Title: PSI 2019/2020 - Chern-Simons Theory Part 1 - Lecture 11

Speakers: Kevin Costello

Collection: PSI 2019/2020 - Chern-Simons Theory (Part 1)

Date: March 06, 2020 - 11:30 AM

URL: http://pirsa.org/20030019

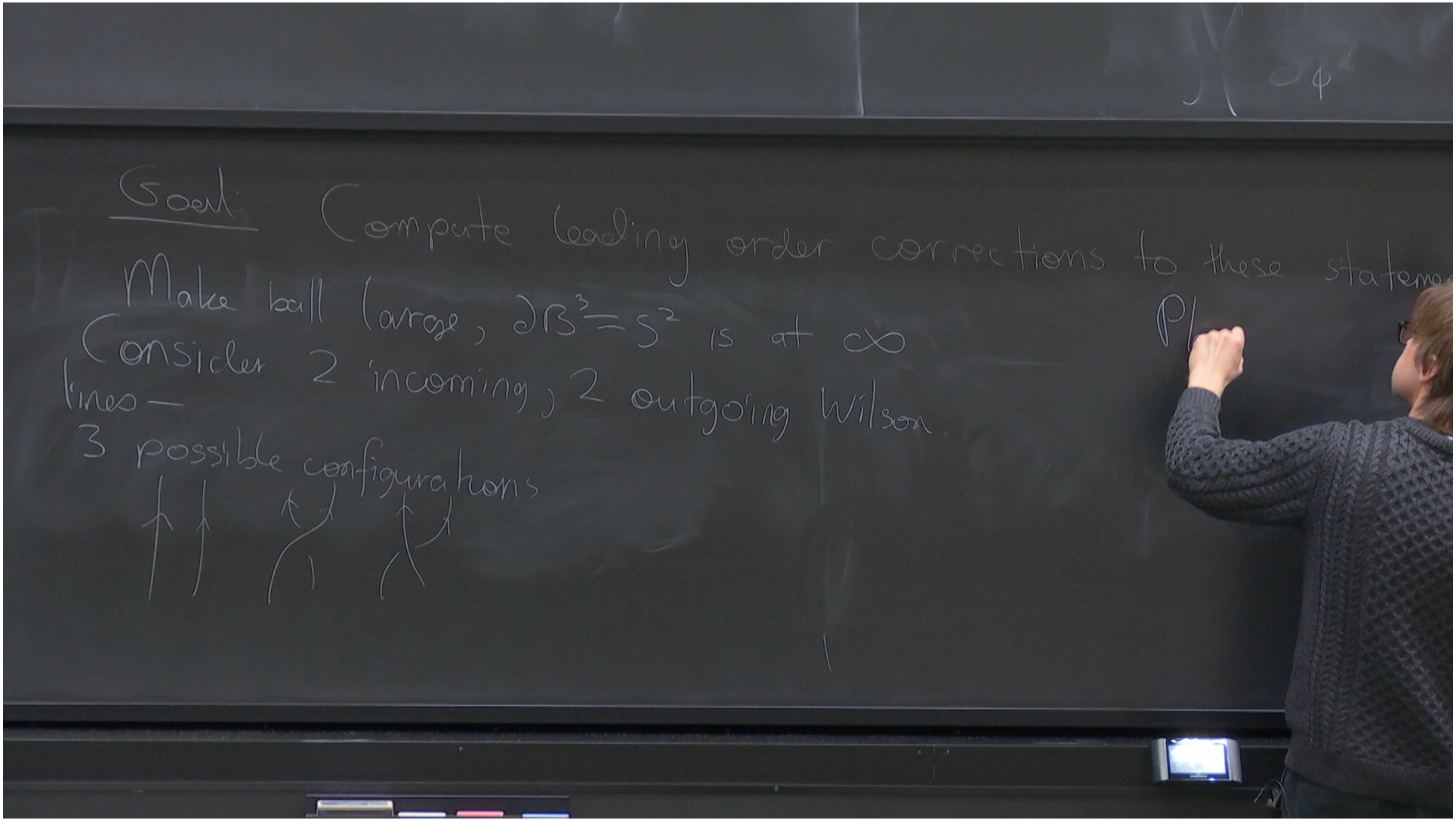
Last time:

$G = su(n)$ or $u(n)$

Hilbert space $H(S^2, p_1^+, p_2^+, p_1^-, p_2^-) = \mathbb{C}^2$

If $V = \mathbb{C}^n =$ fun. rep

The basis for Hilbert space is


Goal: Compute leading order corrections to these stateme
Make ball large, ∂B³ = S² is at ∞
Consider 2 incoming, 2 outgoing Wilson lines —
3 possible configurations

ing order corrections to these statements

S^2 is at ∞

outgoing Wilson

Place states at beginnings of Wilson lines

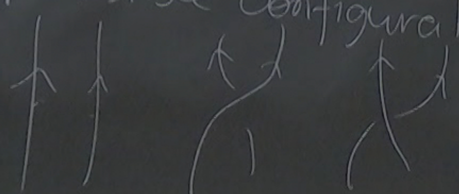
What do they become at the end?

Goal: Compute leading order corrections to these statemen

Make ball large, $\partial B^3 = S^2$ is at ∞

Consider 2 incoming, 2 outgoing Wilson lines —

3 possible configurations



Place states a

What do they

Goal: Reproduce rela

t^{-1} ⤫ $-\, t$ ⤫ $= (t^{1/2} - t^{-1/2})$

We'll compute order $\hbar$ scattering.

$P =$

Propagator for $u(1)$ case is

$$P = \varepsilon_{ijk} \frac{(x_i - y_i)\, d(x_j - y_j)\, d(x_k - y_k)}{\|x-y\|^3}$$

For $u(n)$ (or $gl(n)$) it's the same, decorated by a flavour index.

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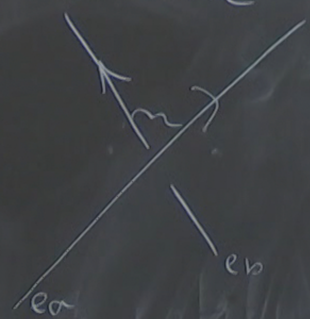
$P = $ (Same thi

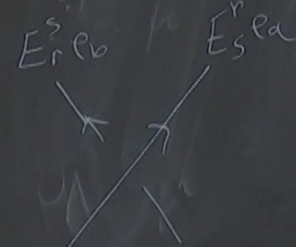
If we study the exchange of 1 gluon,

we have

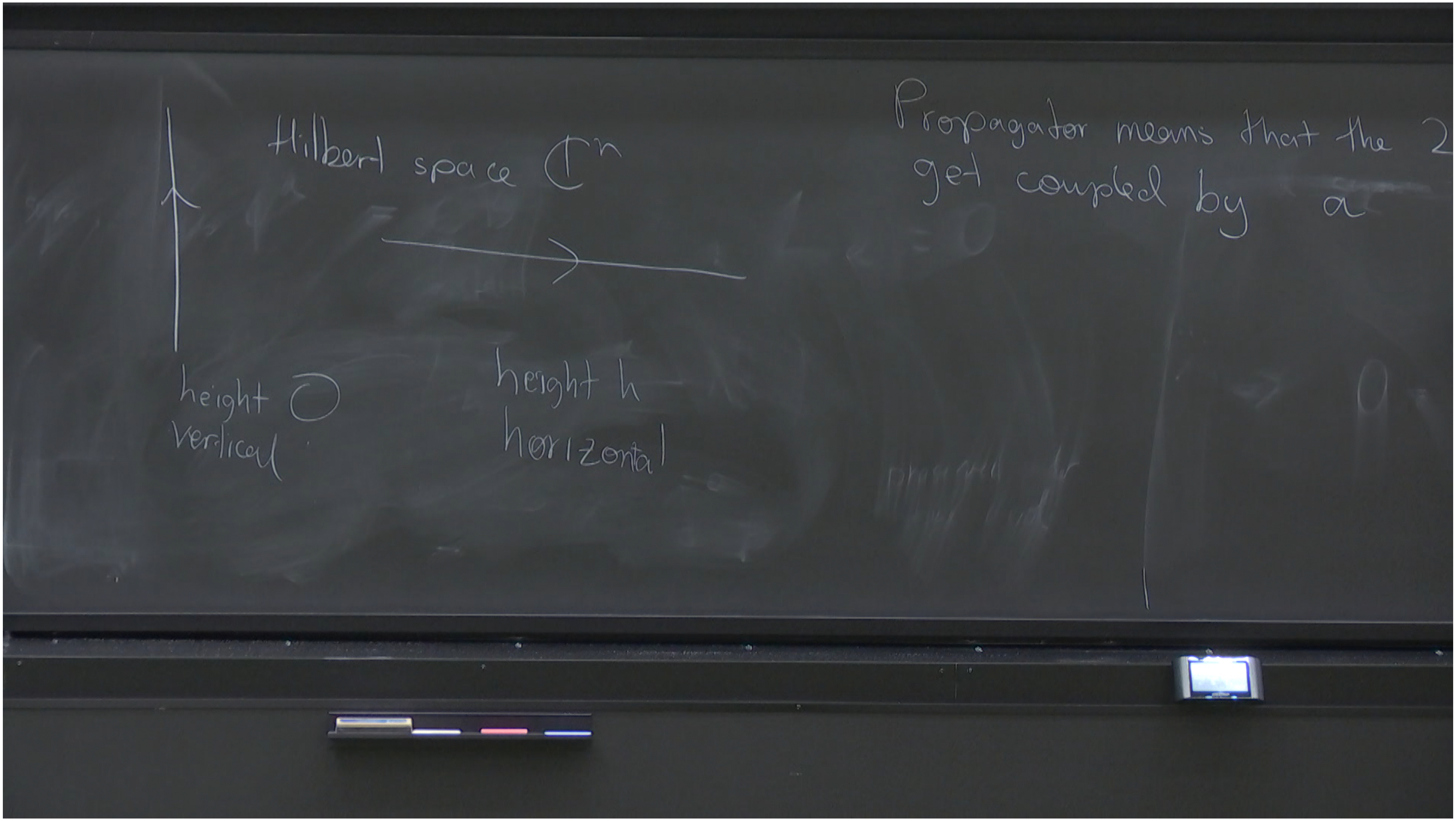


this gives final states:

$\sum E^r_s \otimes E^s_r$ get, colour factor



$= \delta^s_b e_r \otimes \delta^r_a e_s$


Hilbert space $\mathbb{C}^n$
height 0
vertical
height h
horizontal
Propagator means that the 2
get coupled by a

Propagator means that the 2 QM systems get coupled by a Hamiltonian which is

$$\sum E^{r}_{s} \otimes E^{s}_{r} \left(h \quad d \right)$$

Propagator means that the 2 QM systems get coupled by a Hamiltonian which is

$$\sum E_S^r \otimes E_r^s \left(\frac{h\, dx_1\, dy_2}{(h^2 - y_2^2 + x_1^2)^{3/2}} \right)$$

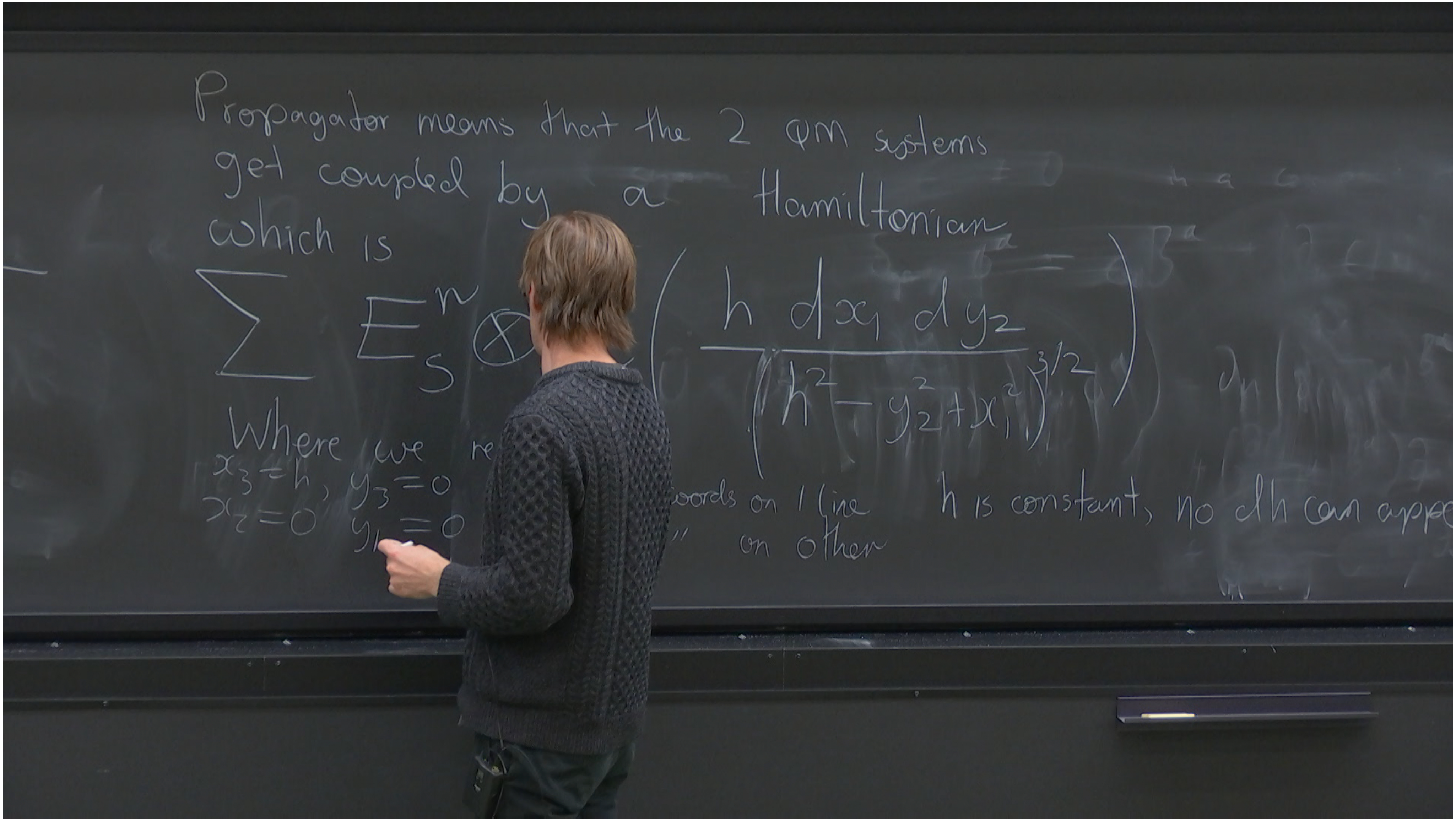
Where we restrict

$x_3 = h,\ y_3 = 0$

$x_2 = 0\ \ y_1 = 0$

y - coords on 1 line

x - " on other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Where we
$x_3 = h, \; y_3 = 0$
$x_2 = 0 \quad y_1 = 0$
on 1 line
h is constant, no dh can
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Parallel lines

$x_2 - y_2$ and $x_3 - y_3$

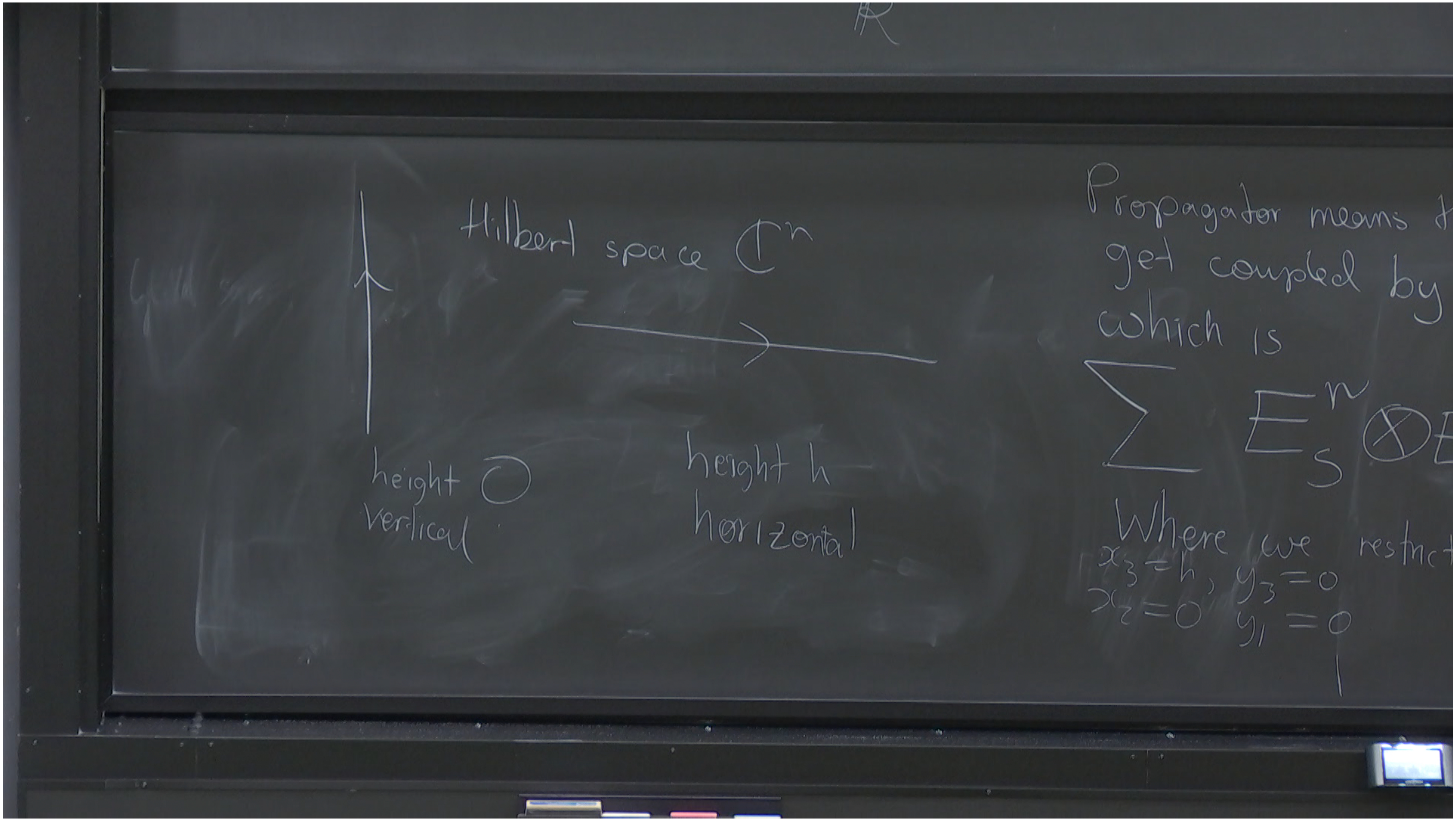
are constant

$d(x_2 - y_2) = 0 \quad d(x_3 - y_3) = 0$

$\Rightarrow$ propagator is zero, as

in $(x_i - y_i)\, d(x_j - y_j)\, d(x_k - y_k)\, \varepsilon_{ijk}$

will have $d(x_2 - y_2)$ or $d(x_3 - y_3)$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...ans that the 2 QM systems

... by a Hamiltonian

$$\otimes E^s_r \left(\frac{h\, dx_1\, dy_2}{(h^2 - y_2^2 + x_1^2)^{3/2}} \right)$$

restrict

y - coords on 1 line
x - " on other

h is constant, no dh can appear

$\mathrm{Tr}\, A\, dA$

$A = A^r{}_s E^r_s$

$\sum A^r{}_s\, dA^s_r$

states get sent to

$$\left(E^s_r e_b \otimes E^r_s e_a\right) \iint_{x,y} \frac{h\, dx\, dy}{(h^2+x^2+y^2)^{3/2}}$$

ompute the integral:

x/h, then

$$\overline{\quad(\ldots x^2+y^2)^{3/2}} = \overline{|h|^3 (1+\tilde{x}^2+\tilde{y}^2)}$$

$$dx\, dy \to h^2\, d\tilde{x}\, d\tilde{y}$$

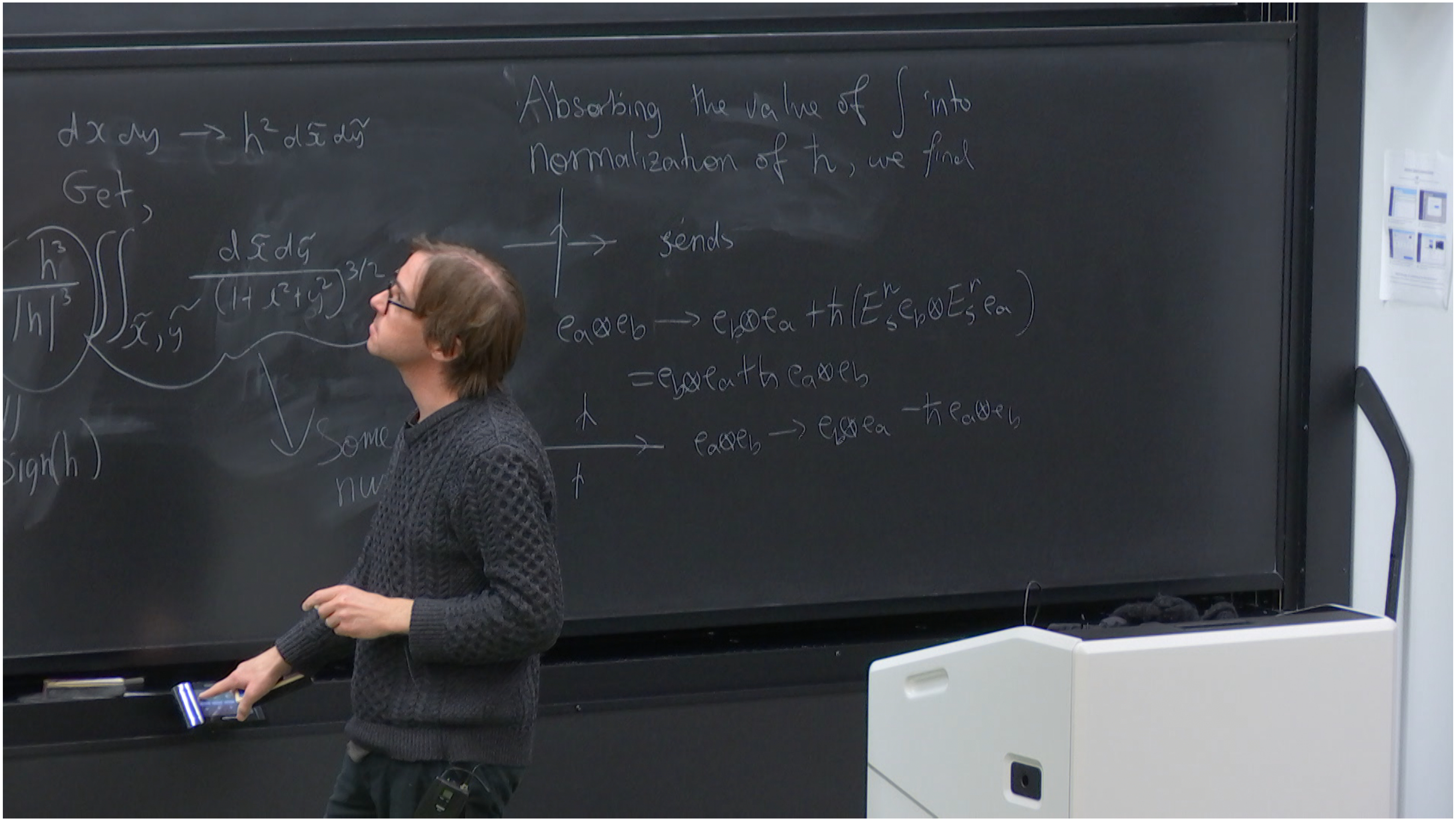
Get,

$$\frac{h^3}{|h|^3} \iint_{\tilde{x},\tilde{y}} \frac{d\tilde{x}\, d\tilde{y}}{(1+\tilde{x}^2+\tilde{y}^2)^{3/2}}$$

$\|$ Sign(h)

Some +ve number

Absorbing the normaliz



Absorbing the value of ∫ into
normalization of ħ, we find
sends
Get,
Some

Skein Relation:

$\rho = \text{flip}$

$L_+ = \quad = \rho + \hbar \, \mathrm{Id}$

$L_- = \quad = \rho - \hbar \, \mathrm{Id}$

$L_0 = \quad = \mathrm{Id}$

mod $\hbar^2$

$L_+ - L_- = L_0 \, 2\hbar$

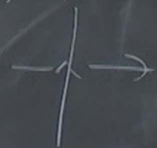
Compare Jones Polynomial

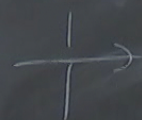
$t^{-1} L_+ - t L_- = (t^{1/2} - t^{-1/2}) L_0$

Very Close!

Skein Relation

ρ = flip

$L_+ =$  $= \rho + \hbar\,\mathrm{Id}$

$L_- =$  $= \rho - \hbar\,\mathrm{Id}$

$L_0 =$  $= \mathrm{Id}$

mod $\hbar^2$

$$L_+ - L_- = L_0\, 2\hbar$$

Compare: Jones Polynomial

$$t^{-1}L_+ - tL_- = (t^{1/2} - t^{-1/2})L_0$$

Very Close!

If we use sl_n, rather than gl_n, the colour factors are modified a little to give (I think)

$$= \rho + \hbar\,\mathrm{Id} - \frac{\hbar}{n}\rho$$

$$= \rho - \hbar\,\mathrm{Id} + \frac{\hbar}{n}\rho$$

So we get

$$L_+ - L_- = 2\hbar L_0 + \frac{2\hbar}{n}\rho$$

So,

$$L_+\left(1 + \frac{\hbar}{n}\right) - L_-\left(1 - \frac{\hbar}{n}\right) = 2\hbar L_0$$

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