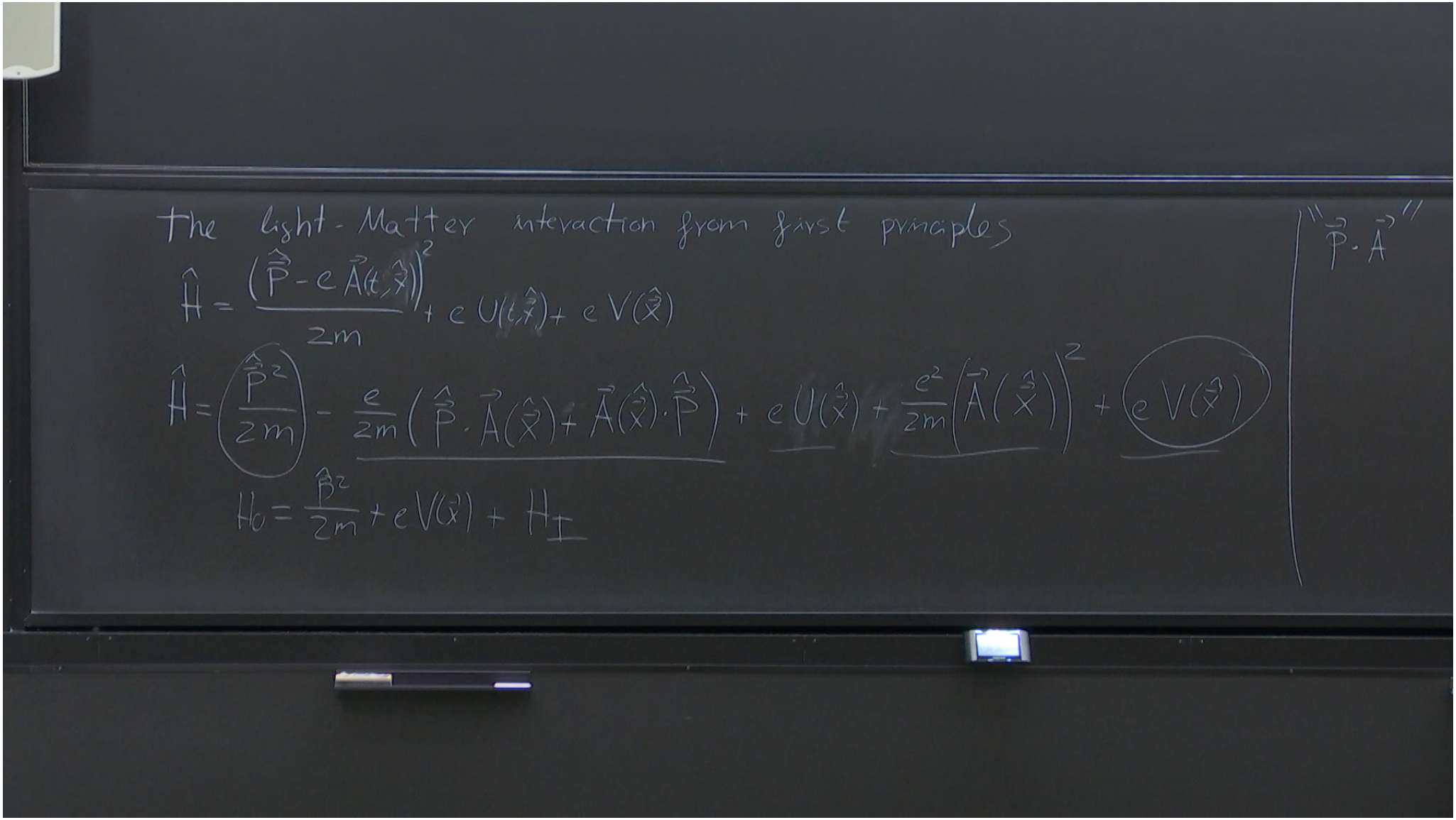
Title: PSI 2019/2020 - Relativistic Quantum Information Part 1 - Lecture 2

Speakers: Eduardo Martin-Martinez

Collection: PSI 2019/2020 - Relativistic Quantum Information Part 1

Date: March 04, 2020 - 9:00 AM

URL: http://pirsa.org/20030044

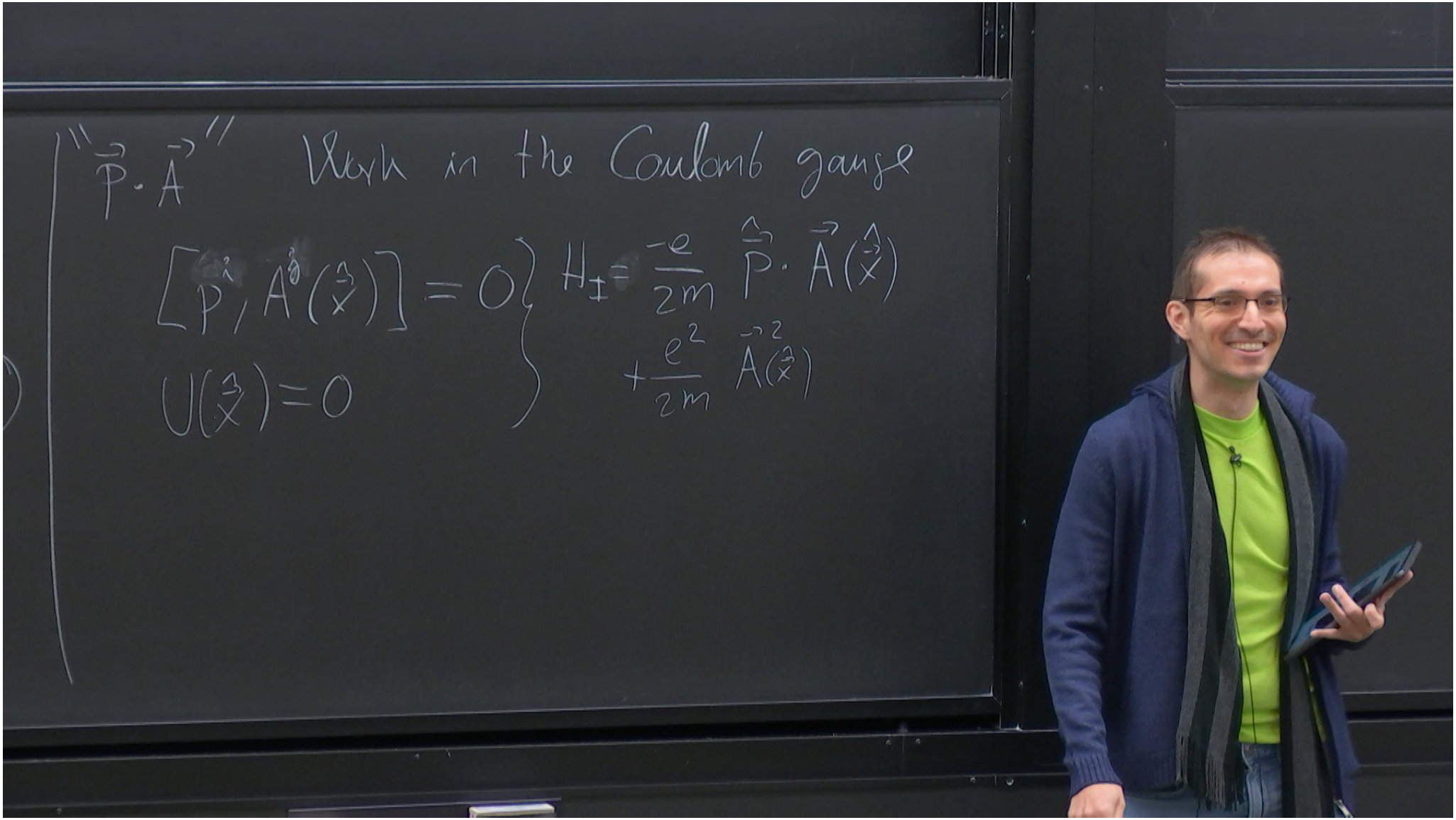

the light-Matter interaction from first principles
$\hat{H} = \frac{(\hat{\vec{p}} - e\vec{A}(t,\hat{\vec{x}}))^2}{2m} + eU(t,\hat{\vec{x}}) + eV(\hat{\vec{x}})$
$\hat{H} = \frac{\hat{\vec{p}}^2}{2m} - \frac{e}{2m}(\hat{\vec{p}}\cdot\vec{A}(\hat{\vec{x}}) + \vec{A}(\hat{\vec{x}})\cdot\hat{\vec{p}}) + eU(\hat{\vec{x}}) + \frac{e^2}{2m}\vec{A}(\hat{\vec{x}})^2 + eV(\hat{\vec{x}})$
$H_0 = \frac{\hat{p}^2}{2m} + eV(\hat{x}) + H_I$
"$\vec{p}\cdot\vec{A}$"

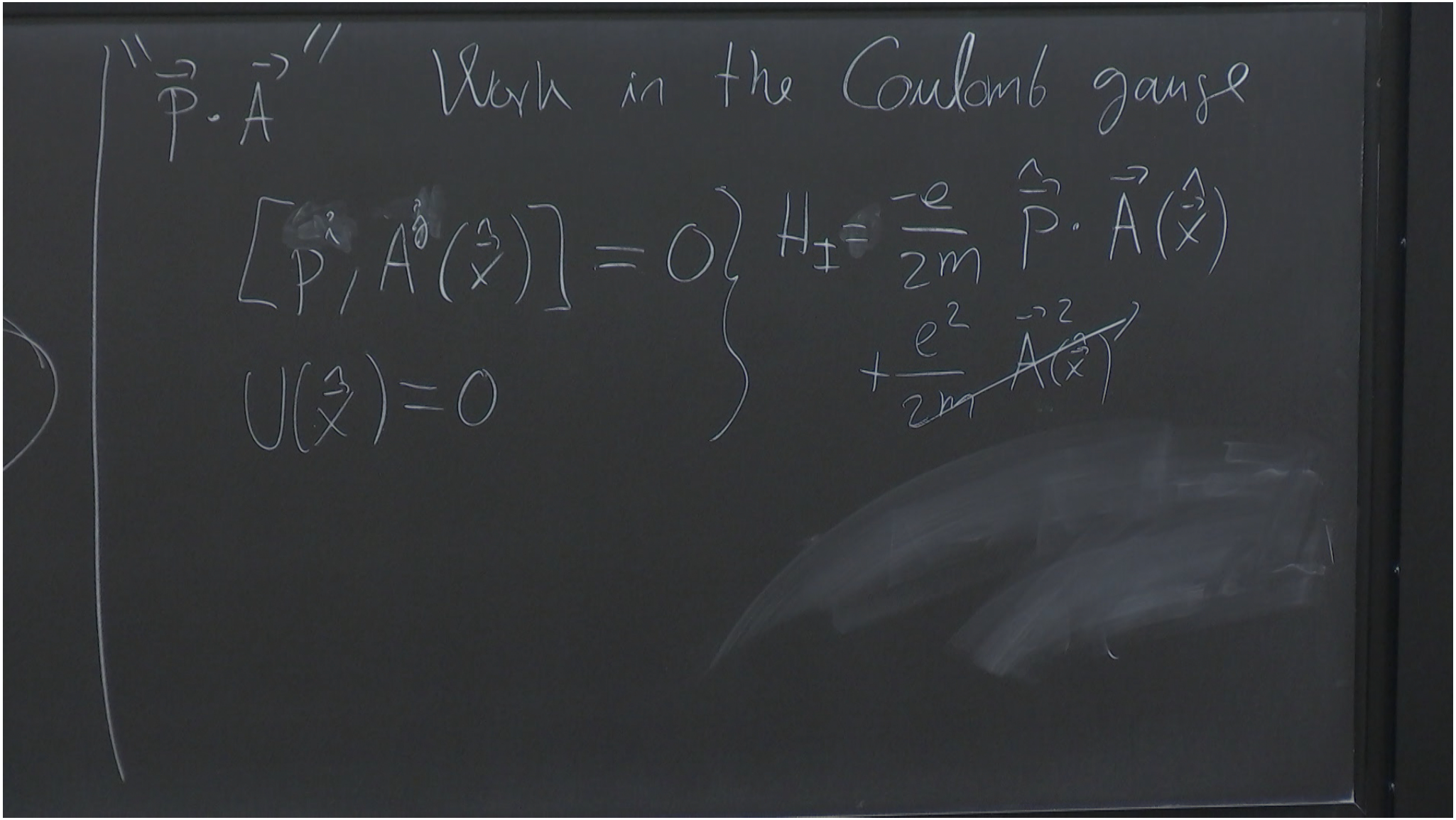
The light-Matter interaction from first principles

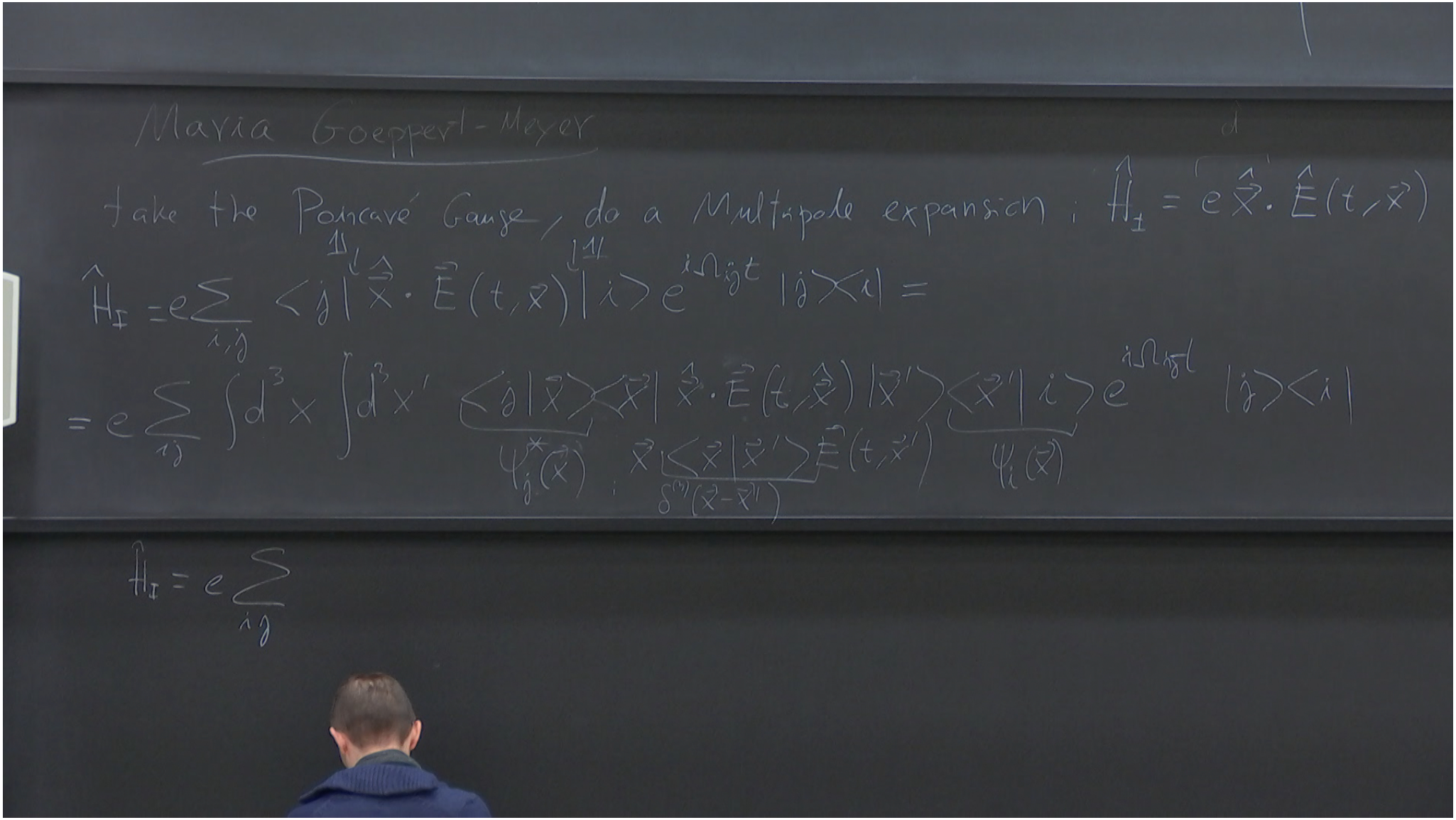
$$\hat{H} = \frac{\left(\hat{\vec{p}} - e\vec{A}(t,\hat{\vec{x}})\right)^2}{2m} + eU(t,\hat{\vec{x}}) + eV(\hat{\vec{x}})$$

$$\hat{H} = \left(\frac{\hat{\vec{p}}^2}{2m}\right) - \frac{e}{2m}\left(\hat{\vec{p}}\cdot\vec{A}(\hat{\vec{x}}) + \vec{A}(\hat{\vec{x}})\cdot\hat{\vec{p}}\right) + eU(\hat{\vec{x}}) + \frac{e^2}{2m}\left(\vec{A}(\hat{\vec{x}})\right)^2 + eV(\hat{\vec{x}})$$

$$H_0 = \frac{\hat{p}^2}{2m} + eV(\hat{x}) + H_I$$


"$\vec{P}\cdot\vec{A}$" Work in the Coulomb gauge
$[P^i, A^j(\vec{x})] = 0$
$U(\vec{x}) = 0$
$H_I = \frac{-e}{2m}\vec{P}\cdot\vec{A}(\vec{x}) + \frac{e^2}{2m}\vec{A}^2(\vec{x})$


"$\vec{P}\cdot\vec{A}$"
Work in the Coulomb gauge
$[\hat{P}^i, A^j(\vec{x})] = 0$
$U(\vec{x}) = 0$
$H_I = \frac{-e}{2m}\hat{\vec{P}}\cdot\vec{A}(\hat{\vec{x}})$
$+\frac{e^2}{2m}\vec{A}^2(\hat{\vec{x}})$


Maria Goeppert-Mayer
take the Poincaré Gauge, do a Multipole expansion: $\hat{H}_I = e\hat{\vec{x}}\cdot\hat{\vec{E}}(t,\vec{x})$
$\hat{H}_I = e\sum_{i,j} \langle j|\hat{\vec{x}}\cdot\vec{E}(t,\vec{x})|i\rangle e^{i\Omega_{ij}t}|j\rangle\langle i| =$
$= e\sum_{ij}\int d^3x \int d^3x' \underbrace{\langle j|\vec{x}\rangle}_{\psi_j^*(\vec{x})}\langle\vec{x}|\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}})|\vec{x}'\rangle\underbrace{\langle\vec{x}'|i\rangle}_{\psi_i(\vec{x})} e^{i\Omega_{ij}t}|j\rangle\langle i|$
$\vec{x}\underbrace{\langle\vec{x}|\vec{x}'\rangle}_{\delta^{(3)}(\vec{x}-\vec{x}')}\vec{E}(t,\vec{x}')$
$\hat{H}_I = e\sum_{ij}$

$$\hat{H}_I = e\sum_{i,j} \langle j|\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}})|i\rangle e^{i\Omega_{ij}t}\,|j\rangle\langle i| =$$

$$= e\sum_{ij}\int d^3x \int d^3x' \underbrace{\langle j|\vec{x}\rangle}_{\psi_j^*(\vec{x})}\langle\vec{x}|\,\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}})\,|\vec{x}'\rangle \underbrace{\langle\vec{x}'|i\rangle}_{\psi_i(\vec{x})} e^{i\Omega_{ij}t}\,|j\rangle\langle i|$$

$$\vec{x}\underbrace{\langle\vec{x}|\vec{x}'\rangle}_{\delta^{(3)}(\vec{x}-\vec{x}')}\vec{E}(t,\vec{x}')$$

$$\hat{H}_I = e\sum_{ij}\int d^3x\, \underbrace{\psi_j^*(\vec{x})\,\vec{x}\,\psi_i(\vec{x})}_{\vec{F}_{ij}(\vec{x})\ \leftarrow \text{smearing vector}}\cdot\vec{E}(t,\vec{x})\,e^{i\Omega_{ij}t}\,|j\rangle\langle i|$$

$$\hat{\vec{d}} := e\sum_{ij}\vec{F}_{ij}(\vec{x})\,e^{i\Omega_{ij}t}\,|j\rangle\langle i| \quad\Rightarrow\quad \hat{H}_I = e\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}}) = \int d^3x\,\hat{\vec{d}}(\vec{x})\cdot$$

take the Poincaré Gauge, do a Multipole expansion; $H_I = e\vec{x}\cdot\vec{E}(t,\vec{x})$

$$\hat{H}_I = e\sum_{i,j} \langle j|\hat{\vec{x}}\cdot\vec{E}(t,\vec{x})|i\rangle e^{i\Omega_{ij}t}\,|j\rangle\langle i| =$$

$$= e\sum_{ij}\int d^3x \int d^3x' \underbrace{\langle j|\vec{x}\rangle}_{\psi_j^*(\vec{x})}\langle\vec{x}|\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}})|\vec{x}'\rangle\underbrace{\langle\vec{x}'|i\rangle}_{\psi_i(\vec{x})} e^{i\Omega_{ij}t}\,|j\rangle\langle i|$$

$$\vec{x}\underbrace{\langle\vec{x}|\vec{x}'\rangle}_{\delta^{(3)}(\vec{x}-\vec{x}')}\vec{E}(t,\vec{x}')$$

$$\hat{H}_I = e\sum_{ij}\int d^3x \underbrace{\psi_j^*(\vec{x})\,\vec{x}\,\psi_i(\vec{x})}_{\vec{F}_{ij}(\vec{x})\ \leftarrow \text{smearing vector}}\cdot\vec{E}(t,\vec{x})\,e^{i\Omega_{ij}t}\,|j\rangle\langle i|$$

$$\hat{\vec{d}} := e\sum_{ij}\vec{F}_{ij}(\vec{x})\,e^{i\Omega_{ij}t}\,|j\rangle\langle i| \Rightarrow H_I = e\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}}) = \int d^3x\,\hat{\vec{d}}(t,\vec{x})\cdot\vec{E}(t,\vec{x})$$

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take the Poincaré Gauge, do a Multipole expansion: $\hat{H}_I = e\hat{\vec{x}}\cdot\vec{E}(t,\vec{x})$

$$\hat{H}_I = e\sum_{i,j} \langle j|\hat{\vec{x}}\cdot\vec{E}(t,\vec{x})|i\rangle e^{i\Omega_{ij}t}\,|j\rangle\langle i| =$$

$$= e\sum_{ij}\int d^3x \int d^3x' \underbrace{\langle j|\vec{x}\rangle}_{\psi_j^*(\vec{x})}\langle\vec{x}|\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}})|\vec{x}'\rangle\underbrace{\langle\vec{x}'|i\rangle}_{\psi_i(\vec{x})} e^{i\Omega_{ij}t}\,|j\rangle\langle i|$$

$$\vec{x}\underbrace{\langle\vec{x}|\vec{x}'\rangle}_{\delta^{(3)}(\vec{x}-\vec{x}')}\vec{E}(t,\vec{x}')$$

$$\hat{H}_I = e\sum_{ij}\int d^3x \underbrace{\psi_j^*(\vec{x})\,\vec{x}\,\psi_i(\vec{x})}_{\vec{F}_{ij}(\vec{x})\ \leftarrow \text{smearing vector}}\cdot\vec{E}(t,\vec{x})\,e^{i\Omega_{ij}t}\,|j\rangle\langle i|$$

$$\hat{\vec{d}} := e\sum_{ij}\vec{F}_{ij}(\vec{x})\,e^{i\Omega_{ij}t}\,|j\rangle\langle i| \Rightarrow H_I = e\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}}) = \int d^3x\,\hat{\vec{d}}(t,\vec{x})\cdot\vec{E}(t,\vec{x})$$

$$H_I = \chi(t)\int d^3x\,\hat{\vec{d}}(t,\vec{x})\cdot\vec{E}(t,\vec{x})$$

$\chi(t)$? Switching function

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take the Poincaré Gauge, do a Multipole expansion: $\hat{H}_I = e\hat{\vec{x}}\cdot\vec{E}(t,\vec{x})$

$$\hat{H}_I = e\sum_{i,j} \underbrace{\langle j|}_{\Psi_j^*(\vec{x})}\hat{\vec{x}}\cdot\vec{E}(t,\vec{x})\underbrace{|i\rangle}_{\Psi_i(\vec{x})} e^{i\Omega_{ij}t}\,|j\rangle\langle i| =$$

$$= e\sum_{ij}\int d^3x\int d^3x'\,\underbrace{\langle j|\vec{x}\rangle\langle\vec{x}|}_{\Psi_j^*(\vec{x})}\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}})|\vec{x}'\rangle\underbrace{\langle\vec{x}'|i\rangle}_{\Psi_i(\vec{x})} e^{i\Omega_{ij}t}\,|j\rangle\langle i|$$

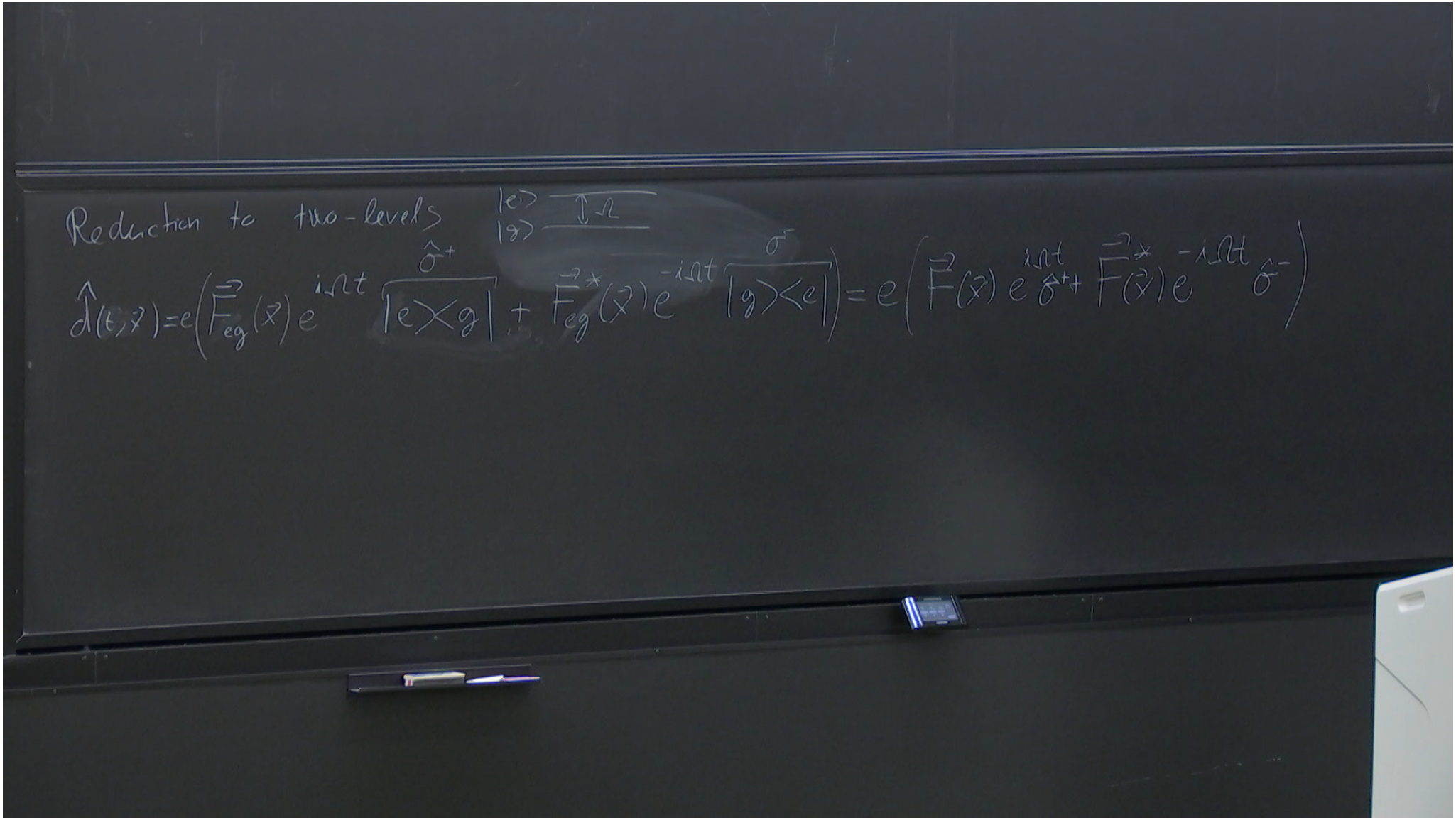
$$\vec{x}\underbrace{\langle\vec{x}|\vec{x}'\rangle}_{\delta^{(3)}(\vec{x}-\vec{x}')}\vec{E}(t,\vec{x}')$$

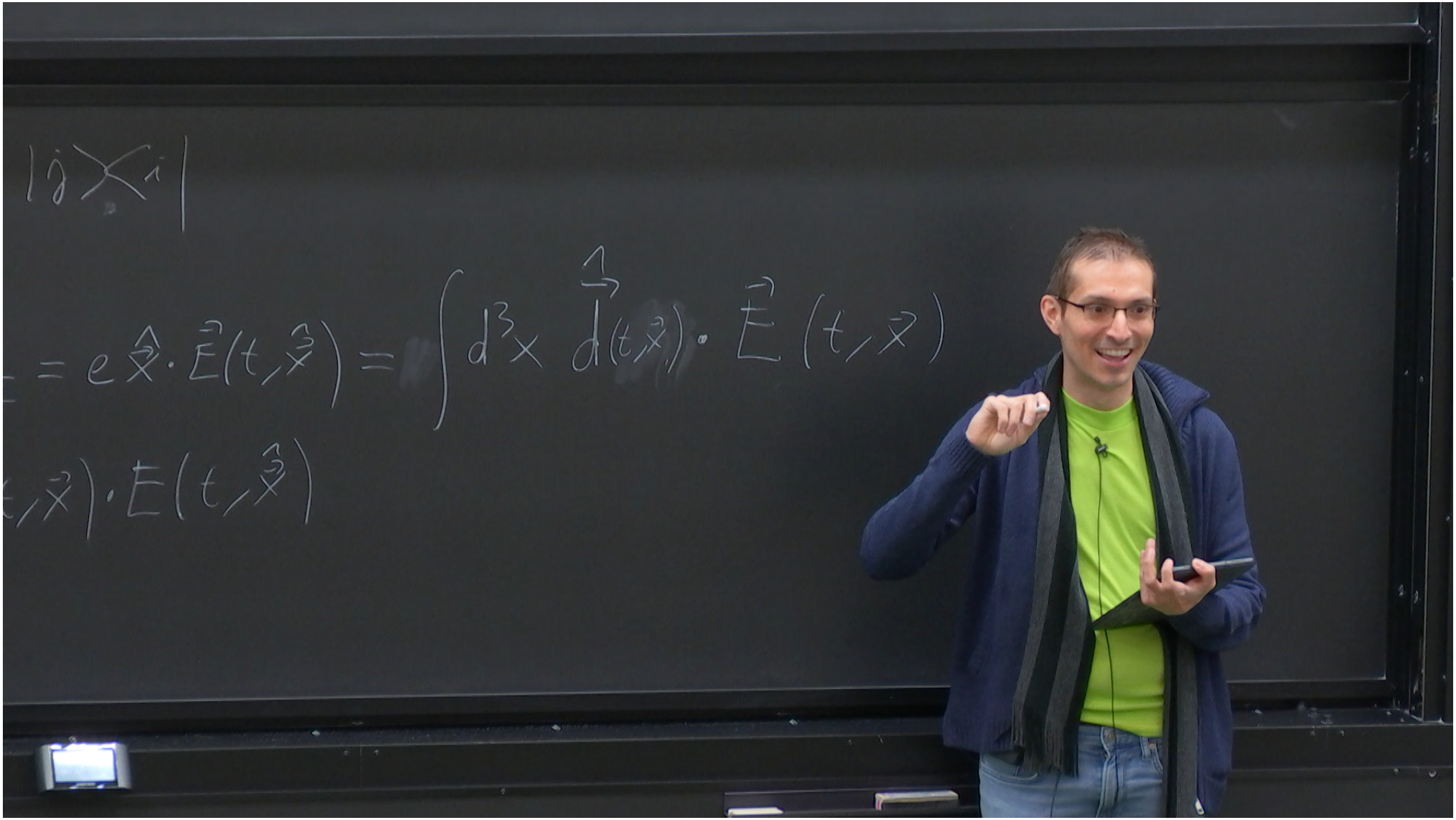
$$\hat{H}_I = e\sum_{ij}\int d^3x\,\underbrace{\Psi_j^*(\vec{x})\,\vec{x}\,\Psi_i(\vec{x})}_{\vec{F}_{ij}(\vec{x})\ \leftarrow\ \text{smearing vector}}\cdot\vec{E}(t,\vec{x})\,e^{i\Omega_{ij}t}\,|j\rangle\langle i|$$

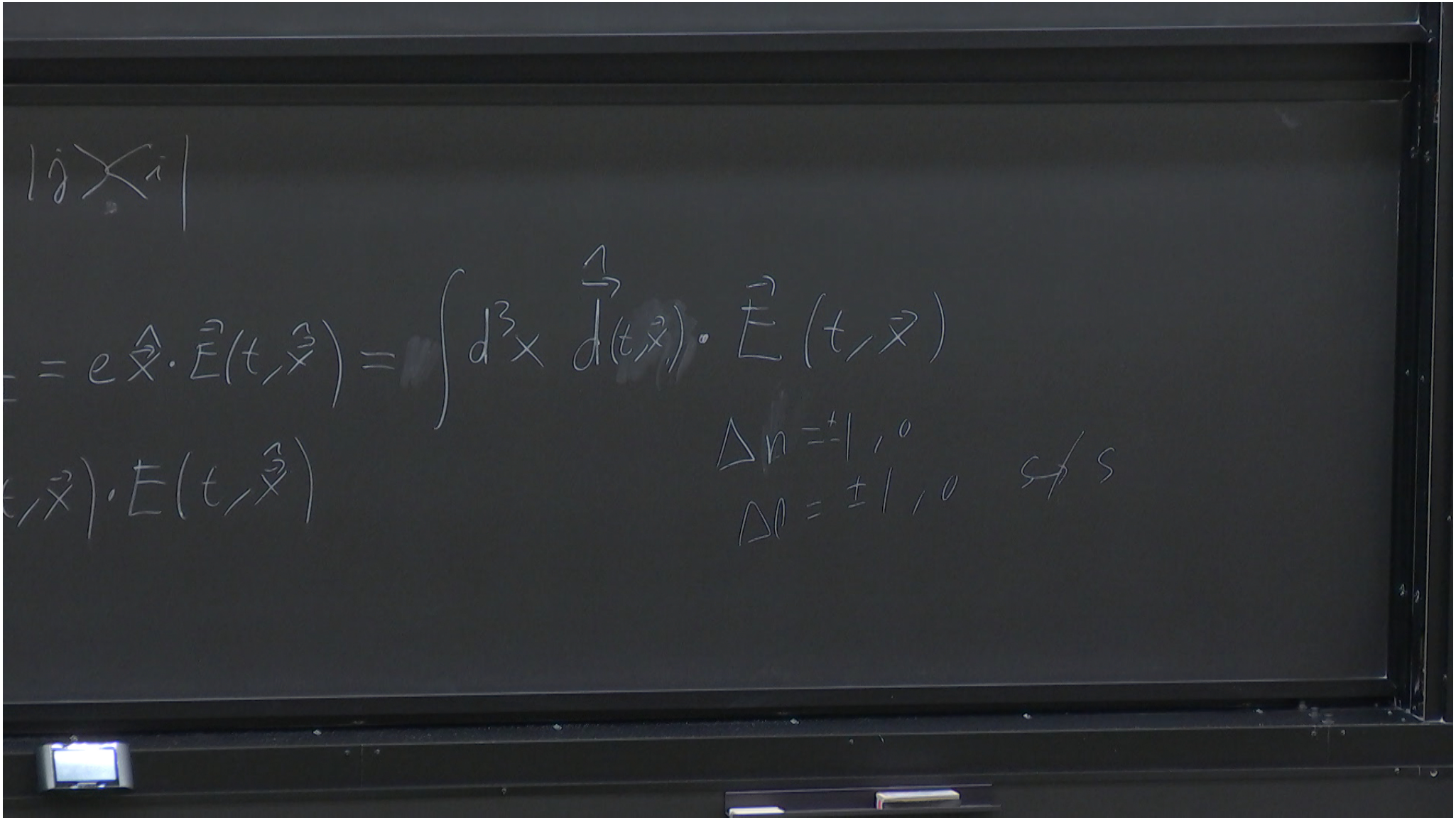
$$\hat{\vec{d}} := e\sum_{ij}\vec{F}_{ij}(\vec{x})\,e^{i\Omega_{ij}t}\,|j\rangle\langle i| \Rightarrow H_I = e\hat{\vec{x}}\cdot\vec{E}(t,\hat{\vec{x}}) = \int d^3x\,\hat{\vec{d}}(t,\vec{x})\cdot\vec{E}(t,\vec{x})$$

$$H_I = \chi(t)\int d^3x\,\hat{\vec{d}}(t,\vec{x})\cdot\vec{E}(t,\vec{x})$$

$\chi(t)$? Switching function


Reduction to two-levels
|e>
|g>
Ω
$\hat{d}(t,\vec{x}) = e\left(\vec{F}_{eg}(\vec{x})\, e^{i\Omega t}\, \overbrace{|e\rangle\langle g|}^{\hat{\sigma}^+} + \vec{F}^{*}_{eg}(\vec{x})\, e^{-i\Omega t}\, \overbrace{|g\rangle\langle e|}^{\sigma^-}\right) = e\left(\vec{F}(\vec{x})\, e^{i\Omega t}\, \hat{\sigma}^+ + \vec{F}^{*}(\vec{x})\, e^{-i\Omega t}\, \hat{\sigma}^-\right)$

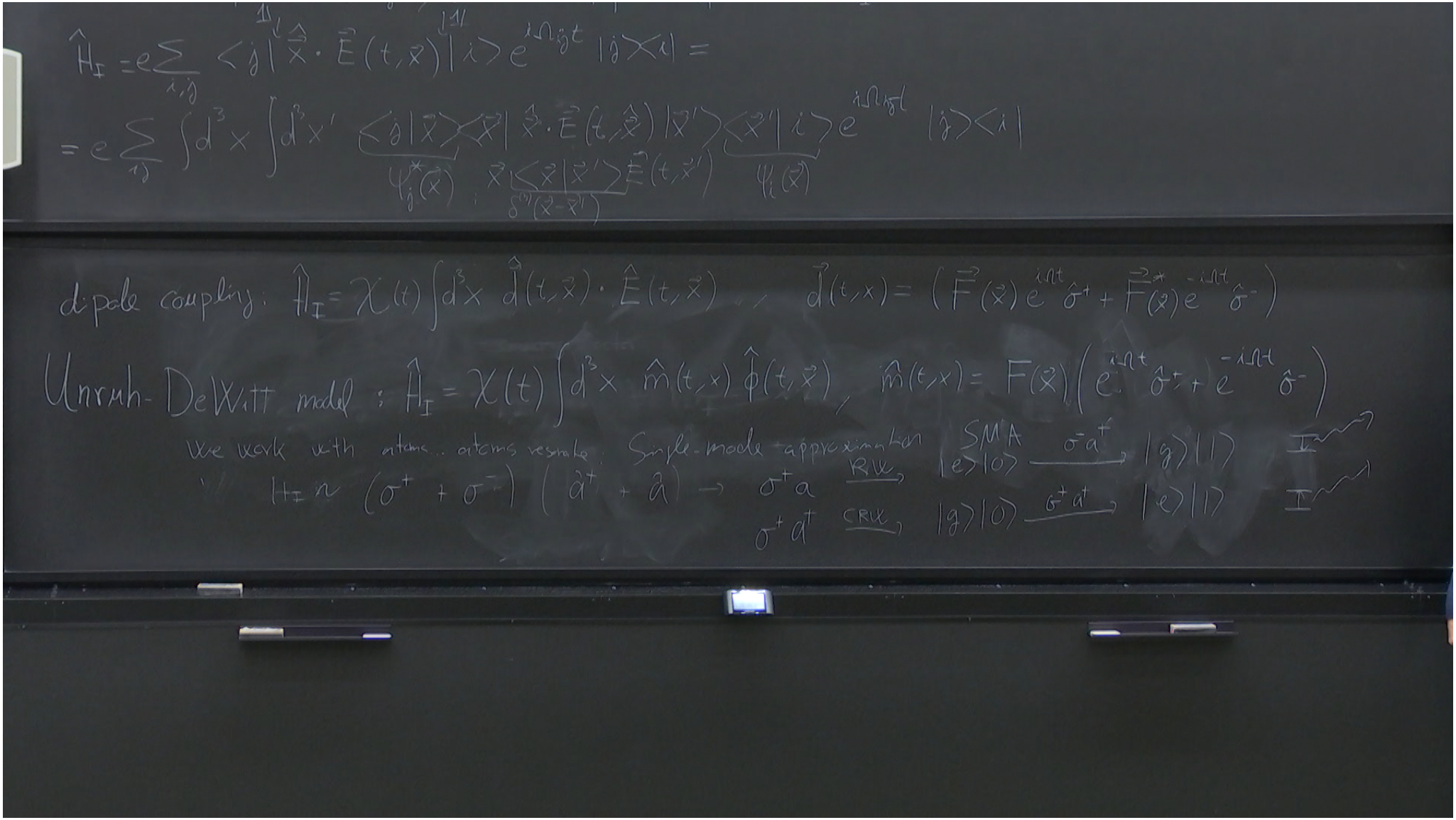

$|\vec{y}-\vec{x}_i|$
$= e\hat{\vec{x}} \cdot \vec{E}(t,\vec{x}) = \int d^3x \; \vec{j}(t,\vec{x}) \cdot \vec{E}(t,\vec{x})$
$(t,\vec{x}) \cdot E(t,\vec{x})$

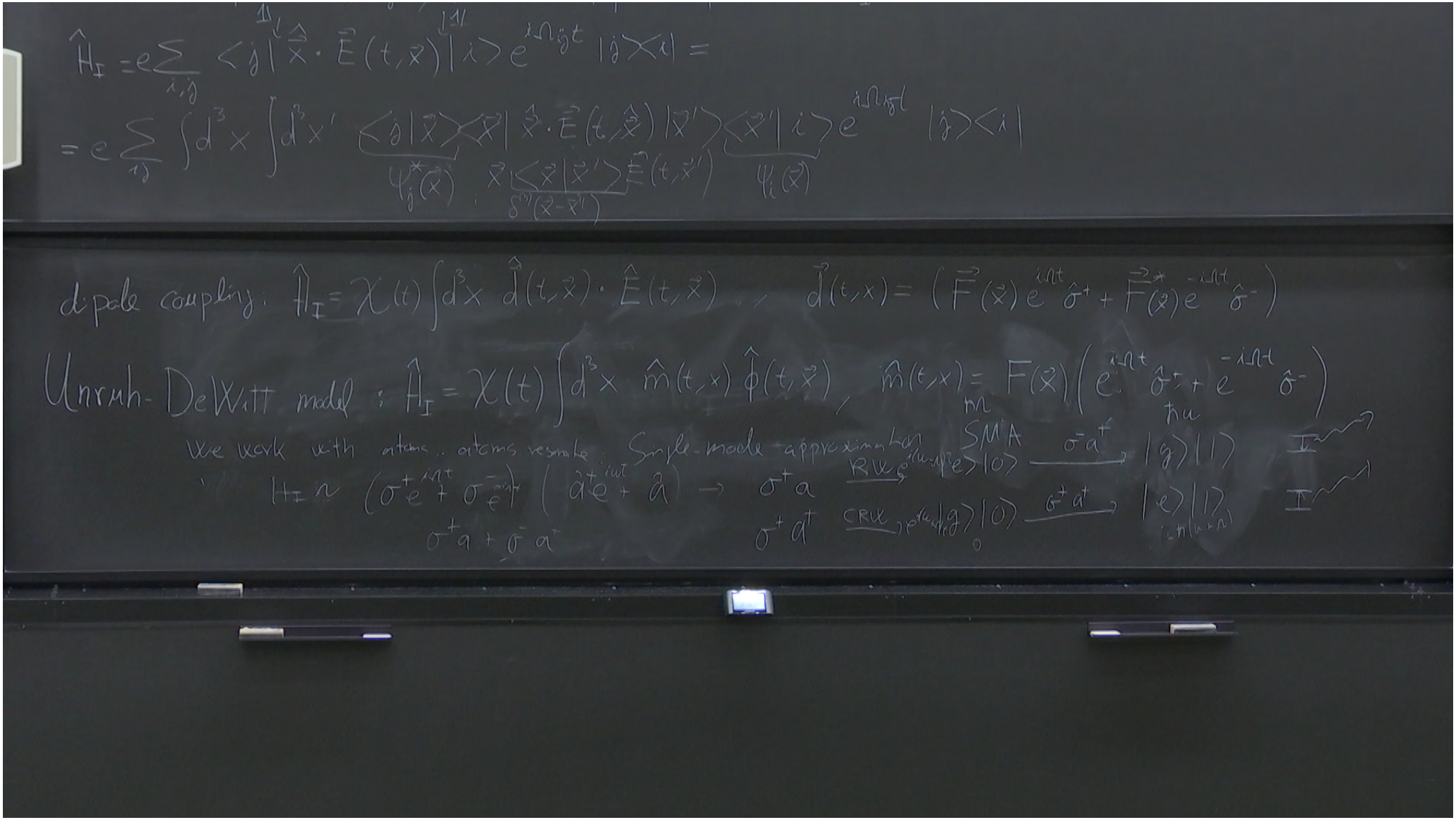

$|g\rangle \quad |i\rangle$
$= e\hat{\vec{x}} \cdot \vec{E}(t, \vec{x}) = \int d^3x \; \vec{j}(t, \vec{x}) \cdot \vec{E}(t, \vec{x})$
$(t, \vec{x}) \cdot E(t, \vec{x})$
$\Delta n = \pm 1, 0$
$\Delta l = \pm 1, 0$ $s \not\to s$

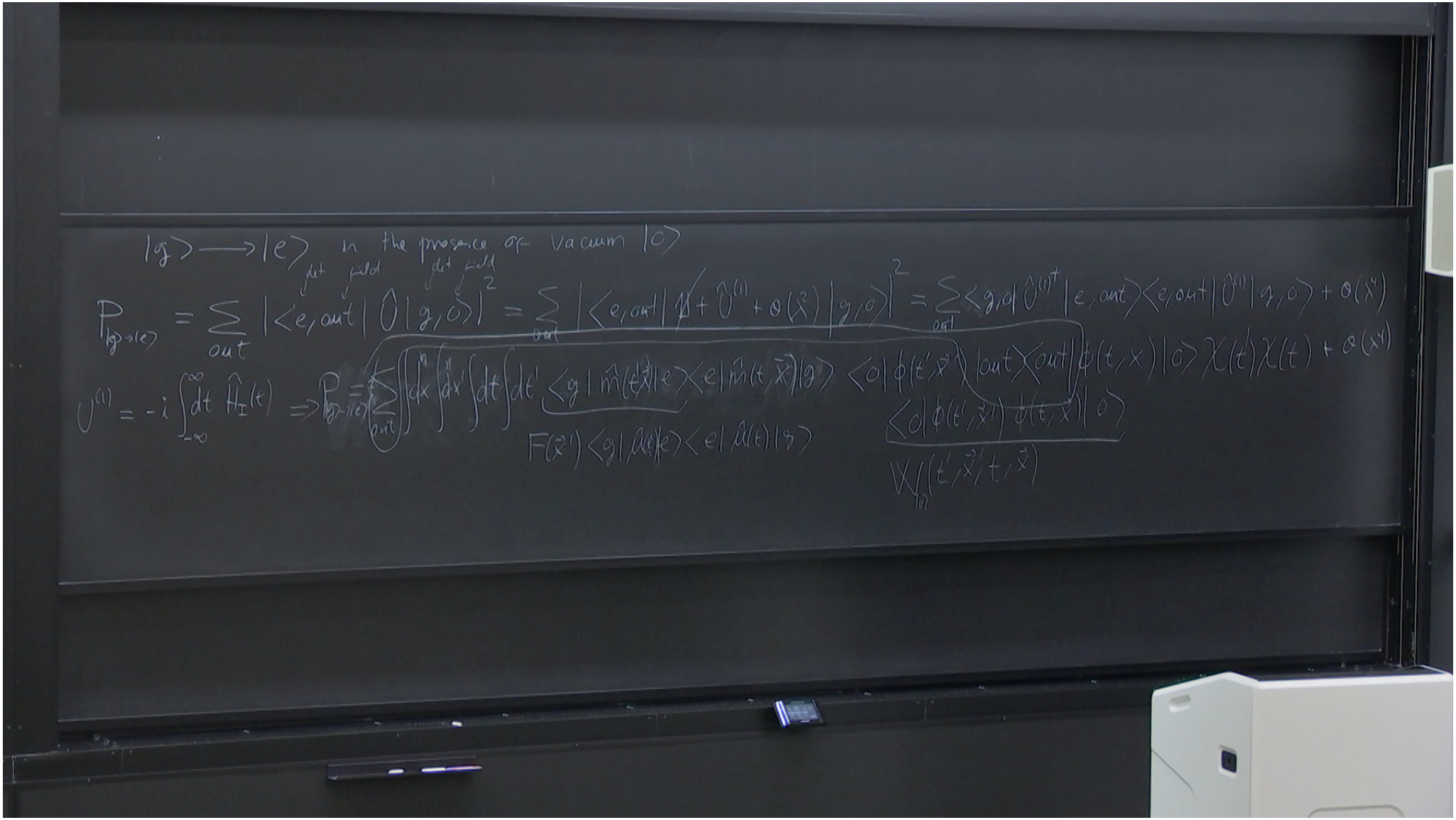
$$H = \frac{p^2}{2m}$$

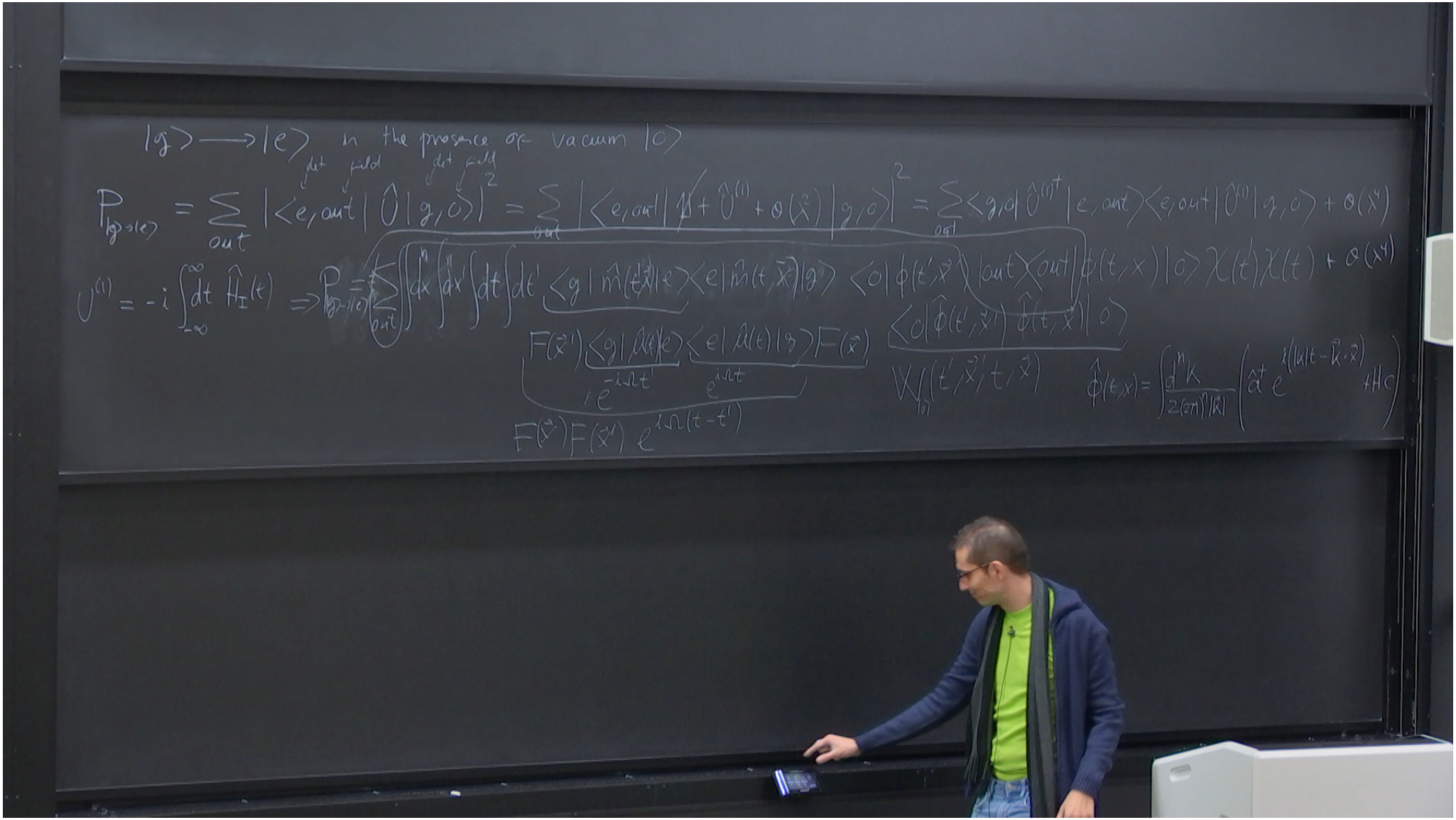
$$\hat{H} = \frac{\hat{p}^2}{2m}$$

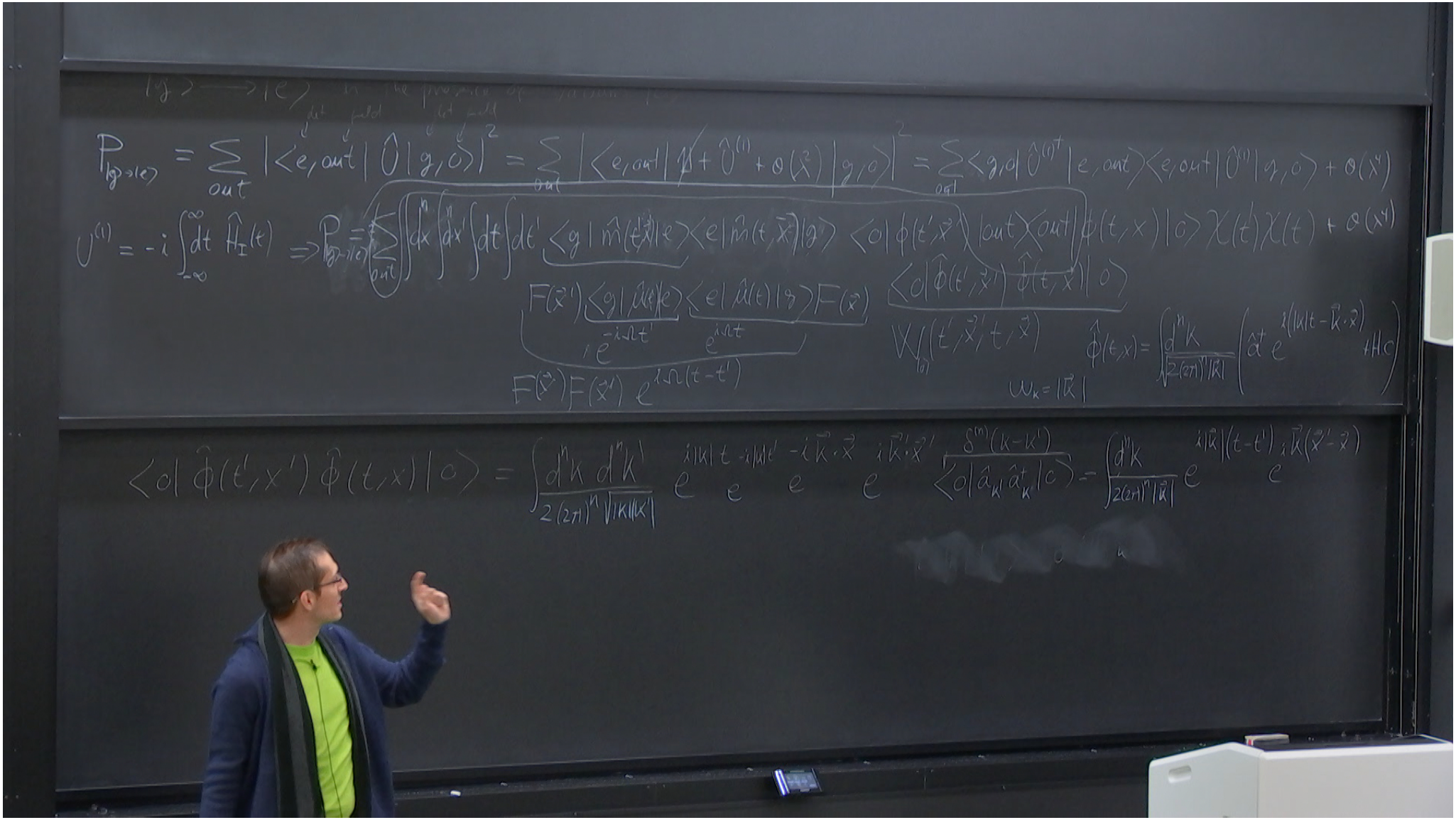
$$\hat{H} = \frac{\hat{p}^2}{2m} + i\underbrace{\{\hat{x}, \hat{p}\}}_{i\hbar\mathbb{1}} \sin\left[\sinh(\hat{x}^2)\right]$$

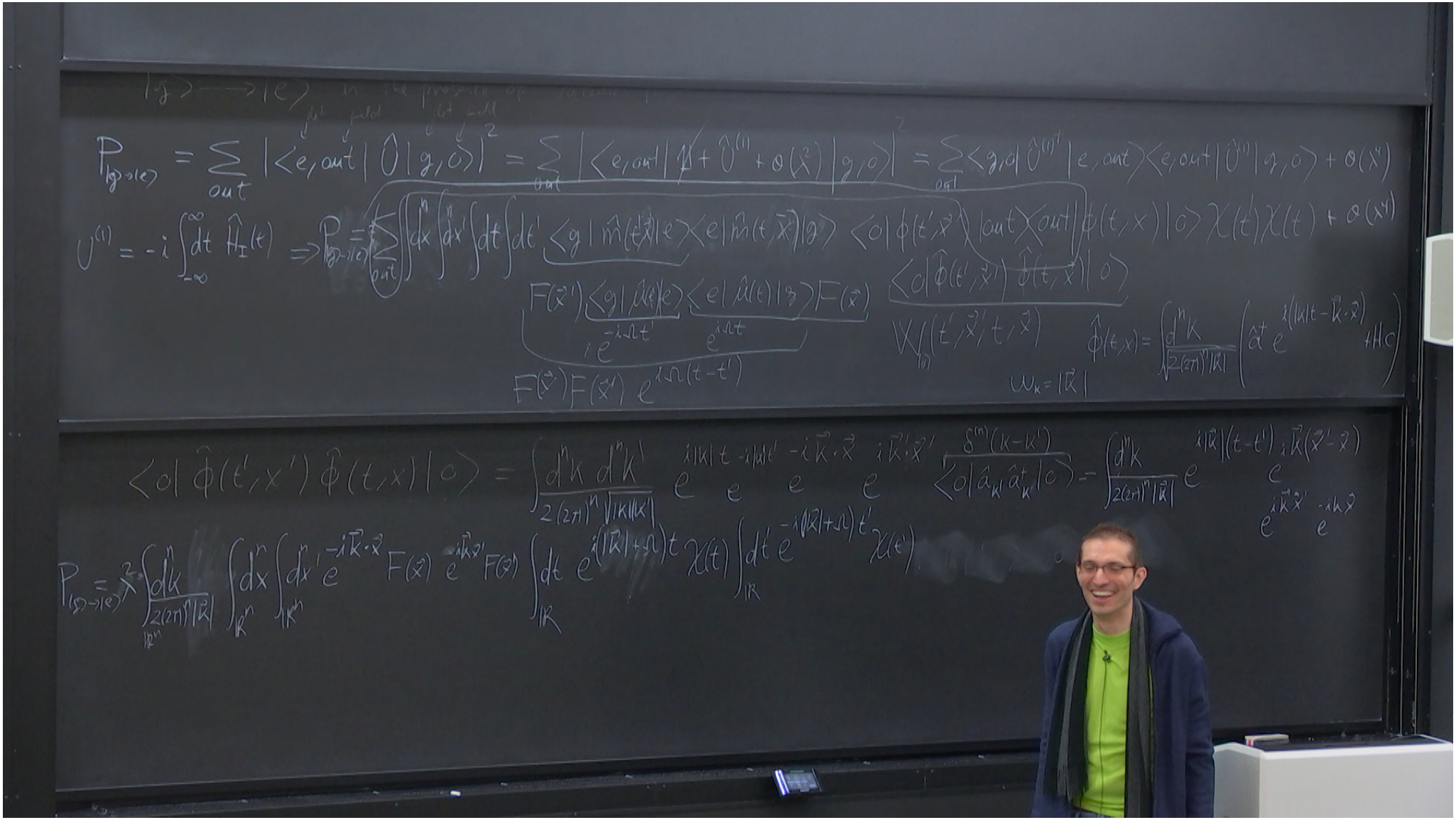

$\hat{H}_I = e \sum_{i,j} \langle j | \hat{\vec{x}} \cdot \vec{E}(t,x) | i \rangle e^{i\Omega_{ij} t} |j\rangle\langle i| =$
$= e \sum_{ij} \int d^3x \int d^3x' \langle j|\vec{x}\rangle\langle\vec{x}| \hat{\vec{x}} \cdot \vec{E}(t,\hat{\vec{x}}) |\vec{x}'\rangle\langle\vec{x}'|i\rangle e^{i\Omega_{ij} t} |j\rangle\langle i|$
$\psi_j^*(\vec{x})$
$\vec{x}\langle\vec{x}|\vec{x}'\rangle = \delta^{(3)}(\vec{x}-\vec{x}')$
$\vec{E}(t,\vec{x}')$
$\psi_i(\vec{x})$
dipole coupling: $\hat{H}_I = \chi(t) \int d^3x \, \hat{\vec{d}}(t,\vec{x}) \cdot \hat{\vec{E}}(t,\vec{x})$, $\hat{\vec{d}}(t,x) = (\vec{F}(\vec{x}) e^{i\Omega t} \hat{\sigma}^+ + \vec{F}^*(\vec{x}) e^{-i\Omega t} \hat{\sigma}^-)$
Unruh-DeWitt model: $\hat{H}_I = \chi(t) \int d^3x \, \hat{m}(t,x) \hat{\phi}(t,\vec{x})$, $\hat{m}(t,x) = F(\vec{x}) (e^{i\Omega t} \hat{\sigma}^+ + e^{-i\Omega t} \hat{\sigma}^-)$
We work with atoms.. atoms resonate. Single-mode approximation
$H_I \propto (\sigma^+ + \sigma^-)(\hat{a}^\dagger + \hat{a}) \to \sigma^+ a$ RWA
$\sigma^+ a^\dagger$ CRVK
SMA
$|e\rangle|0\rangle \xrightarrow{\sigma^- a^\dagger} |g\rangle|1\rangle$ I
$|g\rangle|0\rangle \xrightarrow{\sigma^+ a^\dagger} |e\rangle|1\rangle$ I

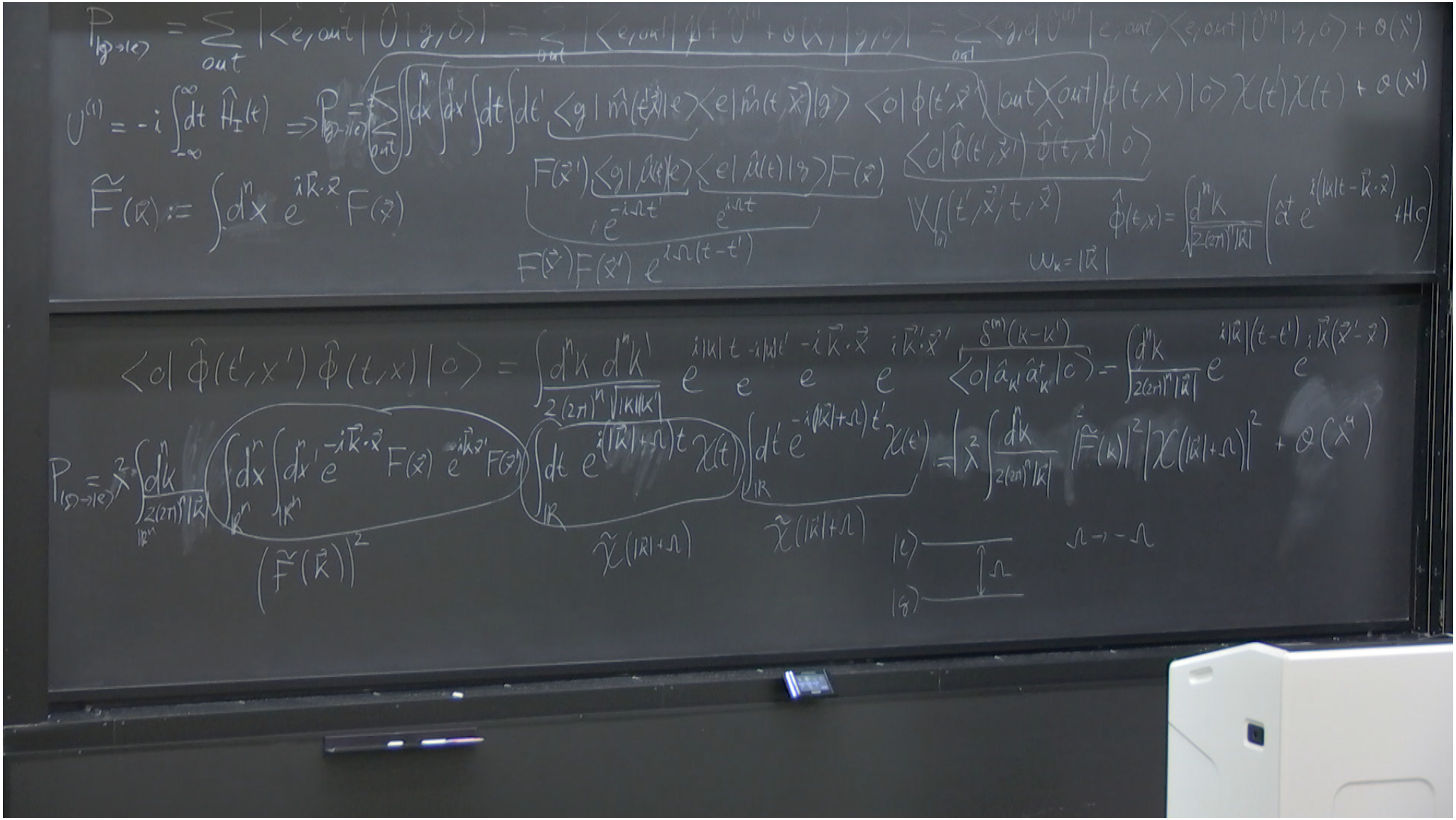

dipole coupling
Unruh-DeWitt model
We work with atoms .. atoms resonate
Single-mode approximation
SMA
RWA
CRVK


$|g\rangle \longrightarrow |e\rangle$ in the presence of vacuum $|0\rangle$
dot field
dot field
$P_{|g\rangle\to|e\rangle} = \sum_{out} |\langle e, out|\, \hat{U}|g,0\rangle|^2 = \sum_{out} |\langle e,out|\,\mathbb{1} + \hat{U}^{(1)} + \mathcal{O}(\lambda^2)\,|g,0\rangle|^2 = \sum_{out} \langle g,0|\hat{U}^{(1)\dagger}|e,out\rangle\langle e,out|\hat{U}^{(1)}|g,0\rangle + \mathcal{O}(\lambda^4)$
$U^{(1)} = -i\int_{-\infty}^{\infty} dt\, \hat{H}_I(t) \Rightarrow P_{|g\rangle\to|e\rangle} = \sum_{out} \int d^n\vec{x} \int d^n\vec{x}' \int dt \int dt' \langle g|\hat{m}(t,\vec{x})|e\rangle\langle e|\hat{m}(t',\vec{x}')|g\rangle \langle 0|\phi(t',\vec{x}')|out\rangle\langle out|\phi(t,x)|0\rangle \chi(t')\chi(t) + \mathcal{O}(\lambda^4)$
$F(\vec{x}')\langle g|\hat{\mu}(t')|e\rangle\langle e|\hat{\mu}(t)|g\rangle$
$\langle 0|\phi(t',\vec{x}')\,\phi(t,\vec{x})|0\rangle$
$W_{|0\rangle}(t',\vec{x}',t,\vec{x})$


$|g\rangle \longrightarrow |e\rangle$ in the presence of vacuum $|0\rangle$
$P_{|g\rangle\to|e\rangle} = \sum_{out} |\langle e, out|\hat{U}|g,0\rangle|^2 = \sum_{out} |\langle e,out|\mathbb{1} + \hat{U}^{(1)} + \mathcal{O}(\hat{x}^2)|g,0\rangle|^2 = \sum_{out} \langle g,0|\hat{U}^{(1)\dagger}|e,out\rangle\langle e,out|\hat{U}^{(1)}|g,0\rangle + \mathcal{O}(\hat{x}^4)$
$U^{(1)} = -i\int_{-\infty}^{\infty} dt\, \hat{H}_I(t)$
$\Rightarrow P_{|g\rangle\to|e\rangle} = \sum_{out} \int d^n x \int d^n x' \int dt \int dt' \langle g|\hat{m}(t',\vec{x}')|e\rangle\langle e|\hat{m}(t,\vec{x})|g\rangle \langle 0|\hat{\phi}(t',\vec{x}')|out\rangle\langle out|\hat{\phi}(t,x)|0\rangle \chi(t)\chi(t) + \mathcal{O}(x^4)$
$F(\vec{x}')\langle g|\hat{\mu}(e)|e\rangle\langle e|\hat{\mu}(e)|g\rangle F(\vec{x})$
$i e^{-i\Omega t'}$ $e^{i\Omega t}$
$F(\vec{x}')F(\vec{x})\, e^{i\Omega(t-t')}$
$\langle 0|\hat{\phi}(t',\vec{x}')\hat{\phi}(t,\vec{x})|0\rangle$
$W_{|0\rangle}(t',\vec{x}',t,\vec{x})$
$\hat{\phi}(t,x) = \int \frac{d^n k}{2(2\pi)^n |k|}\left(\hat{a}^\dagger e^{i(|k|t - \vec{k}\cdot\vec{x})} + Hc\right)$







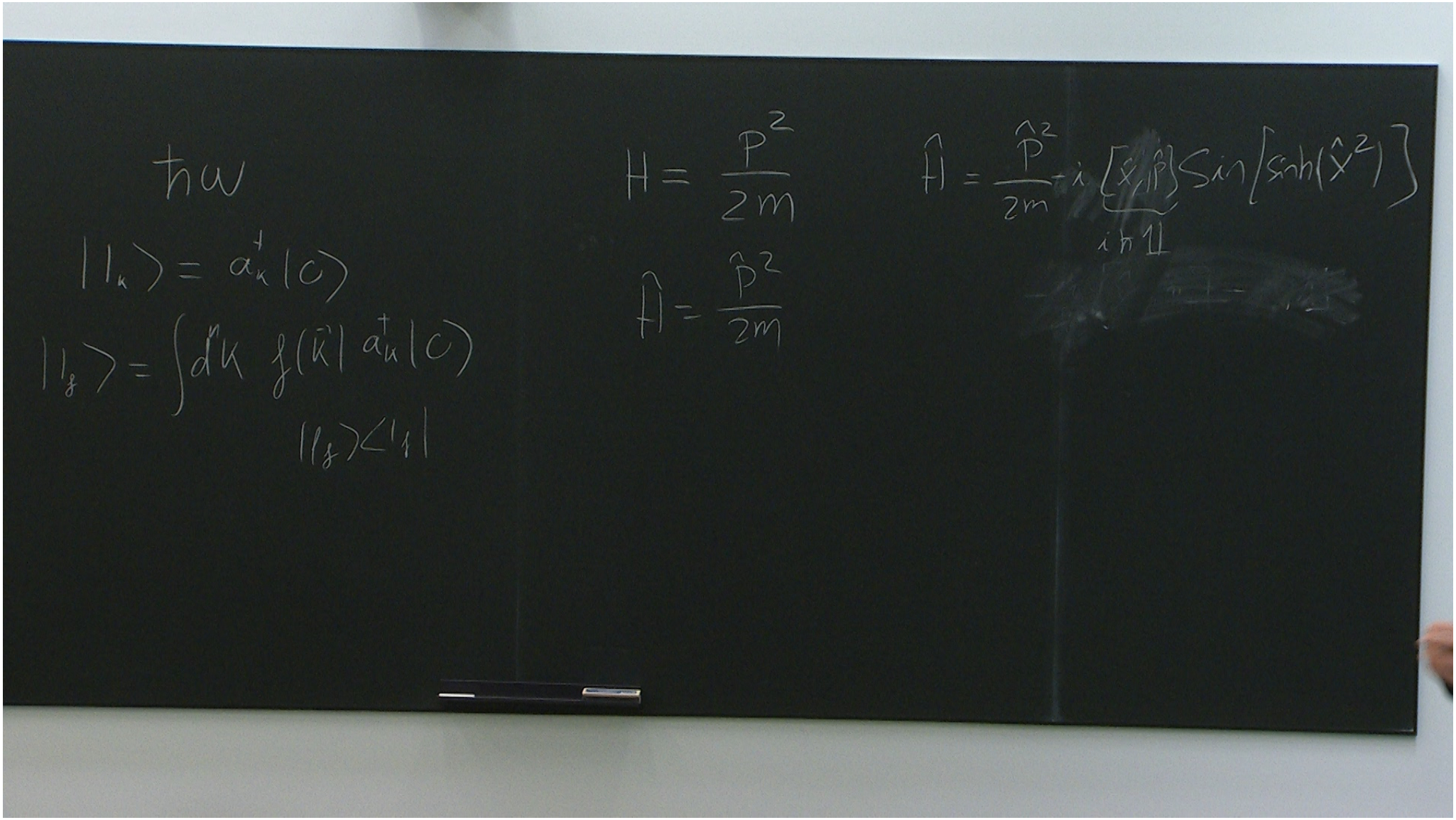
$$\hat{H}_I = e \sum_{i,j} \langle j | \hat{\vec{x}} \cdot \vec{E}(t,x) | i \rangle e^{i\Omega_{ji} t} |j\rangle\langle i| =$$

$$e \sum_{ij} \int d^3x \int d^3x' \underbrace{\langle j|\vec{x}\rangle}_{\psi_j^*(\vec{x})} \langle \vec{x}| \hat{\vec{x}} \cdot \vec{E}(t,\hat{\vec{x}}) |\vec{x}'\rangle \underbrace{\langle \vec{x}'|i\rangle}_{\psi_i(\vec{x})} e^{i\Omega_{ji} t} |j\rangle\langle i|$$

$$\underbrace{\langle \vec{x}|\vec{x}'\rangle}_{\delta^{(3)}(\vec{x}-\vec{x}')} \vec{E}(t,\vec{x}')$$

Prob. of vacuum excitation: $P_{|g\rangle \to |e\rangle} \equiv P(\Omega) = \lambda^2 \int_{\mathbb{R}^n} \frac{d^n k}{2(2\pi)^n |\vec{k}|} |\tilde{F}(\vec{k})|^2 \left|\tilde{\chi}(|\vec{k}| + \Omega)\right|^2 + \mathcal{O}(\lambda^4)$

Prob of spontaneous emission $P_{|e\rangle \to |g\rangle} = P(-\Omega) = \lambda^2 \int_{\mathbb{R}^n} \frac{d^n k}{2(2\pi)^n |\vec{k}|} |\tilde{F}(\vec{k})|^2 \left|\tilde{\chi}(|\vec{k}| - \Omega)\right|^2 + \mathcal{O}(\lambda^4)$


$\hbar\omega$
$|1_k\rangle = a_k^\dagger |0\rangle$
$|1_g\rangle = \int d^n k\, g(\vec{k})\, a_k^\dagger |0\rangle$
$|1_g\rangle\langle 1_g|$
$H = \frac{p^2}{2m}$
$\hat{H} = \frac{\hat{p}^2}{2m}$
$\hat{H} = \frac{\hat{p}^2}{2m} - i\underbrace{[\hat{x},\hat{p}]}_{i\hbar\mathbb{1}} \sin[\sinh(\hat{x}^2)]$