Title: PSI 2019/2020 - Relativistic Quantum Information Part 1 - Lecture 3

Speakers: Eduardo Martin-Martinez

Collection: PSI 2019/2020 - Relativistic Quantum Information Part 1

Date: March 09, 2020 - 11:30 AM

URL: http://pirsa.org/20030045

# Particle detectors: Covariant formulation

QM Recipe: $\tau\ \hat{H}_{0,d} = \Omega\, \hat{\sigma}^{+} \hat{\sigma}^{-}$ $\qquad |e\rangle,\ \Omega,\ |g\rangle$

Problem: Hamiltonian Formulation

$$t\ \hat{H}_{0,\phi} = \int_{\mathbb{R}^n} d^n k\ \omega_k\, \hat{a}_k^{\dagger} \hat{a}_k$$

$$\tau\ \hat{H}_I = \chi \cdot \hat{\mu} \cdot \hat{\phi}$$

$(t, \vec{x}) \equiv$ quantization frame

$(\tau, \xi) \equiv$ detector c.o.m frame

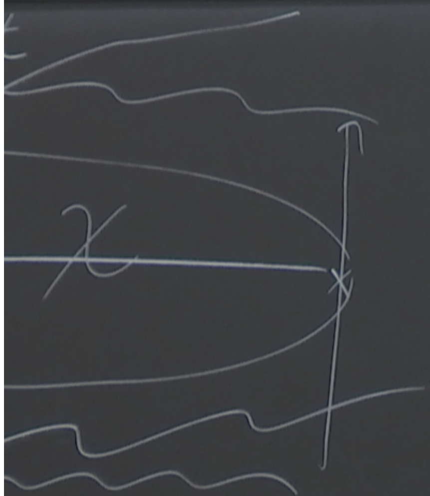
Give
res
how
one q
$i \frac{d}{dt} |$
$\Rightarrow$

Given a Hamiltonian ${}^{t}\hat{H}$ that generates time evolution with respect to $t$, and a time reparametrization $t = t(\tau)$ how does $\hat{H}$ change, ${}^{t}\hat{H}(t) \to {}^{t}\hat{H}(t(\tau))$

one quick way to see it:

$$i\frac{d}{dt}|\psi\rangle = {}^{t}\hat{H}(t)\,|\psi\rangle \Rightarrow i\,\frac{d\tau}{dt}\frac{d}{d\tau}|\psi\rangle = {}^{t}\hat{H}(t(\tau))\,|\psi\rangle \Rightarrow$$

$$\Rightarrow i\frac{d}{d\tau}|\psi\rangle = \frac{dt}{d\tau}\,{}^{t}\hat{H}(t(\tau)) \Rightarrow \boxed{{}^{\tau}\hat{H}(\tau) = \frac{dt}{d\tau}\,{}^{t}\hat{H}(t(\tau))}$$



$$\hat{U} = \mathcal{T} \exp\left(-i\int_{\mathbb{R}} d\tau\, {}^{\tau}H_I(\tau)\right) = \mathcal{T} \exp\left(-i\int_{\mathbb{R}} dt\, \frac{d\tau}{dt}\, {}^{t}H_I(t)\right)$$

$${}^{\tau}H_I = \lambda\, \chi(\tau) \int_{\mathbb{R}^n} d^n\xi\; F(\vec{\xi})\, \hat{\mu}(\tau)\, \hat{\phi}\big(t(\tau,\vec{\xi}), \vec{x}(\tau,\vec{\xi})\big)$$

$(\tau, \vec{\xi}) \rightarrow$ c.o.m. detector

$\vec{\xi} = \vec{0}$ traj on the c.o.m, $\vec{x}(t)$ traj on the lab frame

$$\int_{\mathbb{R}} dt \, \frac{dz}{dt} \, {}^{t}H_I(t) \Big)$$

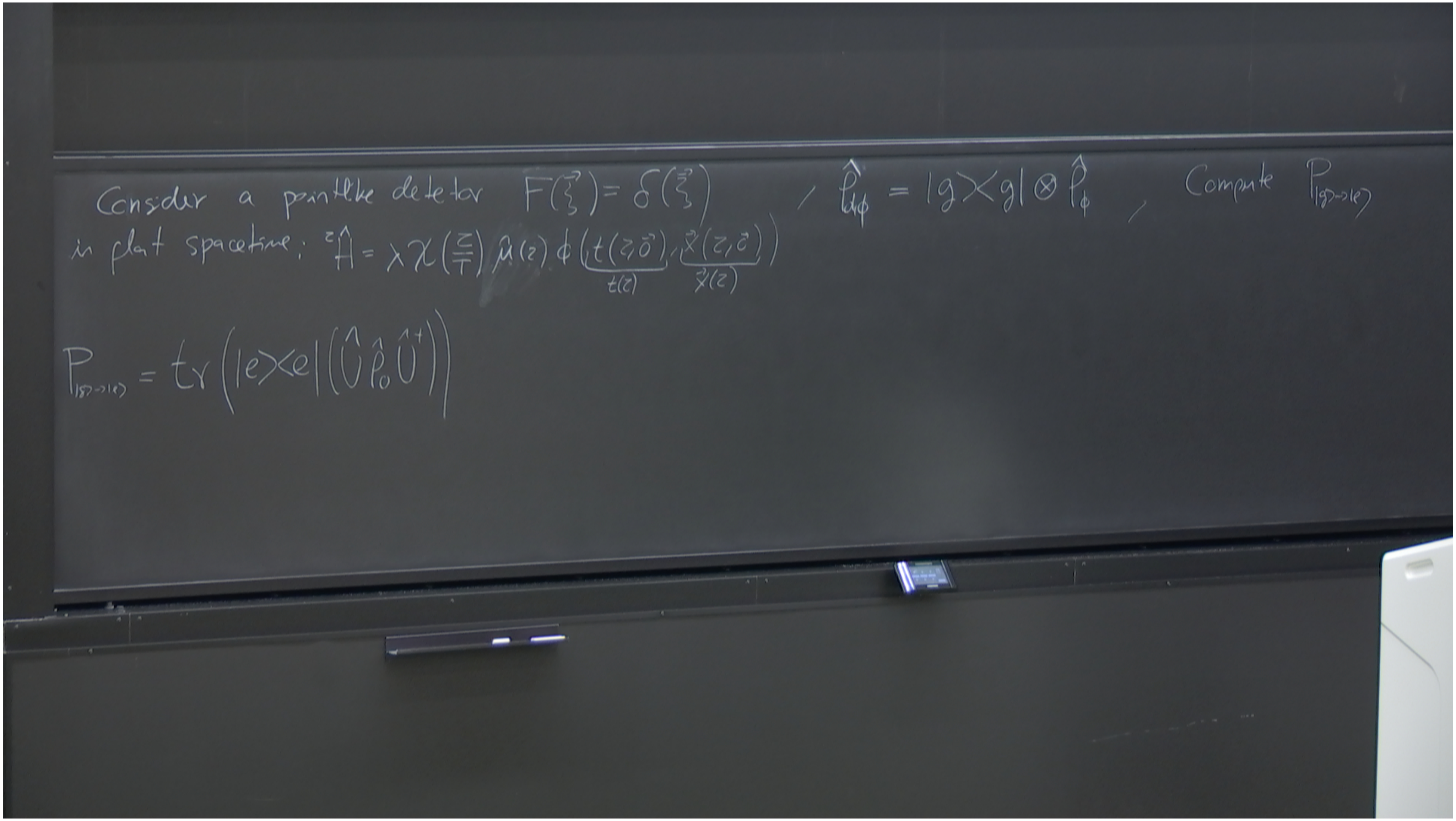
$$(z, \vec{\xi}), \vec{x}(z, \vec{\xi}) \Big)$$

$$\hat{U} = \mathcal{T} \exp\left( -i \int_{\mathbb{R}} dz \int_{\mathbb{R}^n} d^n\xi \, \boxed{\sqrt{-g} \, {}^{z}\hat{h}(z, \vec{\xi})} \right)$$

$$ {}^{z}\hat{h}(z, \vec{\xi}) = \lambda \chi(z) F(\vec{\xi}) \hat{\mu}(z) \phi(t(z, \vec{\xi}), \vec{x}(z, \vec{\xi}))$$

$$ {}^{t'}\hat{H} = \lambda \int d^n x' \sqrt{-g} \, \chi(z(t', \vec{x}')) F(\vec{\xi}(t', \vec{x}')) \hat{\mu}(z(t', \vec{x}')) \phi(t(t', \vec{x}'), \vec{x}(t', \vec{x}'))$$

in an arbitrary frame $(t', \vec{x}')$


Consider a pointlike detector $F(\vec{\xi}) = \delta(\vec{\xi})$, $\hat{\rho}_{d\phi} = |g\rangle\langle g| \otimes \hat{\rho}_\phi$, Compute $P_{|g\rangle\to|e\rangle}$
in flat spacetime: ${}^{\tau}\hat{H} = \lambda \chi\left(\frac{\tau}{T}\right) \hat{\mu}(\tau)\, \phi\big(\underbrace{t(\tau,\vec{0})}_{t(\tau)}, \underbrace{\vec{x}(\tau,\vec{0})}_{\vec{x}(\tau)}\big)$
$P_{|g\rangle\to|e\rangle} = \mathrm{tr}\left(|e\rangle\langle e| \left(\hat{U} \hat{\rho}_0 \hat{U}^\dagger\right)\right)$

$\hat{\rho}_{d,\phi} = |g\rangle\langle g| \otimes \hat{\rho}_{\phi}$

Compute $P_{|g\rangle \to |e\rangle}$

Consider a pointlike detector $F(\vec{\xi}) = \delta(\vec{\xi})$

in flat spacetime: ${}^{\tau}\hat{H} = \lambda \chi\left(\frac{\tau}{T}\right) \hat{\mu}(\tau)\, \phi\Big(\underbrace{t(\tau,\vec{0})}_{t(\tau)}, \underbrace{\vec{x}(\tau,\vec{0})}_{\vec{x}(\tau)}\Big)$

$$P_{|g\rangle \to |e\rangle} = \mathrm{tr}\left(|e\rangle\langle e| \left(\hat{U}\hat{\rho}_0 \hat{U}^{\dagger}\right)\right)$$

$${}^{\tau}\hat{H}_I = \chi \cdot \hat{\mu} \cdot \hat{\phi}$$

$(t, \vec{x}) \equiv$ quantization frame

$(\tau, \vec{\xi}) \equiv$ detector c.o.m frame

One quick way to see it:

$$i\frac{d}{dt}|\psi\rangle = {}^{t}\hat{H}(t)|\psi\rangle \Rightarrow i\frac{d\tau}{dt}\frac{d}{d\tau}|\psi\rangle = {}^{t}\hat{H}(t(\tau))|\psi\rangle \Rightarrow$$

$$\Rightarrow i\frac{d}{d\tau}|\psi\rangle = \frac{dt}{d\tau}\,{}^{t}\hat{H}(t(\tau)) \Rightarrow \boxed{{}^{\tau}\hat{H}(\tau) = \frac{dt}{d\tau}\,{}^{t}\hat{H}(t(\tau))}$$

$$\hat{\rho}_0, \qquad \hat{\rho}_T = \hat{U}\hat{\rho}_0\hat{U} = \hat{\rho}_0 + \hat{U}^{(1)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(1)\dagger} + \underline{\hat{U}^{(1)}\hat{\rho}_0\hat{U}^{(1)\dagger}} + \hat{U}^{(2)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(2)\dagger} + \mathcal{O}(\chi^3)$$

$\sigma^+\sigma^-$

$\sigma^-\sigma^+$

$$\hat{U}^{(1)} = -i\int_{-\infty}^{\infty} dt\, {}^{\tau}\hat{H}_I(\tau)$$

Consider a pointlike detector $F(\vec{\xi}) = \delta(\vec{\xi})$ , $\hat{\rho}_{d\phi} = |g\rangle\langle g| \otimes \hat{\rho}_\phi$ , Compute

in flat spacetime: ${}^{\tau}\hat{H} = \lambda\, \chi\!\left(\frac{\tau}{T}\right) \hat{\mu}(\tau)\, \phi\big(\underbrace{t(\tau,\vec{0})}_{t(\tau)}, \underbrace{\vec{x}(\tau,\vec{0})}_{\vec{x}(\tau)}\big)$

$$P_{|g\rangle\to|e\rangle} = \mathrm{tr}\Big(|e\rangle\langle e| \big(\hat{U}\hat{\rho}_0\hat{U}^\dagger\big)\Big) = \mathrm{tr}_\phi\Big(\langle e|\hat{U}^{(1)}\hat{\rho}_0\hat{U}^{(1)\dagger}|e\rangle\Big) =$$

$$= \lambda^2 \int d\tau \int d\tau'\, \chi\!\left(\frac{\tau}{T}\right)\chi\!\left(\frac{\tau'}{T}\right) \langle e|\hat{\mu}(\tau)|g\rangle\langle g|\hat{\mu}(\tau')|e\rangle\, \underbrace{\mathrm{tr}_\phi\Big(\hat{\rho}_\phi\, \phi(t(\tau'),\vec{x}(\tau'))\, \phi(t(\tau),\vec{x}(\tau))\Big)}_{\mathrm{tr}_\phi\left(\phi(t(\tau),\vec{x}(\tau))\, \rho\, \phi(t(\tau'),\vec{x}(\tau'))\right)}$$

detector c.o.m frame

$$\Rightarrow i\frac{d}{d\tau}|\psi\rangle = \left(\frac{dt}{d\tau}\,{}^{t}\hat{H}(t(\tau))\right) \Rightarrow \boxed{{}^{\tau}\hat{H}(\tau) = \frac{dt}{d\tau}\,{}^{t}\hat{H}(t(\tau))}$$

$$\hat{\rho} = \hat{\rho}_0 + \hat{U}^{(1)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(1)\dagger} + \underline{\hat{U}^{(1)}\hat{\rho}_0\hat{U}^{(1)\dagger}} + \hat{U}^{(2)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(2)\dagger} + \mathcal{O}(\lambda^3)$$

$\sigma^+\sigma^-$

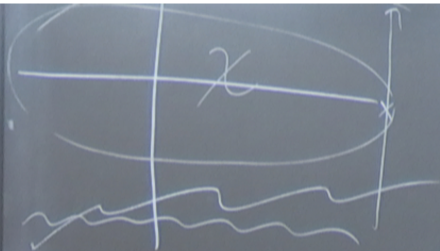
$\sigma^-\sigma^+$

$$\hat{U}^{(1)} = -i\int_{-\infty}^{\infty} dt\,{}^{\tau}H_I(\tau)$$

$$\hat{\mu} = \left(\sigma^+ e^{i\Omega t} + \sigma^- e^{-i\Omega t}\right)$$

$\sigma^+\sigma^-$

$\sigma^-\sigma^+$



$$\ldots\exp\left(-i\int_{\mathbb{R}} dz\, H_z\right) = \mathcal{T}\exp\left(-i\int_{\mathbb{R}} dt\, \frac{dz}{dt} H_I(t)\right)$$

$${}^{z}H_I = \lambda\, \chi(z)\int_{\mathbb{R}^n} d^n\xi\;\, F(\vec\xi)\,\hat\mu(z)\,\hat\phi\big(t(z,\vec\xi),\, \vec x(z,\vec\xi)\big)$$

$(z,\vec\xi) \to$ c.o.m detector

$\vec\xi=\vec 0$ traj on the c.o.m, $\vec x(t)$ traj on the lab frame

$$\hat U = \mathcal{T}\exp\left(-i\int_{\mathbb{R}} dz\int_{\mathbb{R}^n} d^n\xi\,\sqrt{-g}\;\hat h(z,\ldots\right.$$

$$\hat h(z,\vec\xi) = \lambda\chi(z)\,F(\vec\xi)\,\hat\mu(z)\,\phi\big(t(z,\vec\xi),\vec x(z,\ldots$$

$${}^{t'}\hat H = \lambda\int d^n x'\sqrt{-g}\;\chi(z(t',\vec x'))\,F(\vec\xi(t',\vec x'))\,\hat\mu(z(t',\vec x'))\,\phi\big(t(t',\vec x'),\vec x(t',\vec x')\big)$$

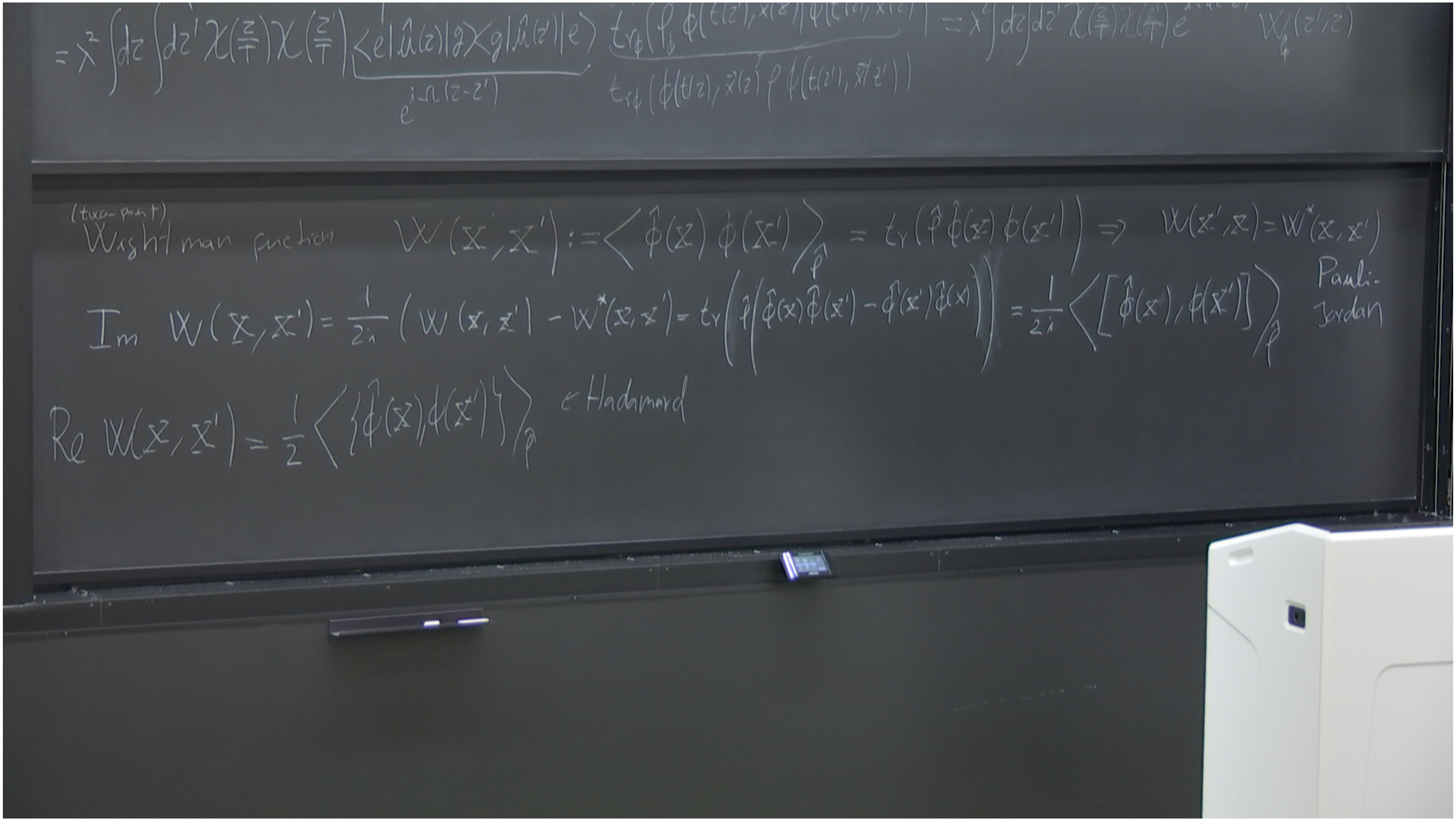
in an arbitrary frame $(t',\vec x')$

in flat spacetime: ${}^{z}\hat H = \lambda\chi\left(\frac{z}{T}\right)\hat\mu(z)\,\phi\big(\underbrace{t(z,\vec 0)}_{t(z)},\underbrace{\vec x(z,\vec 0)}_{\vec x(z)}\big)$

$$P_{|g\rangle\to|e\rangle} = \mathrm{tr}\left(|e\rangle\langle e|\left(\hat U\hat\rho_0\hat U^\dagger\right)\right) = \mathrm{tr}_\phi\left(\langle e|\hat U^{(1)}\hat\rho_0\hat U^{(1)\dagger}|e\rangle\right) =$$

$$= \lambda^2\int dz\int dz'\,\chi\left(\frac{z}{T}\right)\chi\left(\frac{z'}{T}\right)\underbrace{\langle e|\hat\mu(z)|g\rangle\langle g|\hat\mu(z')|e\rangle}_{e^{i\Omega(z-z')}}\;\underbrace{\mathrm{tr}_\phi\left(\hat\rho_\phi\,\phi(t(z'),\vec x(z'))\,\phi(t(z),\vec x(z))\right)}_{\mathrm{tr}_\phi\left(\phi(t(z),\vec x(z))\,\rho\,\phi(t(z'),\vec x(z'))\right)} = \lambda^2\int dz\int dz'\,\chi\left(\frac{z}{T}\right)\chi\left(\frac{z'}{T}\right)e^{i\Omega(z-z')}\,W_{\hat\rho_\phi}(z',z)$$

(two-point) Wightman function $W(x,x') := \left\langle\hat\phi(x)\,\phi(x')\right\rangle_{\hat\rho} = \mathrm{tr}\left(\hat\rho\,\hat\phi(x)\,\phi(x')\right) \Rightarrow W(x',x) = W^*(x,x')$


$= \lambda^2 \int dz \int dz' \chi(\frac{z}{T})\chi(\frac{z'}{T}) \underbrace{\langle e|\hat{\mu}(z)|g\rangle\langle g|\hat{\mu}(z')|e\rangle}_{e^{i\Omega(z-z')}} \underbrace{\mathrm{tr}_\phi(\rho_\phi \phi(t(z'),x(z'))\phi(t(z),x(z)))}_{\mathrm{tr}_\phi(\phi(t(z),x(z))\rho\phi(t(z'),x(z')))} = \lambda^2 \int dz \int dz' \chi(\frac{z}{T})\chi(\frac{z'}{T}) e^{\dots} W_\phi(z',z)$
(two-point)
Wightman function $W(x,x') := \langle \hat{\phi}(x)\phi(x')\rangle_{\hat{\rho}} = \mathrm{tr}(\hat{\rho}\hat{\phi}(x)\phi(x')) \Rightarrow W(x',x) = W^*(x,x')$
$\mathrm{Im}\, W(x,x') = \frac{1}{2i}(W(x,x') - W^*(x,x')) = \mathrm{tr}(\hat{\rho}(\hat{\phi}(x)\hat{\phi}(x') - \hat{\phi}(x')\hat{\phi}(x))) = \frac{1}{2i}\langle[\hat{\phi}(x),\phi(x')]\rangle_\rho$
Pauli-Jordan
$\mathrm{Re}\, W(x,x') = \frac{1}{2}\langle\{\hat{\phi}(x),\phi(x')\}\rangle_\rho$
← Hadamard

$$\ldots \hat{\rho}_0 \hat{U}^{(1)\dagger} + \underline{\hat{U}^{(1)} \hat{\rho}_0 \hat{U}^{(1)\dagger}} + \hat{U}^{(2)} \hat{\rho}_0 + \hat{\rho}_0 \hat{U}^{(2)\dagger} + \mathcal{O}(\lambda^3)$$

$\sigma^+\sigma^-$

$\sigma^-\sigma^+$

$$\hat{U}^{(1)} = -i \int_{-\infty}^{\infty} dt\, \hat{H}_I(\tau)$$

$$\hat{\mu} = \left(\sigma^+ e^{i\Omega t} + \sigma^- e^{-i\Omega t}\right)$$

$$\rho = \frac{1}{Z} e^{-\beta \hat{H}}$$

$\sigma^+\sigma^-$

$\sigma^-\sigma^+$