Title: PSI 2019/2020 - Quantum Gravity Part 1 - Lecture 1

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Collection: PSI 2019/2020 - Quantum Gravity Part 1

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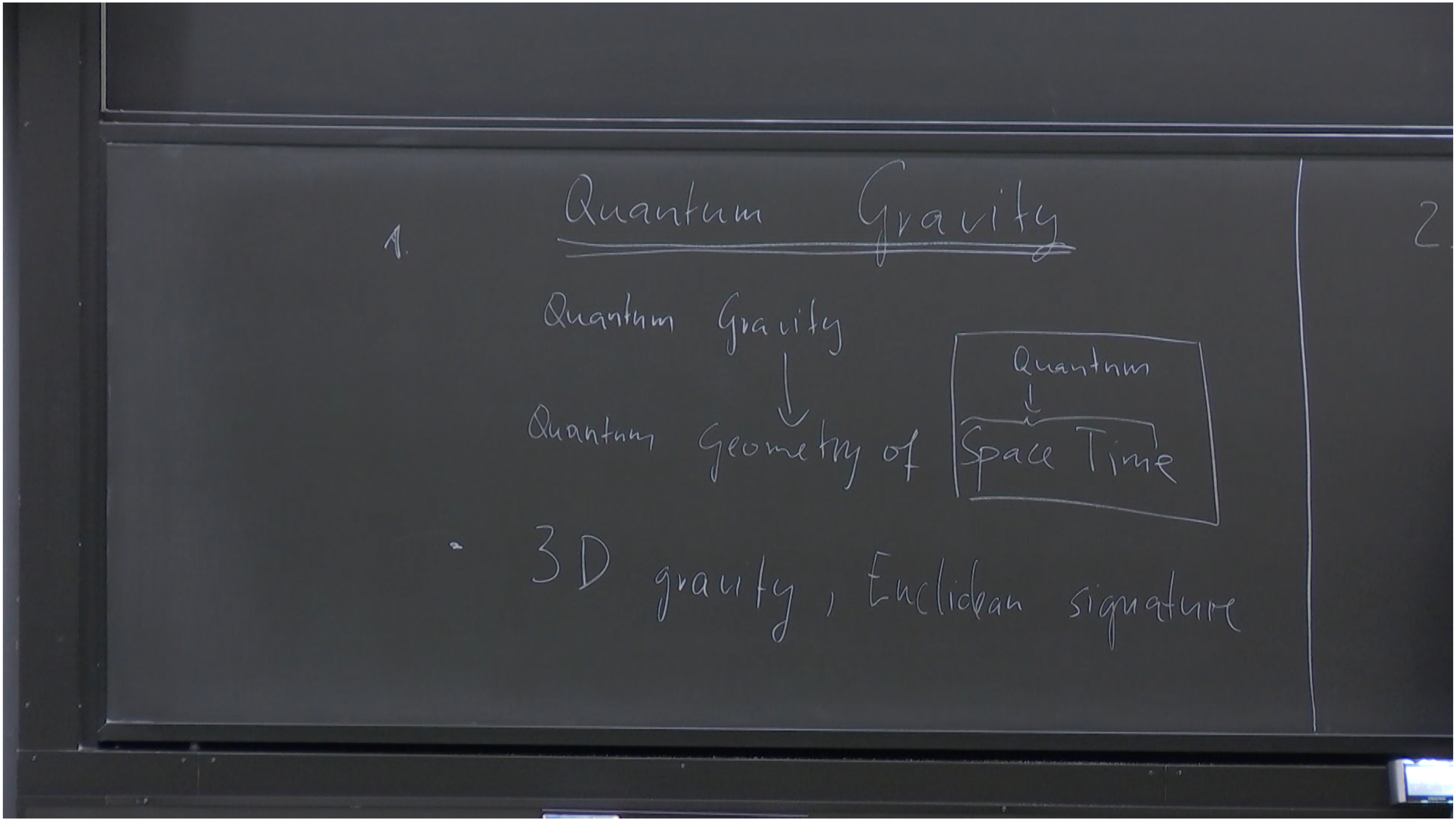

1.
Quantum Gravity
Quantum Gravity
Quantum Geometry of Space Time
Quantum
3D gravity, Euclidean signature
2

# 2. Actions for Gravity

## Einstein-Hilbert action

$$S = \frac{1}{\kappa} \int \overbrace{R}^{= R_{\mu\nu} g^{\mu\nu}} \sqrt{g}\, d^4 x \qquad \kappa = 1$$

- basic variable $g^{\mu\nu}$

$$\delta S = \int \left( R_{\mu\nu} (\delta g^{\mu\nu}) \sqrt{g} + (\delta R_{\mu\nu}) g^{\mu\nu} \sqrt{g} + R\, \delta\sqrt{g} \right) d^4 x$$

$$\delta S = \int_M \left( R_{\mu\nu}(\delta g^{\mu\nu})\sqrt{g} + (\delta R_{\mu\nu}) g^{\mu\nu}\sqrt{g} + R\,\delta\sqrt{g} \right) d^4x$$

(a) (b)

$$(\delta R_{\mu\nu}) g^{\mu\nu} \underset{\substack{\uparrow \\ \text{Palatini} \\ \text{equations}}}{=} \nabla^\mu \left( \nabla^\nu \delta g_{\mu\nu} - g^{\nu\rho} \nabla_\mu \delta g_{\nu\rho} \right)$$

$\longrightarrow$ integrate to zero

(b) $$\delta\sqrt{g} = \frac{1}{2}\frac{1}{\sqrt{g}}\delta g = -\frac{1}{2}\frac{1}{\sqrt{g}} g\, g_{\mu\nu}\delta g^{\mu\nu}$$

$$\delta S = \int \left[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right] \delta g^{\mu\nu} \sqrt{g}\, d^4x \Rightarrow$$

$$R_{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 0$$

$$\Rightarrow \boxed{R_{\mu\nu} = 0}$$

## 2.1. 3D gravity

$$S = \int R \sqrt{g}\, d^3x$$

$$\underbrace{R_{\mu\nu}}_{S} = 0 \quad \underset{3D}{\Rightarrow} \quad R_{\underbrace{\mu\nu}_{AS}\underbrace{\sigma\rho}_{AS}} = 0$$

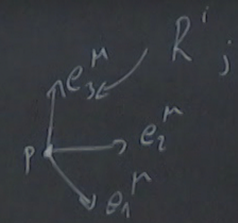
3D: $6$ $\qquad$ $S$, $6$

$$R_{\mu\nu\sigma\rho} = \left(-R_{\mu\nu}\, g_{\rho\sigma}\right) + \left(R\, g_{\mu\nu} g_{\rho\sigma}\right)$$

3D : 6    $\frac{5}{6}$

## 2.2 (Triad) Palatini action

### Triads




$g_{\mu\nu} \longrightarrow e^\mu_i$    $i = 1,2,3$

DOF: 6    9

$R^k{}_l$

Require: $(g_{\mu\nu} e^\mu_i) e^\nu_j = e_{\nu i} e^\nu_j = \delta_{ij}$

$e^i_\nu = e^\mu_j g_{\mu\nu} \delta^{ij}$ ;    $\boxed{e^i_\mu e_{\nu j} = g_{\mu\nu}}$

## Spin connection

$$\nabla_\rho v^\mu = \Gamma^{\mu}_{\rho\sigma} v^\sigma + \partial_\rho v^\mu$$

$$D_\rho \phi^j = \omega_{\rho k}^{\ \ j} \phi^k + \partial_\rho \phi^j$$

$$\nabla_\rho(\phi^j \lambda_j) = \partial_\rho(\phi^j \lambda_j) \quad \Rightarrow \quad D_\rho \lambda_j = \partial_\rho \lambda_j - \omega_{\mu j}^{\ \ k} \phi_k$$

$$D_\rho v_\mu^{\ j} = \partial_\rho v_\mu^{\ j} - \Gamma^{\sigma}_{\rho\mu} v_\sigma^{\ j} + \omega_{\rho k}^{\ \ j} v_\mu^{\ k}$$

Condition: $e_\mu^{\ i}, e_i^{\ \mu}$ translate between tangent space and internal space

$$\boxed{D_\mu e_\nu^{\ j} = 0} \Rightarrow \partial_\mu e_\nu^{\ j} - \Gamma^{\rho}_{\mu\nu} e_\rho^{\ j} + \omega_{\mu k}^{\ \ j} e^{k}_{\ \nu} = 0$$

$$\omega_{\mu k}^{\ \ j} = \boxed{-e_k^{\ \nu} \nabla_\mu e_\nu^{\ j}} = e_k^{\ \nu} \Gamma^{\rho}_{\mu\nu} e_\rho^{\ j} - e_k^{\ \nu} \partial_\mu e_\nu^{\ j}$$