Title: PSI 2019/2020 - Quantum Gravity Part 1 - Lecture 3

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Collection: PSI 2019/2020 - Quantum Gravity Part 1

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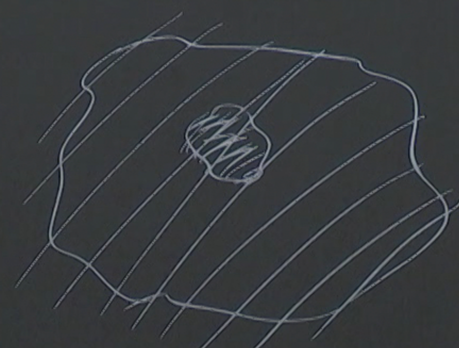
3 Global and gauge symmetries and

Gauge trafo
(local symmetries)



→ non-uniqueness of solutions (for fixed bdry-values) ies and Noether's theorems

$$S = \int \mathcal{L}(\phi^a, \partial_\mu \phi^a)\, d^n x$$

Symmetry trafo $\quad \delta_S \phi^\alpha$, s.t.

$$\delta_S S = \int (\delta_S \mathcal{L})\, d^n x \doteq \int \partial_\mu (\epsilon \lambda^\mu_S)\, d^n x$$

rems

$d^n x$

s.t. (Symmetry only acting on the fields)

$= \int \partial_\mu (\epsilon \lambda^\mu_s) \, d^n x$

and Noether's theorems

$$S = \int \mathcal{L}(\phi^a, \partial_\mu \phi^\alpha) \, d^n x$$

Symmetry trafo $\delta_s \phi^\alpha$, s.t. (Symmetry o

$$\delta_s S = \int (\delta_s \mathcal{L}) d^n x \doteq \int \partial_\mu (\epsilon \lambda^\mu_s) \, d^n x$$

$$= \int \left(\frac{\partial \mathcal{L}}{\partial \phi^\alpha} \delta_s \phi^\alpha + \frac{\partial \mathcal{L}}{\delta_s (\partial_\mu \phi^\alpha)} \partial_\mu \delta_s \phi^\alpha \right) d^n x$$

$$= \int \left(\frac{\partial \mathcal{L}}{\partial \phi^a} \delta_s \phi^a + \frac{\partial \mathcal{L}}{\delta_s (\partial_\mu \phi^a)} \partial_\mu \delta_s \phi^a \right) d^n x$$

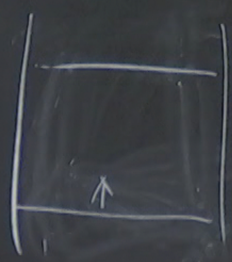
$$= \int d^n x \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \phi^a} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \right)}_{L_a \quad (\mathrm{EOM})} \delta_s \phi + \int d^n x \; \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^a)} \delta_s \phi^a \right)$$

$$0 = \int d^n x \; \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^a)} \delta_s \phi^a - \epsilon \lambda^\mu_s \right) + \int d^n x \, L_a \delta_s \phi^a$$

Generalized Bianchi - identities

Global sym.: $\delta_{glob}\,\phi^a = \underset{\substack{\uparrow \\ \text{constant}}}{\epsilon}\, B^a \quad , \quad L_a = 0$

$$j^\mu_{glob} := \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)} B^a - X^\mu_s \right)$$



$$Q_{glob} = \int d^{n-1}x \; j^0_{glob}$$

preserved under time evol.

Generalized Bianchi - Identities

l symm.: $\epsilon \neq 0$ in a compact region, $L_\alpha = 0$

$$\delta_{gauge} \phi^\alpha = \epsilon(x) B^\alpha + (\partial_\mu \epsilon(x)) C^{\alpha\mu}$$

Generalized Bianchi - identities

symm.: $\epsilon \neq 0$ in a compact region, $L_a = 0$

$$\delta_{gauge} \phi^a = \epsilon(x) B^a + (\partial_\mu \epsilon(x)) C^{a\mu}$$

$$0 = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)} \delta_{gauge} \phi^a \right) = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)} B^a \right) \epsilon$$

$$+ \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)} B^a \right) \partial_\mu \epsilon + \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi^a)} C^{a\mu} \right) \partial_\mu \epsilon$$

$$+ \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi^a)} C^{a\mu} \right) \partial_\nu \partial_\mu \epsilon$$

ϵ arbitrary:

$$\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)} B^a\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)} B^a + \partial_\nu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi^a)} C^{a\mu}\right)\Big) = 0$$

$C = 0$

$\Rightarrow$ constraint

$$\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi^a)} C^{a\mu} + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)} C^{a\nu}\right) = 0 \quad \Rightarrow$$

$(C \neq 0)$

constraints between ϕ, $\partial_0 \phi$

3.1. Symmetries of the first order gravity action

$$S = \frac{1}{2}\int e_{\sigma\ell}\, F^{\ell}_{\mu\nu}\, \tilde{\epsilon}^{\sigma\mu\nu}\, d^3x$$

Internal rotations:

$$e'^{i}_{\mu} = R^{i}{}_{j}\, e^{j}_{\mu} \quad , \quad R_{ij} = \delta_{ij} + \varepsilon\, \epsilon_{ijk}\, \Lambda^{k}$$

$$\delta^{R}_{\Lambda} e^{i}_{\mu} = \frac{d}{d\varepsilon} e'^{i}_{\mu} = \epsilon^{i}{}_{kl}\, e^{k}_{\mu}\, \Lambda^{l}$$

$$\delta^R_\lambda \, \omega^m_\mu = \partial_\mu \lambda^m + \epsilon^m{}_{li} \, \omega^l_\mu \, \Lambda^i$$

$$= D_\mu \Lambda^m$$