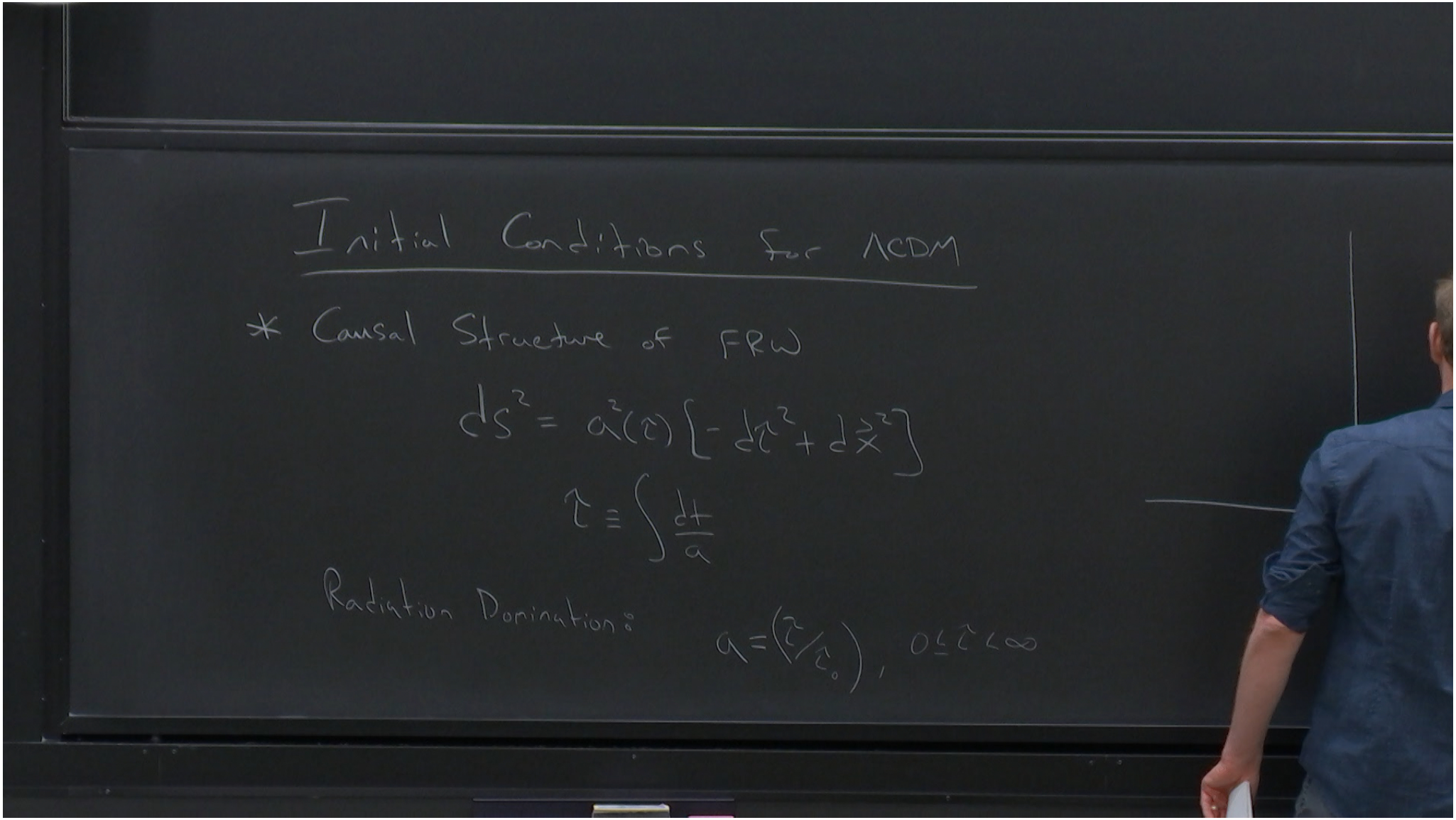
Title: PSI 2019/2020 - Cosmology Part 1 - Lecture 8

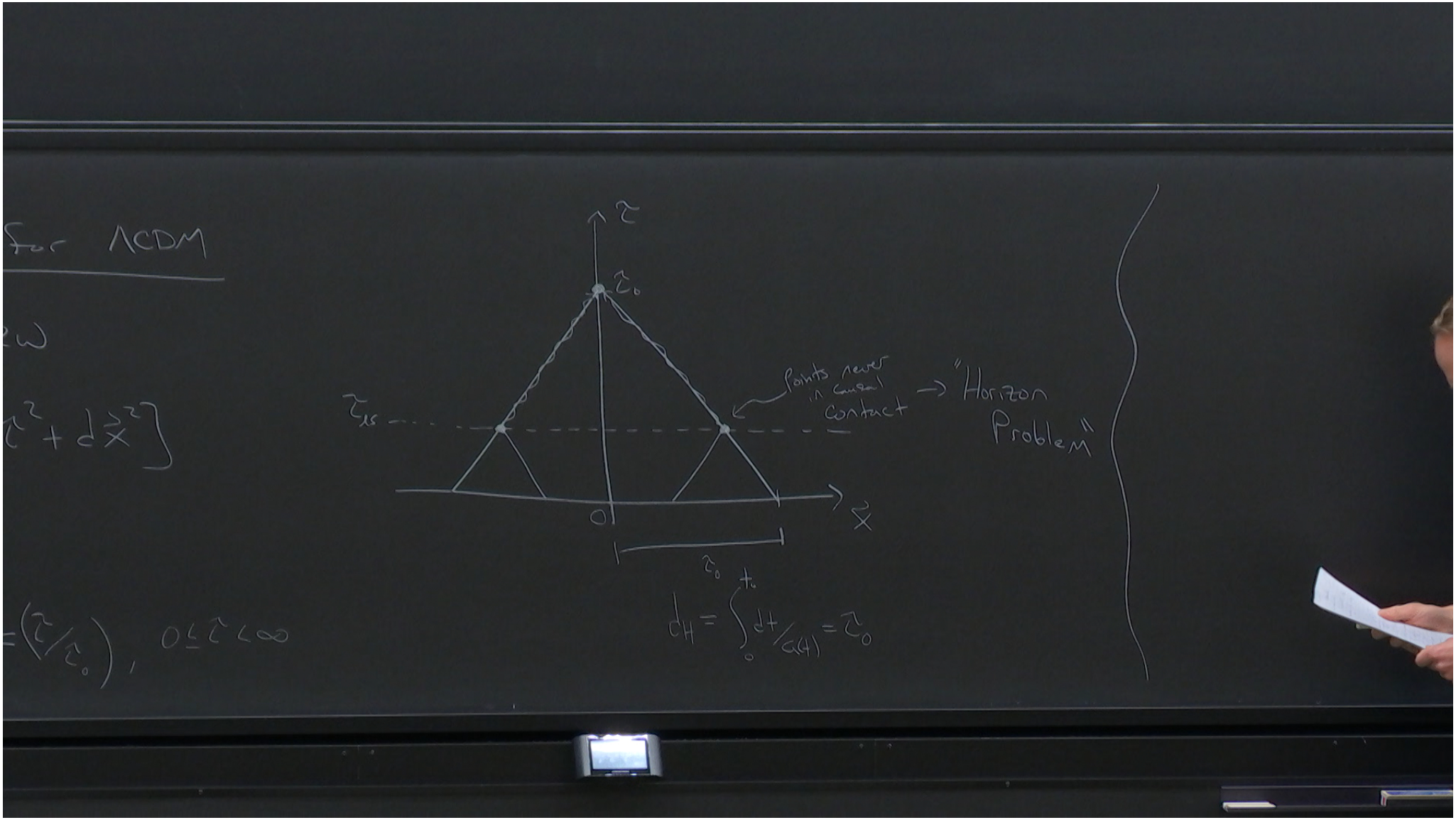
Speakers: Matthew Johnson

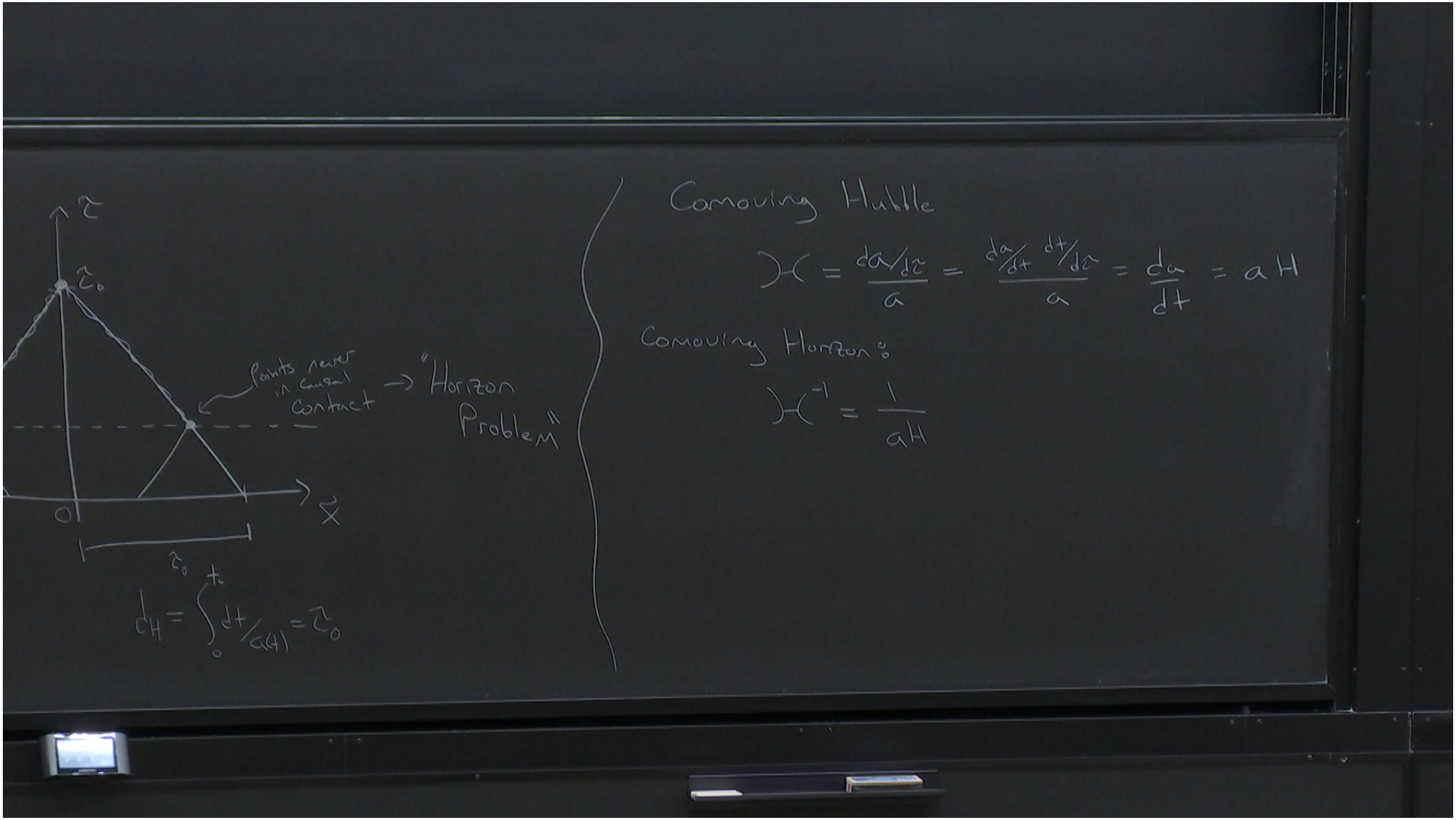
Collection: PSI 2019/2020 - Cosmology (Part 1)

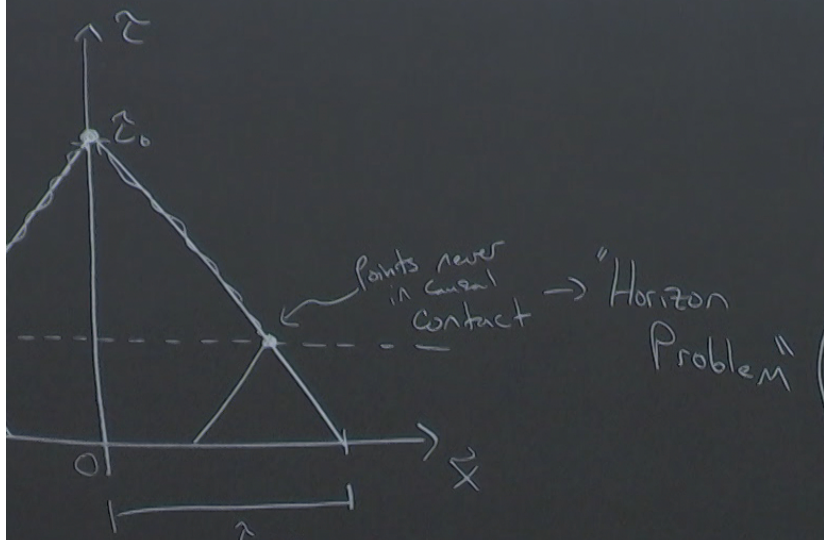
Date: March 05, 2020 - 1:00 PM

URL: http://pirsa.org/20030025


Initial Conditions for ΛCDM
* Causal Structure of FRW
$ds^2 = a^2(\tau)\left[-d\tau^2 + d\vec{x}^2\right]$
$\tau \equiv \int \frac{dt}{a}$
Radiation Domination: $a = \left(\tau/\tau_0\right), \quad 0 \leq \tau < \infty$


for ΛCDM
τ
τ₀
τ_ls
Points never in causal contact → "Horizon Problem"
0
χ
τ₀
$d_H = \int_0^{t_0} dt/a(t) = \tau_0$


τ
τ₀
Points never in Causal Contact → "Horizon Problem"
0
χ
τ₀
$d_H = \int_0^{t_0} dt/a(t) = \tau_0$
Comoving Hubble
$\mathcal{H} = \frac{da/d\tau}{a} = \frac{da/dt \; dt/d\tau}{a} = \frac{da}{dt} = aH$
Comoving Horizon:
$\mathcal{H}^{-1} = \frac{1}{aH}$



$$d_H = \int_0^{t_0} dt/a(t) = \tau_0$$

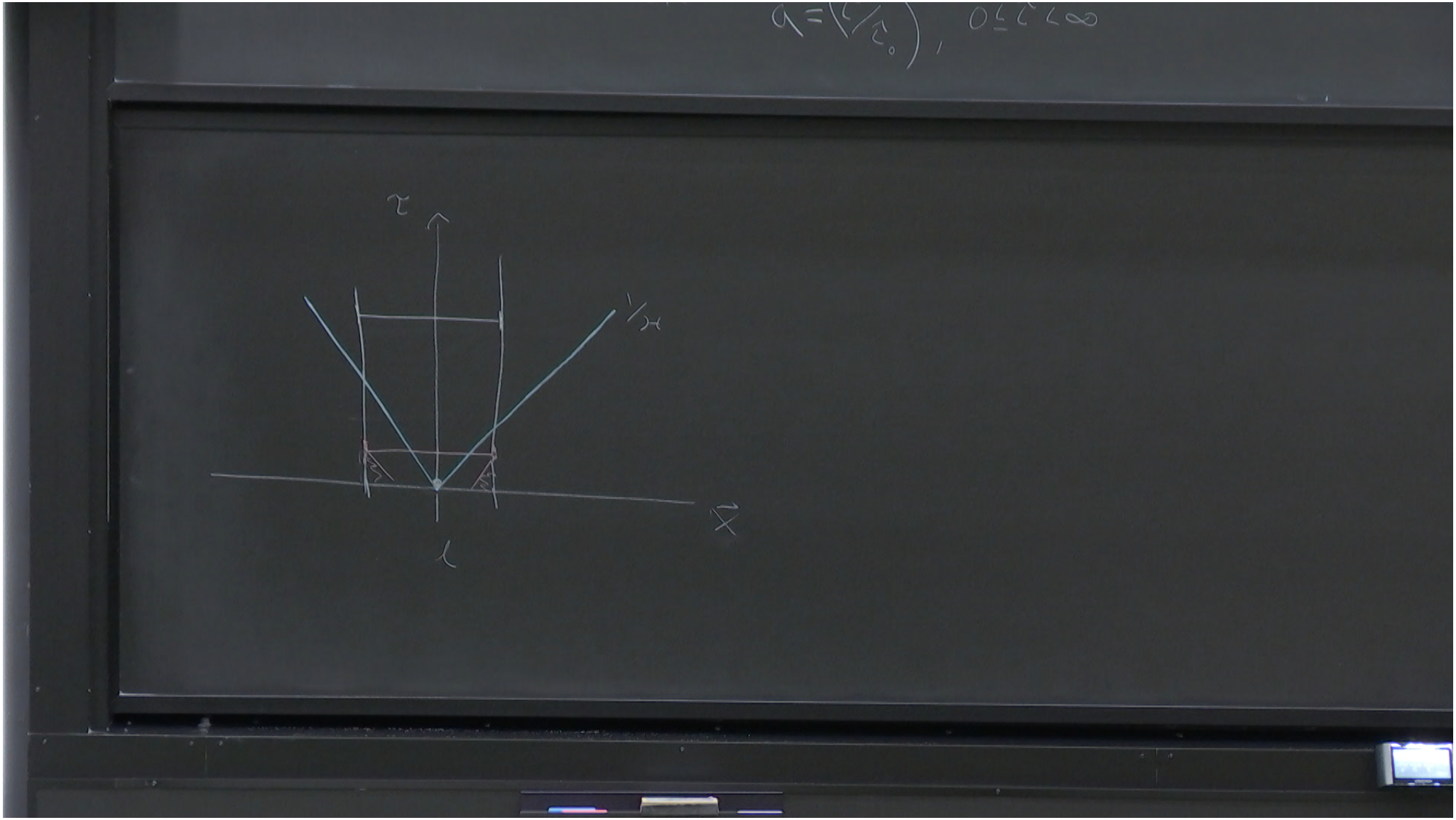
Comoving Hubble

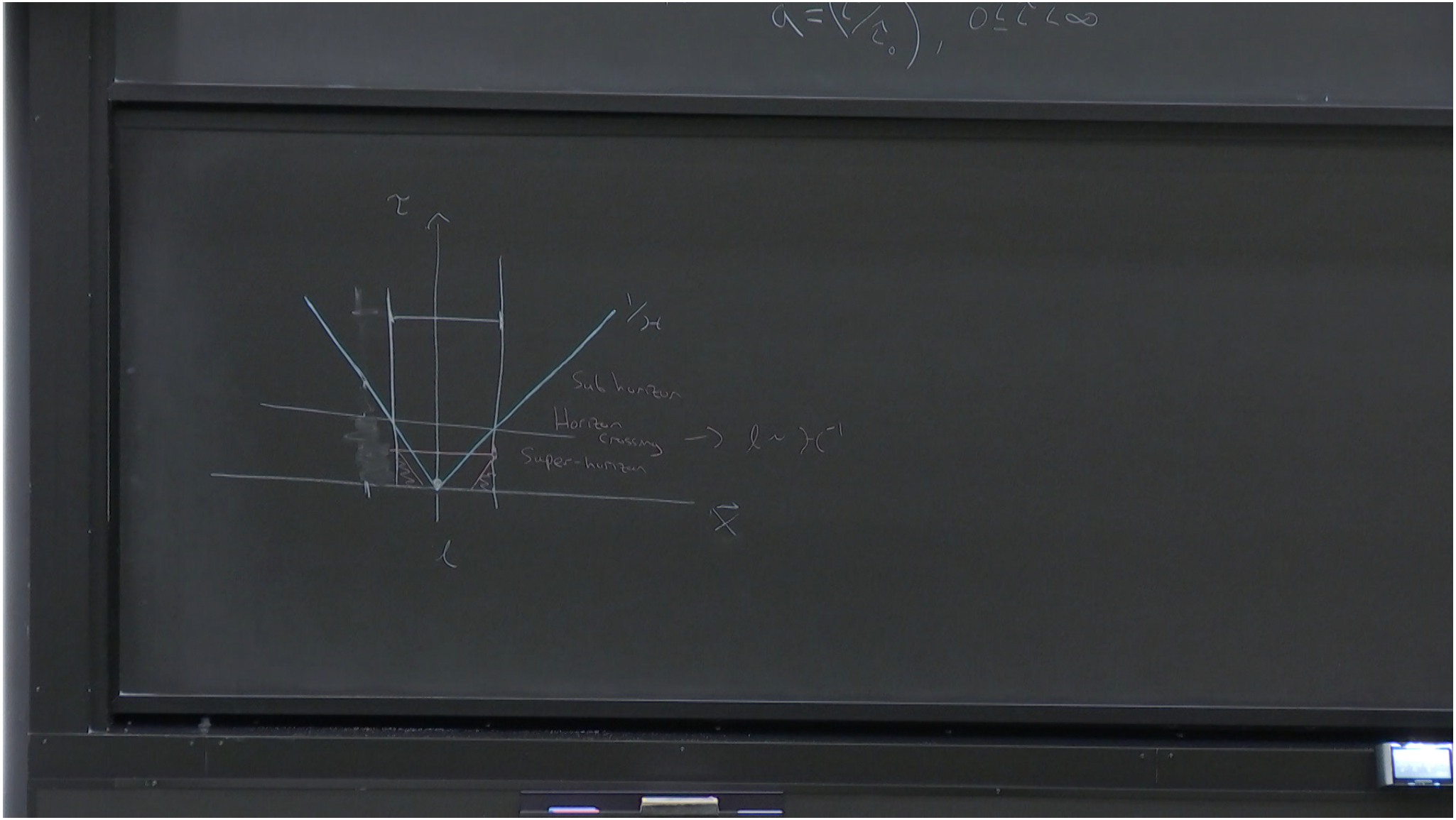
$$\mathcal{H} = \frac{da/d\tau}{a} = \frac{da/dt \; dt/d\tau}{a} = \frac{da}{dt} = aH$$

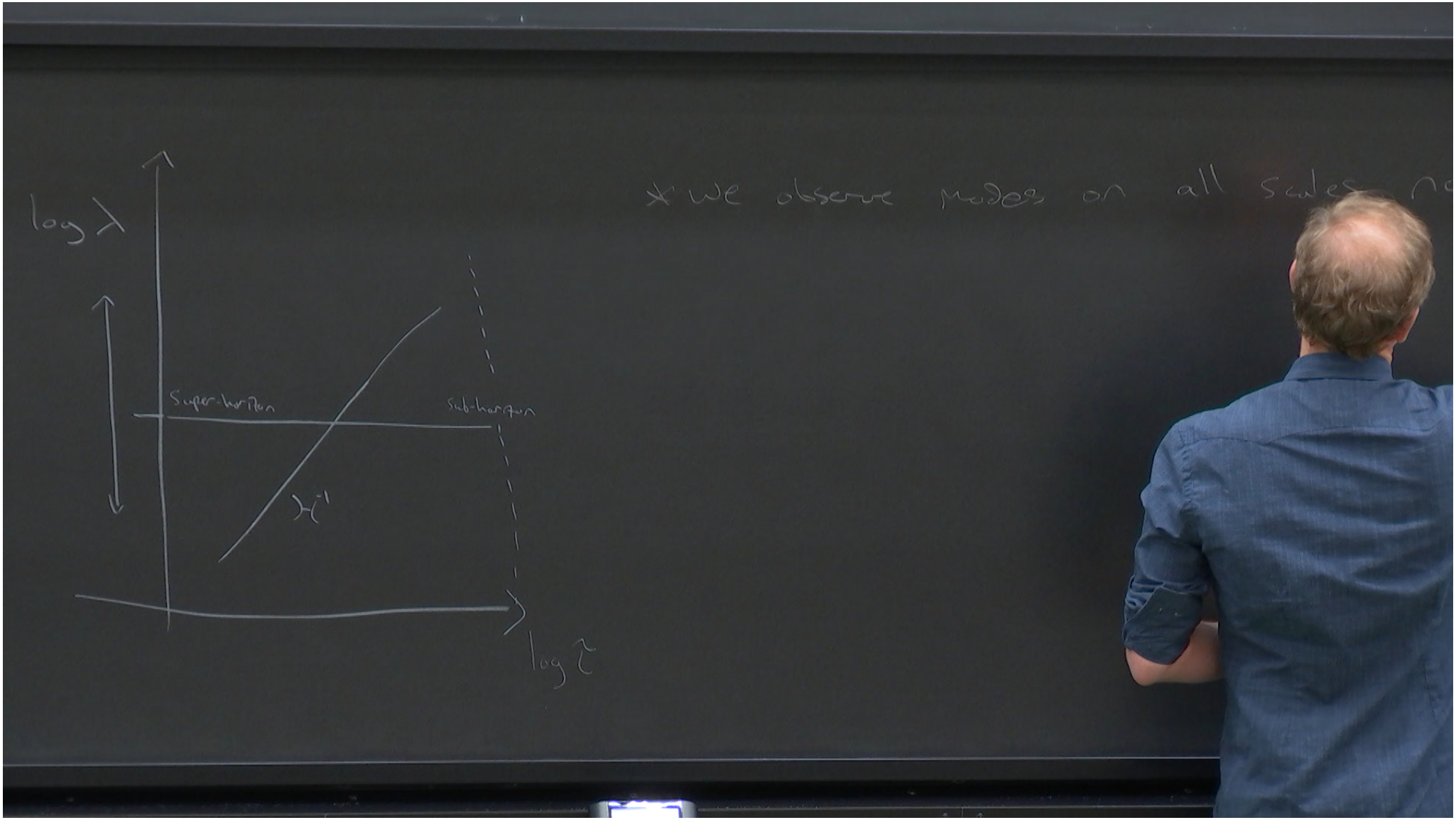
Comoving Horizon ≈

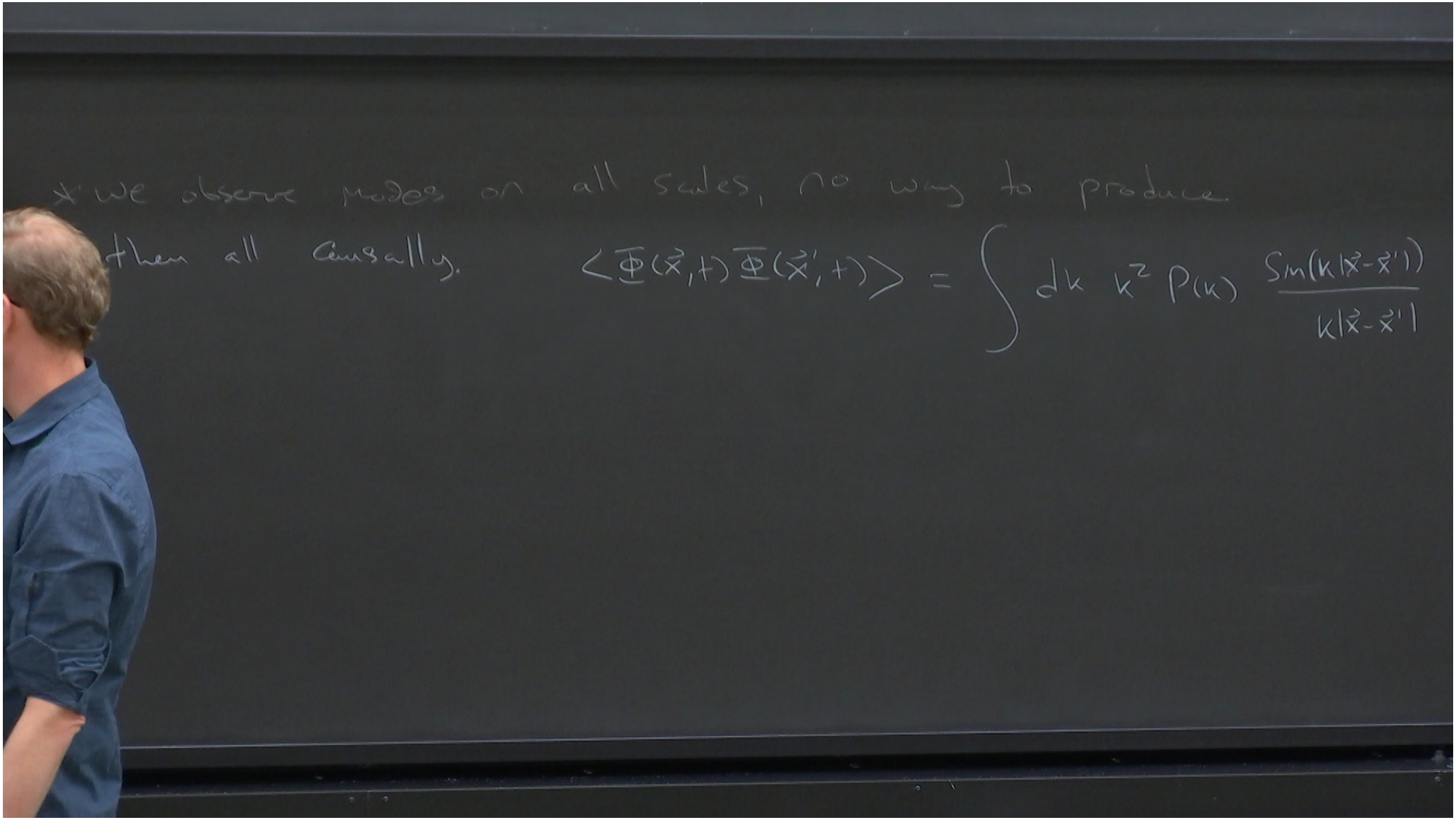
$$\mathcal{H}^{-1} = \frac{1}{aH}$$

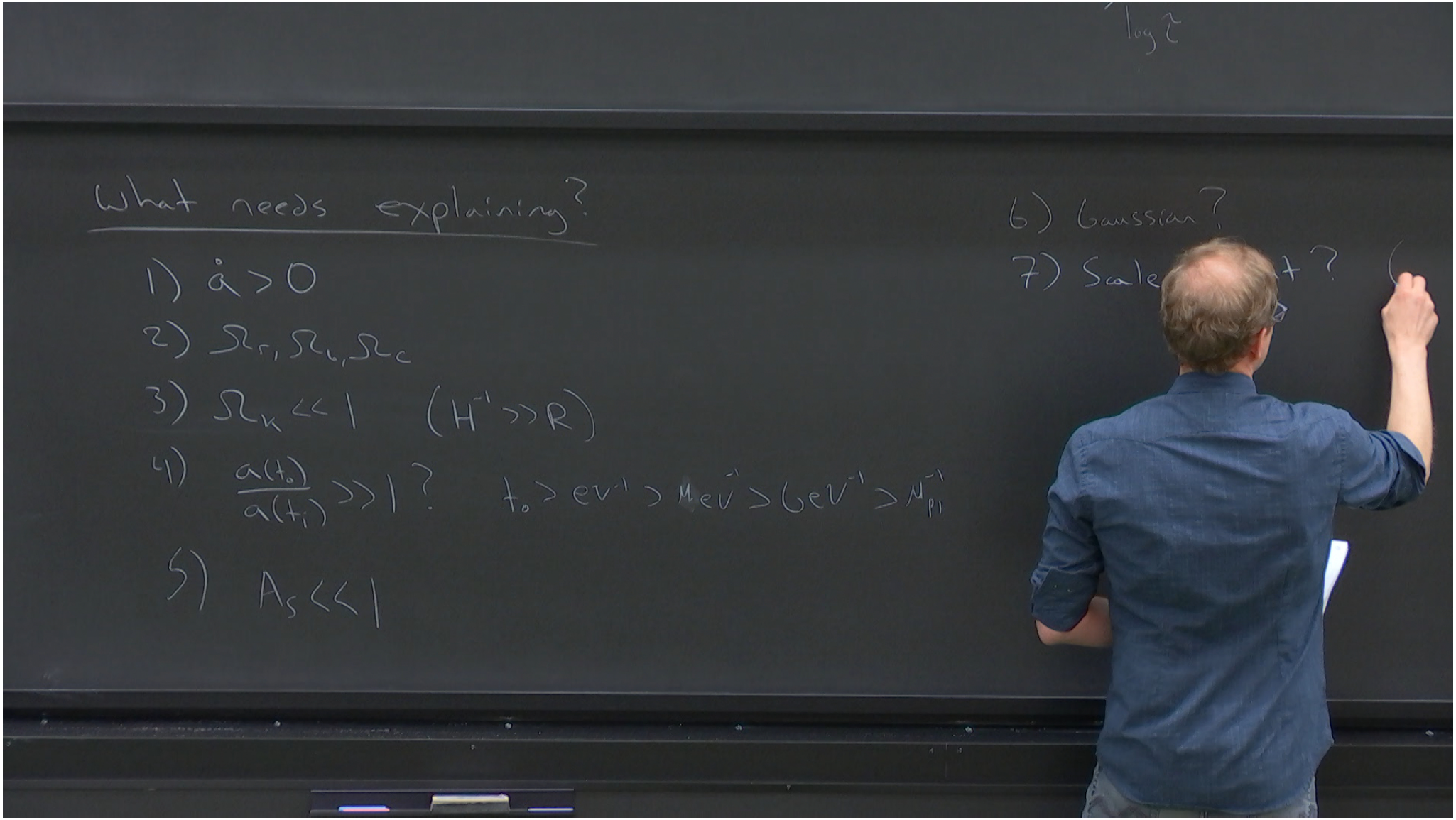
$= \tau$ in radiation dom.

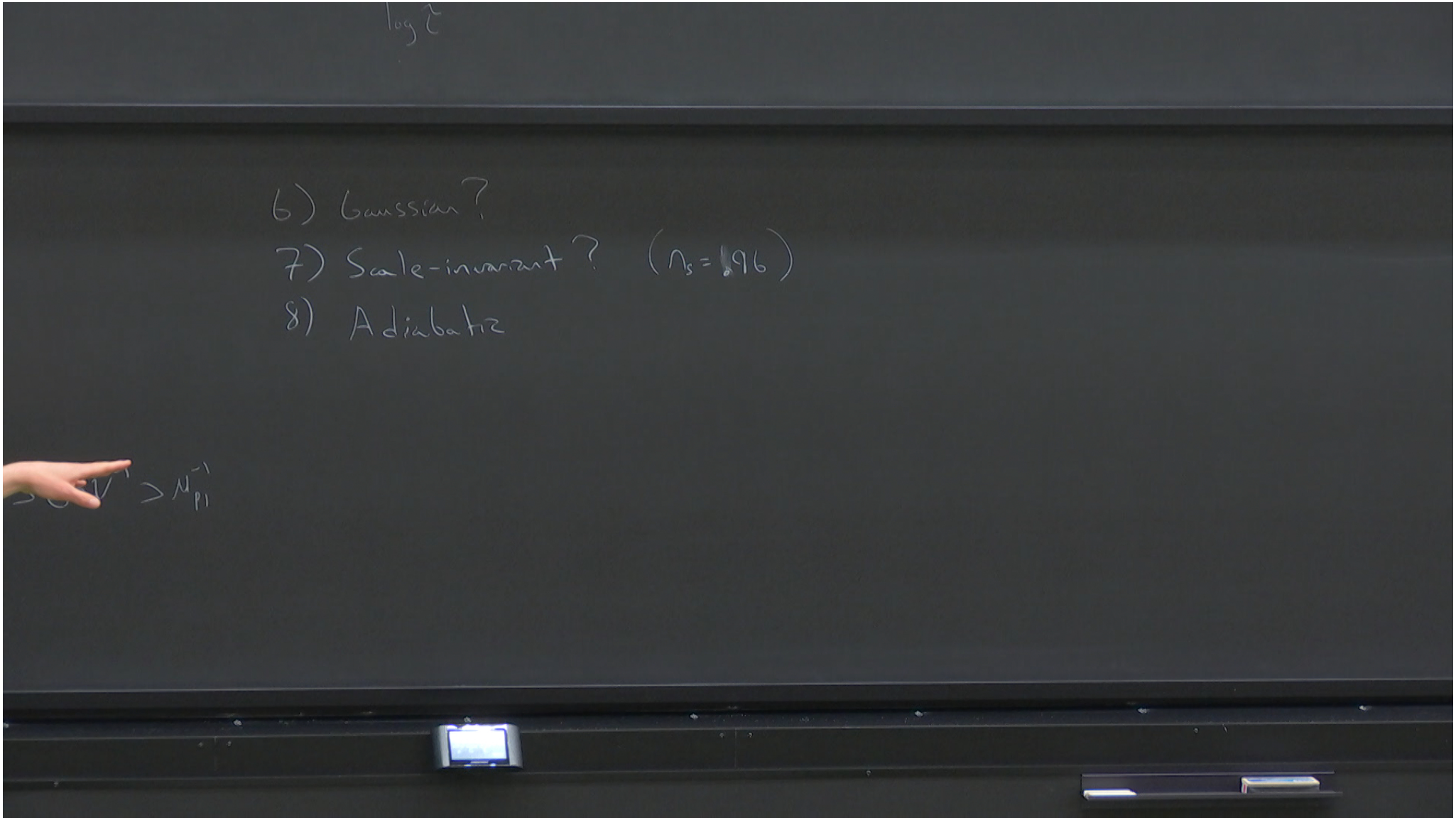

τ
1/2
x
1

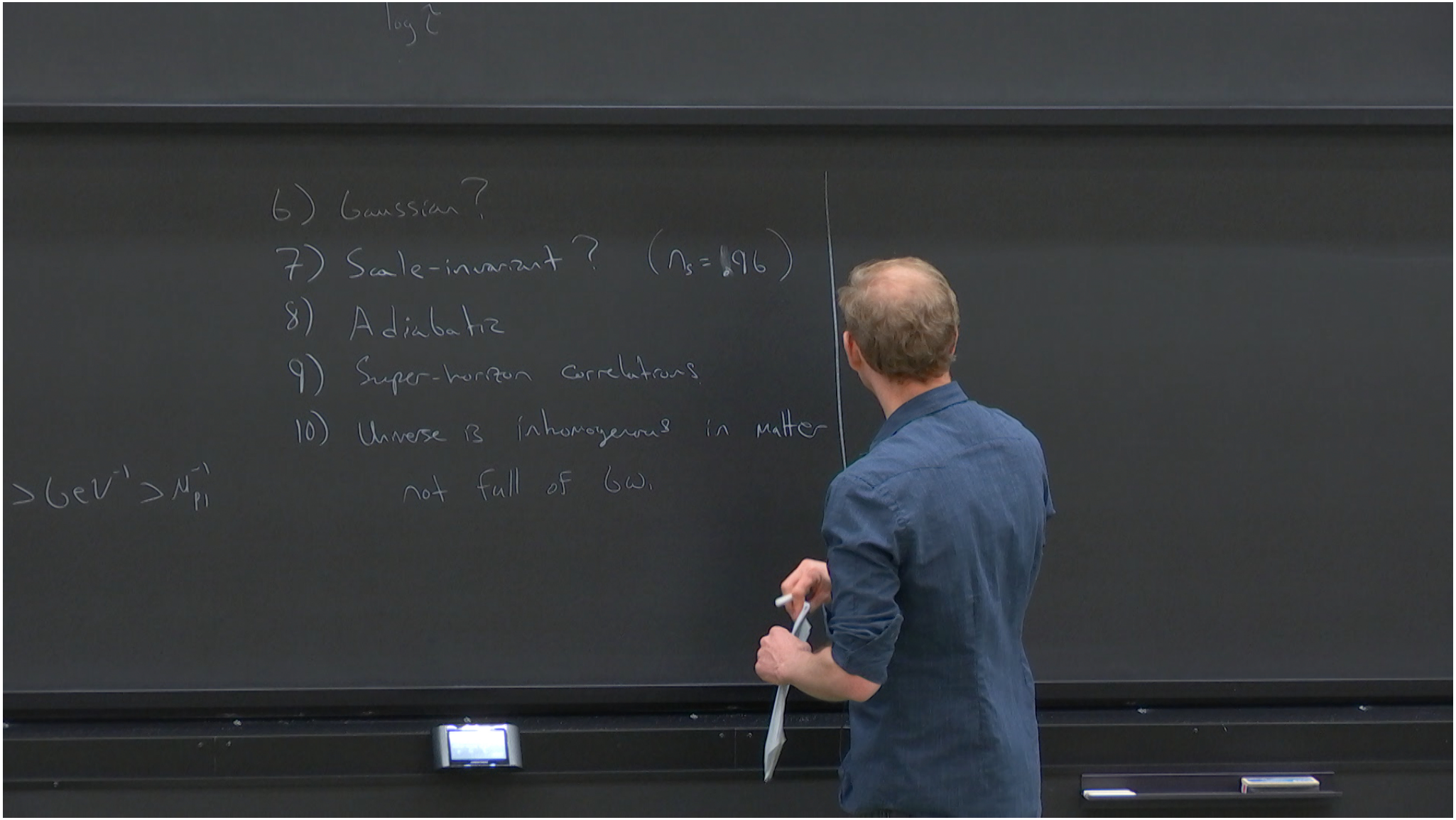

τ
1/H
Sub horizon
Horizon
Crossing
Super-horizon
x
t

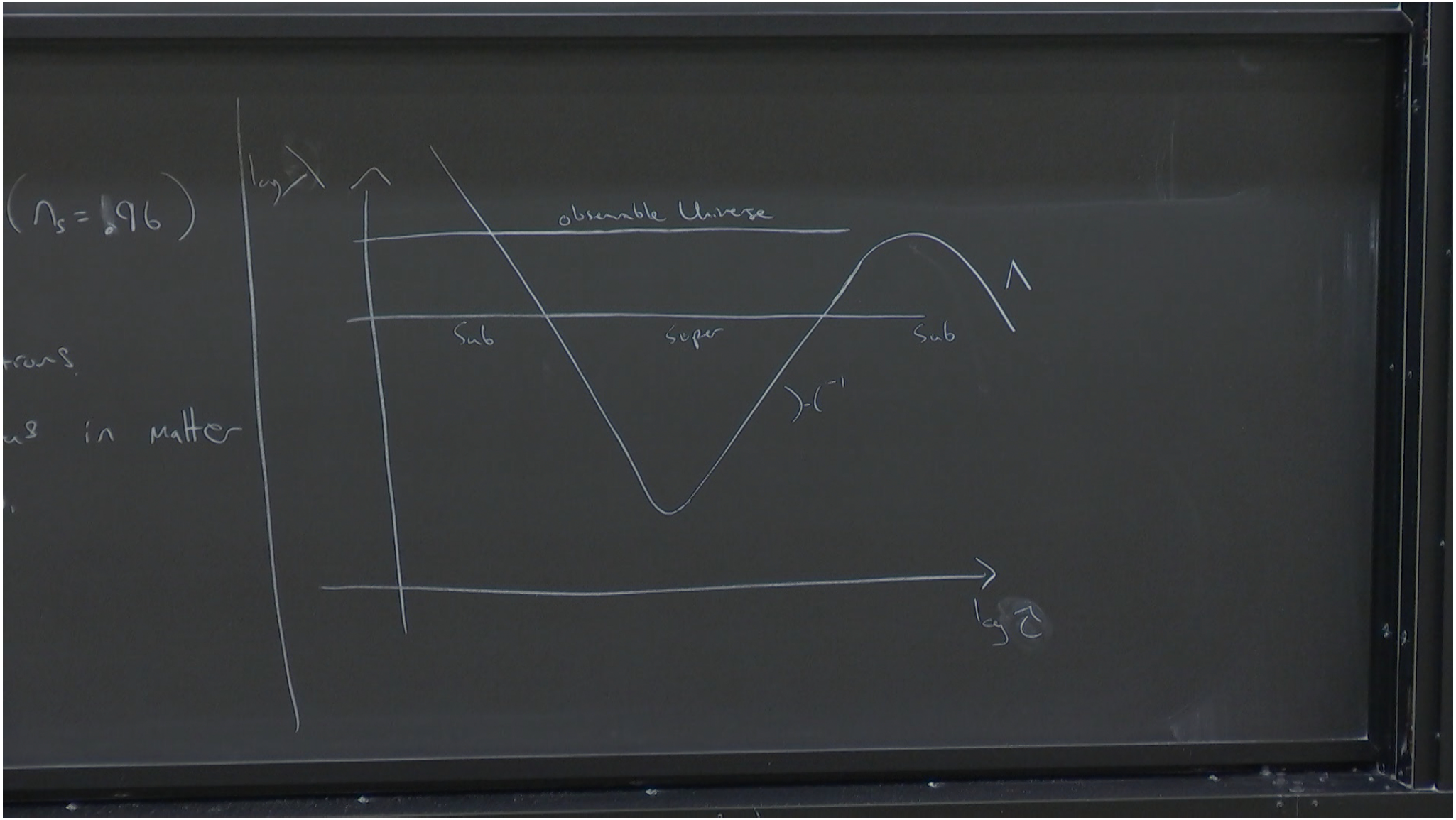

log λ
Super-horizon
sub-horizon
λ^-1
log τ
* we observe modes on all scales

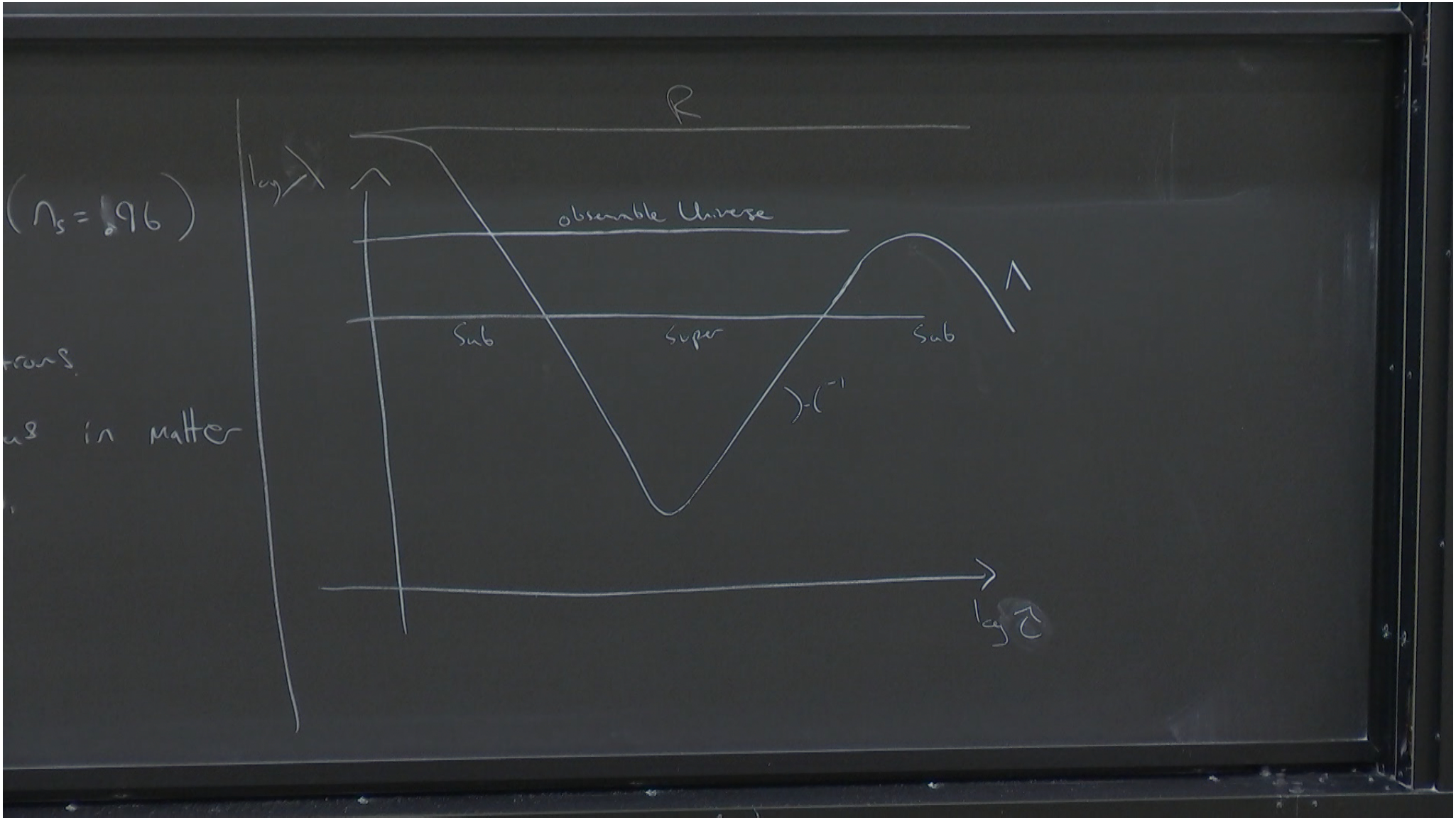

* we observe modes on all scales, no way to produce
them all causally.
$\langle \Phi(\vec{x},t)\,\Phi(\vec{x}',t)\rangle = \int dk\; k^2\, P(k)\, \frac{\sin(k|\vec{x}-\vec{x}'|)}{k|\vec{x}-\vec{x}'|}$


log t
What needs explaining?
1) $\dot{a} > 0$
2) $\Omega_r, \Omega_b, \Omega_c$
3) $\Omega_K \ll 1$ $(H^{-1} \gg R)$
4) $\frac{a(t_0)}{a(t_i)} \gg 1$? $t_0 > eV^{-1} > MeV^{-1} > GeV^{-1} > M_{Pl}^{-1}$
5) $A_S \ll 1$
6) Gaussian?
7) Scale
t ?

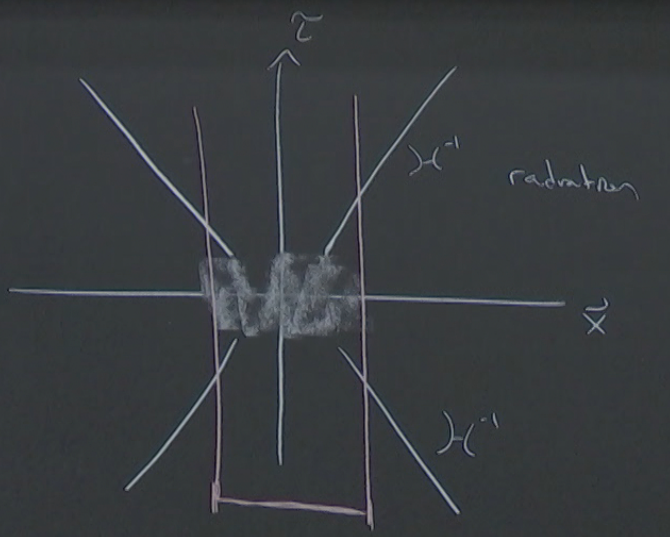

log t
6) Gaussian?
7) Scale-invariant? ($n_s = .96$)
8) Adiabatic
$> M_{Pl}^{-1}$

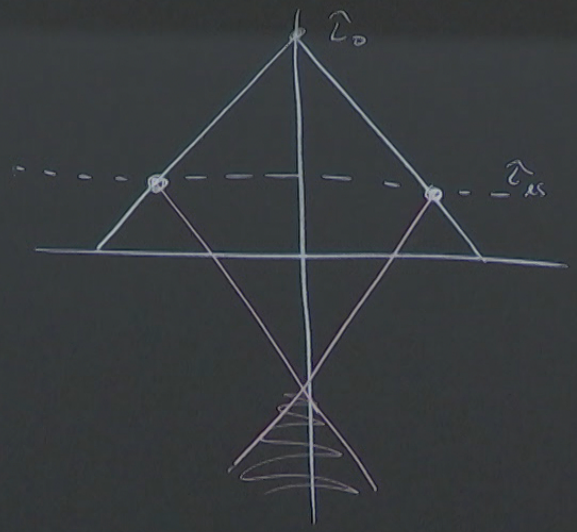

log ℓ
6) Gaussian?
7) Scale-invariant? ($n_s = .96$)
8) Adiabatic
9) Super-horizon correlations
10) Universe is inhomogeneous in matter
not full of GW.
$> GeV^{-1} > M_{Pl}^{-1}$

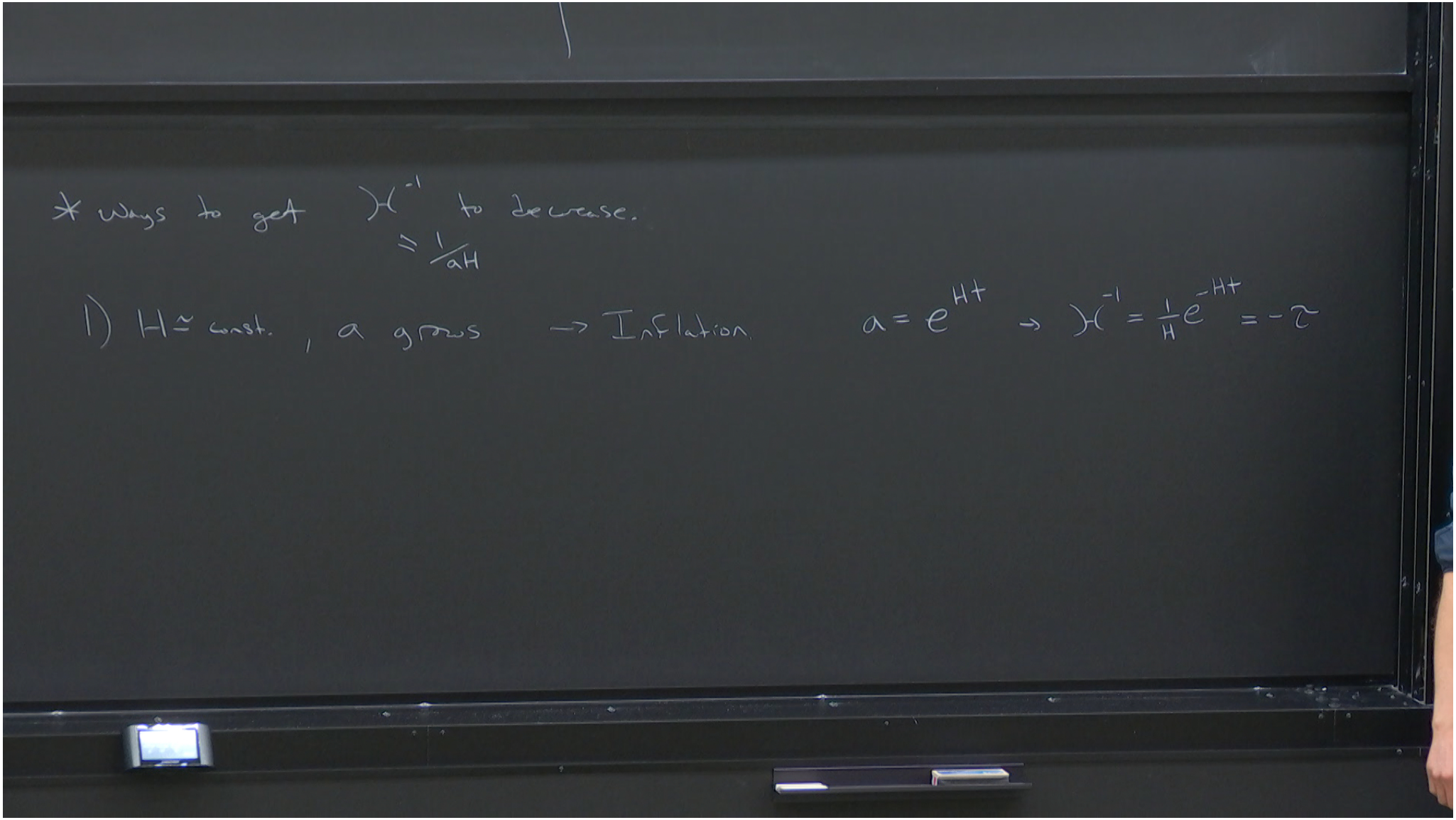

observable Universe
Sub
super
Sub


R
observable Universe
Sub
super
Sub
Λ

5) $A_s << 1$






* ways to get $\mathcal{H}^{-1}$ to decrease.
$\simeq \frac{1}{aH}$
1) $H \simeq$ const., a grows → Inflation
$a = e^{Ht}$ → $\mathcal{H}^{-1} = \frac{1}{H} e^{-Ht} = -\tau$

* ways to get $\mathcal{H}^{-1}$ to decrease.
$\simeq \frac{1}{aH}$

1) $H \simeq$ const., a grows $\rightarrow$ Inflation $\quad a = e^{Ht} \rightarrow \mathcal{H}^{-1} = \frac{1}{H} e^{-Ht} = -\tau$

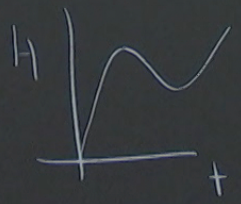
2) $a \simeq$ const., H grows $\rightarrow$ Ekpyrotic Universe

* ways to get $\mathcal{H}^{-1}$ to decrease.

$\simeq \frac{1}{aH}$

→ 1) $H \simeq$ const., a grows → Inflation $\quad a = e^{Ht} \rightarrow \mathcal{H}^{-1} = \frac{1}{H} e^{-Ht} = -\tau$

2) $a \simeq$ const., H grows → Ekpyrotic Universe



Inflationary Cosmology

1) Homogeneous nearly dS phase. - Inflation
2) Transition to radiation domination. - Reheating
3) Mechanism to produce perturbations - Particle production

Scalar fields

Inflaton

$$\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)$$

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}_\varphi \quad (=) \; 1$$

$$\mathcal{L}_\varphi \; (=) \; M^4$$

$$\varphi \; (=) \; M$$

EM-tensor

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_\varphi}{\delta g^{\mu\nu}}$$

FRW - $T^t_{\ t} =$

Scalar fields

Inflaton

$$\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)$$

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}_\varphi \quad (=) \; 1$$

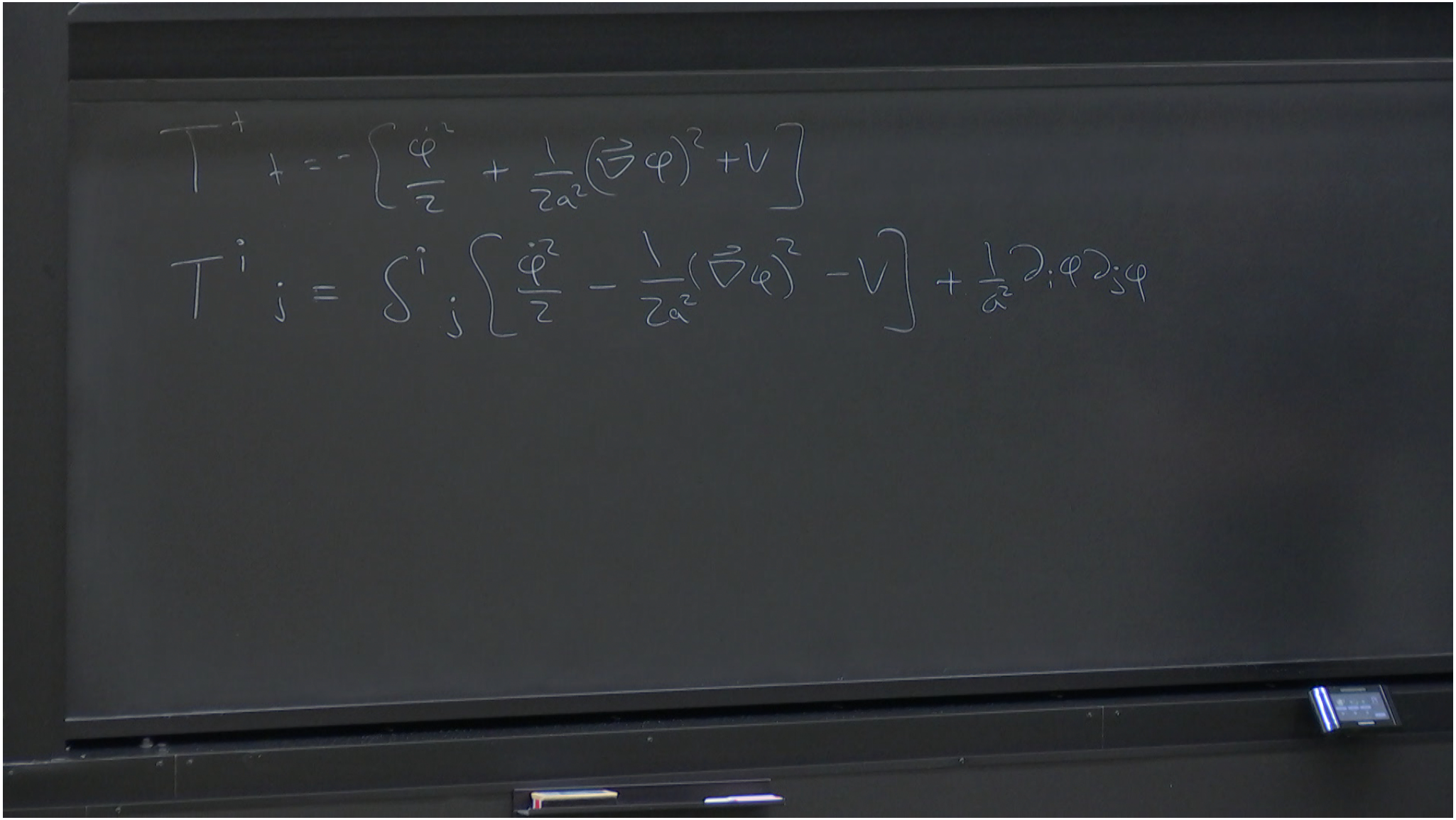
$\mathcal{L}_\varphi \; (=) \; M^4$

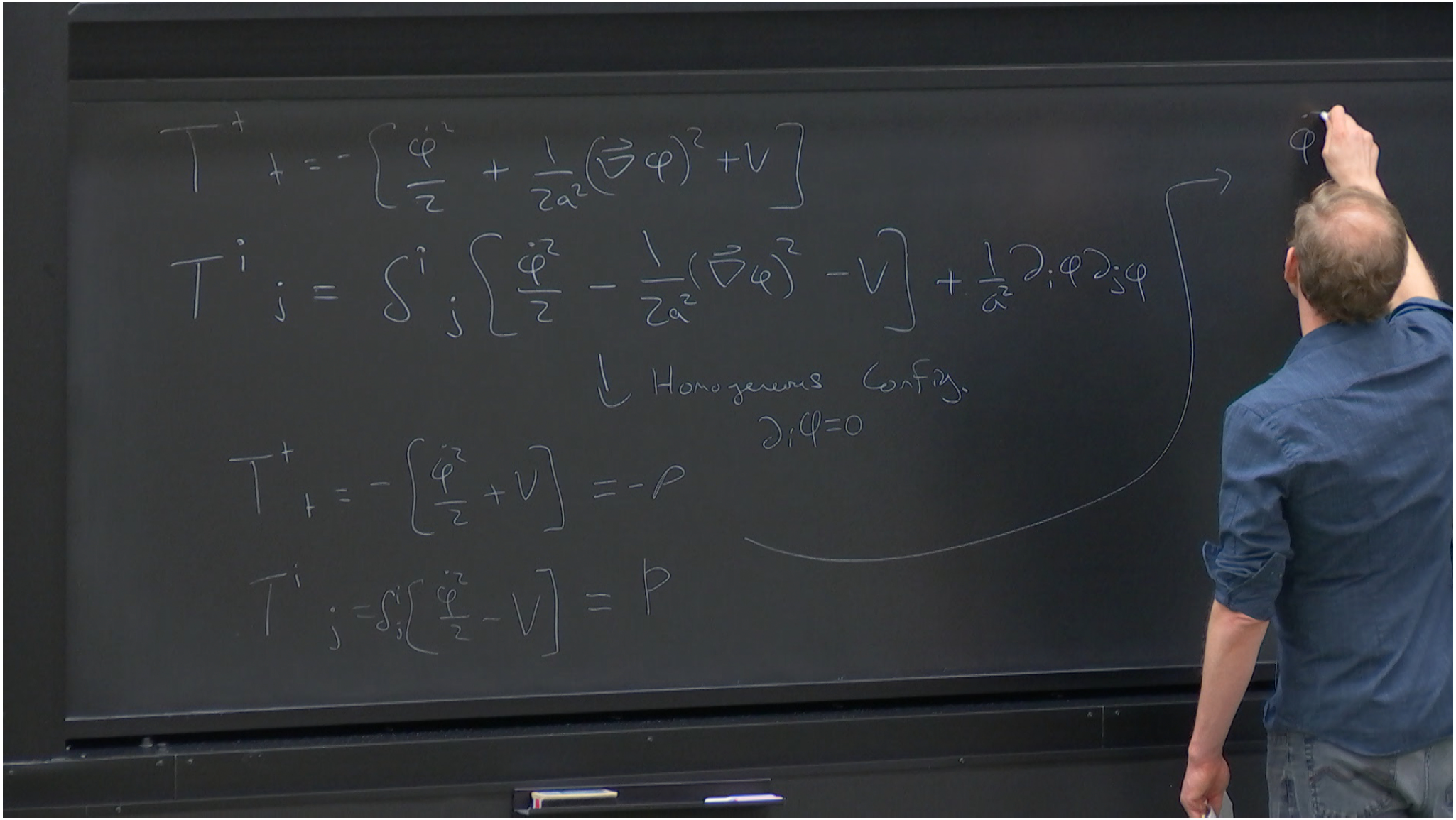
$\varphi \; (=) \; M$

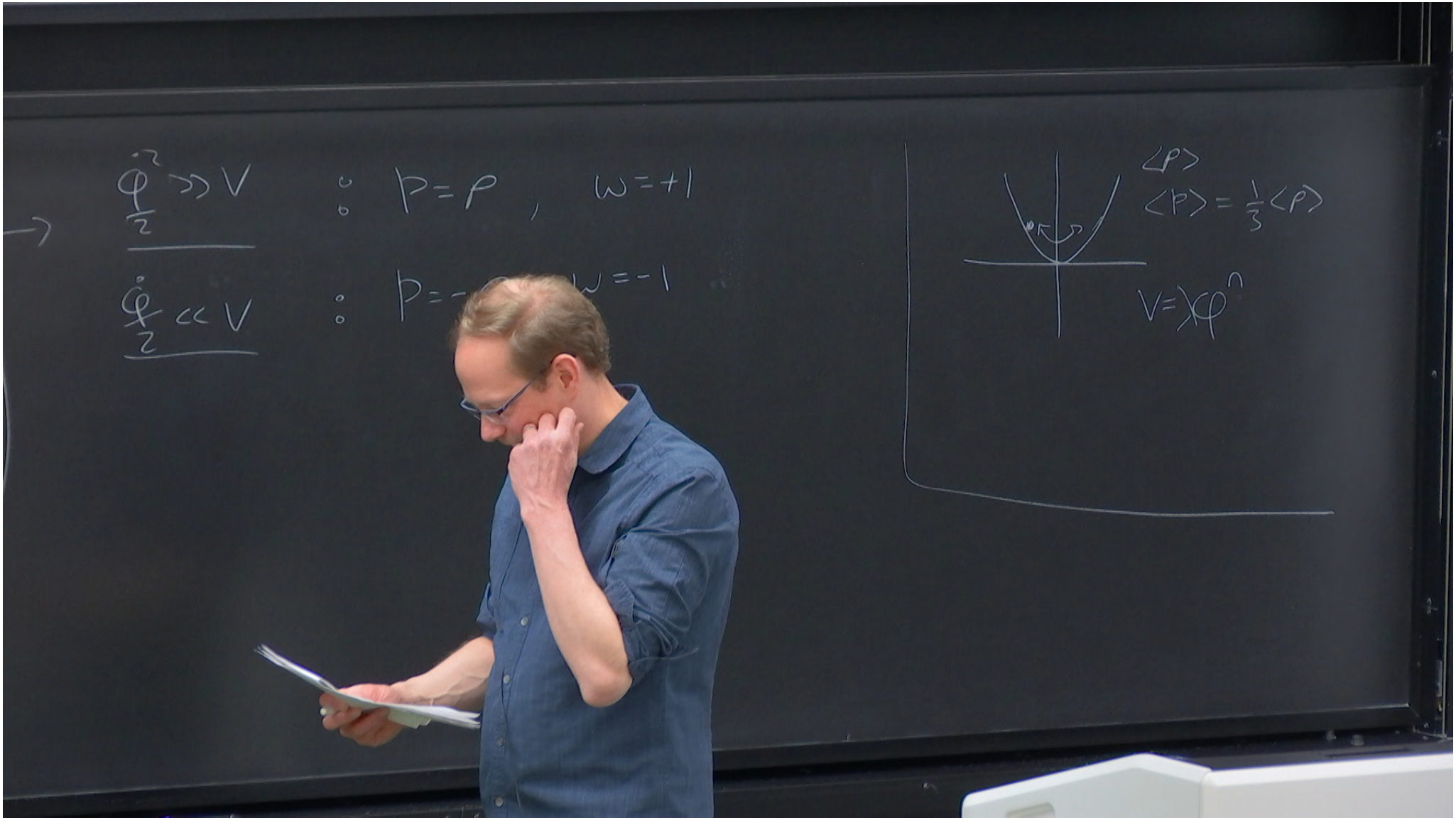
EM-tensor

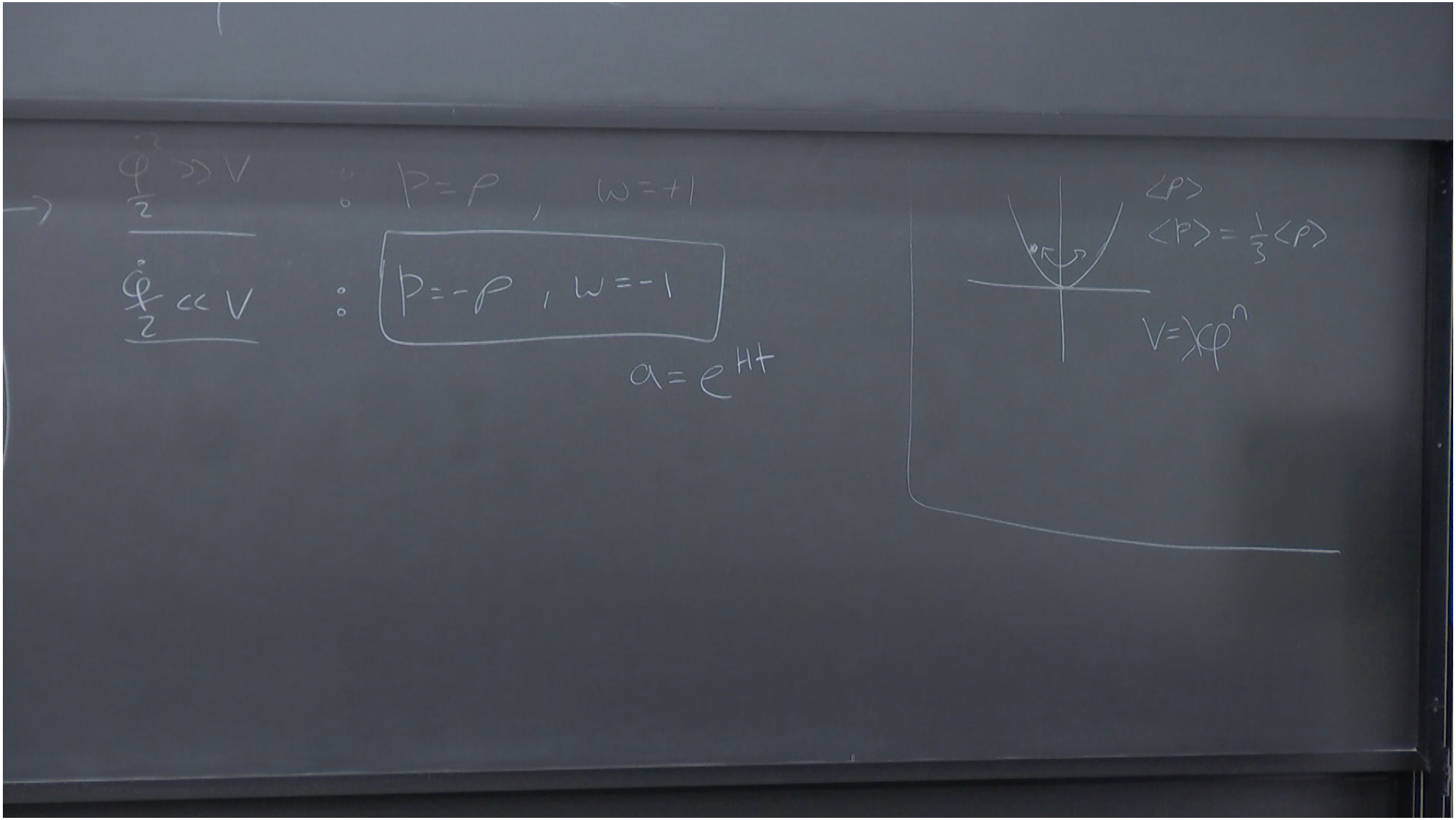
$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_\varphi}{\delta g^{\mu\nu}}$$

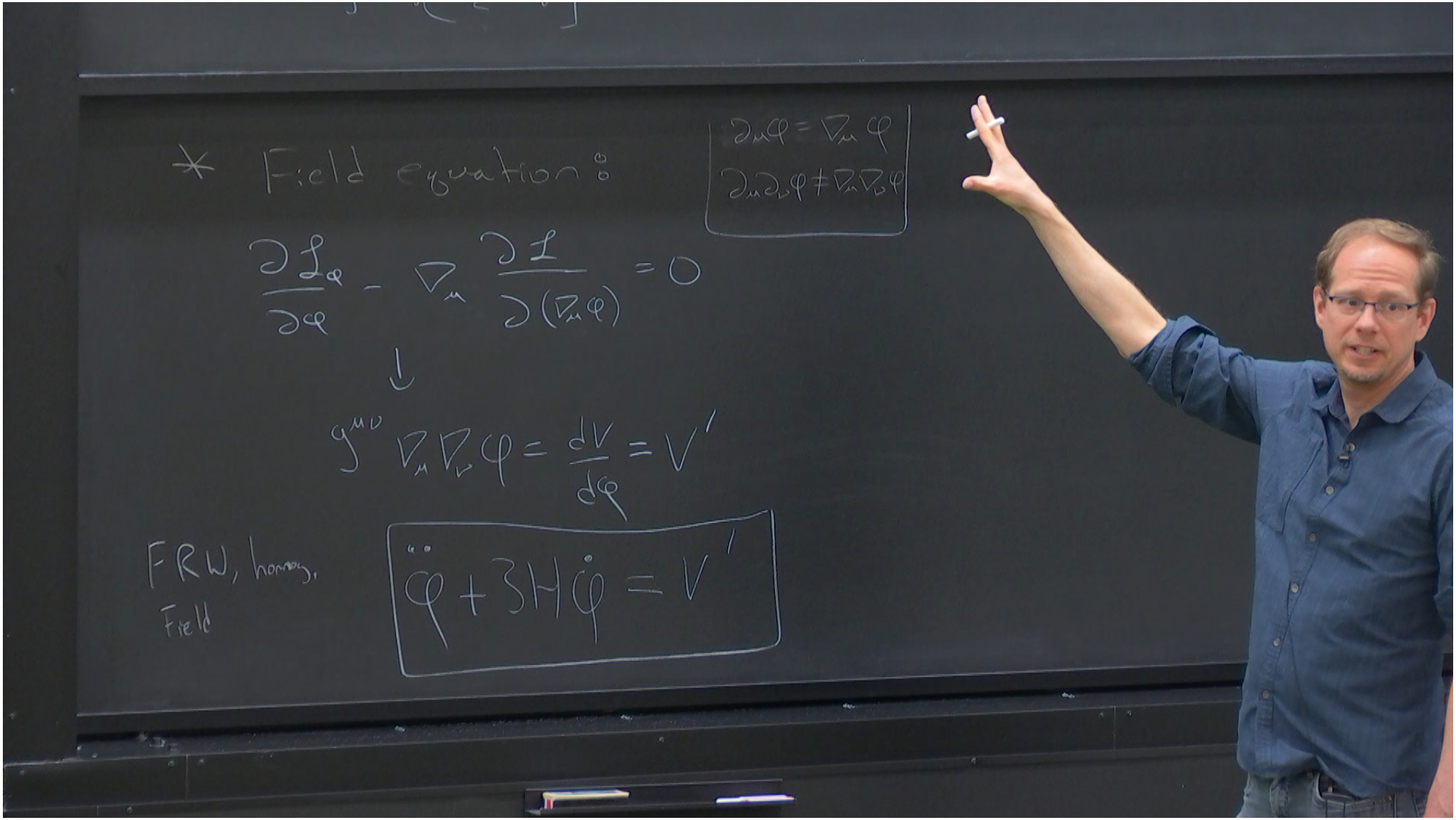
$$\text{FRW} - T^t_{\ t} = -\left[\frac{\dot{\varphi}^2}{2} + \frac{1}{2a^2} (\vec{\nabla}\varphi)^2 + V(\varphi) \right]$$


$T^0{}_0 = -\left[\frac{\dot{\varphi}^2}{2} + \frac{1}{2a^2}(\vec{\nabla}\varphi)^2 + V\right]$
$T^i{}_j = \delta^i{}_j\left[\frac{\dot{\varphi}^2}{2} - \frac{1}{2a^2}(\vec{\nabla}\varphi)^2 - V\right] + \frac{1}{a^2}\partial_i\varphi\partial_j\varphi$


$T^0{}_0 = -\left[\frac{\dot\varphi^2}{2} + \frac{1}{2a^2}(\vec\nabla\varphi)^2 + V\right]$
$T^i{}_j = \delta^i{}_j\left[\frac{\dot\varphi^2}{2} - \frac{1}{2a^2}(\vec\nabla\varphi)^2 - V\right] + \frac{1}{a^2}\partial_i\varphi\partial_j\varphi$
↓ Homogeneous Config.
$\partial_i\varphi = 0$
$T^0{}_0 = -\left[\frac{\dot\varphi^2}{2} + V\right] = -\rho$
$T^i{}_j = \delta^i{}_j\left[\frac{\dot\varphi^2}{2} - V\right] = p$
φ

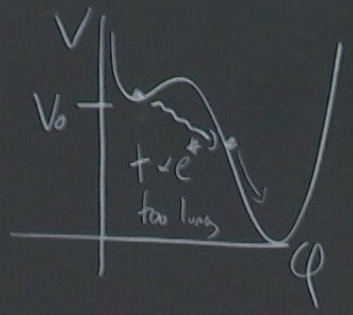

$\frac{\dot{\vec{\varphi}}}{2} \gg V$: $p = \rho$, $\omega = +1$
$\frac{\dot{\vec{\varphi}}}{2} \ll V$: $\omega = -1$
$\langle \rho \rangle$
$\langle p \rangle = \frac{1}{3} \langle \rho \rangle$
$V = \lambda \varphi^n$


$\frac{\dot\varphi^2}{2} \gg V$: $p = \rho$, $w = +1$
$\frac{\dot\varphi^2}{2} \ll V$: $p = -\rho$, $w = -1$
$a = e^{Ht}$
$\langle \rho \rangle$
$\langle p \rangle = \frac{1}{3}\langle \rho \rangle$
$V = \lambda\varphi^n$

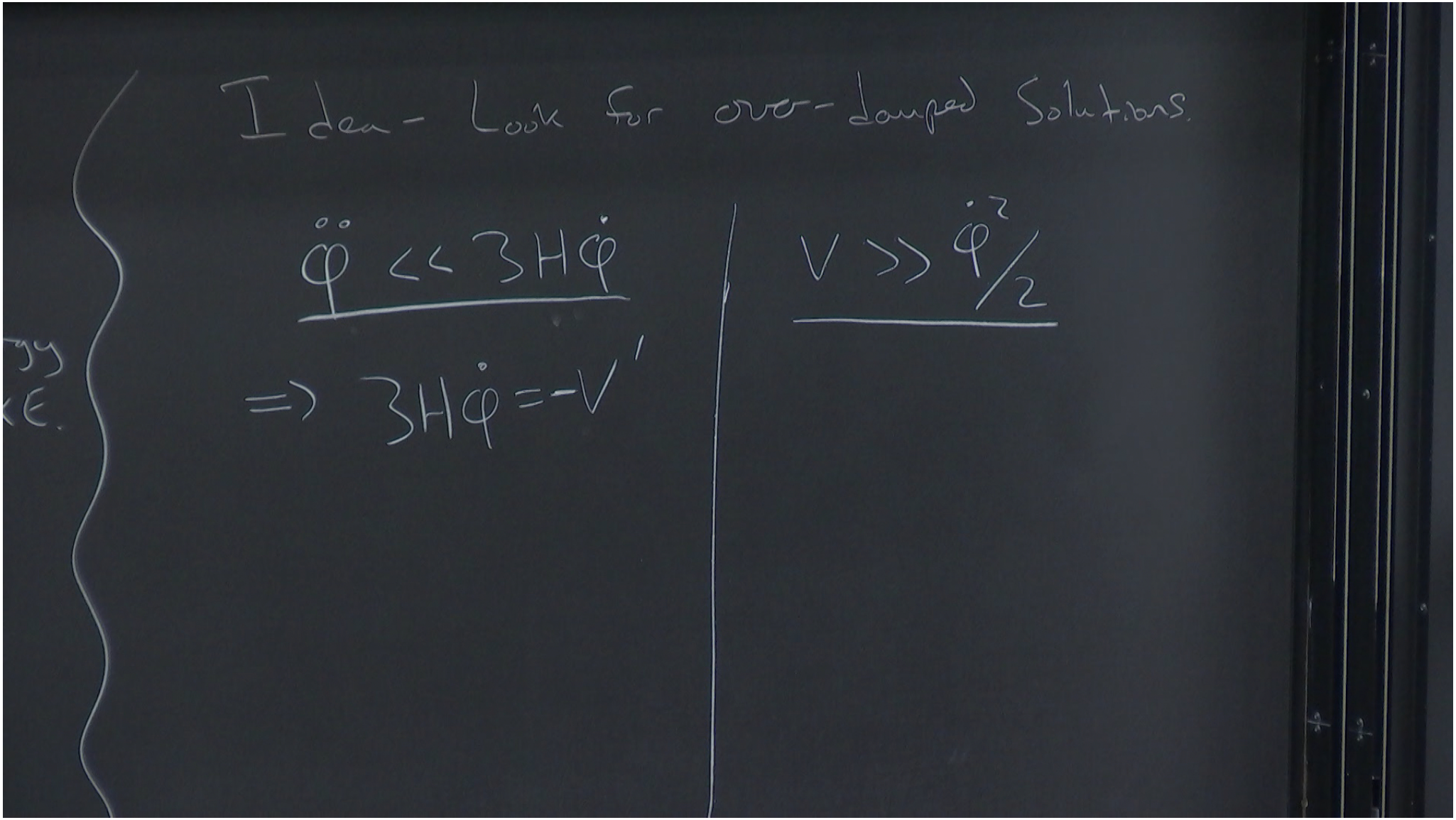

* Field equation:
$\partial_\mu \varphi = \nabla_\mu \varphi$
$\partial_\mu \partial_\nu \varphi \neq \nabla_\mu \nabla_\nu \varphi$
$\frac{\partial \mathcal{L}_\varphi}{\partial \varphi} - \nabla_\mu \frac{\partial \mathcal{L}}{\partial (\nabla_\mu \varphi)} = 0$
$g^{\mu\nu} \nabla_\mu \nabla_\nu \varphi = \frac{dV}{d\varphi} = V'$
FRW, homog.
Field
$\ddot{\varphi} + 3H\dot{\varphi} = V'$

$$H^2 = \frac{8\pi}{3}\rho_\varphi = \frac{8\pi}{3M_{Pl}^2}\left[\frac{\dot{\varphi}^2}{2} + V(\varphi)\right]$$

* Slow-roll inflation – Need Potential energy to dominate KE.



"Old Inflation"
Inflation never ends
Guth '80.


Idea - Look for over-damped Solutions.
$\ddot{\varphi} \ll 3H\dot{\varphi}$
$V \gg \dot{\varphi}^2/2$
$\Rightarrow 3H\dot{\varphi} = -V'$

Field

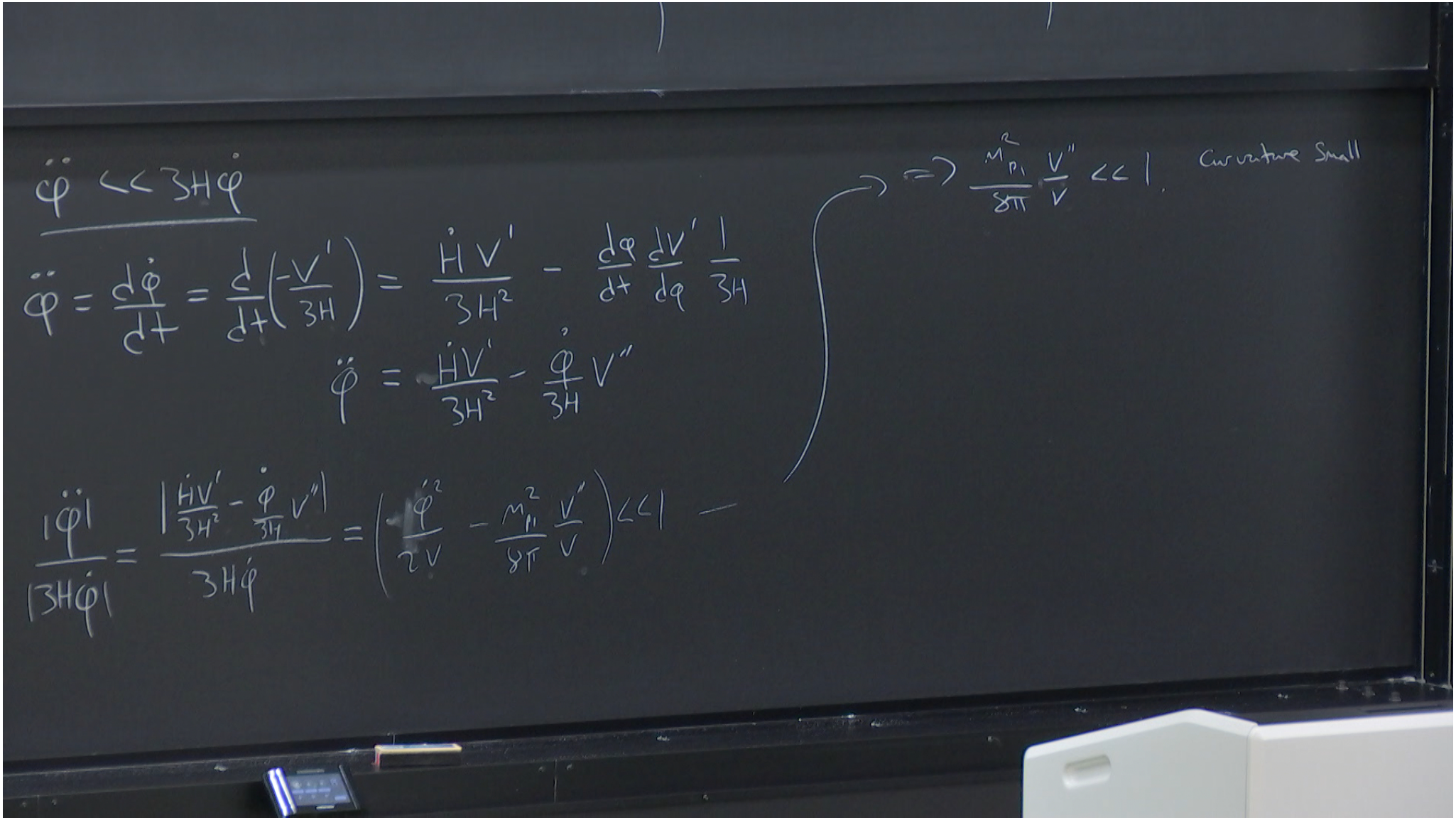
$$\ddot{\varphi} + 3H\dot{\varphi} = -V'$$

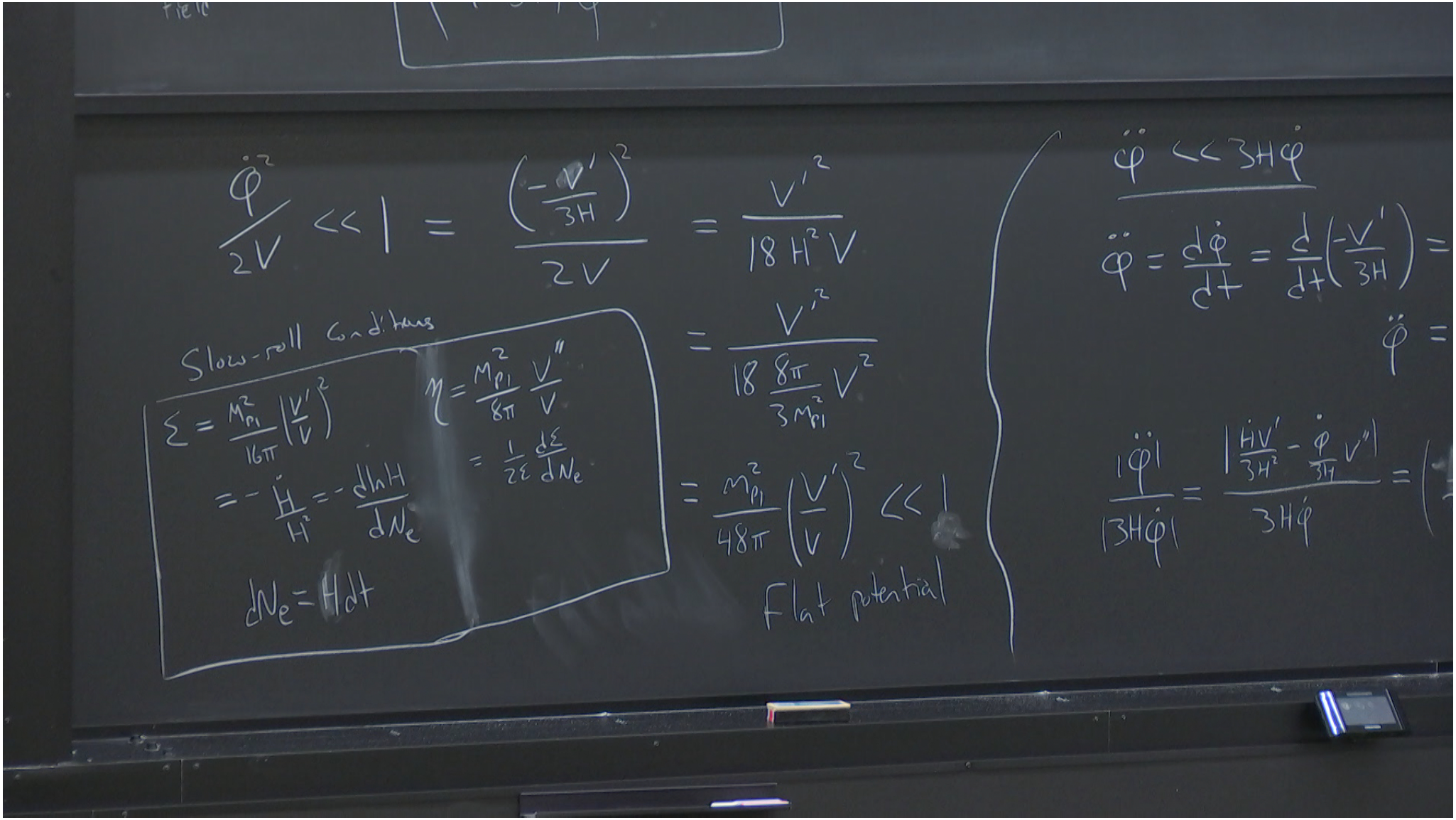
$$\frac{\dot{\varphi}^2}{2V} \ll 1 = \frac{\left(\frac{-V'}{3H}\right)^2}{2V} = \frac{V'^2}{18H^2V}$$

$$= \frac{V'^2}{18\frac{8\pi}{3M_{Pl}^2}V^2}$$

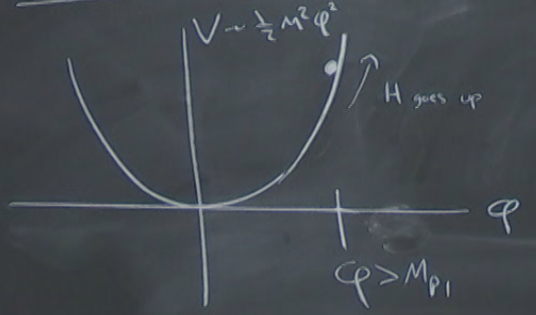
$$= \frac{M_{Pl}^2}{48\pi}\left(\frac{V'}{V}\right)^2 \ll 1$$

Flat potential


$\ddot{\varphi} \ll 3H\dot{\varphi}$
$\ddot{\varphi} = \frac{d\dot{\varphi}}{dt} = \frac{d}{dt}\left(\frac{-V'}{3H}\right) = \frac{\dot{H}V'}{3H^2} - \frac{d\varphi}{dt}\frac{dV'}{d\varphi}\frac{1}{3H}$
$\ddot{\varphi} = \frac{\dot{H}V'}{3H^2} - \frac{\dot{\varphi}}{3H}V''$
$\Rightarrow \frac{M_{Pl}^2}{8\pi}\frac{V''}{V} \ll 1$ Curvature Small
$\frac{|\ddot{\varphi}|}{|3H\dot{\varphi}|} = \frac{\left|\frac{\dot{H}V'}{3H^2} - \frac{\dot{\varphi}}{3H}V''\right|}{3H\dot{\varphi}} = \left(\frac{\dot{\varphi}^2}{2V} - \frac{M_{Pl}^2}{8\pi}\frac{V''}{V}\right) \ll 1$ —


$\frac{\dot\varphi^2}{2V} \ll 1 = \frac{\left(-\frac{V'}{3H}\right)^2}{2V} = \frac{V'^2}{18H^2V}$
$= \frac{V'^2}{18\frac{8\pi}{3M_{Pl}^2}V^2}$
$= \frac{M_{Pl}^2}{48\pi}\left(\frac{V'}{V}\right)^2 \ll 1$
flat potential
Slow-roll conditions
$\varepsilon = \frac{M_{Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2$
$= -\frac{\dot H}{H^2} = -\frac{d\ln H}{dN_e}$
$dN_e = H dt$
$\eta = \frac{M_{Pl}^2}{8\pi}\frac{V''}{V}$
$= \frac{1}{2\varepsilon}\frac{d\varepsilon}{dN_e}$
$\ddot\varphi \ll 3H\dot\varphi$
$\ddot\varphi = \frac{d\dot\varphi}{dt} = \frac{d}{dt}\left(-\frac{V'}{3H}\right) =$
$\ddot\varphi =$
$\frac{|\ddot\varphi|}{|3H\dot\varphi|} = \frac{\left|\frac{\dot H V'}{3H^2} - \frac{\dot\varphi}{3H}V''\right|}{3H\dot\varphi} =$

Types of Inflaton Potentials:



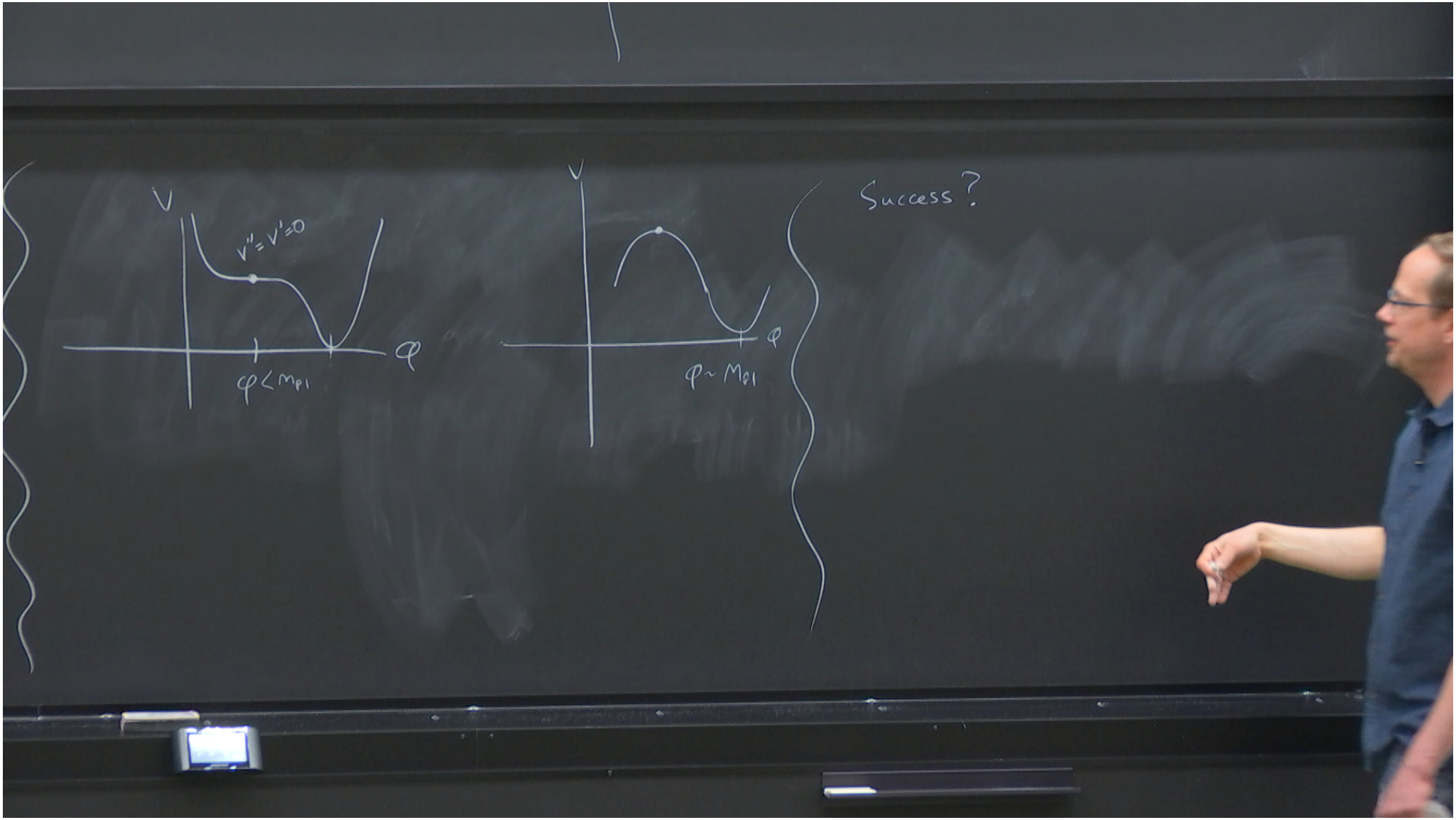
$$V = \lambda \varphi^n$$

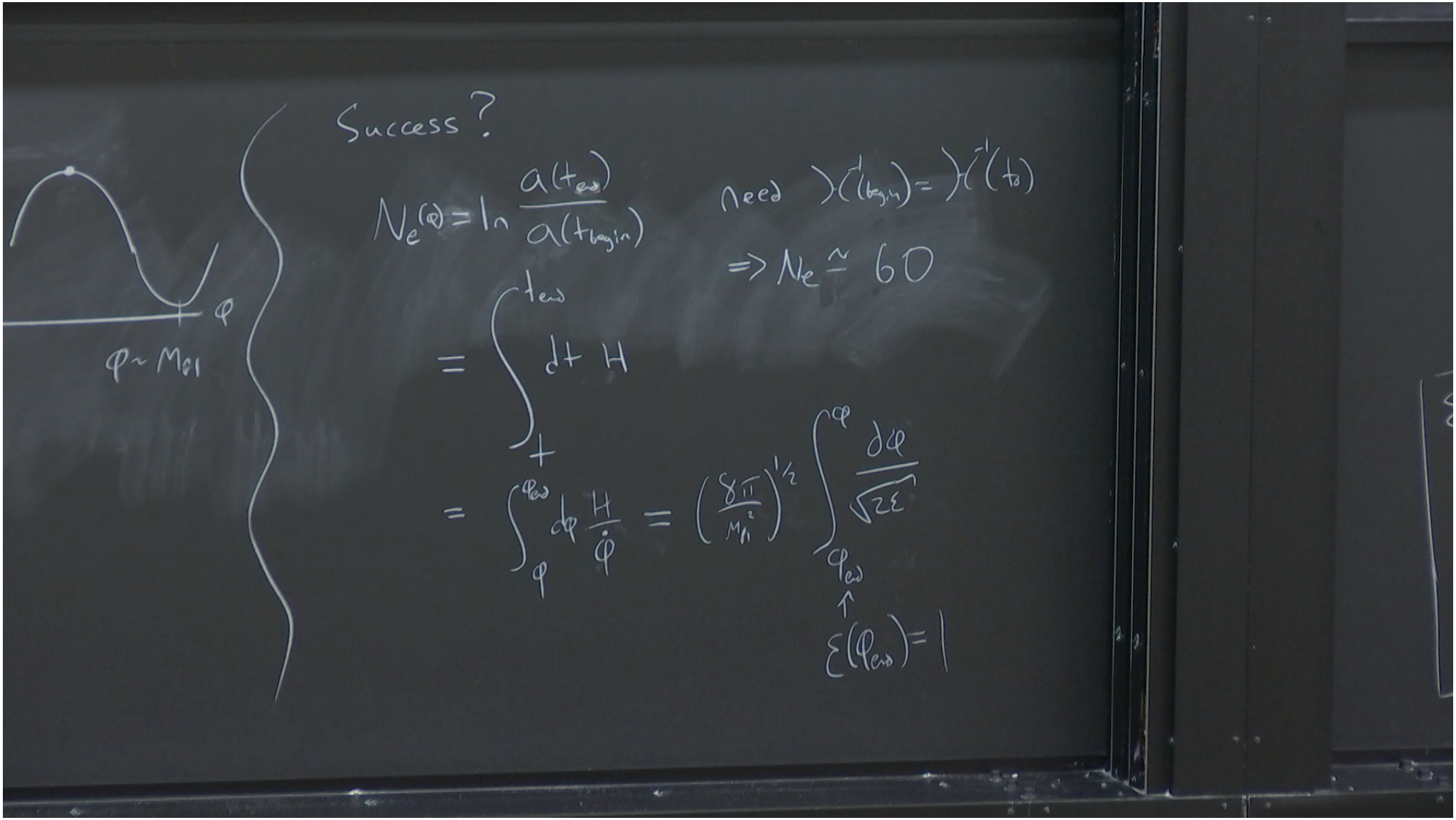
$$\varepsilon = \frac{M_{Pl}^2}{16\pi} \left(\frac{n \lambda \varphi^{n-1}}{\lambda \varphi^n} \right)^2 = \frac{M_{Pl}^2}{16\pi} \frac{n^2}{\varphi^2}$$

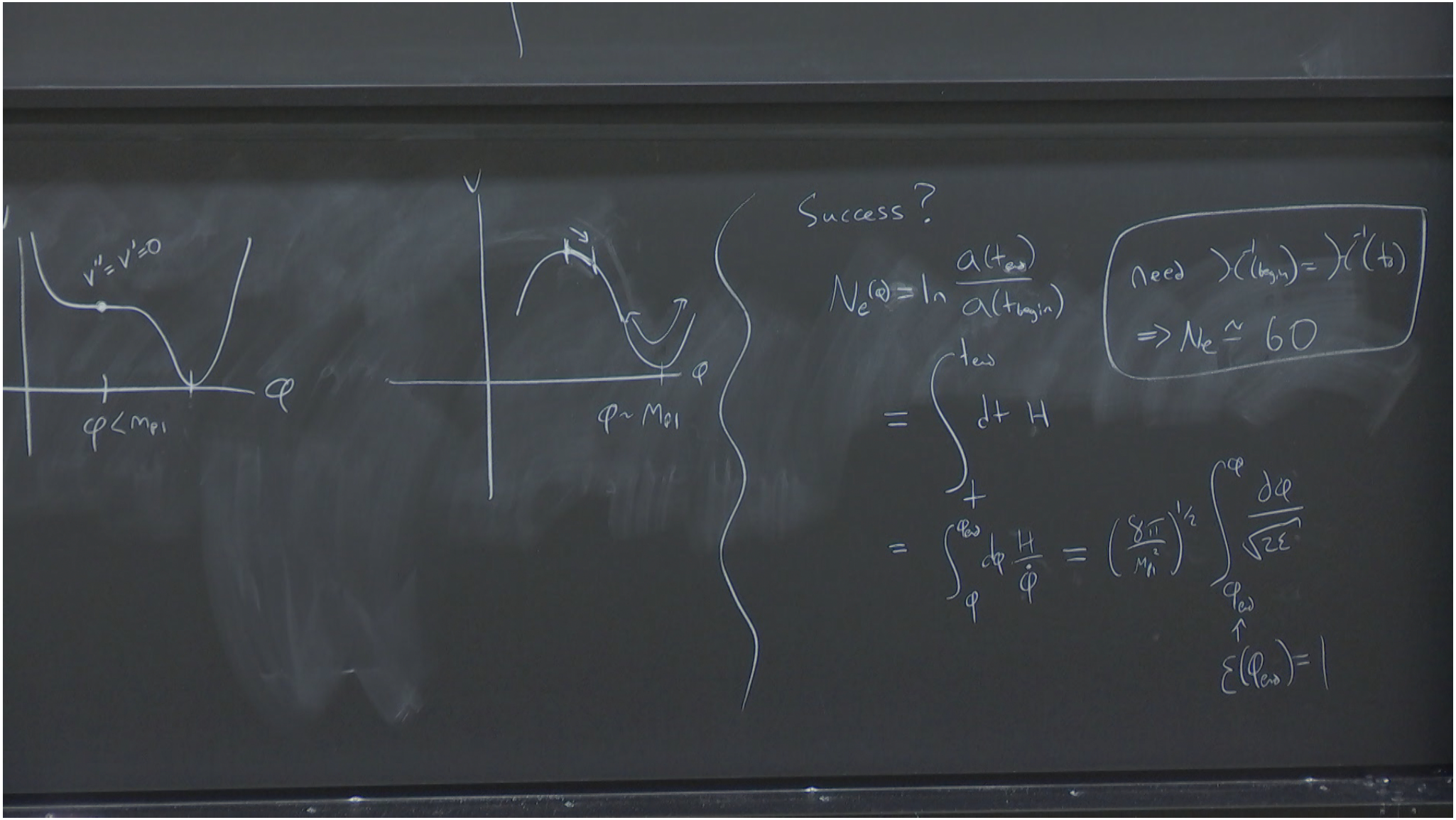
* ways

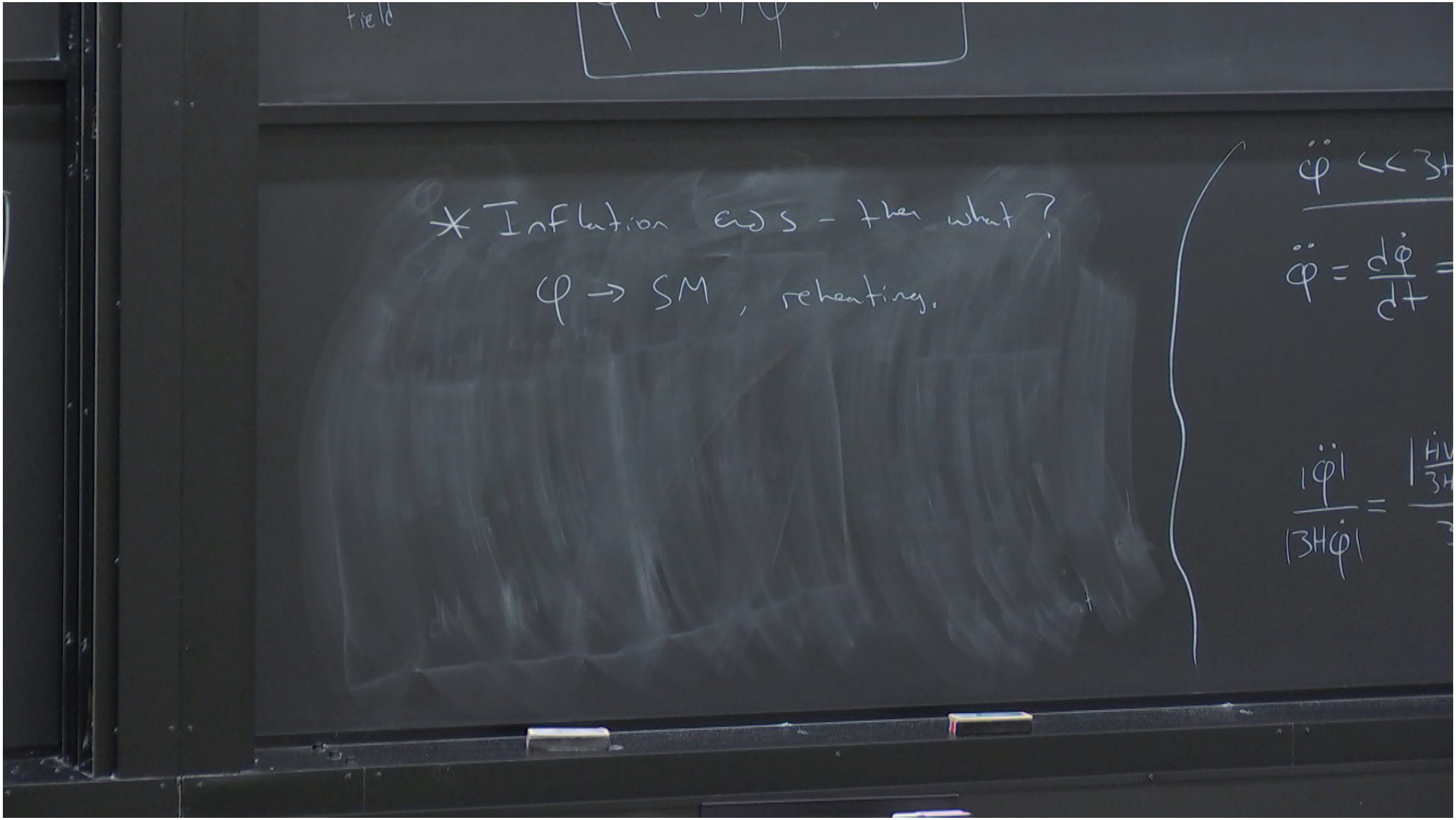
→ 1) $H \simeq$

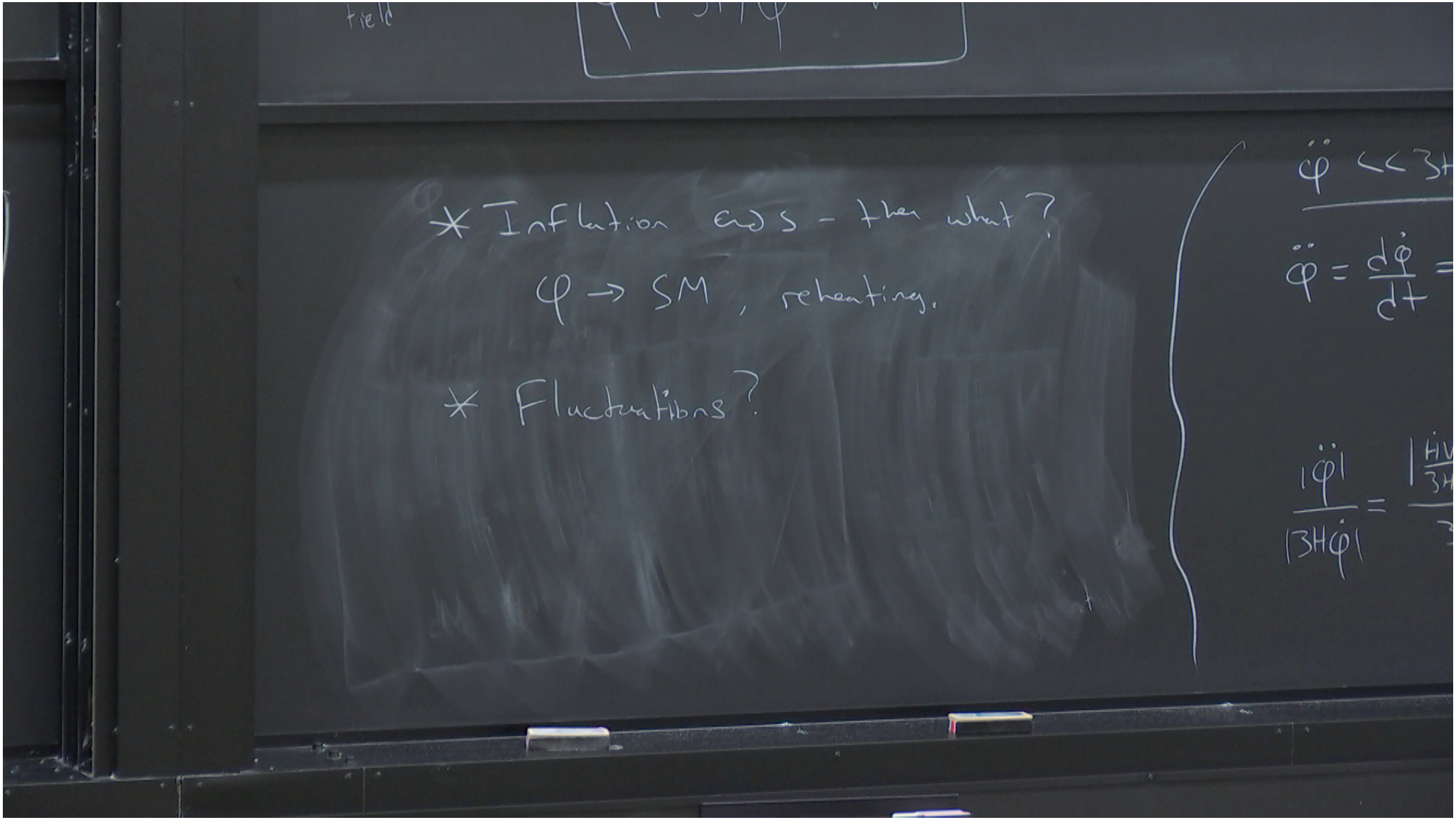
2) $a \simeq$


V
V'=V''=0
φ
φ<M_Pl
V
φ
φ~M_Pl
Success?


Success?
$N_e(\varphi) = \ln \frac{a(t_{end})}{a(t_{begin})}$
need $\chi^{-1}(t_{begin}) = \chi^{-1}(t_0)$
$\Rightarrow N_e \simeq 60$
$= \int_{t}^{t_{end}} dt\, H$
$= \int_{\varphi}^{\varphi_{end}} d\varphi \frac{H}{\dot\varphi} = \left(\frac{8\pi}{M_{Pl}^2}\right)^{1/2} \int_{\varphi_{end}}^{\varphi} \frac{d\varphi}{\sqrt{2\epsilon}}$
$\epsilon(\varphi_{end}) = 1$
φ
$\varphi \sim M_{Pl}$


V'' = V' = 0
φ
φ < M_Pl
V
φ
φ ~ M_Pl
Success?
N_e(φ) = ln a(t_end) / a(t_beginning)
= ∫ dt H
= ∫ dφ H/φ̇ = (8π/M_Pl^2)^{1/2} ∫ dφ/√(2ε)
need χ(t_beginning) = χ(t_0)
⇒ N_e ≃ 60
ε(φ_end) = 1


field
* Inflation ends - then what?
φ → SM, reheating.
φ̈ << 3H
φ̈ = dφ̇/dt =
|φ̈|/|3Hφ̇| =


field
* Inflation ends - then what?
φ → SM, reheating.
* Fluctuations?
φ̈ <<
φ̈ = dφ̇/dt