m_\mu = \partial_\mu \Lambda^m + \epsilon^m{}_{l i}\, \omega^l_\mu \Lambda^i = D_\mu \Lambda^m$$

$\int (d_\omega N) \wedge F^j = \int N_j \wedge \underbrace{d_\omega F^j}_{\text{Bianchi identity}} = 0$

- diffeomorphism:

$$\delta^D_v e^j_\mu = \mathcal{L}_v e^j_\mu = v^\nu \partial_\nu e^j_\mu + e^j_\nu \partial_\mu v^\nu$$

$$\delta^D_v \omega^m_\mu = \mathcal{L}_v \omega^m_\mu = v^\nu \partial_\nu \omega^m_\mu + \omega^m_\nu \partial_\mu v^\nu$$

Translations

$$\delta^T_N e^j_\mu =$$

space time scale internal

$$\delta^T_N \omega^j_\mu =$$

Dependence of symmetries

$$\delta^R_{\omega(v)} e^j_\mu + \delta^T_{e(v)} e^j_\mu = \delta^D_v e^j_\mu + T^j_{\mu\nu} v^\nu$$

$\omega(v) = \omega^i_\nu v^\nu = \Lambda^i \qquad e(v) = e^i_\nu v^\nu = N^i$

$$\delta^R_{\omega(v)} \omega^j_\mu + \delta^T_{e(v)} e^j_\mu = \delta^D_v \omega^j_\mu + F^j_{\mu\nu} v^\nu$$

$\omega(v) = \omega^i_\nu v^\nu = \Lambda^i \qquad e(v) = e^i_\nu v^\nu = N^i$

$$\delta^R_{\omega(v)} \omega^i_\mu + \delta^T_{e(v)} e^i_\mu = \delta^D_v \omega_\mu^i + F^i_{\mu\nu} v^\nu$$

5 Canonical Formalism

Review:

$$S[q(t)] = \int L(q,\dot q)\,dt = \int (p\dot q - H(q,p))\,dt$$

$p = \frac{\partial L(q,\dot q)}{\partial \dot q}$ need to invert $p(q,\dot q)$ to $\dot q(p,q)$

Poisson brackets: $\{q,p\} = 1$, $\{f,g\} = \frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial q} \Rightarrow \{\{f,g\},h\} + \{\{h,f\},g\} + \{\{g,h\},f\} = 0$

$$\dot{q} = \{q, H\} = \frac{\partial H}{\partial p}$$

$$\dot{p} = \{p, H\} = -\frac{\partial H}{\partial q}$$

"H generates time evolution flow"

$$\dot{f}(q,p) = \frac{\partial f}{\partial q}\dot{q} + \frac{\partial f}{\partial p}\dot{p} = \{f, H\}$$

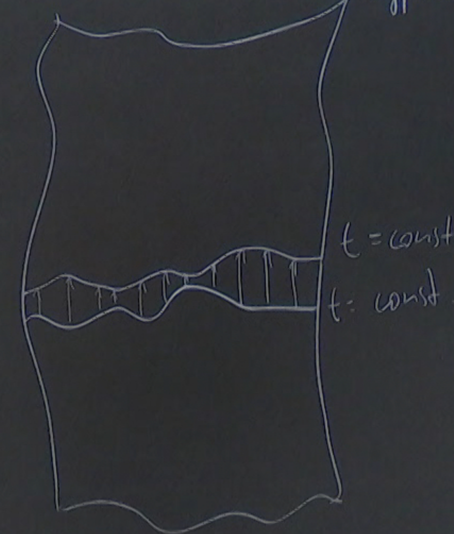
$$f(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \{f, H\}_r \quad ; \quad \{f, H\}_{r+1} = \{\{f, H\}_r, H\}$$

5.2. Canonical analysis of Palatini action

$$S = \frac{1}{2}\int e^{j}_{\mu}\, F_{\nu\rho j}\, \tilde{\epsilon}^{\mu\nu\rho}\, d^3x$$

$t = x^0, x^1, x^2$

$t = x^0 = \text{const}$ constant time hypersurface



$$S = \int \Big(e^j_b\, \partial_0 A_{aj}\, \tilde{\epsilon}^{ab} + e_{0j}\, \tfrac{1}{2} F^j_{ab}\, \tilde{\epsilon}^{ab} + A^j_0 \left(\partial_a e_{bj} + \epsilon_{jlm} A^l_a e^m_b \right) \tilde{\epsilon}^{ab} \Big)\, d^2x\, dt$$

(from integration by parts)

$$E^a_j := \frac{\partial \mathcal{L}}{\partial(\partial_0 A^j_a(x,t))} = \tilde{\epsilon}^{ab} e_{bj}$$

vector densities of weight one

$$\{ A^j_a(x), E^b_k(y) \} = \delta^b_a\, \delta^j_k\, \delta(x,y)$$

density

5.2. Canonical analysis of Palatini action

$t = x^0, x^1, x^2$

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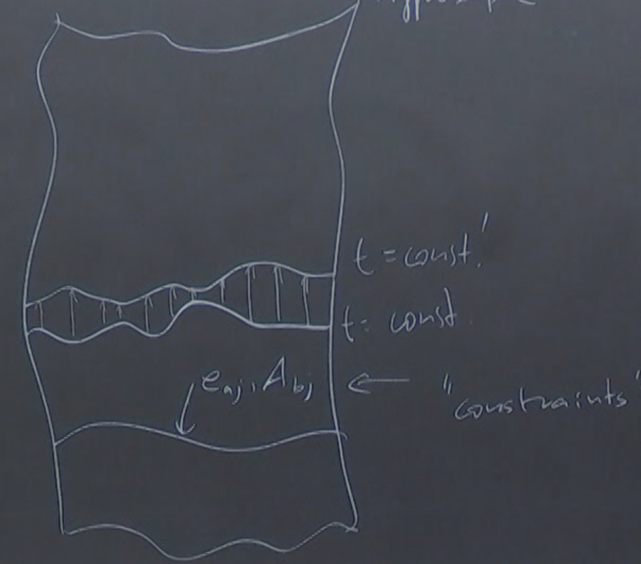


$$S = \frac{1}{2}\int e_\mu^j F_{\nu\rho j}\,\tilde{\epsilon}^{\mu\nu\rho}\, d^3x$$

$\nu,\mu = 0,1,2 \qquad a,b = 1,2$

$A_a^j := \omega_a^j,\; A_0^j = \omega_0^j \;;\; \tilde{\epsilon}^{ab} = \tilde{\epsilon}^{0ab}$

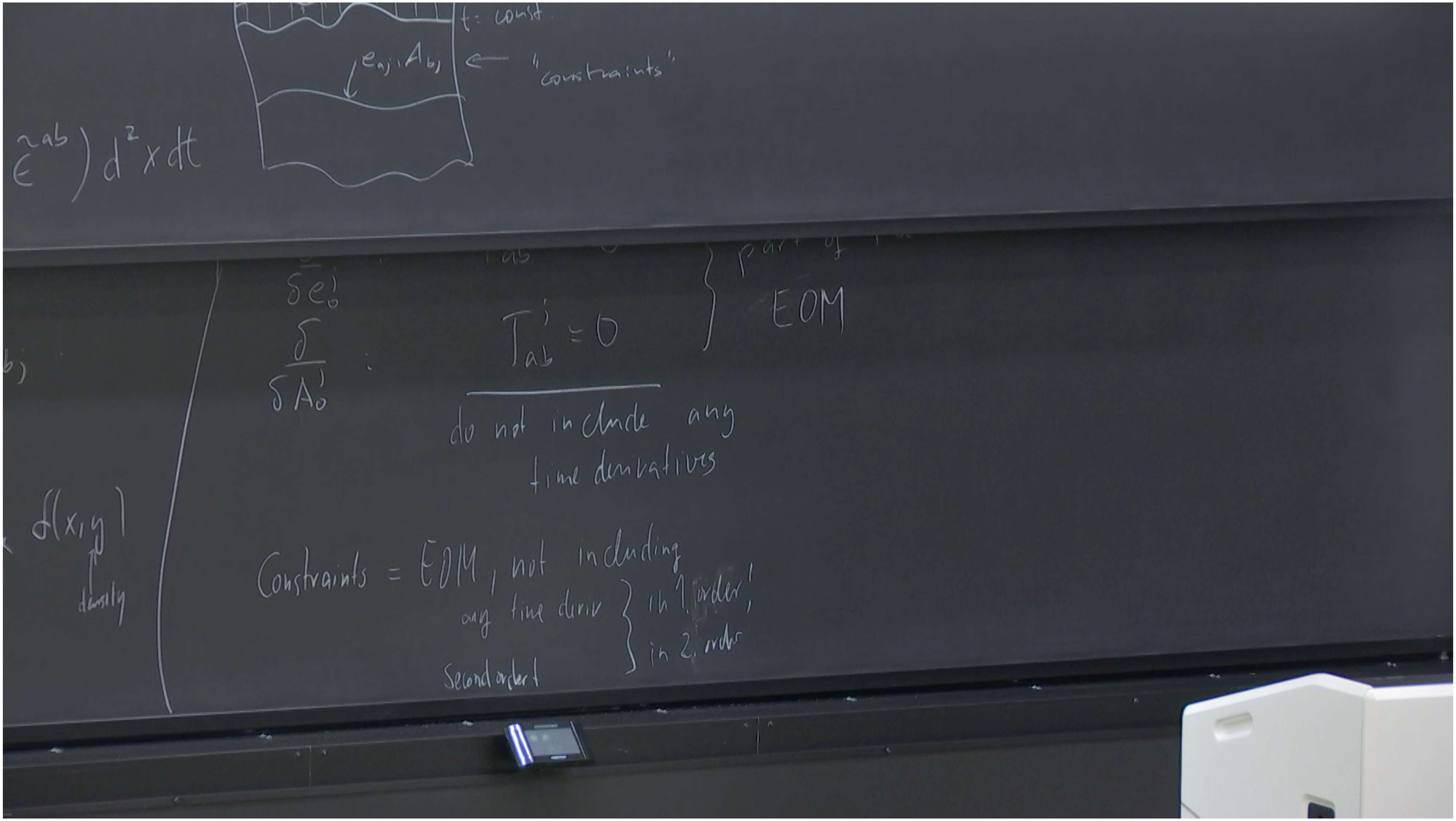
$$S = \int\Big(e_b^j\, \partial_0 A_{aj}\,\tilde{\epsilon}^{ab} + \boxed{e_{0j}}\,\frac{1}{2} F_{ab}^j\,\tilde{\epsilon}^{ab} + \boxed{A_0^j}\big(\partial_a e_{bj} + \epsilon_{jlm} A_a^l e_b^m\big)\tilde{\epsilon}^{ab}\Big)\, d^2x\, dt$$

(from integration by parts)

$$E_j^a := \frac{\partial \mathcal{L}}{\partial(\partial_0 A_a^j(x,t))} = \tilde{\epsilon}^{ab} e_{bj}$$

$\frac{\delta}{\delta e_0^j}$ ... F_{ab} ...

$\frac{\delta}{\delta A_0^j} : \quad T_{ab}^j = 0$



t = const
e_{aj}, A_{bj}
"constraints"
$\tilde{\epsilon}^{ab}) d^2x\,dt$
δe_0^i
$\frac{\delta}{\delta A_0^i}$: $T_{ab}^i = 0$
EOM
do not include any time derivatives
$d(x,y)$
density
Constraints = EOM, not including any time deriv
in 1. order!
in 2. order
Second order t