

Title: Fundamental Constraints for Fundamental Theories

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Series: Colloquium

Date: March 11, 2020 - 2:00 PM

URL: <http://pirsa.org/20030016>

Abstract:

As our understanding of the universe and its fundamental building blocks extends to shorter and shorter distances, experiments capable of probing these scales are becoming increasingly difficult to construct. Fundamental particle physics faces a potential crisis: an absence of data at the shortest possible scales. Yet remarkably, even in the absence of experimental data, the requirement of theoretical consistency puts stringent constraints on viable models of fundamental particles and their interactions. In this talk I'll discuss a variety of criteria that constrain theories of particles in flat spacetime and de Sitter. Such criteria have the possibility to address questions such as: What low energy theories admit consistent UV completions? Which massive particles are allowed in an interacting theory? Is string theory the unique weakly coupled UV completion of General Relativity?

Fundamental Constraints for Fundamental Theories



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March 11, 2020

Outline

- **What Particles?**
- **What Interactions?**
- **What Else?**



1

What Particles?

Starting Assumption: Poincaré Invariance

Poincaré symmetry is the full symmetry
of **Special Relativity**

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Poincaré transformations leave invariant
the interval between two events

Starting Assumption: Poincaré Invariance

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

10 Such Transformations: $P_\mu, J_{\mu\nu}$

- 4 Translations P_μ
- 3 Rotations $\vec{J} = \{J^{23}, J^{31}, J^{12}\}$
- 3 Boosts $\vec{K} = \{J^{01}, J^{02}, J^{03}\}$

Starting Assumption: Poincaré Invariance

Classify one-particle states according to their transformations under the Poincaré Group

$$C_1 = -P_\mu P^\mu$$

$$C_2 = W_\mu W^\mu$$

Pauli-Lubanski $W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}J_{\nu\rho}P_\sigma$

massive

$$C_1 = m^2 > 0$$

$$C_2 = m^2 s(s+1)$$

$$\text{spin } s = 0, \frac{1}{2}, 1, \dots$$

massless

$$C_1 = 0$$

$$J_3 = \sigma$$

$$\text{helicity } \sigma = 0, \pm \frac{1}{2}, \pm 1, \dots$$

Assume: Poincaré Invariance

free *massless* Lagrangians

$$\text{spin-0} \quad \mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\text{spin-1} \quad \mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$$\text{spin-2} \quad \mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h$$

$$\text{spin} \geq 3 \quad \mathcal{L} = \dots$$

free *massive* Lagrangians

$$\text{spin-0} \quad \mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

$$\text{spin-1} \quad \mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2}m^2 A_\mu A^\mu$$

$$\text{spin-2} \quad \mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

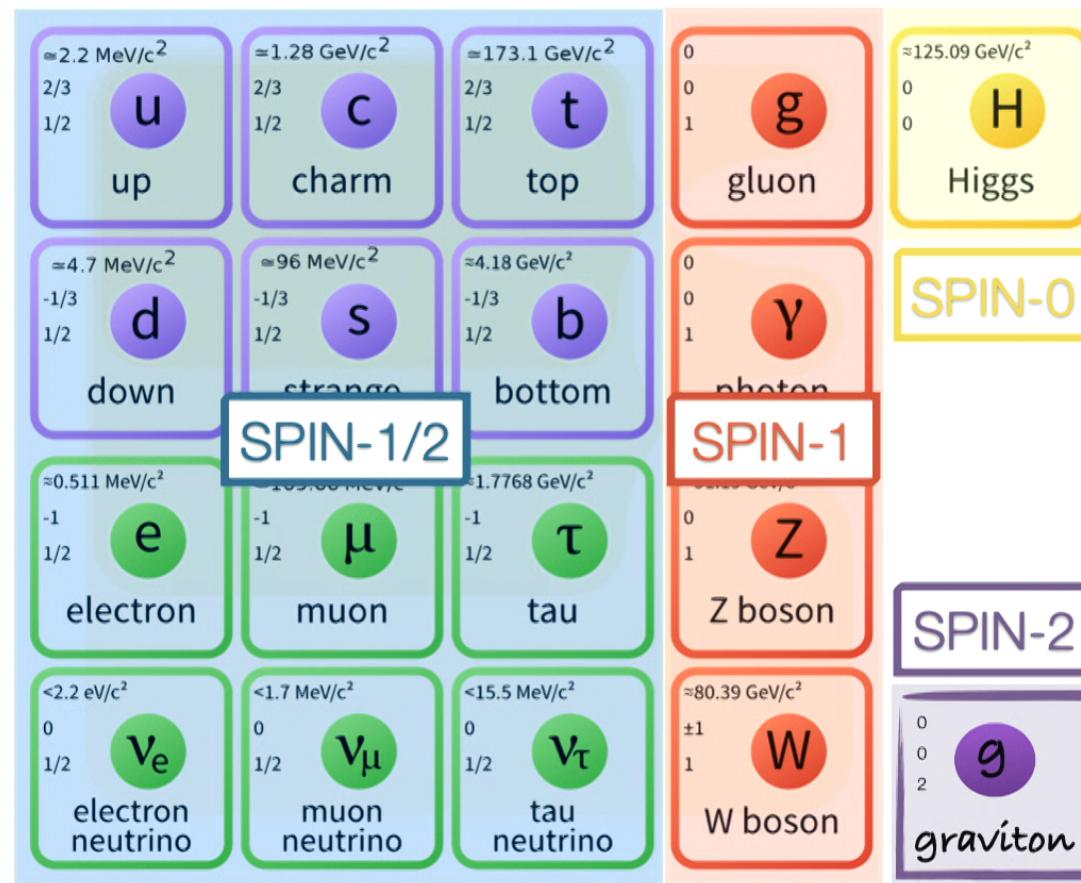
$$\text{spin} \geq 3 \quad \mathcal{L} = \dots$$

Fierz, Pauli (1939) Singh, Hagen (1974) Fronsdal (1978)

Standard Model of Elementary Particles

three generations of matter (fermions)					
	I	II	III		
QUARKS	mass $\approx 2.2 \text{ MeV}/c^2$ charge 2/3 spin 1/2	mass $\approx 1.28 \text{ GeV}/c^2$ charge 2/3 spin 1/2	mass $\approx 173.1 \text{ GeV}/c^2$ charge 2/3 spin 1/2	mass 0 charge 0 spin 1	mass $\approx 125.09 \text{ GeV}/c^2$ charge 0 spin 0
	u up	c charm	t top	g gluon	H Higgs
	mass $\approx 4.7 \text{ MeV}/c^2$ charge -1/3 spin 1/2	mass $\approx 96 \text{ MeV}/c^2$ charge -1/3 spin 1/2	mass $\approx 4.18 \text{ GeV}/c^2$ charge -1/3 spin 1/2	mass 0 charge 0 spin 1	mass 0 charge 0 spin 0
	d down	s strange	b bottom	γ photon	
	mass $\approx 0.511 \text{ MeV}/c^2$ charge -1 spin 1/2	mass $\approx 105.66 \text{ MeV}/c^2$ charge -1 spin 1/2	mass $\approx 1.7768 \text{ GeV}/c^2$ charge -1 spin 1/2	mass $\approx 91.19 \text{ GeV}/c^2$ charge 0 spin 1	mass 0 charge 0 spin 0
	e electron	μ muon	τ tau	Z Z boson	
	mass $< 2.2 \text{ eV}/c^2$ charge 0 spin 1/2	mass $< 1.7 \text{ MeV}/c^2$ charge 0 spin 1/2	mass $< 15.5 \text{ MeV}/c^2$ charge 0 spin 1/2	mass $\approx 80.39 \text{ GeV}/c^2$ charge ± 1 spin 1	mass 0 charge 0 spin 0
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
					SCALAR BOSONS
					GAUGE BOSONS

beyond the Standard Model of Elementary Particles



Can theory tell us *why*?

- Why no particles with spin > 2?
- Why only one spin-0? And one spin-2?
- Why is the spin-2 massless?
- *Are these meaningful questions?*

2

What
Interactions?



Massless Particles

Interactions of *massless* particles are
remarkably constrained

spin-0 $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ +???

spin-1 $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$ +???

spin-2 $\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h$ +???

spin ≥ 3 $\mathcal{L} = \dots$ +???

Massless Particles

Interacting theories must *preserve or extend* the gauge symmetries of the free theories

* spin-0 $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ +???

$$\delta\phi = \sum_{k=0} c_{\mu_1 \dots \mu_k} x^{\mu_1} \cdots x^{\mu_k}$$
 +???

spin-1 $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$ +???

$$\delta A_\mu = \partial_\mu \Lambda$$
 +???

spin-2 $\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h$ +???

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$
 +???

spin ≥ 3 $\mathcal{L} = \dots$ +???

$$\dots$$
 +???

Massless Particles

Can easily write theories which
preserve the linear symmetry

$$\text{spin-0} \quad \mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \sum_n c_n f(X^n)$$
$$\delta\phi = c \quad X \equiv \partial_\mu\phi$$

$$\text{spin-1} \quad \mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + \sum_n c_n f(F^n)$$
$$\delta A_\mu = \partial_\mu \Lambda \quad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\text{spin-2} \quad \mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h + \sum_n c_n f(R^n)$$
$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \quad R_{\mu\nu\rho\sigma} \equiv \partial_\sigma \partial_{[\mu} h_{\nu]\rho} - \partial_\rho \partial_{[\mu} h_{\nu]\sigma}$$

$$\text{spin} \geq 3 \quad \mathcal{L} = \dots$$

...

Massless Particles

“Interesting” theories are the ones where the gauge symmetry is extended by the interactions

$$\delta\phi = c + \text{???} \quad \delta A_\mu = \partial_\mu \Lambda + \text{???} \quad \delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + \text{??}$$

CONSTRAINT:
symmetry algebra must *close*

simple analogue: translations

$$T_a f(x) = f(x + a) \rightarrow T_a T_b f(x) = T_{a+b} f(x)$$

Massless Particles

“Interesting” theories are the ones where the gauge symmetry is extended by the interactions

$$\text{spin-1: } \delta A_\mu^a = \partial_\mu \Lambda^a + g f^{abc} A_\mu^b \Lambda^c \quad \text{Yang-Mills!}$$

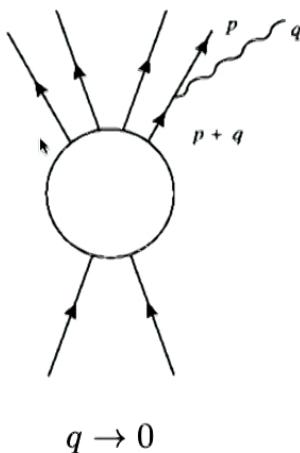
$$\text{spin-2: } \delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu \quad \text{General Relativity!}$$

spin ≥ 3 : *none*

(Wald 1986)

Weinberg (1964)

soft theorems for massless particles



- **spin-1: electric charge is conserved**
- **spin-2: coupling is the same for all forms of energy and momentum**
- **spin ≥ 3 : couplings don't survive at low energies**

$$\sum_n e_n = 0$$

$$\sum_n g_n p_n^\mu = 0$$

$$\sum_n g_n p_n^\mu p_n^\nu \dots = 0$$

Summary: Massless Particles

only assumed Poincaré invariance

- spin-0: no problem
- spin-1: only have E&M, Yang-Mills;
electric charge is conserved
- spin-2: only have GR; can derive
equivalence principle
- spin ≥ 3 : no non-trivial theories at
low energies

consistent with Standard Model + gravity

Massive Particles

$$\text{spin-0: } \mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \dots$$

Exists! E.g., Higgs boson...

Massive Particles

spin-0: $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \dots$

Exists! E.g., Higgs boson...

spin-1: $\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2}m^2 A_\mu A^\mu + e^2 \phi^2 A_\mu A^\mu + \dots$

Exists! E.g., W & Z bosons...

spin-2:

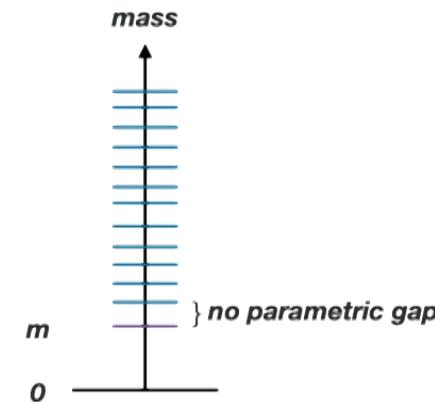
$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + ???$$

Exists???

Massive Particles

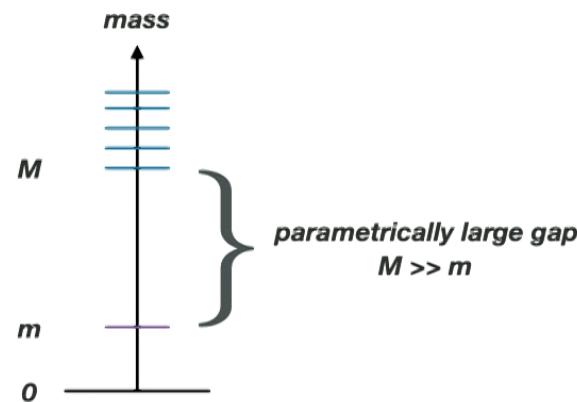
Interacting, massive high-spin particles *do* exist

- **QCD:** $m^2 \sim \Lambda_{QCD}^2$
- **Kaluza-Klein Theory:** $m^2 \sim \frac{1}{R^2}$
- **String Theory:** $m^2 \sim \frac{1}{\alpha'}$



Massive Particles

Can the spectrum look like this?



spin-0: yes!

spin-1: yes!

spin ≥ 2 : ???

Need to construct a low energy EFT with cut-off $>>$ mass

Massive Particles

Massive Spin-2: the Free Theory

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h + m_1^2 h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2$$



add a mass!

Massive Particles

Massive Spin-2: the Free Theory

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h + m_1^2 h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2$$

add a mass!

$h_{\mu\nu}$ has **10** independent components

for the *massless* spin-2, gauge invariance

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

leaves **2** physical degrees of freedom: 

BREAK DIFFEOMORPHISM INVARIANCE → extra DOF!

Massive Particles

Massive Spin-2: the Free Theory

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h + m_1^2 h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2$$

- 4 Bianchi constraints: $m_1^2\partial^\mu h_{\mu\nu} + m_2^2\partial_\nu h = 0$

$$10 - 4 = \underline{6} \text{ DOF}$$

massive graviton should have $2s + 1 = 5$ DOF

Massive Particles

Massive Spin-2: the Free Theory

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

 FIERZ-PAULI
MASS TERM
(1939)

- 4 Bianchi constraints: $m_1^2\partial^\mu h_{\mu\nu} + m_2^2\partial_\nu h = 0$
- additional constraint: $m^2 h = 0$

$$10 - 4 - 1 = 5 \text{ DOF } \checkmark$$

Massive Particles

Massive Spin-2: the Interacting Theory

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$
$$+ \mathcal{O}(h^3) + \mathcal{O}(h^4) + \dots$$

add interaction terms

- Is there a *non-linear* theory of a massive spin-2 particle that maintains a constraint at the fully non-linear level and thus avoids an extra, pathological DOF?

Boulware, Deser (1972)



YES!

Massive Particles

Massive Spin-2: the Interacting Theory

$$\mathcal{S} = \frac{M_{Pl}^2}{2} \left\{ \int d^4x \det e R[e] - m^2 \int \sum_{n=0}^4 \beta_n S_n[e] \right\}$$

$$S_0[e] = \epsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge e^d$$

$$S_1[e] = \epsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge \mathbf{1}^d$$

$$S_2[e] = \epsilon_{abcd} e^a \wedge e^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

$$S_3[e] = \epsilon_{abcd} e^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

$$S_4[e] = \epsilon_{abcd} \mathbf{1}^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$



Free parameters: M_{Pl} , Λ , m , β_2 , β_3

C. de Rham, G. Gabadadze, A. J. Tolley (2010) S. F. Hassan, RAR (2011) K. Hinterbichler, RAR (2012)

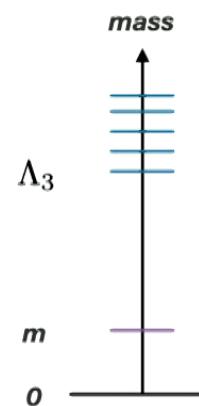
What about the cut-off?

$$\mathcal{S} = \frac{M_{Pl}^2}{2} \left\{ \int d^4x \det e R[e] - m^2 \int \sum_{n=0}^4 \beta_n S_n[e] \right\}$$

EFT cut-off:

$$\Lambda_3 = (M_{Pl} m^2)^{1/3}$$

$$\Lambda_3 \gg m$$



hierarchy is stable under quantum corrections

Beyond Effective Field Theory

High Energy Constraints on Low Energy Physics

LAGRANGIAN FORMULATION:

master expression specifying
particles and their interactions
lots of redundancies

S-MATRIX FORMULATION:

relates initial and final states of
asymptotically free particles
undergoing scattering
eliminates redundancies

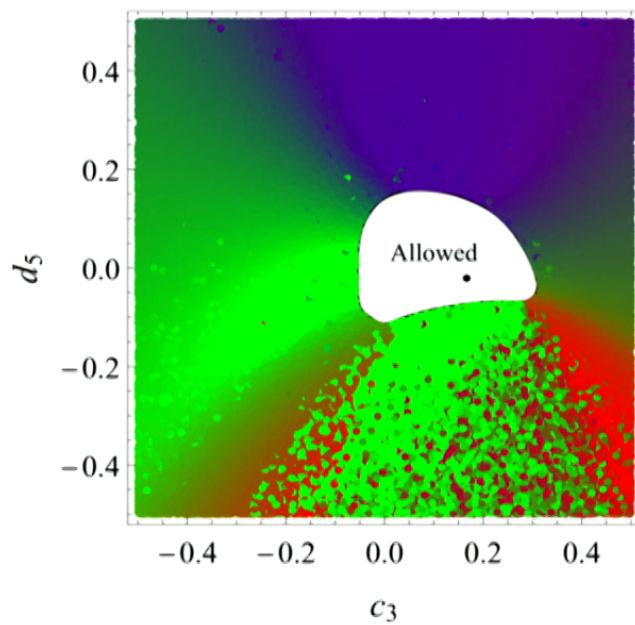
CAUSALITY



ANALYTICITY

Beyond Effective Field Theory

Analyticity of the S-Matrix



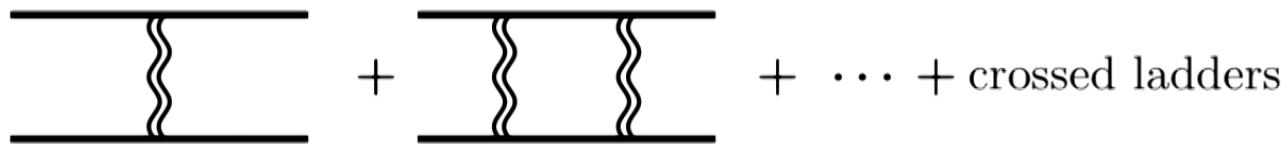
Consistency of scattering amplitudes in the forward limit constrains the free parameters of a massive spin-2 particle

Cheung, Remmen (2016)

Beyond Effective Field Theory

Absence of superluminality in the eikonal limit

Consider scattering at high
energy, large impact parameter $t/s \rightarrow 0$

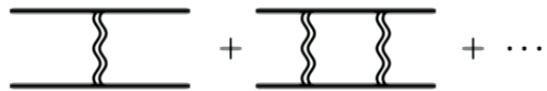


$$i\mathcal{M}_{\text{eik}}(s, t) = 2s \int d^{D-2} \vec{b} e^{i\vec{q} \cdot \vec{b}} \left(e^{i\delta(s, \vec{b})} - 1 \right)$$

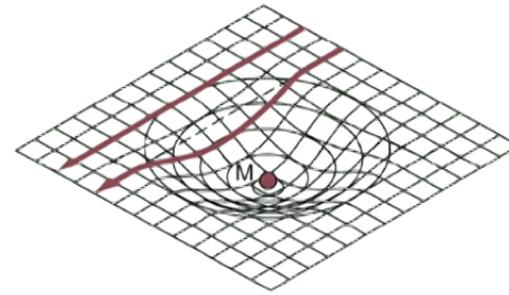
$$\text{time delay: } \Delta x^- = \frac{1}{|p^-|} \delta(s, b) \quad \delta(s, b) > 0$$

Beyond Effective Field Theory

Absence of superluminality in the eikonal limit



eikonal scattering



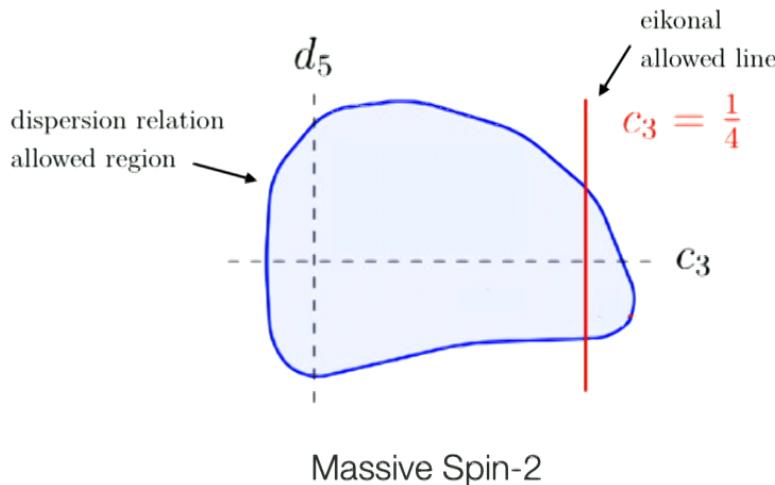
Shapiro time delay

image: Heinkelmann & Schuh

Dray, 't Hooft (1985); 't Hooft (1987); Kabat, Ortiz (1992)

Beyond Effective Field Theory

Absence of superluminality in the eikonal limit



- isolates a particular choice of cubic coupling for massive graviton
- rules out cubic couplings of massless high spin particles
- can be used to look for UV completions of General Relativity

Camanho, Edelstein, Maldacena, Zhiboedov (2014)

Joyce, Hinterbichler, RAR (2017 & 2018)

Bonifacio, Joyce, Hinterbichler, RAR (2018)

Summary: Massive Particles

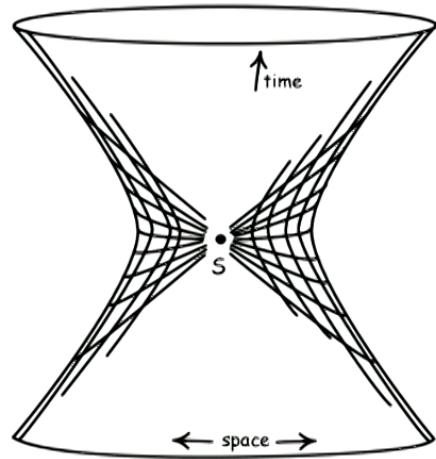
- spin-0: no problem
- spin-1: need a Higgs mechanism
- spin-2: non-trivial
- spin ≥ 3 : well, there's no no-go...

consistent with Standard Model + gravity

3

What Else?

Representations of the de Sitter Group



$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

Minkowski: Poincare
de Sitter: $\text{SO}(1,4)$

Maximally symmetric spaces
10 isometries

Representations of the de Sitter Group

flat spacetime:

$$ds^2 = -dt^2 + d\vec{x}^2$$

massive

$$m^2 > 0$$

massless

$$m^2 = 0$$

gauge symmetry

de Sitter spacetime:

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

massive

$$m^2 > s(s-1)H^2$$

Higuchi bound (1987)

massless

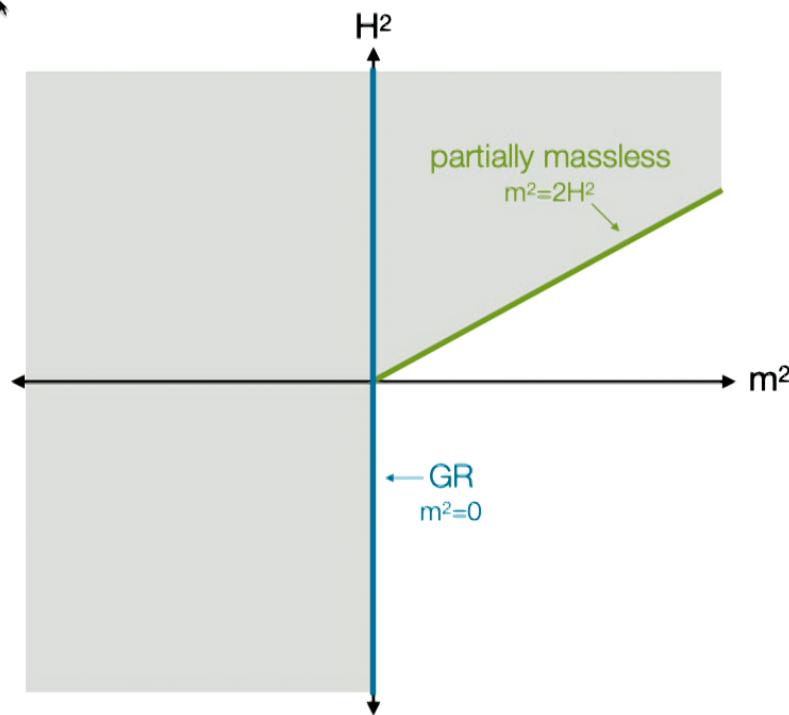
$$m^2 = (s+t)(s-1-t)H^2$$

$$t = 0, 1, \dots, s-1$$

gauge symmetry

Representations of the de Sitter Group

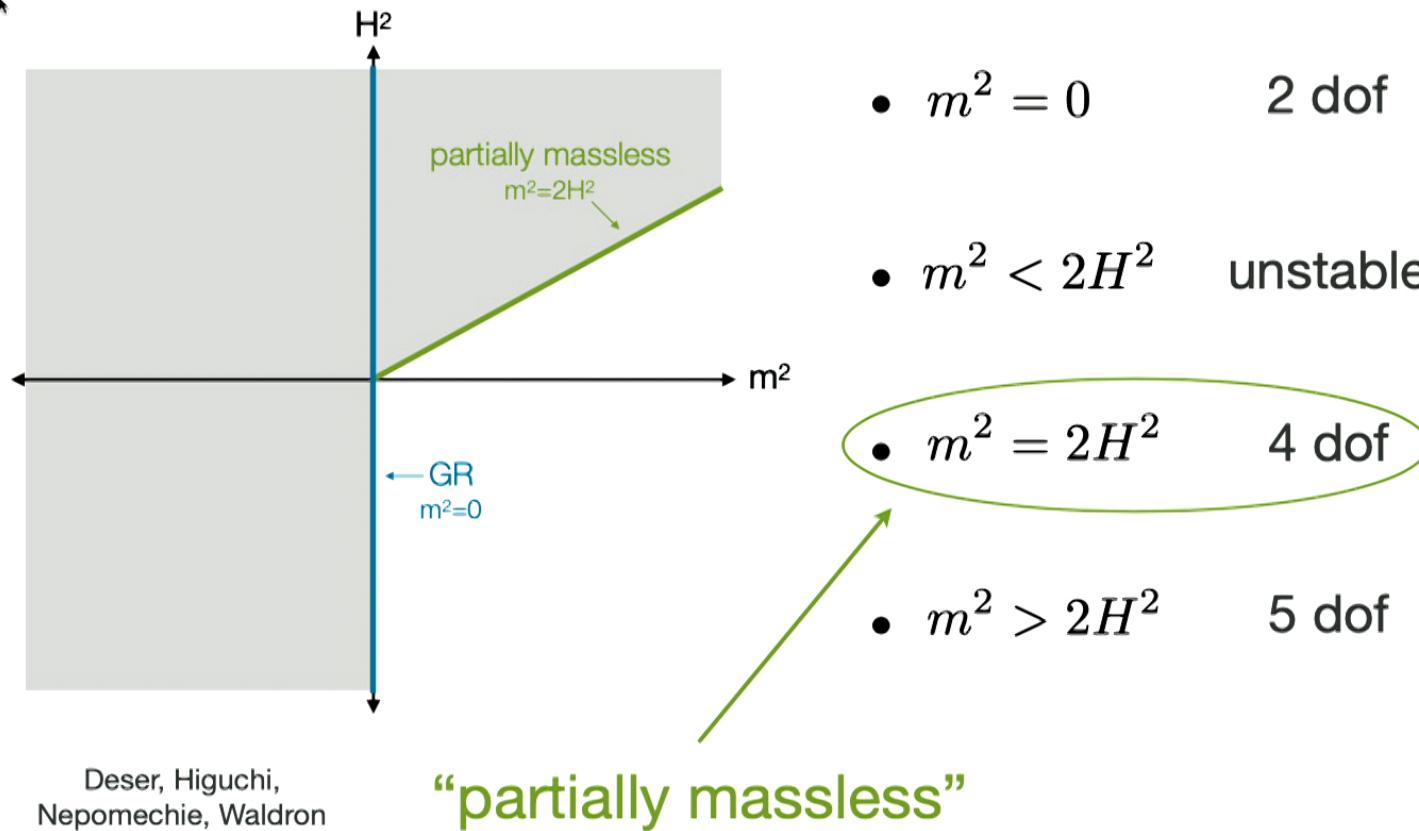
Spin-2 Particles



- $m^2 = 0$ 2 dof
- $m^2 < 2H^2$ unstable
- $m^2 = 2H^2$ 4 dof
- $m^2 > 2H^2$ 5 dof

Representations of the de Sitter Group

Spin-2 Particles



Representations of the de Sitter Group

Partially Massless Spin-2 Particles

$$\mathcal{L}_2 = \sqrt{-\bar{g}} \left[-\frac{1}{2} \bar{\nabla}_\alpha h_{\mu\nu} \bar{\nabla}^\alpha h^{\mu\nu} + \bar{\nabla}_\alpha h_{\mu\nu} \bar{\nabla}^\nu h^{\mu\alpha} - \bar{\nabla}_\mu h \bar{\nabla}_\nu h^{\mu\nu} + \frac{1}{2} \bar{\nabla}_\mu h \bar{\nabla}^\mu h \right. \\ \left. + \frac{\bar{R}}{4} (h^{\mu\nu} h_{\mu\nu} - \frac{1}{2} h^2) - \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) \right]$$

$$\text{gauge symmetry: } \delta h_{\mu\nu} = (\bar{\nabla}_\mu \bar{\nabla}_\nu + \frac{1}{12} \bar{R} \bar{g}_{\mu\nu}) \phi$$

Representations of the de Sitter Group

Partially Massless Spin-2 Particles

$$\begin{aligned}\mathcal{L}_2 = \sqrt{-\bar{g}} & \left[-\frac{1}{2} \bar{\nabla}_\alpha h_{\mu\nu} \bar{\nabla}^\alpha h^{\mu\nu} + \bar{\nabla}_\alpha h_{\mu\nu} \bar{\nabla}^\nu h^{\mu\alpha} - \bar{\nabla}_\mu h \bar{\nabla}_\nu h^{\mu\nu} + \frac{1}{2} \bar{\nabla}_\mu h \bar{\nabla}^\mu h \right. \\ & \left. + \frac{\bar{R}}{4} (h^{\mu\nu} h_{\mu\nu} - \frac{1}{2} h^2) - \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) \right]\end{aligned}$$

$$\text{gauge symmetry: } \delta h_{\mu\nu} = (\bar{\nabla}_\mu \bar{\nabla}_\nu + \frac{1}{12} \bar{R} \bar{g}_{\mu\nu}) \phi$$

- *no non-trivial extensions of the gauge symmetry*

$$[\delta_\phi, \delta_\psi] h_{\mu\nu} = \delta_{\chi(\phi, \psi)} h_{\mu\nu}$$

Garcia-Saenz, RAR (2014)

Hinterbichler, Joyce, Garcia-Saenz, Mitsou, RAR (2015)

Representations of the de Sitter Group

Spin-0 Particles

flat spacetime:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\delta\phi = \sum_{k=0} c_{\mu_1\dots\mu_k} x^{\mu_1}\cdots x^{\mu_k}$$

**Admits interactions
for k=0,1,2**

de Sitter spacetime:

$$\mathcal{L} = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}m_k^2\phi^2$$

$$m_k^2 = -k(k+3)H^2$$

$$\delta\phi = C_{A_1\dots A_k} x^{A_1}\cdots x^{A_k}$$

**Admits interactions
for k=0,1,2**

Bonifacio, Hinterbichler, Joyce, RAR (2018)

Representations of the de Sitter Group

Spin-0 Particles

flat spacetime:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\delta\phi = \sum_{k=0} c_{\mu_1\dots\mu_k} x^{\mu_1}\cdots x^{\mu_k}$$

Interacting theories: arise generically in theories with spontaneous symmetry breaking, enjoy enhanced soft limits, non-renormalization theorems...

de Sitter spacetime:

$$\mathcal{L} = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}m_k^2\phi^2$$

$$m_k^2 = -k(k+3)H^2$$

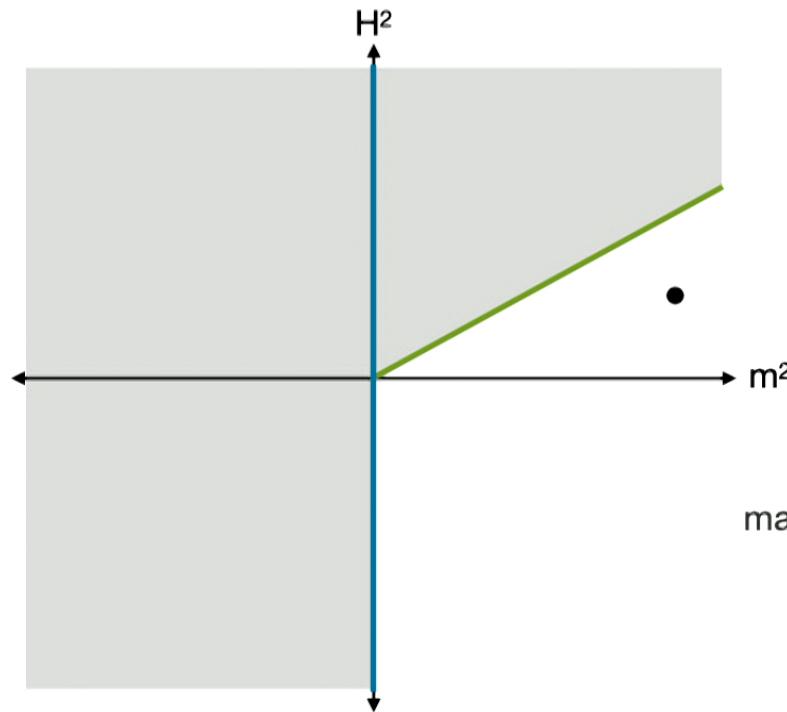
$$\delta\phi = C_{A_1\dots A_k} x^{A_1}\cdots x^{A_k}$$

tachyonic masses!
but unitary irreps??

Bonifacio, Hinterbichler, Joyce, RAR (2018)

Representations of the de Sitter Group

Spin-2 Particles

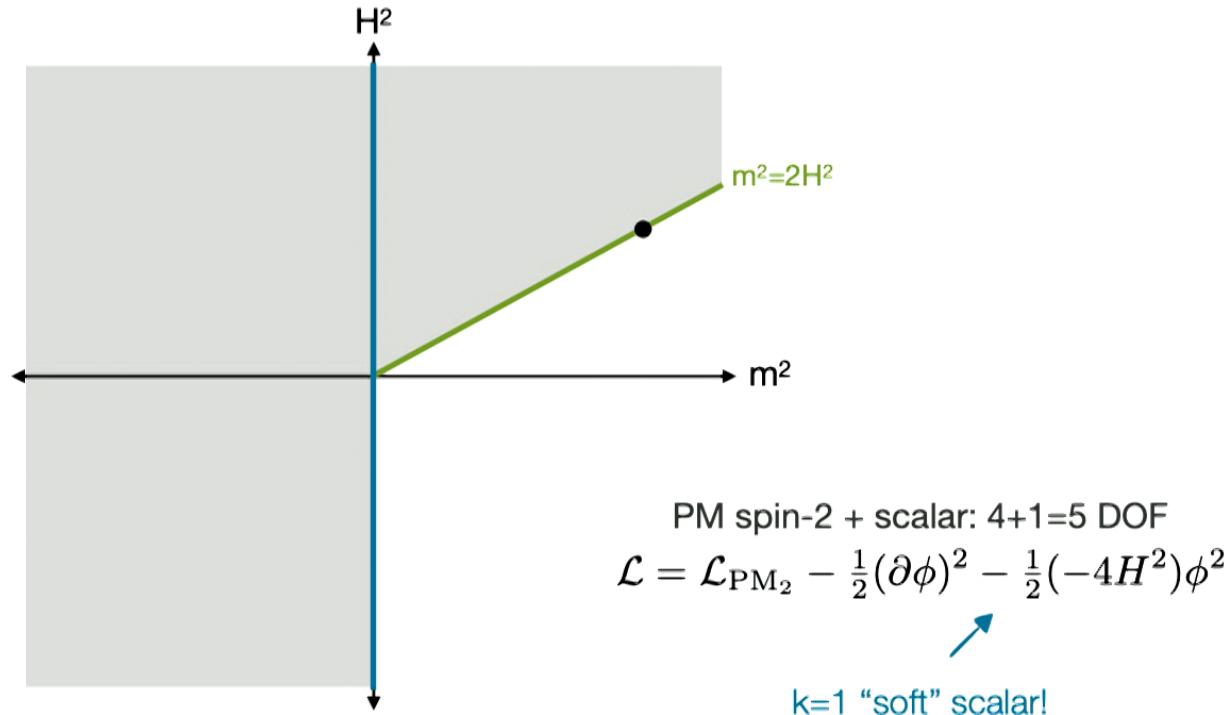


massive spin-2: 5 DOF

$$\mathcal{L} = \mathcal{L}_{FP}$$

Representations of the de Sitter Group

Spin-2 Particles



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Representations of the de Sitter Group

*level “k” scalars are the decoupled helicity-0 modes
of depth $t=0$, spin $s=k+1$ PM particles*

“Cosmological Bootstrap”

Arkani-Hamed, Baumann, Lee, Pimentel (2018)

flat spacetime



de Sitter spacetime

see, e.g., Britto, Cachazo, Feng, Witten (2005);
Bern, Carrasco, Johansson (2008) Page 52 of 55

Summary

Theory is powerful! Can constrain both observable and unobservable physics.

Variety of possible applications: cosmological correlators, massive spinning particles, UV completions of General Relativity...

Thank you!



$$\epsilon_{klm} \epsilon_{lpi} + \epsilon_{kpl} \epsilon_{lmq} + \epsilon_{plm} \epsilon_{lkq} = 0$$

$$\phi \sim \gamma^{\Delta_+} \alpha \gamma^{\Delta_-}$$

$\Delta_+ = 3+k$ $\Delta_- = -k$



$$\partial_b \left(\epsilon^{bc} + \epsilon_{pm} \Lambda^p_b E^{bm} \right) \Lambda^q \Big) \Big|^2 \times$$

$[\Lambda^i, \Lambda_j] = \epsilon_{jk} \underbrace{(\Lambda^i)^j}_{\Lambda^i = \Lambda^i(T)} \Lambda^k$
 T_i