Title: PSI 2019/2020 - Standard Model and Beyond part 2 - Lecture 7

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Collection: PSI 2019/2020 - Standard Model and Beyond - Part 2

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SM+B #25

Grand Unification (SO(10), SU(5), Pati-Salam)
Baez + Huerta arXiv: 0904.1556

H. Freudenthal: "Lie Groups + Foundations of Geom." (1964)

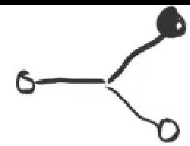
J. Baez: "The Octonions" arXiv: math/0105155

P. Ramond: "Intro to Exceptional Lie Groups + Lie Alg." (1976)

Barton + Sudbery: arXiv: math/0203010

Dubois-Violette + Todorov: arXiv: 1808.08110

Triality: $SO(8)$



V, S^+, S^-

$$g_1 \in SO(8) \begin{cases} \to g_1 V \\ \to g_2 S^+ \\ \to g_3 S^- \end{cases}$$

g_2

$\mathbb{K} \to \mathrm{Tri}(\mathbb{K})$:

$\mathrm{Der}(\mathbb{K})$: $\underline{D}(xy) = \underline{D}(x)y + x\underline{D}(y) \leftarrow D \quad \forall x,y \in \mathbb{K}$

$\mathrm{Tri}(\mathbb{K})$: $\underline{\underline{A}}(xy) = \underline{\underline{B}}(x)y + x\underline{\underline{C}}(y) \leftarrow \overline{(A,B,C)} \quad \forall x,y \in \mathbb{K}$

$(A,B,C) \quad (A',B',C')$ $\qquad\qquad (D,D,D)$

$(A+A', B+B', C+C') \qquad ([A,A'],[B,B'],[C,C'])$

$\mathbb{K} =$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$	
$\mathrm{der}\,\mathbb{K}$	0	0	$su(2)$	g_2	$\leftarrow 14$
$\mathrm{tri}\,\mathbb{K}$	0	$so(2)^2$	$su(2)^3$	$so(8)$	$\leftarrow 28$

$$L(\mathbb{K}_A, \mathbb{K}_B) = \mathrm{Der}(\mathbb{K}_A) \oplus \mathrm{Der}(\mathbb{K}_B) \oplus A_3'(\mathbb{K}_A \otimes \mathbb{K}_B)$$

$$= \mathrm{Tri}(\mathbb{K}_A) \oplus \mathrm{Tri}(\mathbb{K}_B) \oplus (\underline{\mathbb{K}_A \otimes \mathbb{K}_B}) \oplus (\underline{\mathbb{K}_A \otimes \mathbb{K}_B}) \oplus (\mathbb{K}_A \otimes \mathbb{K}_B)$$

$133 \quad \mathbb{H} \otimes \mathbb{O}$

$9 + 28 + 32 \times 3$

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$				F_4
$\mathbb{C}$				E_6
$\mathbb{H}$				E_7
$\mathbb{O}$	F_4	E_6	E_7	E_8

$$L(\mathbb{K}_A, \mathbb{K}_B) = \mathrm{Der}(\mathbb{K}_A) \oplus \mathrm{Der}(\mathbb{K}_B) \oplus A_3'(\mathbb{K}_A \otimes \mathbb{K}_B)$$

$$= \mathrm{Tri}(\mathbb{K}_A) \oplus \mathrm{Tri}(\mathbb{K}_B) \oplus \overset{(a\otimes b)_1}{(\mathbb{K}_A \otimes \mathbb{K}_B)_1} \oplus \overset{(a\times b)_2}{(\mathbb{K}_A \otimes \mathbb{K}_B)_2} \oplus \overset{(a\times b)_3}{(\mathbb{K}_A \otimes \mathbb{K}_B)_3}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$T_A = (T_{A_1}, T_{A_2}, T_{A_3}) \qquad T_B = (T_{B_1}, T_{B_2}, T_{B_3})$$

$$[T_A, (a\otimes b)_i] = ((T_{A_i} a)\otimes b)_i$$

$$[T_B, (a\otimes b)_i] = (a \otimes T_{B_i} b)_i$$

$$[(a\otimes b)_i, (a'\otimes b')_j] = (\bar{a}'\bar{a} \otimes \bar{b}'\bar{b})_k$$

$H_3(\mathbb{O})$ $F_4 \to spin(9)$, $F_4 \to \frac{su(3)\times su(3)}{\mathbb{Z}_3}$ $\Big\}$ $\frac{su(3)\times su(2)\times u(1)}{\mathbb{Z}_6}$

$\mathbb{O}$: $G_2 \to SU(3)$

I

$$x = a_0 1 + a_1 i + a_2 j + a_3 k + a_4 l + a_5 li + a_6 lj + a_7 lk$$

$$= (a_0 + a_4 l) + (a_1 + a_5 l) i + (a_2 + a_6 l) j + (a_3 + a_7 l) k$$

$$= m_0 1 + m_1 i + m_2 j + m_3 k$$

UC

$$m_0 + \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} \longrightarrow m_0 + U \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

$$H_3(\mathbb{O}) \ni \begin{pmatrix} \rho_1 & \mathcal{M}_3 & \bar{\mathcal{M}}_2 \\ \bar{\mathcal{M}}_3 & \rho_2 & \mathcal{M}_1 \\ \mathcal{M}_2 & \bar{\mathcal{M}}_1 & \rho_3 \end{pmatrix} = \begin{pmatrix} \rho_1 & m_3 & \bar{m}_2 \\ \bar{m}_3 & \rho_2 & m_1 \\ m_2 & \bar{m}_1 & \rho_3 \end{pmatrix} + \left(\vec{M}_1 \vec{M}_2 \vec{M}_3\right)$$

$$\mathcal{M}_i = m_i + \begin{pmatrix} M_{i1} \\ M_{i2} \\ M_{i3} \end{pmatrix} \qquad H_3(\mathbb{O}) \qquad H_3(\mathbb{C}) \qquad M_3(\mathbb{C})$$

$$(X) \qquad (Y)$$

$$= m_i + \vec{M}_i \qquad U \to e^{2\pi i/3} U$$

$$V \to e^{2\pi i/3} V$$

$$\underline{X} \to V X V^\dagger \qquad \underline{Y} \to U Y V^\dagger$$

$$U, V \in SU(3) \qquad \underline{SU(3) \times SU(3)}$$

$$H_3(\mathbb{O}) \ni \underbrace{\begin{pmatrix} \rho_1 & \mu_3 & \bar{\mu}_2 \\ \bar{\mu}_3 & \rho_2 & \mu_1 \\ \mu_2 & \bar{\mu}_1 & \rho_3 \end{pmatrix}}_{H_3(\mathbb{O})} = \underbrace{\begin{pmatrix} \rho_1 & m_3 & \bar{m}_2 \\ \bar{m}_3 & \rho_2 & m_1 \\ m_2 & \bar{m}_1 & \rho_3 \end{pmatrix}}_{\substack{H_3(\mathbb{C}) \\ (X)}} + \underbrace{\overline{\left(\vec{M}_1\ \vec{M}_2\ \vec{M}_3 \right)}}_{\substack{M_3(\mathbb{C}) \\ (Y)}}$$

$$\mu_i = m_i + \begin{pmatrix} M_{i1} \\ M_{i2} \\ M_{i3} \end{pmatrix} = m_i + \vec{M}_i$$

$$U \to e^{2\pi i/3} U$$

$$V \to e^{2\pi i/3} V$$

$$\underline{X} \to V X V^\dagger \qquad \underline{Y} \to U Y V^\dagger$$

$$U, V \in SU(3) \qquad \frac{SU(3) \times SU(3)}{\mathbb{Z}_3}$$

$$\begin{pmatrix} \rho_1 & m_3 & \overline{m}_2 \\ \overline{m}_3 & \rho_2 & m_1 \\ m_2 & \overline{m}_1 & \rho_3 \end{pmatrix}$$

$$\left(\begin{array}{c|cc} \xi_1 & m_3 & \overline{m_2} \\ \hline \overline{m_3} & \xi_2 & m_1 \\ m_2 & \overline{m_1} & \xi_3 \end{array}\right)$$

$$H_3(\mathbb{K}) \qquad v = \dim(\mathbb{K})$$

$$H_2(\mathbb{K}) \cong \mathrm{Jspin}(v+1)$$

$$H_2(\mathbb{C}) \cong \mathrm{Jspin}(3)$$

$$H_2(\mathbb{O}) \cong \mathrm{Jspin}(9) \to \mathrm{spin}$$

$$X = \begin{pmatrix} s_1 & m_3 & \bar{m}_2 \\ \bar{m}_3 & s_2 & m_1 \\ m_2 & \bar{m}_1 & s_3 \end{pmatrix} \qquad Y = \begin{pmatrix} \vec{M}_1 & \vec{M}_2 & \vec{M}_3 \end{pmatrix}$$

$$\underline{X \to VXV^\dagger} \qquad Y \underline{\to UXV}^\dagger$$

$$U, V \in SU(3) \qquad V = \begin{pmatrix} e^{-2i\alpha} & \\ & e^{i\alpha} W \end{pmatrix} \leftarrow$$

$$\frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}$$

$$U \to e^{2\pi i/3} U$$
$$V \to e^{2\pi i/3} V$$
$$W \to -W, \; e^{i\alpha} \to -e^{i\alpha}$$

$$H_3(\mathbb{O}) \to \mathbb{O}P^2$$

$$H_2(\mathbb{O}) \to M_{9,1}$$

$$H_2(\mathbb{C}) \to M_{3,1}$$