Title: PSI 2019/2020 - Cosmology Part 1 - Lecture 7

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Collection: PSI 2019/2020 - Cosmology (Part 1)

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## Boltzmann eq'n

$$a^{-3}\frac{d(a^3 n_1)}{dt} = n_1^{(0)}\,\Gamma\left[\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}\right]$$

$\Gamma >> H$ : Saha eqlbm $\quad \dfrac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} = \dfrac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}$

$\Gamma << H$ : Freeze out $\quad \dfrac{d(a^3 n_1)}{dt} = 0$

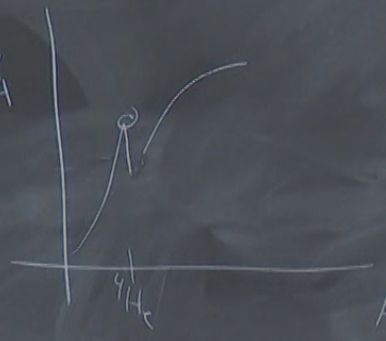
# BBN

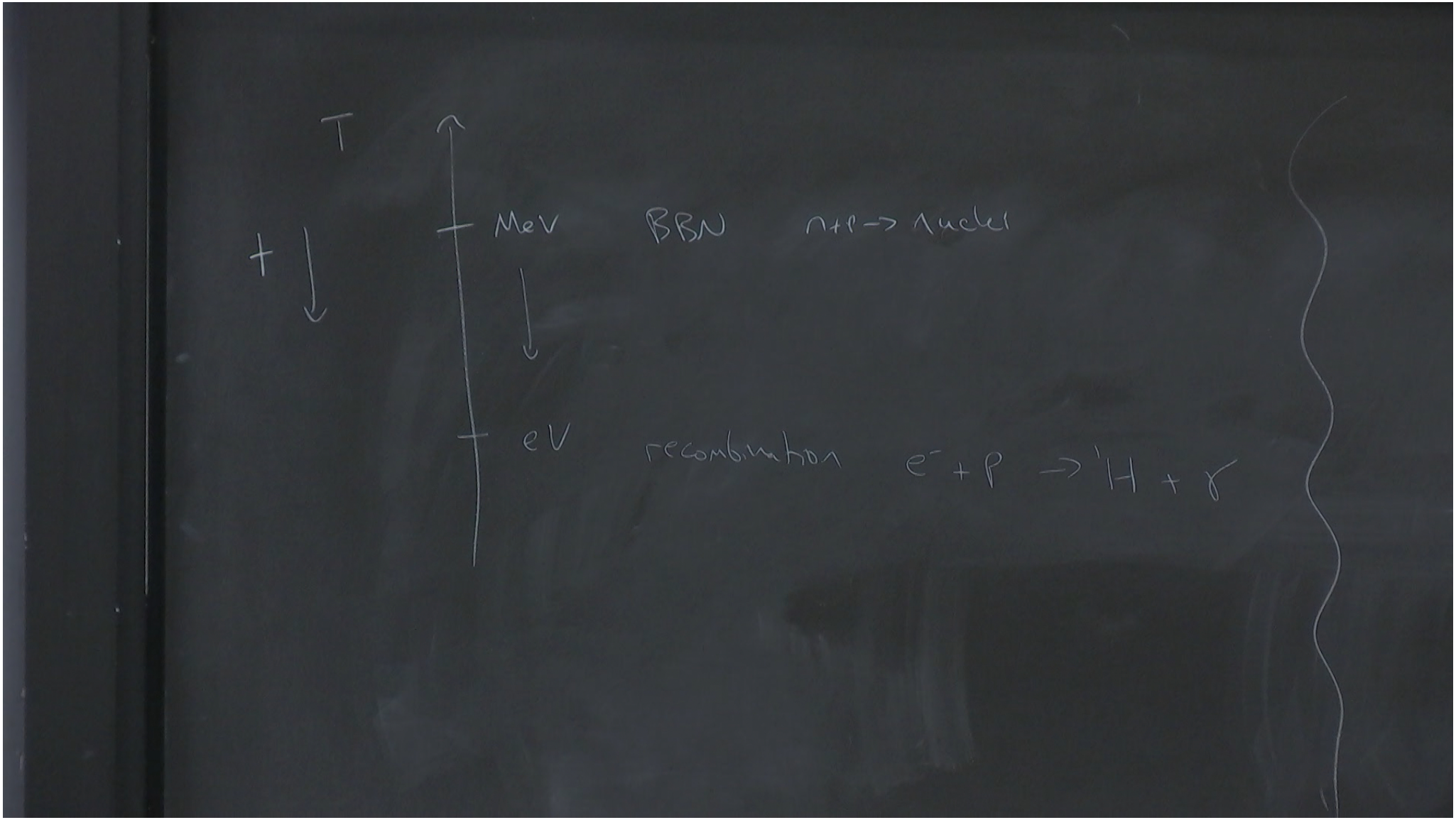
- $n + \nu \leftrightarrow p + e^-$ (Saha $\rightarrow \frac{n_n}{n_p}(T)$, Freeze-out $\frac{n_n(T_{freeze})}{n_p} \simeq .2$)
- $n + p \leftrightarrow D + \gamma$ (Saha $\rightarrow \frac{n_D}{n_p n_n}(T)$, all $n$ go into $D$)
- $D + p \leftrightarrow {}^3He + \gamma$

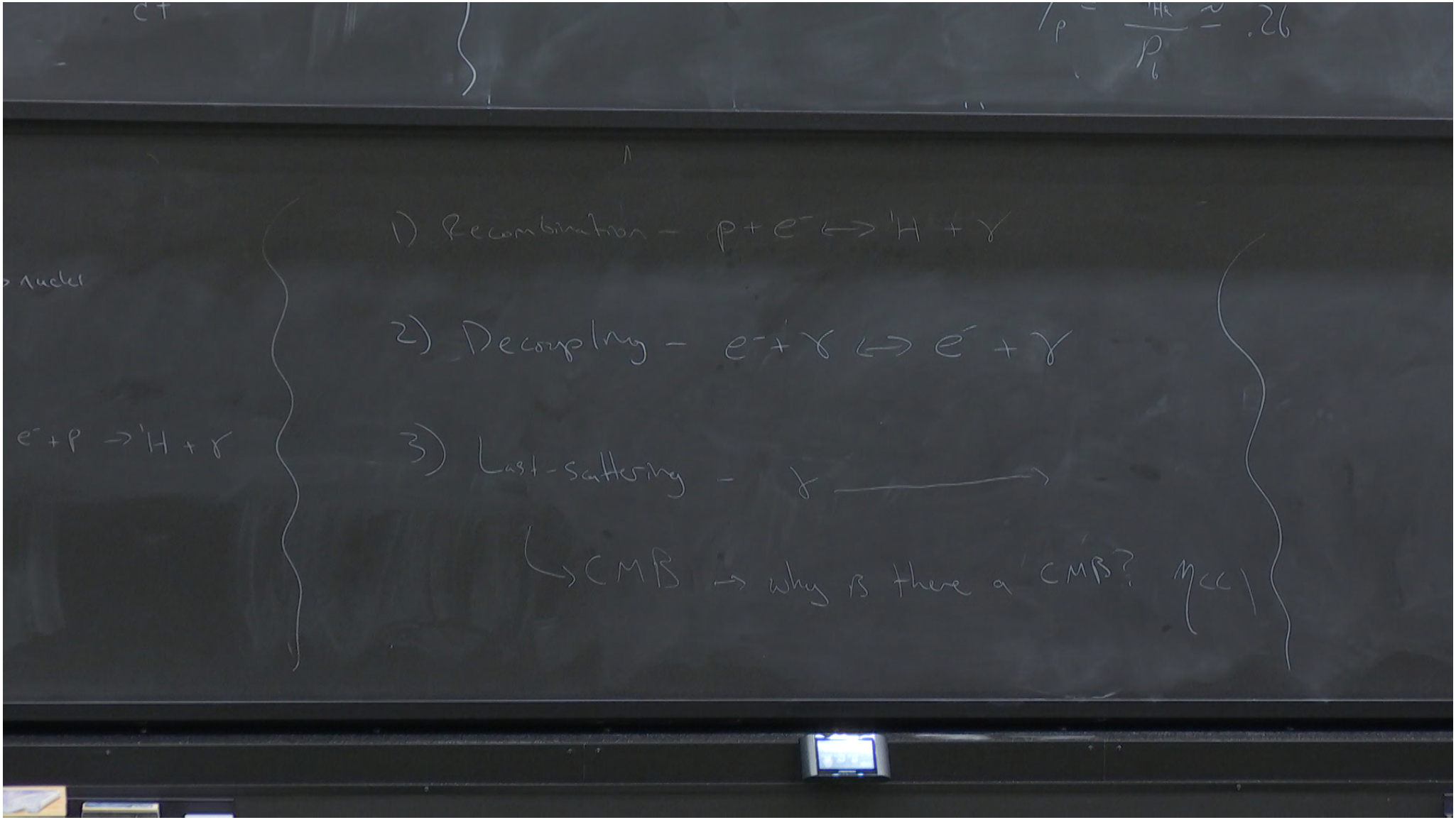
  ${}^3He + n \leftrightarrow {}^4He + \gamma$ (all $n$ end up in ${}^4He$)

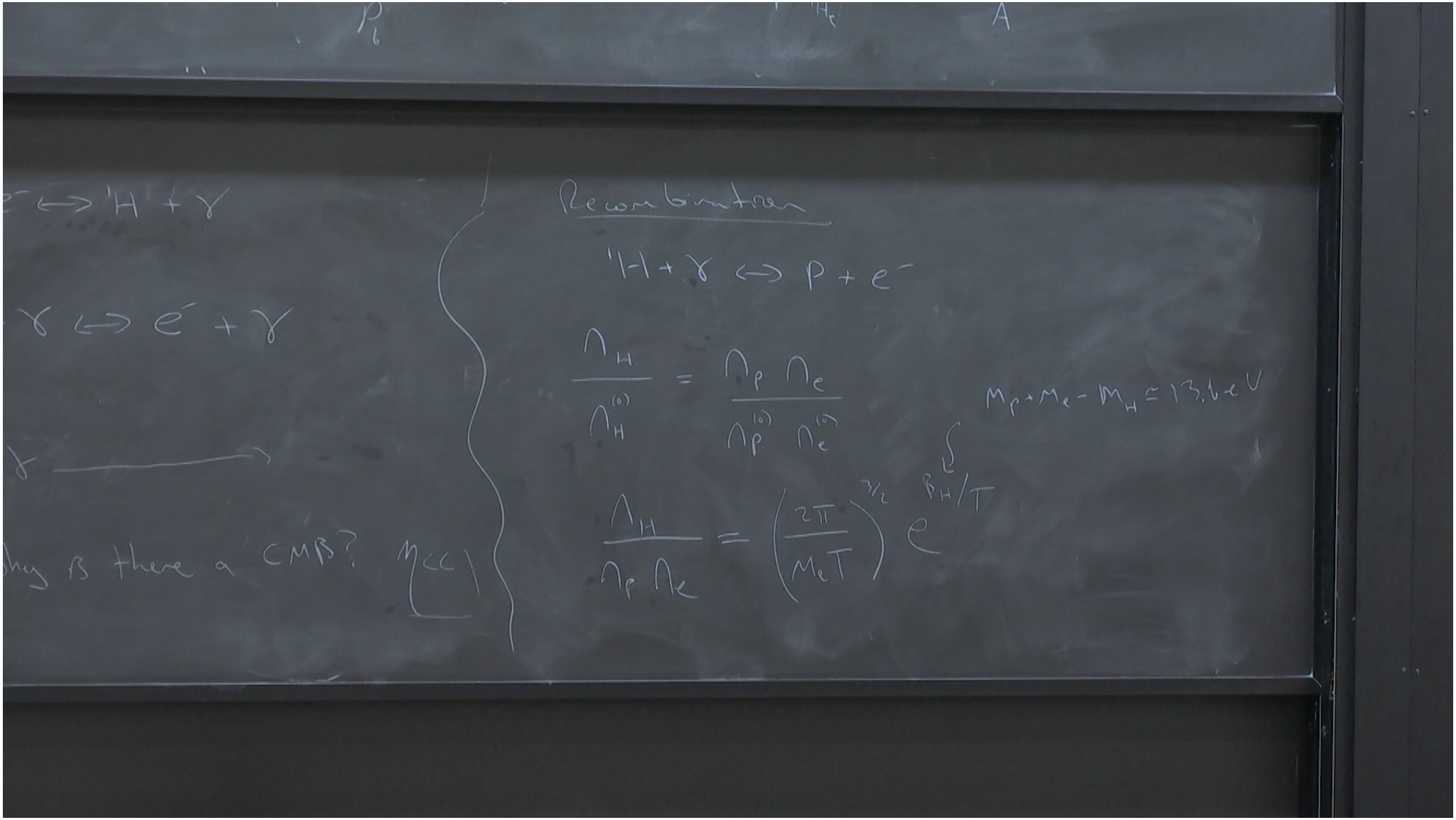
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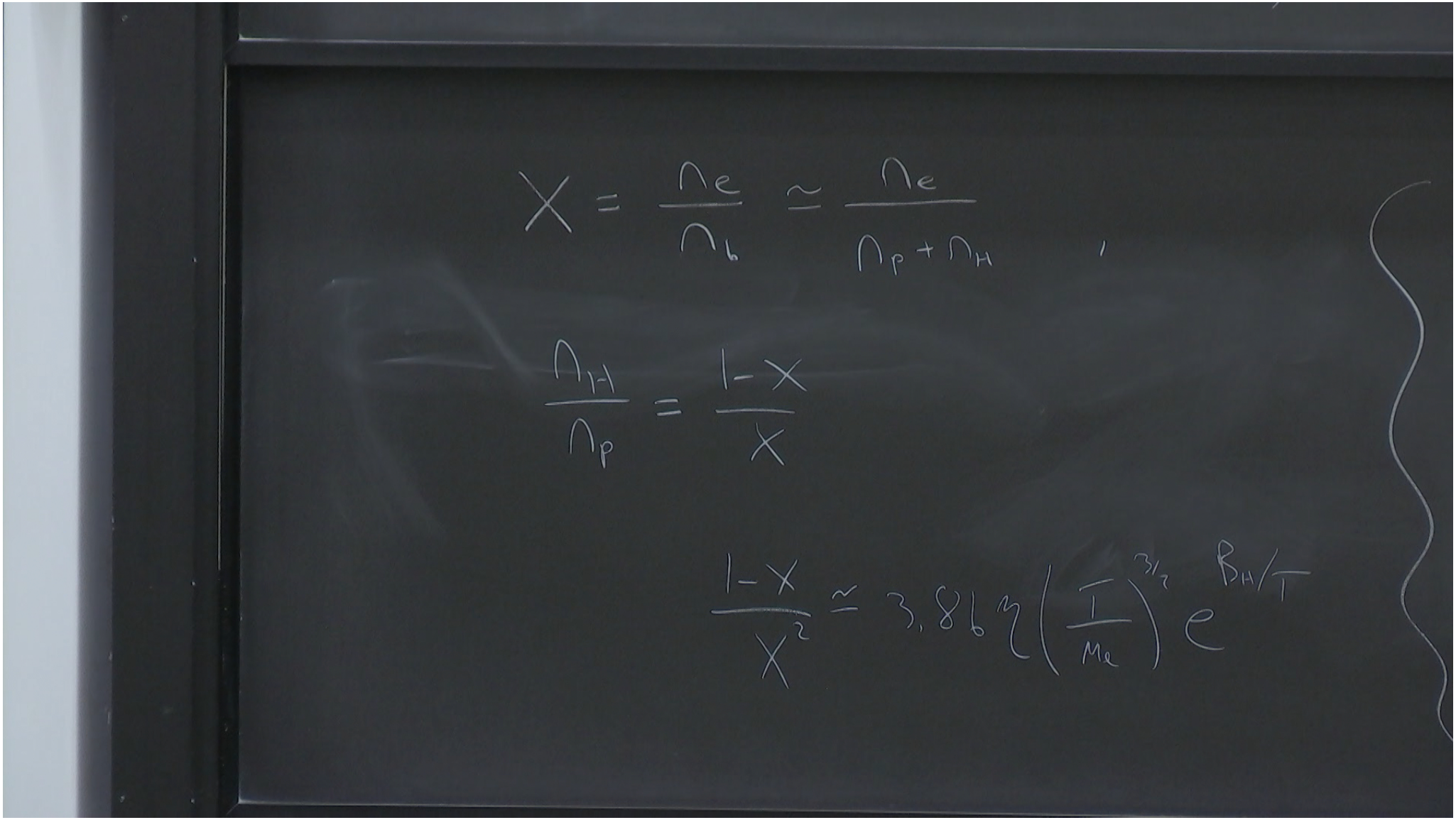
$$Y_p = \frac{\rho_{^4He}}{\rho_b} \simeq .26$$

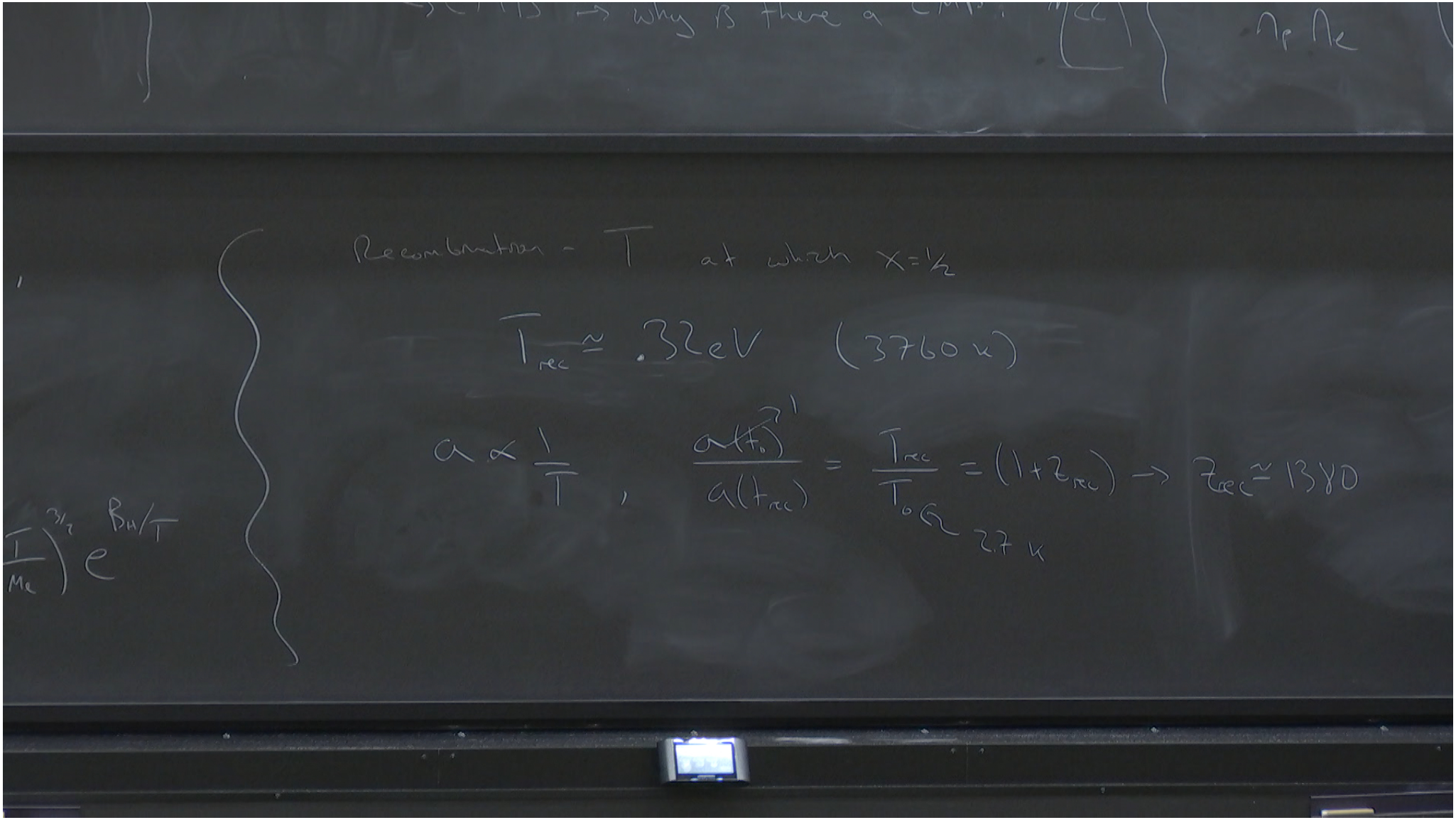


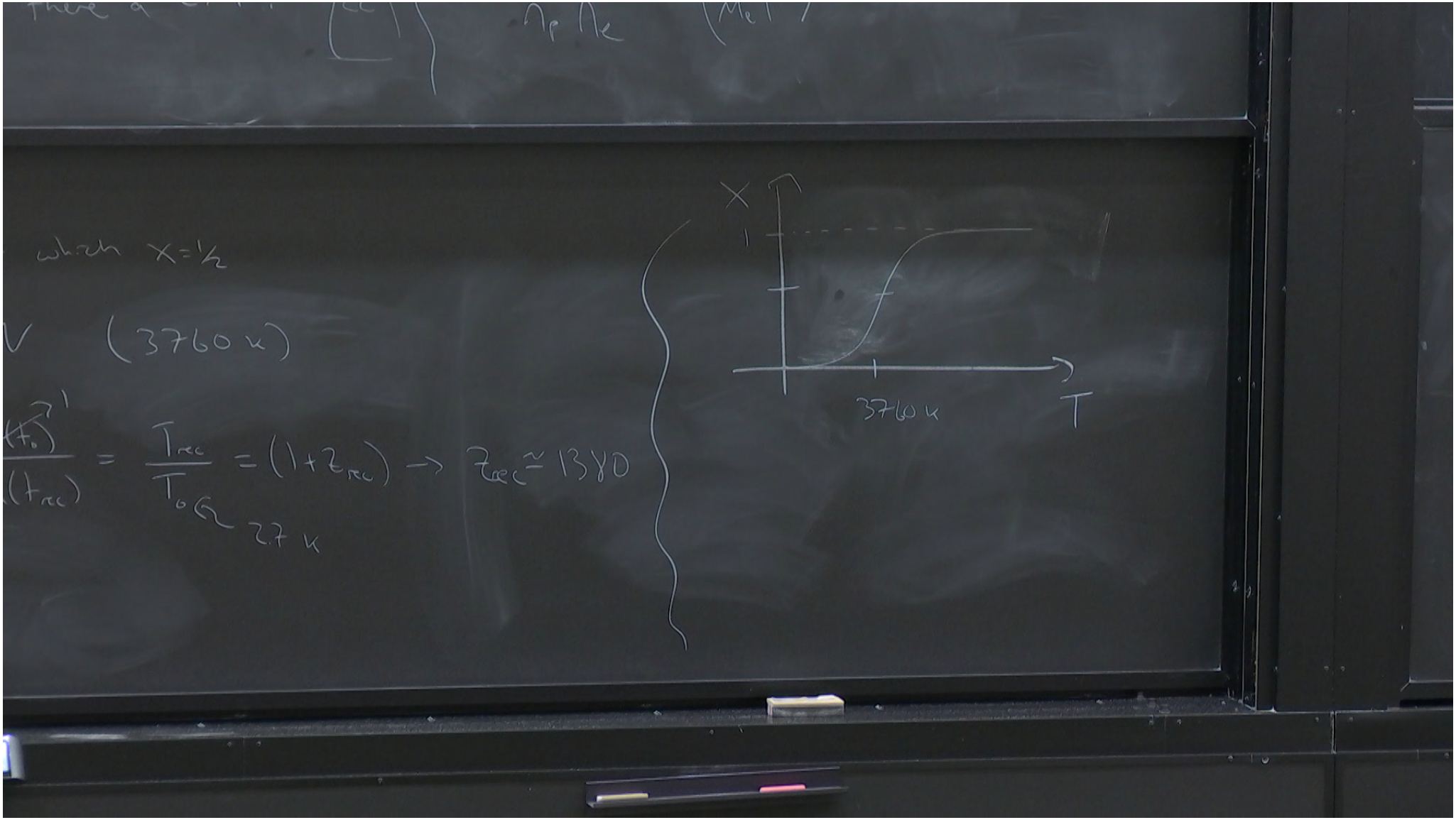

T
t
MeV
BBN
n+p -> nuclei
eV
recombination
e⁻ + p -> ¹H + γ

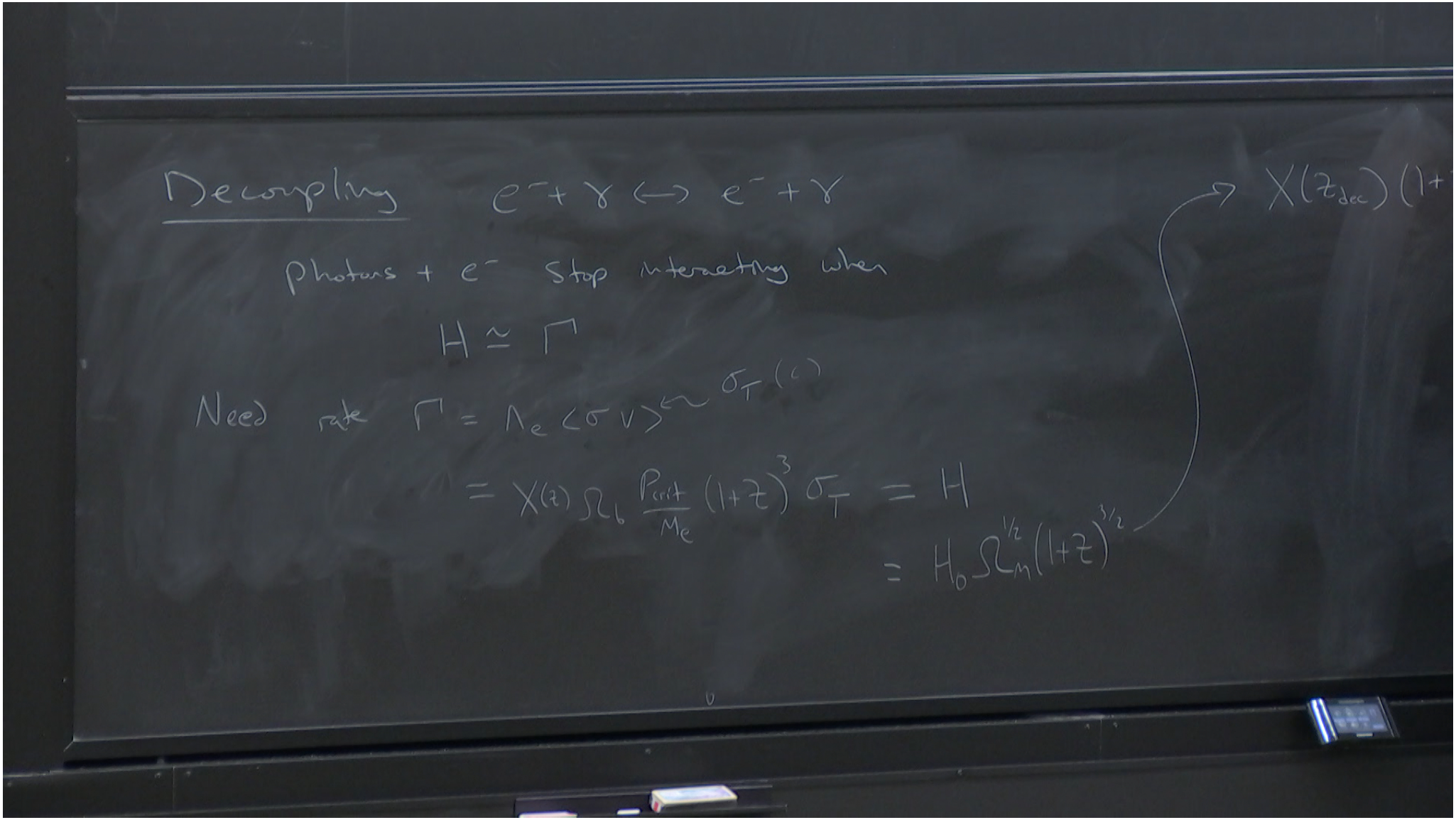

1) Recombination – $p + e^- \leftrightarrow {}^1H + \gamma$
2) Decoupling – $e^- + \gamma \leftrightarrow e^- + \gamma$
3) Last-scattering – $\gamma$ ⟶
↳ CMB → why is there a CMB? $\eta \ll 1$
$e^- + p \rightarrow {}^1H + \gamma$

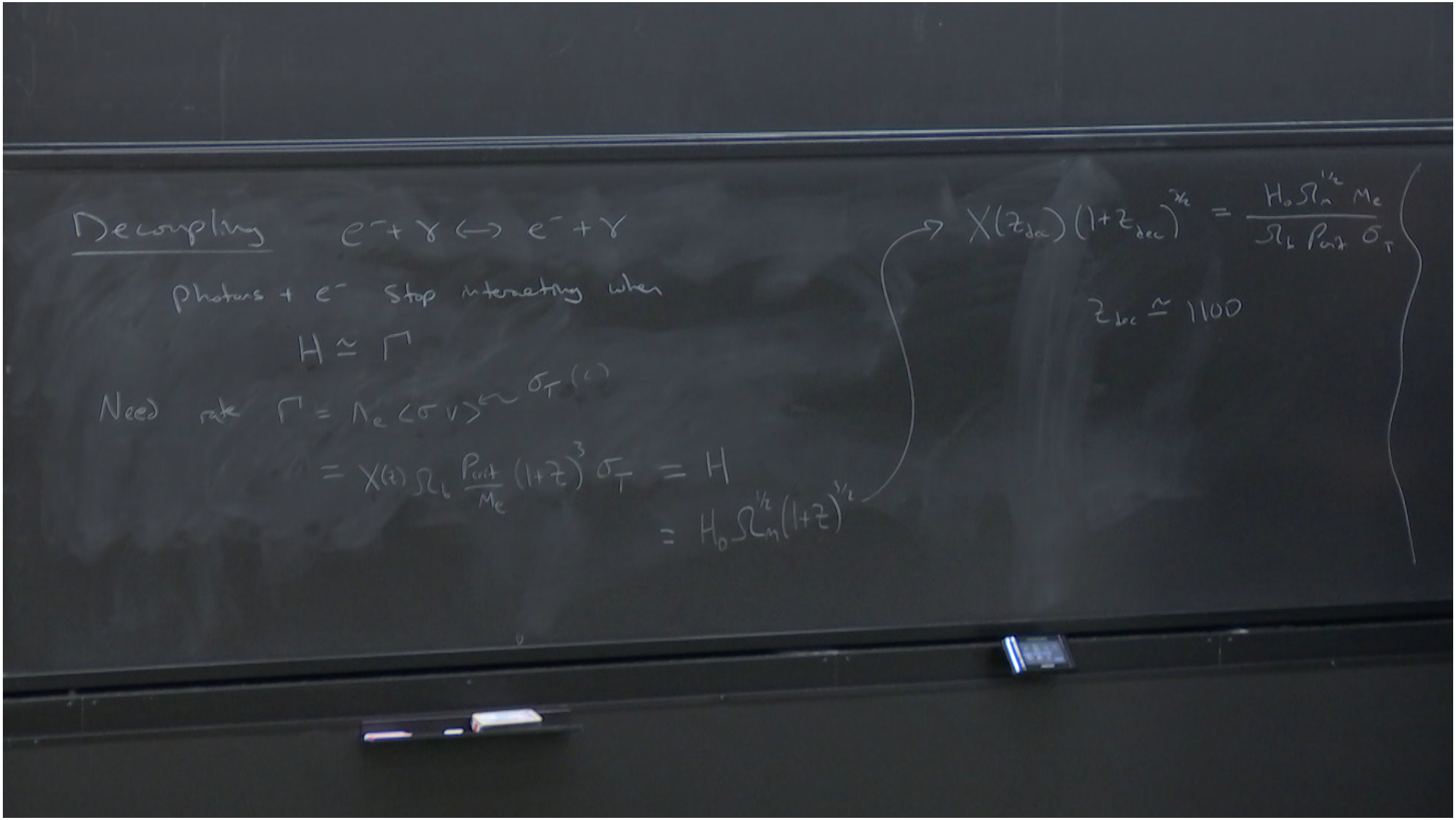

Recombination
$e^- \leftrightarrow {}^1H + \gamma$
$\gamma \leftrightarrow e^- + \gamma$
$M_p + M_e - M_H \simeq 13.6\,eV$
$\beta_H/T$
$\eta \ll 1$

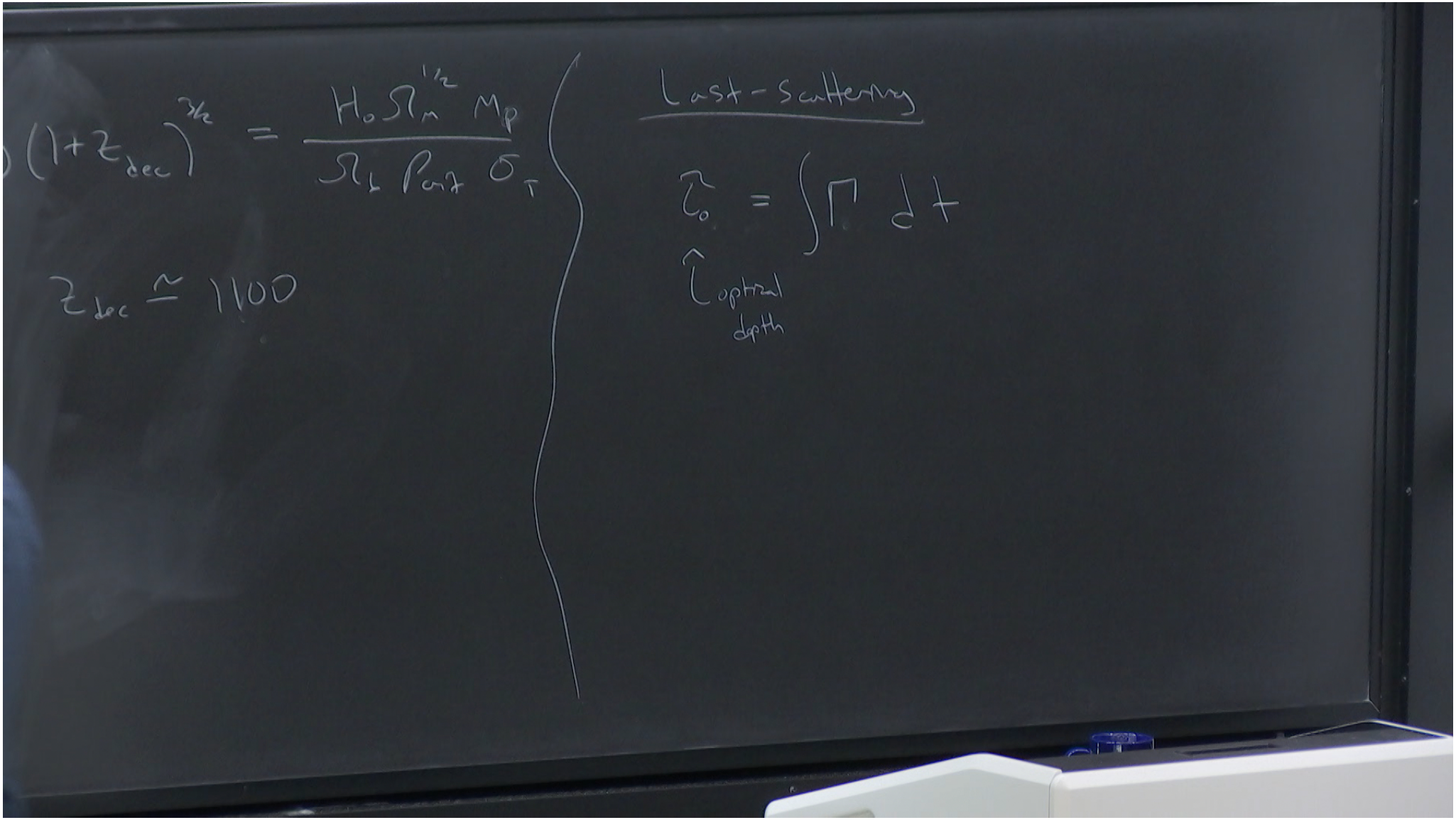

$X = \frac{n_e}{n_b} \simeq \frac{n_e}{n_p + n_H}$ ,
$\frac{n_H}{n_p} = \frac{1-X}{X}$
$\frac{1-X}{X^2} \simeq 3.86 \zeta \left(\frac{T}{m_e}\right)^{3/2} e^{B_H/T}$

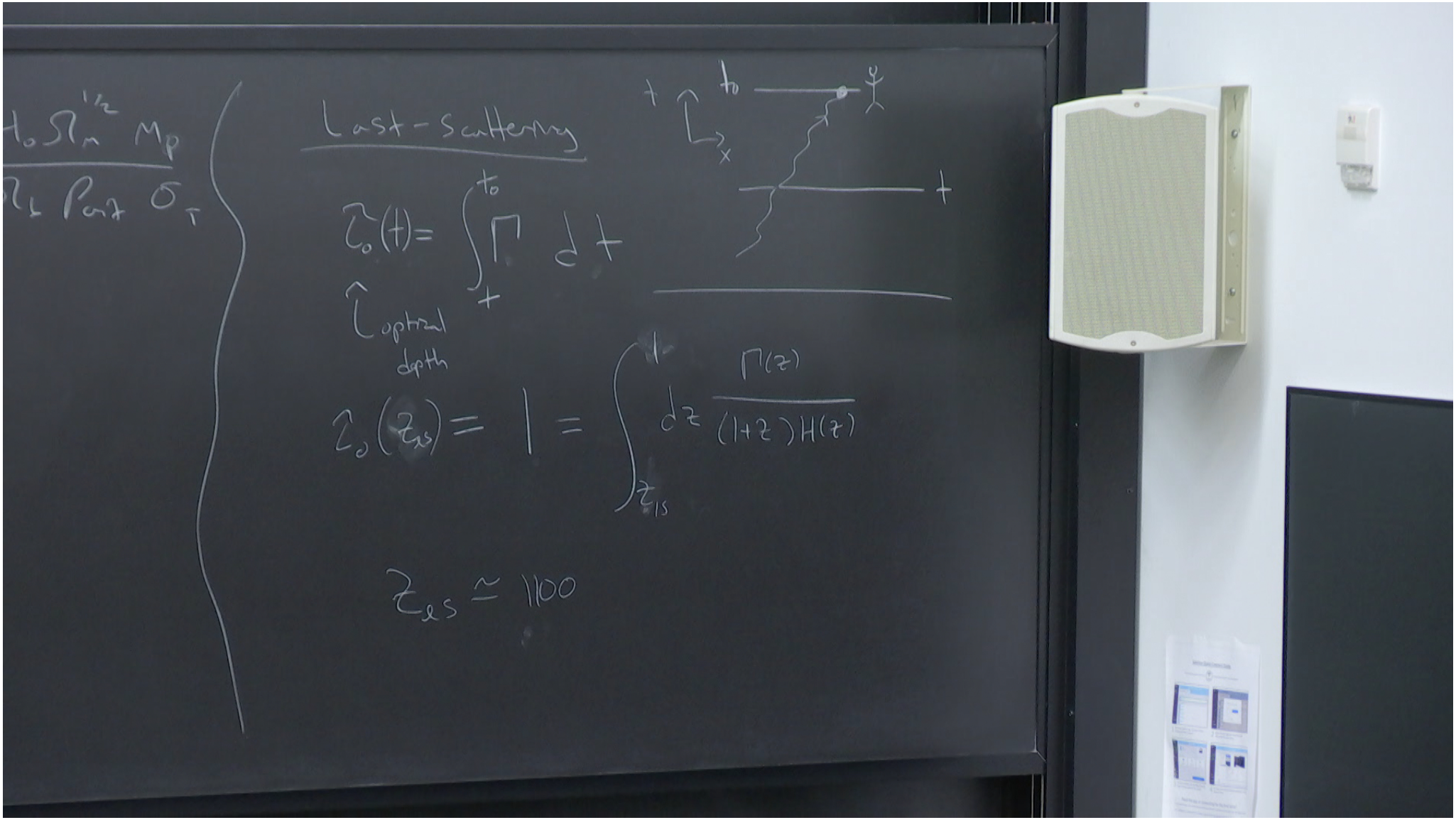

Recombination - T at which X=½
$T_{rec} \simeq .32eV \quad (3760K)$
$a \propto \frac{1}{T}, \quad \frac{a(t_0)}{a(t_{rec})} = \frac{T_{rec}}{T_0} = (1+z_{rec}) \rightarrow z_{rec} \simeq 1380$
2.7 K

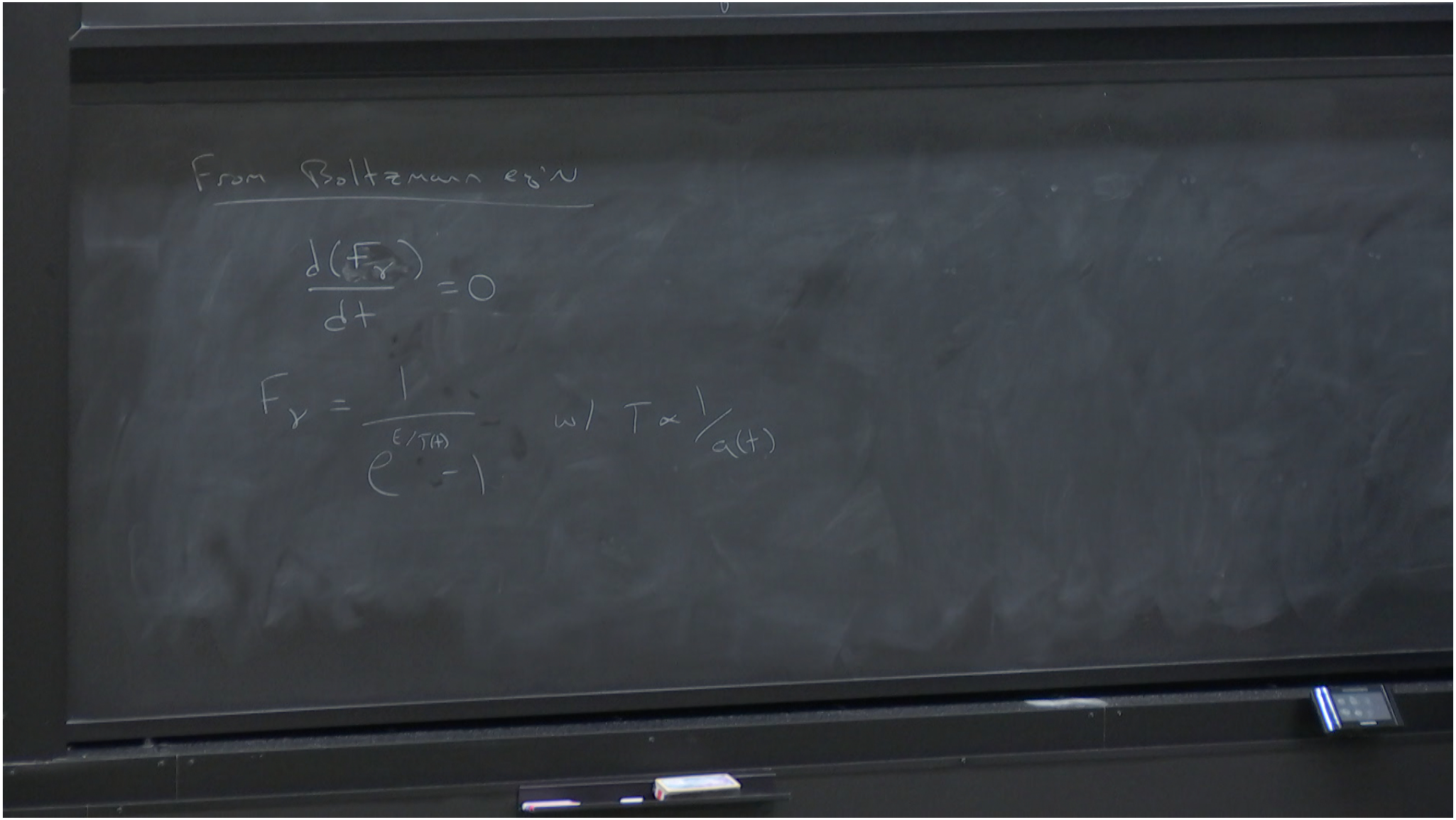

which X = ½
(3760 K)
$\frac{T_{rec}}{T_0} = (1+z_{rec}) \rightarrow z_{rec} \simeq 1380$
2.7 K
X
1
3760 K
T

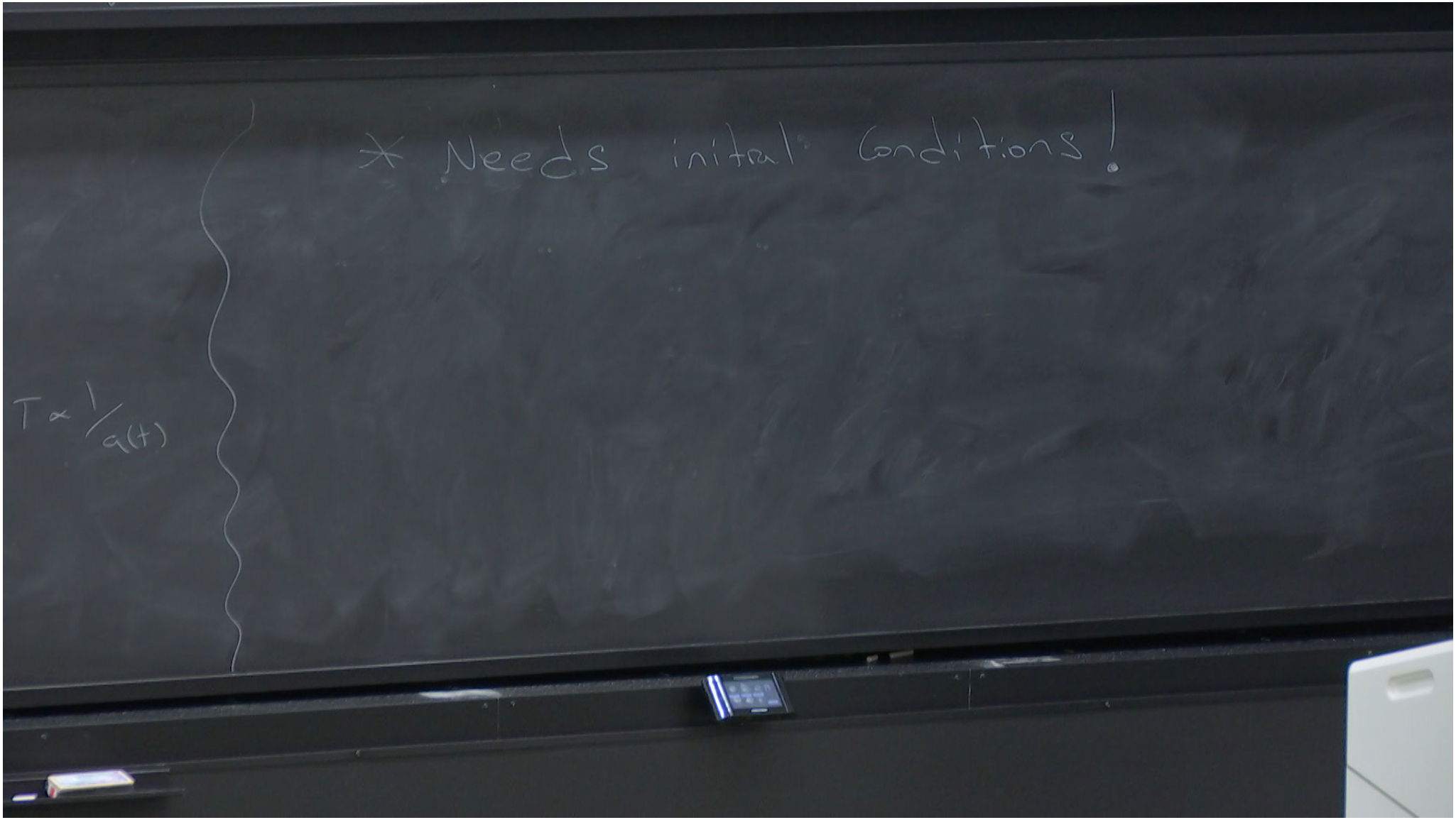

Decoupling $e^- + \gamma \leftrightarrow e^- + \gamma$
Photons + $e^-$ Stop interacting when
$H \simeq \Gamma$
Need rate $\Gamma = n_e \langle \sigma v \rangle \leftarrow \sigma_T$ (c)
$= X(z)\,\Omega_b \frac{\rho_{crit}}{m_e}(1+z)^3 \sigma_T = H$
$= H_0 \Omega_m^{1/2} (1+z)^{3/2}$
$\rightarrow X(z_{dec})(1+$


Decoupling $e^- + \gamma \leftrightarrow e^- + \gamma$
Photons + $e^-$ stop interacting when
$H \simeq \Gamma$
Need rate $\Gamma = n_e \langle \sigma v \rangle \sim \sigma_T$
$= X(z)\,\Omega_b \frac{\rho_{crit}}{m_e}(1+z)^3 \sigma_T = H$
$= H_0 \Omega_m^{1/2} (1+z)^{3/2}$
$X(z_{dec})(1+z_{dec})^{3/2} = \frac{H_0 \Omega_m^{1/2} m_e}{\Omega_b \rho_{crit} \sigma_T}$
$z_{dec} \simeq 1100$


$(1+z_{dec})^{3/2} = \frac{H_0 \Omega_m^{1/2} M_p}{\Omega_\Lambda \rho_{crit} \sigma_T}$
$z_{dec} \simeq 1100$
Last-Scattering
$\tau_0 = \int \Gamma \, dt$
$\tau_{optical\ depth}$


Last-scattering
$\tau_0(t) = \int_t^{t_0} \Gamma \, dt$
$\tau_{optical\ depth}$
$\tau_0(z_{ls}) = 1 = \int_{z_{ls}}^{\infty} dz \frac{\Gamma(z)}{(1+z)H(z)}$
$z_{ls} \simeq 1100$


From Boltzmann eq'n
$\frac{d(F_\gamma)}{dt} = 0$
$F_\gamma = \frac{1}{e^{\epsilon/T(t)} - 1}$ w/ $T \propto \frac{1}{a(t)}$


* Needs inital Conditions!
T ∝ 1/q(t)

$\sigma_T = H$

$= H_0 \Omega_m^{1/2} (1+z)^{3/2}$

$z_{es} \simeq 1100$

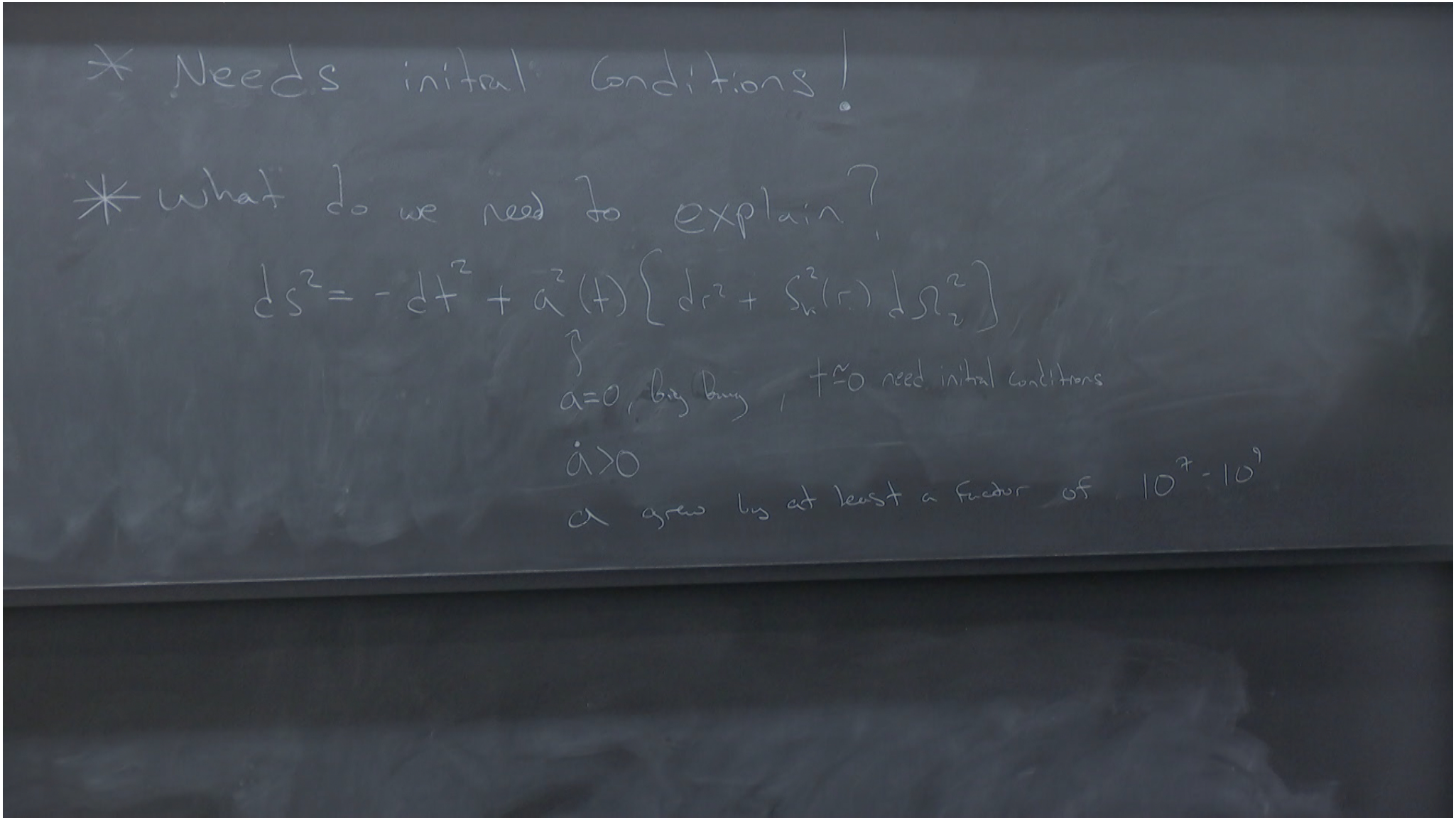
* Needs initial conditions!

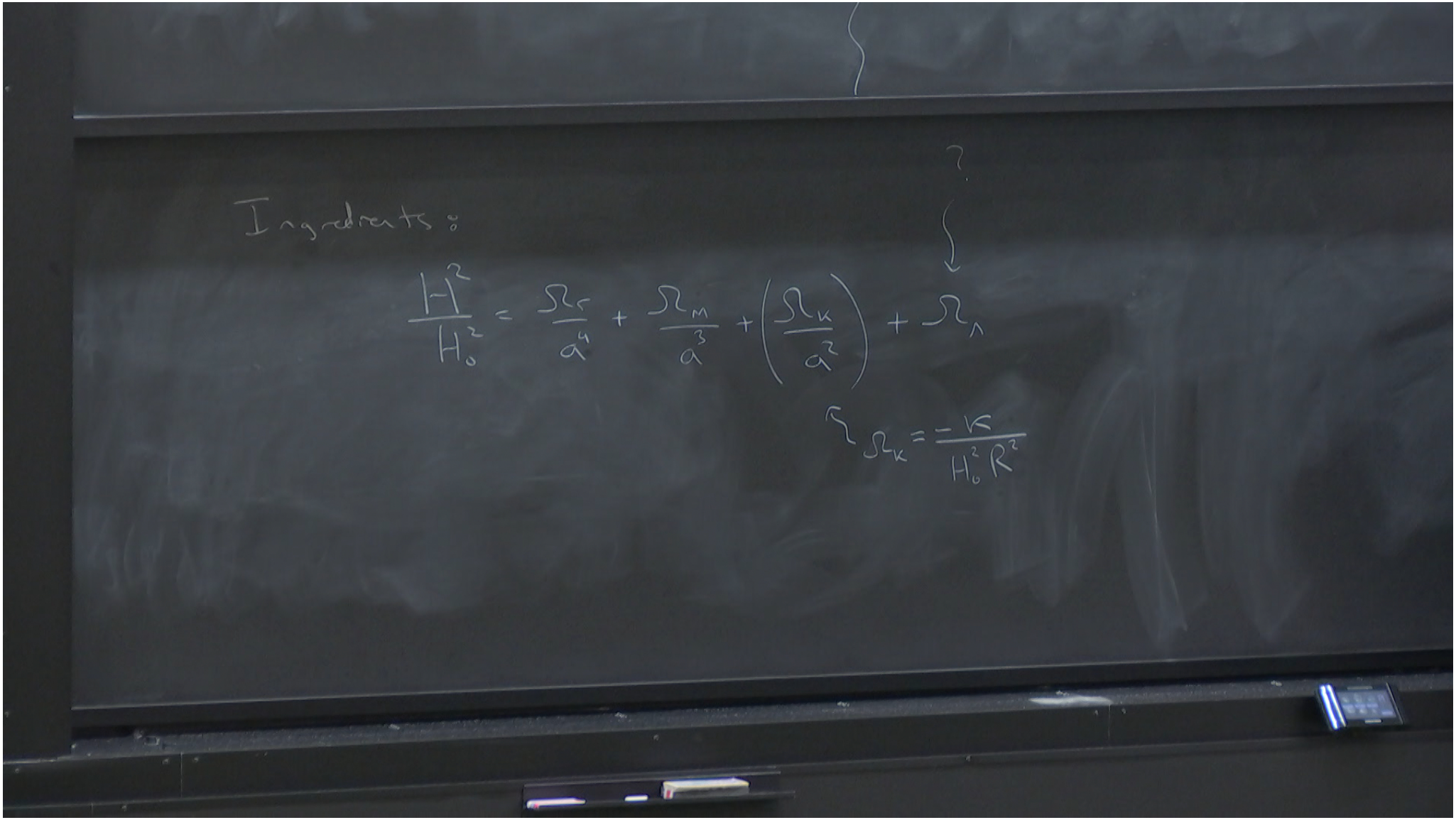
* what do we need to explain?

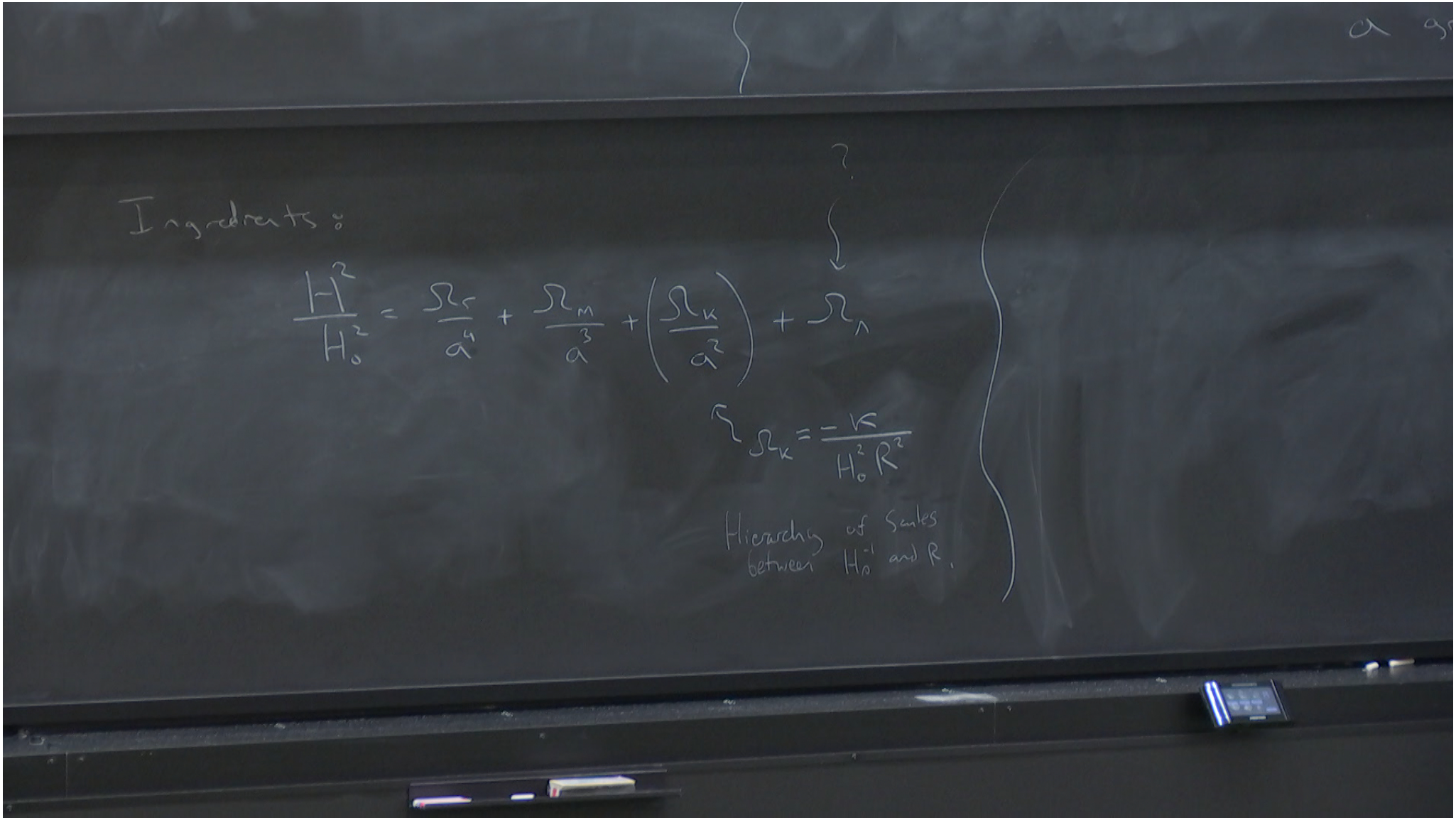
$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + S_k^2(r)\, d\Omega_2^2\right]$$

$a=0$, big bang, $t \simeq 0$ need initial conditions

$\dot{a} > 0$


* Needs inital Conditions!
* what do we need to explain?
$ds^2 = -dt^2 + a^2(t)\left[dr^2 + S_k^2(r)\, d\Omega_2^2\right]$
$a=0$, big bang, $t \simeq 0$ need inital conditions
$\dot{a}>0$
$a$ grew by at least a factor of $10^7 - 10^9$


Ingredients:
$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \left(\frac{\Omega_k}{a^2}\right) + \Omega_\Lambda$
?
$\Omega_k = \frac{-k}{H_0^2 R^2}$

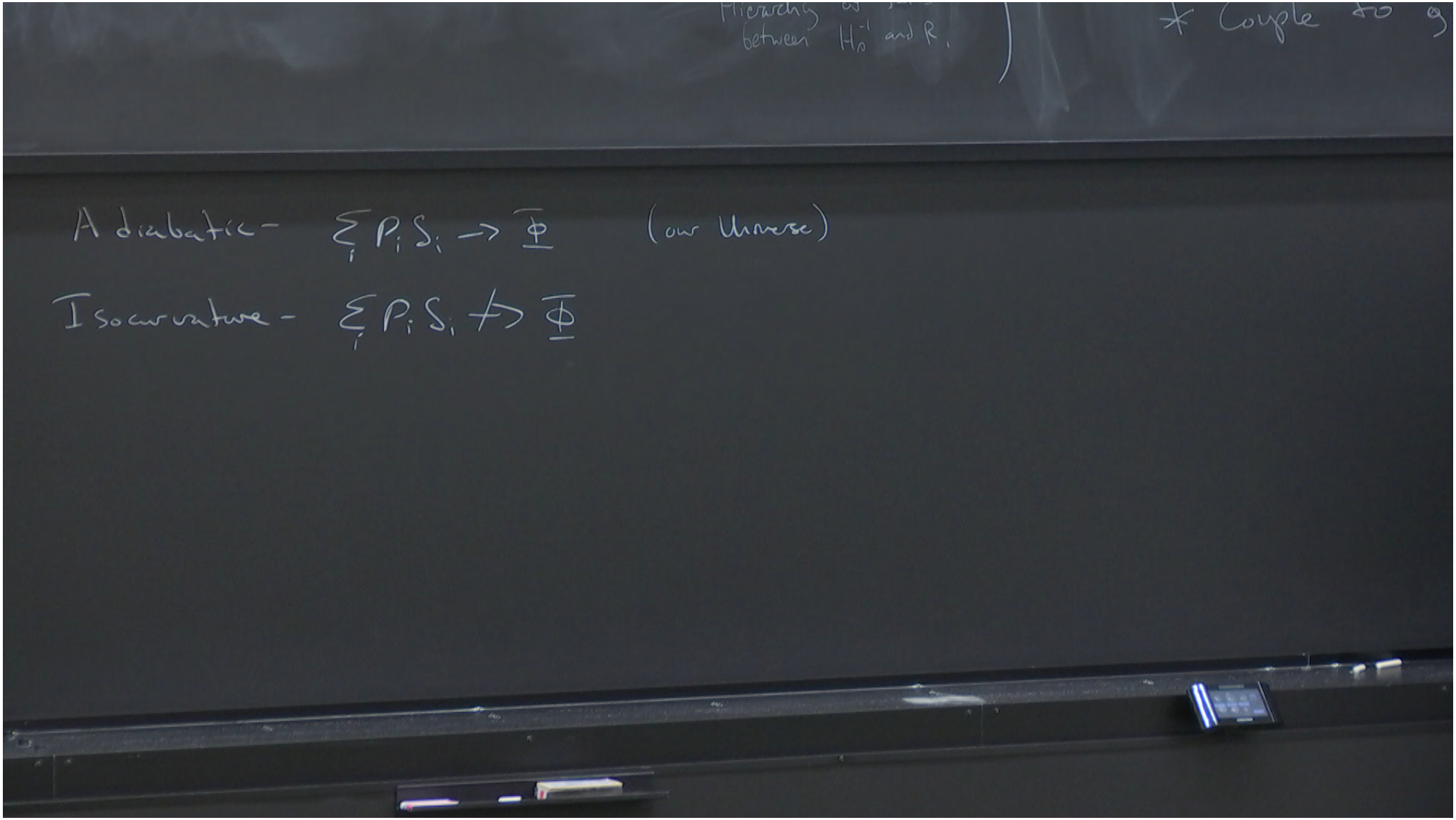

Ingredients:
$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_M}{a^3} + \left(\frac{\Omega_K}{a^2}\right) + \Omega_\Lambda$
?
$\Omega_K = \frac{-K}{H_0^2 R^2}$
Hierarchy of Scales between $H_0^{-1}$ and R.

* Approximately homogeneous + isotropic, inhomogeneities have special properties.

Aside

* density perturbations: $\rho_i(\vec{x},t) = \rho_i(t)\left(1+\delta_i(\vec{x},t)\right)$ $\delta \ll 1$

* Couple to gravity: $\nabla^2\Phi = -4\pi G_N a^2 \sum_i \rho_i \delta_i$


between $H_0^{-1}$ and R.
* Couple to g
Adiabatic - $\sum_i P_i S_i \to \underline{\Phi}$ (our Universe)
Isocurvature - $\sum_i P_i S_i \not\to \underline{\Phi}$

Hierarchy between $H_0^{-1}$ and $R_*$

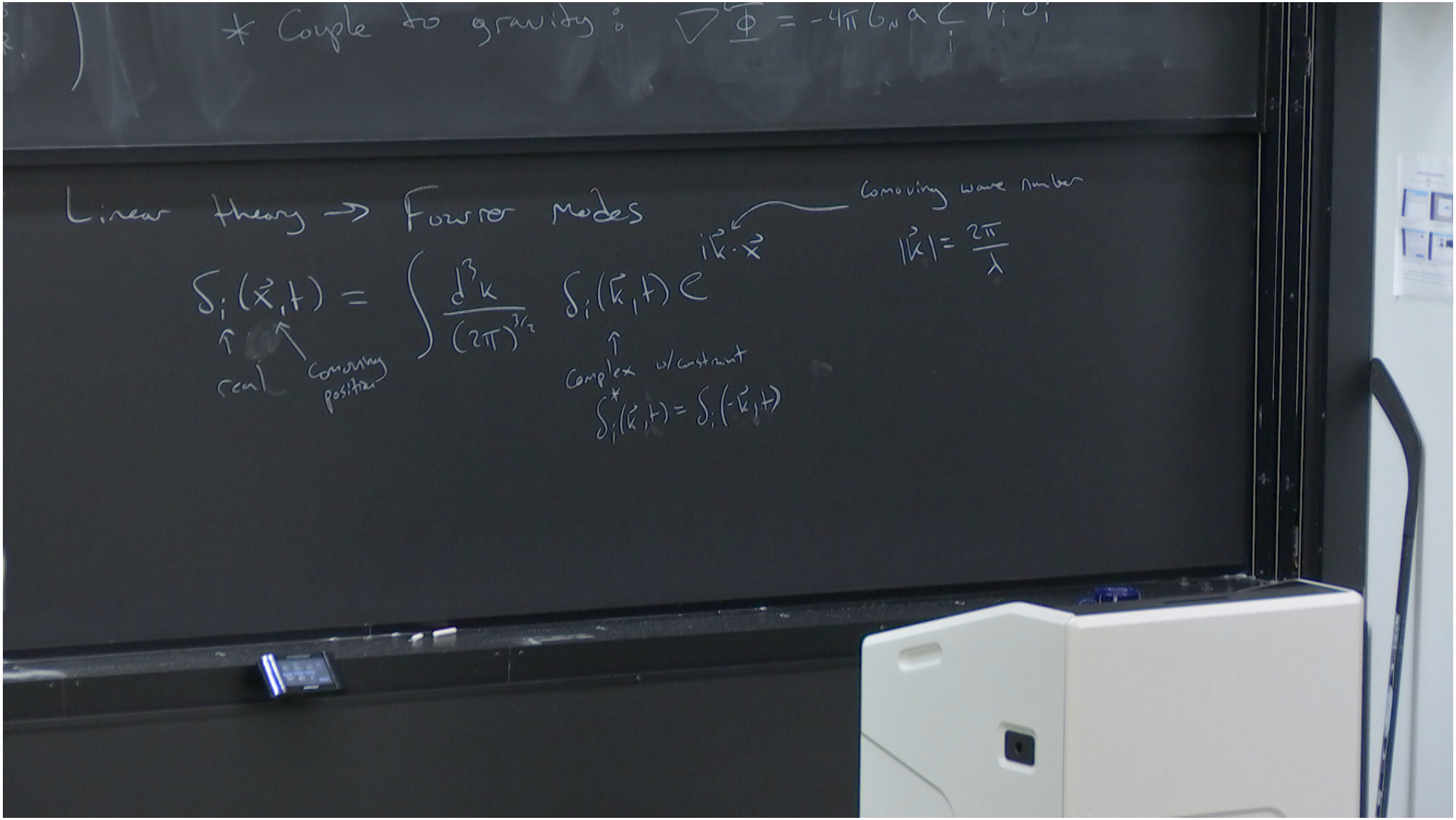
* Couple

Adiabatic - $\sum_i P_i S_i \to \Phi$ (our Universe)

Isocurvature - $\sum_i P_i S_i \not\to \Phi$

2 ways to interpret Poisson eqn

1) $\delta \to \Phi$ 2) $\Phi \to \delta$


* Couple to gravity:
Linear theory → Fourier modes
Comoving wave number
$|\vec{k}| = \frac{2\pi}{\lambda}$
$\delta_i(\vec{x},t) = \int \frac{d^3k}{(2\pi)^{3/2}} \delta_i(\vec{k},t) e^{i\vec{k}\cdot\vec{x}}$
real
Comoving position
Complex w/ constraint
$\delta_i^*(\vec{k},t) = \delta_i(-\vec{k},t)$

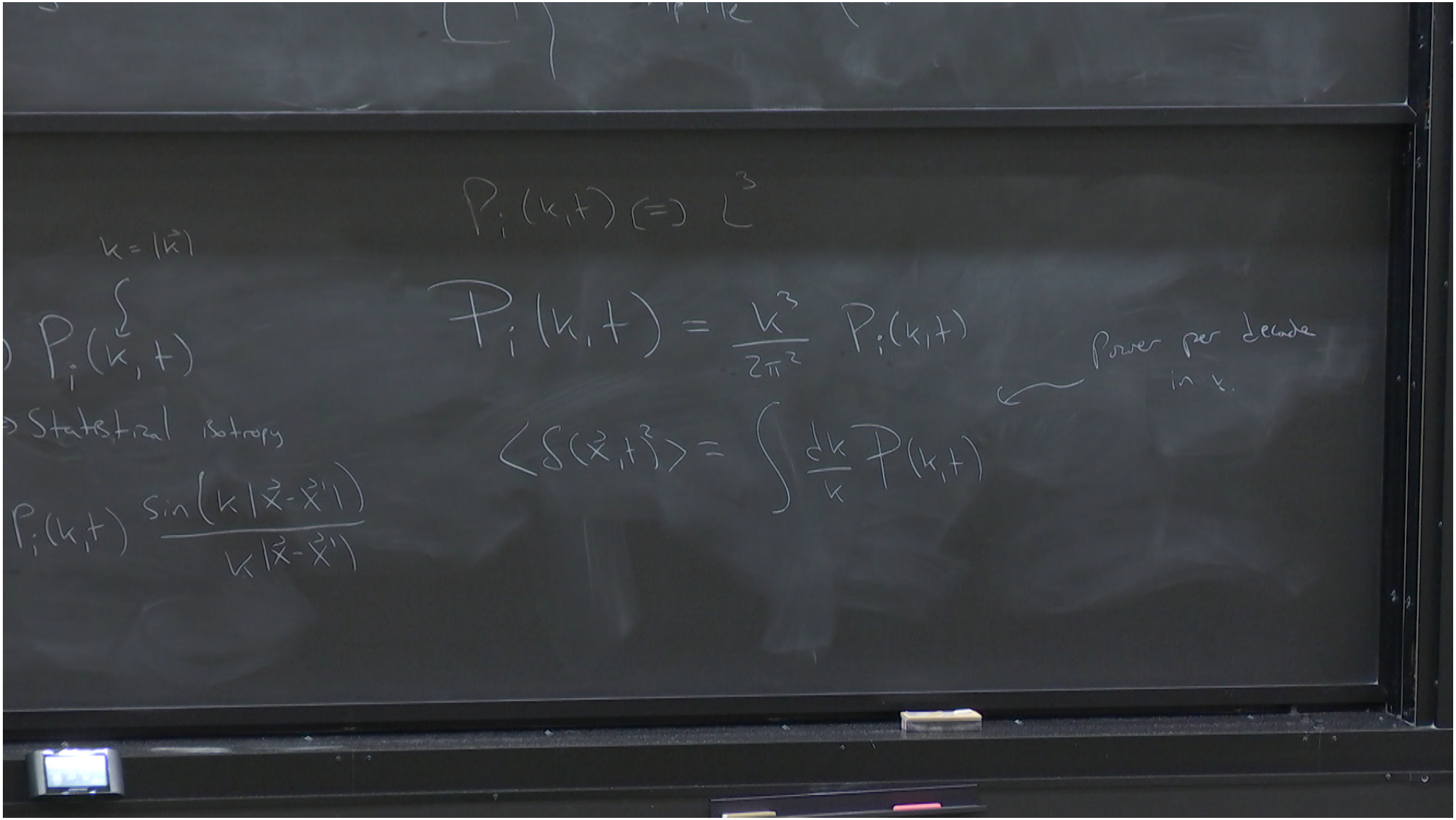
↳ CMB → why is there a CMB?

* Statistics of $\delta_i$ :

Power spectrum : $\langle \delta_i^*(\vec{k},t)\, \delta_i(\vec{k}',t) \rangle = \delta^{(3)}(\vec{k}-\vec{k}')\, P_i(k,t)$ $\quad k=|\vec{k}|$

$\Rightarrow$ Statistical isotropy

$$\langle \delta_i(\vec{x},t)\, \delta_i(\vec{x}',t) \rangle = \int_0^\infty \frac{dk}{2\pi^2} k^2 P_i(k,t) \frac{\sin\left(k|\vec{x}-\vec{x}'|\right)}{k|\vec{x}-\vec{x}'|}$$


$P_i(k,t) \propto L^3$
$k = |\vec{k}|$
$P_i(\vec{k},t)$
$\mathcal{P}_i(k,t) = \frac{k^3}{2\pi^2} P_i(k,t)$
Power per decade in k.
Statistical isotropy
$\langle \delta(\vec{x},t)^2 \rangle = \int \frac{dk}{k} \mathcal{P}(k,t)$
$P_i(k,t) \frac{\sin(k|\vec{x}-\vec{x}'|)}{k|\vec{x}-\vec{x}'|}$

$\int_0^{2\pi^2}$

* Scale-invariant power spectrum:

$$P(k,t) = \text{const.}$$

$$P_{\Phi}(k,t) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \leftarrow .96$$

$\uparrow$

$10^{-9}$

our Universe almost Scale invariant and

$P_{\Phi} \ll 1.$

Gaussian Field: Our Universe

$$P_r\left(\delta_i(\vec{k},t), \delta_i^*(\vec{k},t)\right) = \frac{1}{\sqrt{2\pi P_i(k,t)}} \exp\left[\frac{-\delta_i^*(k,t)\,\delta_i(k,t)}{2 P_i(k,t)}\right]$$

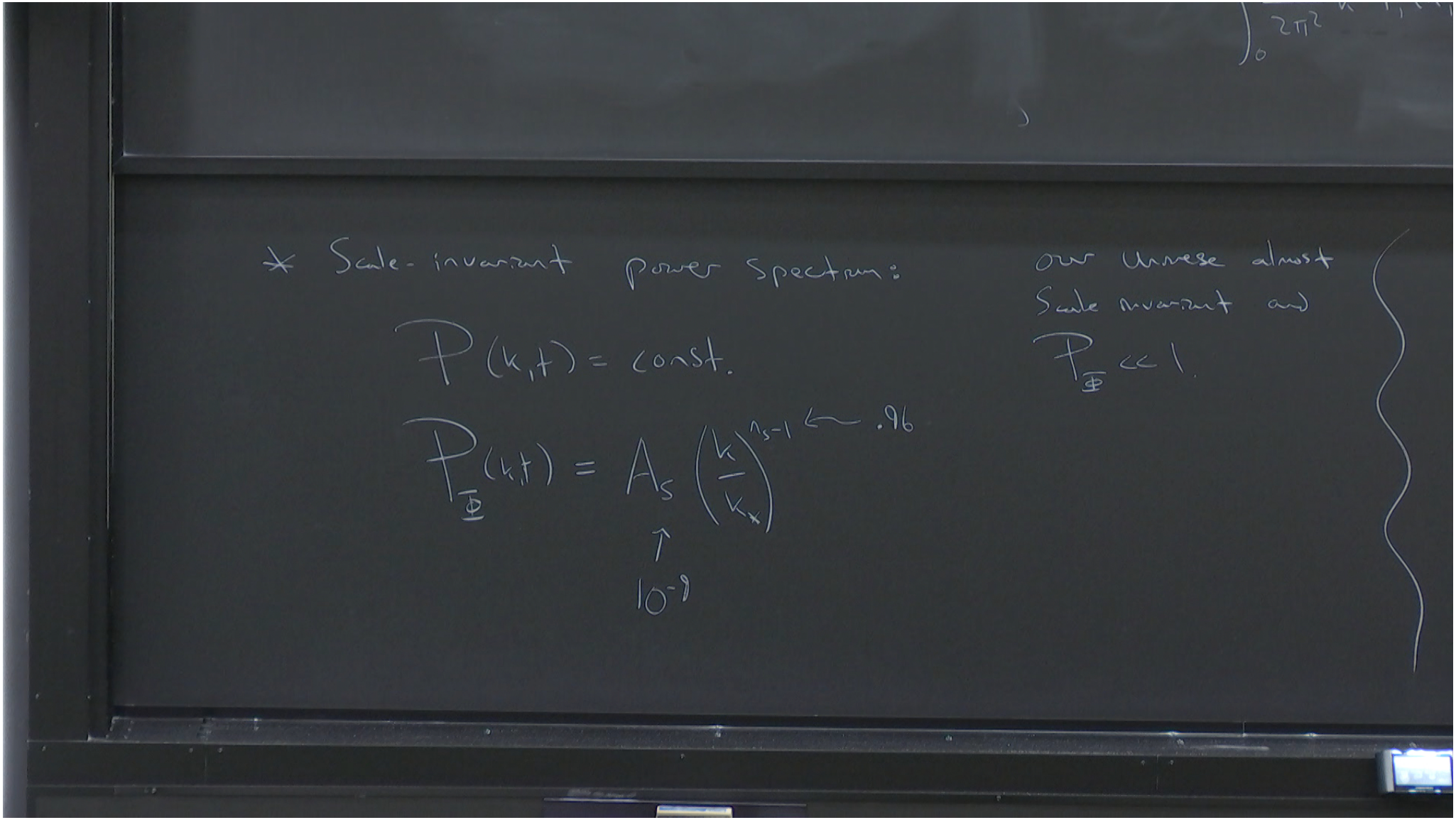
Deviations:

$$\langle \delta\delta\delta \rangle = \int \delta^3 P_r(\,)$$

$$= f_{NL} \int^{(3)} (\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

$P_i^{3/2}$

$f_{NL} \lesssim 5$


* Scale-invariant power spectrum:
$P(k,t) = \text{const.}$
$P_{\Phi}(k,t) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$
$10^{-9}$
$n_s - 1 \leftarrow .96$
our Universe almost Scale invariant and
$P_{\Phi} \ll 1.$