Title: Precision microstate counting of AdS black hole entropy

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Series: Quantum Fields and Strings

Date: March 03, 2020 - 2:30 PM

URL: http://pirsa.org/20030013

Abstract: I will describe how within eleven dimensional supergravity one can compute the logarithmic correction to the Bekenstein-Hawking entropy of certain magnetically charged asymptotically AdS_4 black holes with arbitrary horizon topology. The result perfectly agrees with the dual field theory computation of the topologically twisted index in ABJM theory and in certain theories obtained from M5 wrapping a hyperbolic 3-manifold. The extension to rotating, electrically charged AdS_4 black holes and the dual superconformal index will also be discussed.

Precision Microstate Counting of AdS Black Hole Entropy

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Waterloo, March 3, 2020

arXiv:1909.11612, F. Benini, D. Gang andL PZ
arXiv:1905.01559, D. Gang, N. Kim and LPZ
arXiv:1907.12841, A. González Lezcano and LPZ
PRL 120, 221602 (2018), J. Liu, LPZ, V. Rathee and W. Zhao

Motivation

$$S = \frac{k_B c^3}{\hbar}\frac{A}{4G_N}$$

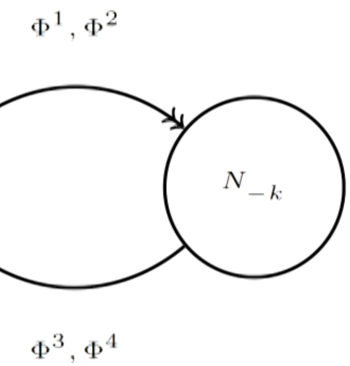
- A confluence of thermodynamical, relativistic, gravitational, and quantum aspects. Hydrogen atom of QG. [Strominger-Vafa '96].
- An explicit example in $\mathrm{AdS}_4/\mathrm{CFT}_3$: The large-$N$ limit of the topologically twisted index of ABJM correctly reproduces the leading term in the entropy of magnetically charged black holes in asymptotically AdS_4 spacetimes [Benini-Hristov-Zaffaroni '15].
- Extended to dyonic black holes, black holes in massive IIA theory and black holes from wrapped M5-branes. Rotating BH's in various dimensions through superconformal indices ['18].
- Agreement has been shown beyond the large N limit by matching the coefficient of $\log N$ [Liu-PZ-Rathee-Zhao, Gang-Kim-PZ, Benini-Gang-PZ]. (Beyond Bohr energies).

Outline

- The Topologically Twisted Index of ABJM Theory beyond large N
- Magnetically Charged Asymptotically AdS_4 Black Holes
- Logarithmic Corrections in Quantum Supergravity
- Entropy Counting for Wrapped M5 Branes:
  (i) Magnetically charged AdS$_4$ black holes
  (ii) Electrically charged, rotating AdS$_4$ black holes
- Entropy Counting for AdS$_5$ black h Holes :
  (i) The superconformal index for $\mathcal{N} = 1$ toric quiver gauge theories
  (ii) Logarithmic corrections in the index
- Conclusions and some open problems.

ABJM Theory

- ABJM: A 3d Chern-Simons-matter theory with $U(N)_k \times U(N)_{-k}$ gauge group.
- Matter sector in bifundamental representations.
- SCFT $\mathcal{N} = 6$ supersymmetry generically but for $k = 1, 2$, the symmetry is enhanced to $\mathcal{N} = 8$.
- Global Symmetry in $\mathcal{N} = 2$ notation is $SU(2)_{1,2} \times SU(2)_{3,4} \times U(1)_T \times U(1)_R$.



The Topologically Twisted Index of ABJM Theory

- The topologically twisted index for three dimensional $\mathcal{N}=2$ field theories was defined in [Benini-Zaffaroni '15] (Honda '15, Closset '15) by evaluating the supersymmetric partition function on $S^1 \times S^2$ with a topological twist on S^2.
- Hamiltonian: The supersymmetric partition function of the twisted theory, $Z(n_a, \Delta_a) = \mathrm{Tr}\,(-1)^F e^{-\beta H} e^{iJ_a\Delta_a}$. It depends on the fluxes and on the chemical potentials: n_a, Δ_a.
- The topologically twisted index for $\mathcal{N} \geq 2$ supersymmetric theories on $S^2 \times S^1$ can be computed via supersymmetric localization.
- The supersymmetric localization computation of the topologically twisted index can be extended to theories defined on $\Sigma_g \times S^1$.

General form of the Index

- Background:

$$ds^2 = R^2(d\theta^2 + \sin^2\theta d\phi^2) + \beta^2 dt^2, \quad A^R = \frac{1}{2}\cos\theta d\phi.$$

- The index can be expressed as a contour integral:

$$Z(n_a, y_a) = \sum_{\mathfrak{m}\in\Gamma_{\mathfrak{h}}} \oint_{\mathcal{C}} Z_{int}(x, \mathfrak{m}; n_a, y_a).$$

- Z_{int} meromorphic form, Cartan-valued complex variables $x = e^{i(A_t + i\beta\sigma)} = e^{iu}$, lattice of magnetic gauge fluxes $\Gamma_{\mathfrak{h}}$.
- Flavor magnetic fluxes n_a and fugacities $y_a = e^{i(A_t^a + i\beta\sigma^a)}$.
- Localization: $Z_{int} = Z_{\text{class}} Z_{\text{one-loop}}$.
- E.G.: $Z^{CS}_{class} = x^{k\mathfrak{m}}$, $Z^{gauge}_{1-loop} = \prod_{\alpha\in G}(1 - x^\alpha)(idu)^r$, r – rank of the gauge group, α – roots of G and $u = A_t + i\beta\sigma$.

- The topologically twisted index for ABMJ theory:

$$Z(y_a, n_a) = \prod_{a=1}^{4} y_a^{-\frac{1}{2}N^2 n_a} \sum_{I \in BAE} \frac{1}{\det \mathbb{B}} \times \frac{\prod_{i=1}^{N} x_i^N \tilde{x}_i^N \prod_{i\neq j}\left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right)}{\prod_{i,j=1}^{N}\prod_{a=1,2}(\tilde{x}_j - y_a x_i)^{1-n_a}\prod_{a=3,4}(x_i - y_a \tilde{x}_j)^{1-n_a}}.$$

- Contour integral $\to$ Evaluation (Poles): $e^{iB_i} = e^{i\tilde{B}_i} = 1$

$$e^{iB_i} = x_i^k \prod_{j=1}^{N} \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})},$$

$$e^{i\tilde{B}_j} = \tilde{x}_j^k \prod_{i=1}^{N} \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})}.$$

- The $2N \times 2N$ matrix $\mathbb{B}$ is the Jacobian relating the $\{x_i, \tilde{x}_j\}$ variables to the $\{e^{iB_i}, e^{i\tilde{B}_j}\}$ variables

Algorithmic Summary:

- Given the chemical potentials Δ_a according to $y_a = e^{i\Delta_a}$, and variables $x_i = e^{iu_i}$, $\tilde{x}_j = e^{i\tilde{u}_j}$, the equations (poles):

$$
\begin{aligned}
0 &= k u_i - i \sum_{j=1}^{N} \left[\sum_{a=3,4} \log\left(1 - e^{i(\tilde{u}_j - u_i + \Delta_a)}\right) - \sum_{a=1,2} \log\left(1 - e^{i(\tilde{u}_j - u_i - \Delta_a)}\right) \right] - 2\pi n_i , \\
0 &= k \tilde{u}_j - i \sum_{i=1}^{N} \left[\sum_{a=3,4} \log\left(1 - e^{i(\tilde{u}_j - u_i + \Delta_a)}\right) - \sum_{a=1,2} \log\left(1 - e^{i(\tilde{u}_j - u_i - \Delta_a)}\right) \right] - 2\pi \tilde{n}_j .
\end{aligned}
$$

- The topologically twisted index: (i) solve these equations for $\{u_i, \tilde{u}_j\}$; (ii) insert the solutions into the expression for Z.
- Exact expression in N.

The large-N limit

- In the large-N limit, the eigenvalue distribution becomes continuous, and the set $\{t_i\}$ may be described by an eigenvalue density $\rho(t)$.

$$u_i = iN^{1/2}\,t_i + \pi - \tfrac{1}{2}\delta v(t_i), \qquad \tilde{u}_i = iN^{1/2}\,t_i + \pi + \tfrac{1}{2}\delta v(t_i),$$

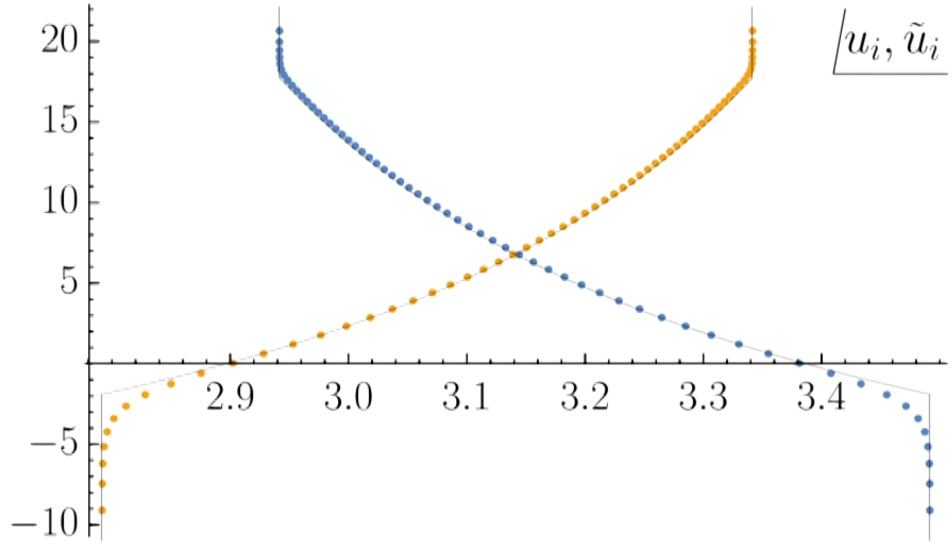


Figure: Eigenvalues for $\Delta_a = \{0.4, 0.5, 0.7, 2\pi - 1.6\}$ and $N = 60$.

- Description of the eigenvalue distribution.

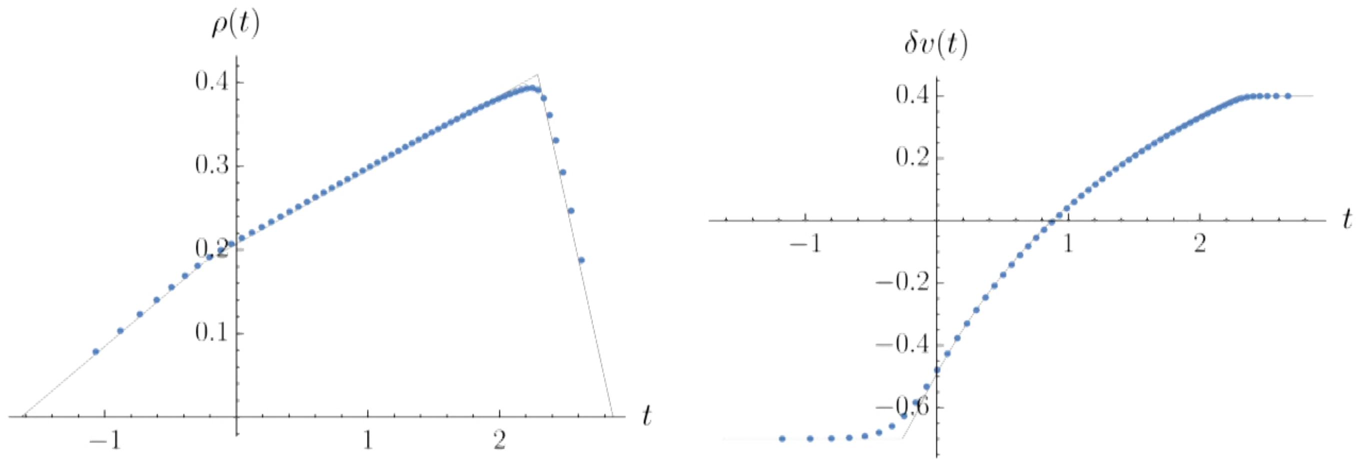


Figure: The eigenvalue density $\rho(t)$ and the function $\delta v(t)$ for $\Delta_a = \{0.4, 0.5, 0.7, 2\pi - 1.6\}$ and $N = 60$, compared with the leading order expression.

$$\mathrm{Re}\log Z = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{n_a}{\Delta_a}$$

Beyond Large N: Numerical Fits

Δ_1	Δ_2	Δ_3	f_1	f_2	f_3
$\pi/2$	$\pi/2$	$\pi/2$	3.0545	-0.4999	-3.0466
$\pi/4$	$\pi/2$	$\pi/4$	$4.2215-0.0491n_1$ $-0.1473n_2-0.0491n_3$	$-0.4996+0.0000n_1$ $+0.0000n_2+0.0000n_3$	$-4.1710-0.2943n_1$ $+0.0645n_2-0.2943n_3$
0.3	0.4	0.5	$7.9855-0.2597n_1$ $-0.5833n_2-0.6411n_3$	$-0.4994-0.0061n_1$ $-0.0020n_2-0.0007n_3$	$-9.8404-0.9312n_1$ $-0.0293n_2+0.3739n_3$
0.4	0.5	0.7	$6.6696-0.1904n_1$ $-0.4166n_2-0.4915n_3$	$-0.4986-0.0016n_1$ $-0.0008n_2-0.0001n_3$	$-7.5313-0.6893n_1$ $-0.1581n_2+0.2767n_3$

- Numerical fit for:

$$\mathrm{Re}\log Z = \mathrm{Re}\log Z_0 + f_1 N^{1/2} + f_2 \log N + f_3 + \cdots$$

- The values of N used in the fit range from 50 to $N_{\max}$ where $N_{\max} = 290, 150, 190, 120$ for the four cases, respectively.
- The index is independent of the magnetic fluxes in the special case $\Delta_a = \{\pi/2, \pi/2, \pi/2, \pi/2\}$

- In the large-N limit, the $k=1$ index takes the form

$$F = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{n_a}{\Delta_a} + N^{1/2} f_1(\Delta_a, n_a)$$
$$-\frac{1}{2}\log N + f_3(\Delta_a, n_a) + \mathcal{O}(N^{-1/2}),$$

where $F = \mathrm{Re}\log Z$.

- The leading $\mathcal{O}(N^{3/2})$ term [BHZ], and exactly reproduces the Bekenstein-Hawking entropy of a family of extremal AdS_4 magnetic black holes admitting an explicit embedding into 11d supergravity, once extremized with respect to the flavor and R-symmetries.
- The $-\frac{1}{2}\log N$ term [Liu-PZ-Rathee-Zhao].

Topologically twisted index on Riemann surfaces

- The topologically twisted index can be defined on Riemann surfaces with arbitrary genus. There is a simple relation between the index on $\Sigma_g \times S^1$ and that on $S^2 \times S^1$:
  $F_{S^2\times S^1}(n_a, \Delta_a) = (1-g) F_{\Sigma_g\times S^1}(\frac{n_a}{1-g}, \Delta_a)$.
- Since the coefficient of the logarithmic term in $F_{S^2\times S^1}$ does not depend on n_a we simply have

$$F_{\Sigma_g\times S^1}(n_a, \Delta_a) = \cdots - \frac{1-g}{2}\log N + \cdots .$$

- The $-\frac{1-g}{2}\log N$ from quantum supergravity [Liu-PZ-Rathee-Zhao].

AdS_4/CFT_3

- Holographically, ABJM describes a stack of N M2-branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity, whose low energy dynamics is effectively described by 11 dimensional supergravity.
- The index is computed for ABJM theory with a topological twist, equivalently, fluxes on S^2. On the gravity side it corresponds to microstate counting of magnetically charged asymptotically AdS_4 black holes.

Supergravity solution

- A solution of four dimensional $\mathcal{N} = 2$ gauged sugra with prepotential $F = -2i\sqrt{X^0X^1X^2X^3}$ coming from M theory on $AdS_4 \times S^7$ with $U(1)^4 \in SO(8)$.
- Background metric :

$$ds^2 = -e^{\mathcal{K}(X)}\left(g\,r - \frac{c}{2g\,r}\right)^2 dt^2 + e^{-\mathcal{K}(X)}\frac{dr^2}{\left(g\,r - \frac{c}{2g\,r}\right)^2} + 2e^{-\mathcal{K}(X)}r^2 d\Omega_2^2$$

- Magnetic charges

$$F^a_{\theta\phi} = -\frac{n_a}{\sqrt{2}}\sin\theta, \qquad F^1_{tr} = 0.$$

Bekenstein-Hawking entropy and Index

- The Bekenstein-Hawking entropy:

$$
\begin{aligned}
S_{BH}(n_a) &= \frac{1}{4G_N}A = \frac{2\pi}{G_N}e^{-\mathcal{K}(X_h)}r_h^2 = \frac{2\pi}{G_N}(F_2+\sqrt{\Theta})^{1/2} \\
F_2 &= \frac{1}{2}\sum_{a<b} n_a n_b - \frac{1}{4}\sum_a n_a^2, \quad \Theta = (F_2)^2 - 4n_1n_2n_3n_4.
\end{aligned}
$$

- Extremize the index $Z(n_a, \Delta_a)$ with respect to Δ_a coincides with the entropy $\mathrm{Re}\ln Z(n_a, \tilde{\Delta}_a) = S_{BH}(n_a)$.
- Goal: Compute one-loop corrections around this sugra background in 11 Sugra and compare with field theory.

Logarithmic terms: An IR window into UV physics

- Logarithmic corrections are determined macroscopically from the massless particle spectrum, insensitive to the UV completion of the theory.
- A consistency check for any enumeration of quantum black hole microstates: Given the field theory answer, use gravity to check its veracity through the logarithmic correction; a litmus test.
- Sen and collaborators have checked many black holes in string theory and challenged loop quantum gravity [Precision Strominger-Vafa].
- Micoscopic realization of Kerr/CFT [Strominger].
- Free energy of ABJM [Bhattacharyya-Grassi-Mariño-Sen]: $F_{S^3} \sim \mathrm{Ai}(N, k_a) \mapsto -\frac{1}{4}\log N$.

Logarithmic terms in one-loop effective actions

- One-loop effective action is equivalent to computations of determinants.
- For a given kinetic operator A one naturally defines the logarithm of its determinant as

$$\frac{1}{2}\,\ln \det{}' A = \frac{1}{2}\sum_n{}'\,\ln \kappa_n$$

where prime denotes that the sum is over non-vanishing eigenvalues, κ_n, of A.

- It is further convenient to define the heat Kernerl of the operator A as

$$K(\tau) = e^{-\tau A} = \sum_n e^{-\kappa_n \tau}\mid \phi_n\rangle\langle\phi_n\mid .$$

Logarithmic terms in one-loop effective actions

- Since, non-zero eigenvalues of a standard Laplace operator A scale as L^{-2}, it is natural to redefine $\bar{\tau} = \tau/L^2$.

$$-\frac{1}{2}\,\ln\det{}'A = \frac{1}{2}\int_{\bar{\epsilon}}^{\infty}\frac{d\bar{\tau}}{\bar{\tau}}\left(\sum_{n=0}^{\infty}\frac{1}{(4\pi)^{d/2}}\,\bar{\tau}^{\,n-d/2}\,L^{2n-d}\int d^dx\,\sqrt{g}\,a_n(x,x) - n_A^0\right).$$

- The logarithmic contribution to $\ln\det{}'A$ comes from the term $n = d/2$,

$$-\frac{1}{2}\,\ln\det{}'A = \left(\frac{1}{(4\pi)^{d/2}}\int d^dx\,\sqrt{g}\,a_{d/2}(x,x) - n_A^0\right)\log L + \dots .$$

- On very general grounds (diffeomorphism), the coefficient $a_{d/2}$ vanishes in odd-dimensional spacetimes,.

Logarithmic terms: Key Facts

- Robustness: Independent of UV cutoff ϵ.
- In odd-dimensional spaces the coefficient of the $\log$ can only come from zero modes or boundary modes.
- Corrections to entropy from one-loop part of the partition function:

$$S_1 = \lim_{\beta\to\infty}(1-\beta\partial_\beta)\left(\sum_D (-1)^D(\tfrac{1}{2}\log \det{}' D) + \Delta F_0\right),$$

- D stands for kinetic operators corresponding to various fluctuating fields and $(-1)^D = -1$ for bosons and 1 for fermions.
- The zero modes are accounted for separately by

$$\Delta F_0 = \log \int [d\phi]|_{D\phi=0},$$

where $\exp(-\int d^dx\sqrt{g}\phi D\phi) = 1$.

Quantum Supergravity: Key Facts

- The structure of the logarithmic term in 11d Sugra:

$$\log Z[\beta, \ldots] = \sum_{\{D\}} (-1)^D (\beta_D - 1) n_D^0 \log L + \Delta F_{\text{Ghost}}.$$

- Subtract the zero modes (-1) and add them appropriately due to integration over zero modes (β_D).
- The ghost contributions are treated separately.

Zero modes

- In the non-extremal case the topology of the black hole is homotopic to its horizon Σ_g due to the contractible (t, r) directions.
- The Euler characteristic of the non-extremal black hole is simply $\chi_{\mathsf{BH}} = 2(1-g)$.
- It also indicates that all but the second relative de-Rham cohomology vanish

$$\dim {}^R\mathcal{H}^2_{L^2}(M, \mathbb{R}) = \int^{\mathsf{Reg}} \mathsf{Pf}(R) = \chi_{\mathsf{BH}} = 2(1-g).$$

- The non-extremal black hole background has only two-form zero modes and their regularized number is:

$$n_2^0 = 2(1-g).$$

- Where are the 2-forms in 11d Sugra?

Quantizing a p-form

- The general action for quantizing a p-form A_p requires a set of $(p-j+1)$-form ghost fields, with $j = 2, 3, \ldots, p+1$.
- The ghost is Grassmann even if j is odd and Grassmann odd if j is even [Siegel '80]

$$\Delta F_{\text{Ghost}} = \sum_j (-1)^j (\beta_{A_{p-j}} - j - 1) n^0_{A_{p-j}} \log L.$$

Quantizing C_3 and zero modes

- The quantization of the three-form $C_{\mu\nu\rho}$ introduces 2 two-form ghosts that are Grassmann odd, 3 one-form ghosts that are Grassmann even and 4 scalar ghosts that are Grassmann odd[Siegel '80].
- Note that $C_{\mu\nu\rho}$ itself can't decompose as a massless two-form in the black hole background and a massless one-form in the compact dimension since S^7 does not admit any non-trivial 1-cycles.
- The only two-form comes from the two-form ghosts when quantizing $C_{\mu\nu\rho}$

Quantizing C_3 and zero modes

- Ghost contribution to one-loop effective action:

$$\Delta F = \Delta F_{\text{Ghost2form}}.$$

- The 2-form ghost A_2 in 11d has action

$$S_2 = \int A_2 \wedge \star(\delta d + d\delta)^2 A_2,$$

- The logarithmic term in the one-loop contribution to the entropy is

$$(2 - \beta_2) n_2^0 \log L,$$

- Recall β_2 comes from integrating the zero modes in the path integral, and the minus sign takes care of the Grassmann odd nature of A_2.

Zero modes: Two form zero modes

- The properly normalized measure is $\int d[A_{\mu\nu}]\exp(-L^7\int d^{11}x\sqrt{g^{(0)}}g^{(0)\mu\nu}g^{(0)\rho\sigma}A_{\mu\rho}A_{\nu\sigma}) = 1$, where we single out the L dependence of the metric, $g^{(0)}_{\mu\nu} = \frac{1}{L^2}g_{\mu\nu}$. Thus the normalized measure is $\prod_x d(L^{\frac{7}{2}}A_{\mu\nu})$. For each zero mode, there is a $L^{\frac{7}{2}}$ factor. Thus in the logarithmic determinant, one has $\beta_2 = \frac{7}{2}$.
- Recall that the $\log L$ correction to the partition function is $((2-\beta_2)n_2^0\log L$; using that $\beta_2 = 7/2$ and $n_2^0 = 2(1-g))$ leads to:

$$\log Z[\beta,\ldots] = -3(1-g)\log L + \cdots .$$

Final Result

- The coefficient of the logarithmic term does not depend on β

$$S_{\text{1-loop}} = (1 - \beta\partial_\beta)(-3(1-g)\log L) + \cdots$$
$$= -3(1-g)\log L + \cdots .$$

- As this is β independent, it is also valid in the extremal limit, $\beta \to \infty$.
- The AdS/CFT dictionary establishes that $L \sim N^{1/6}$ leading to a logarithmic correction to the extremal black hole entropy of the form

$$S = \cdots - \frac{1-g}{2}\log N + \cdots ,$$

- Perfectly agrees with the microscopic result!!!

Wrapped M5 branes in AdS/CFT

- Entries in AdS_4/CFT_3 from M2's and M5's.

$\mathrm{AdS}_4/\mathrm{CFT}_3$	from M2-branes	from M5-branes
M-theory set-up	N M2-branes probing Cone(Y_7)	N M5-branes wrapped on M_3
Dual Field theory	Known only for special examples of Y_7	Systematic algorithm applicable to general M_3
Gravity dual	$AdS_4 \times Y_7$	Warped $AdS_4 \times M_3 \times \tilde{S}^4$
Symmetry	Isometry of Y_7 ($\supset U(1)_R$)	Only $U(1)_R$
L^2/G_4	$\frac{N^{3/2}\pi^2}{\sqrt{27/8\mathrm{vol}(Y_7)}}$	$\frac{2N^3\mathrm{vol}(M_3)}{3\pi^2}$
L/L_p	$\propto N^{1/6}$	$\propto N^{1/3}$

Field theory topologically twisted partition function

- The twisted partition functions for general $\mathcal{N}=2$ theory can be written as [Closset et al. '17, '18].

$$\mathcal{Z}^{\nu_R}_{p,g} = \sum_{\alpha} (\mathcal{H}^{\alpha}_{\nu_R})^{g-1} (\mathcal{F}^{\alpha}_{\nu_R})^{p} \; , \tag{1}$$

- where α labels vacua of the 3d $\mathcal{N}=2$ on $\mathbb{R}^2 \times S^1$, called Bethe-vacua, and $\mathcal{H}$ and $\mathcal{F}$ are called 'handle-gluing' and 'fibering' operators, respectively.

The 3d-3d Correspondence

3D $\mathcal{T}_N[M_3]$ theory on $\mathbb{R}^2 \times S^1$	$PSL(N,\mathbb{C})$ CS theory on M_3
Bethe vacuum α	Irreducible flat connection $\mathcal{A}^\alpha$
Fibering operator $\mathcal{F}^\alpha_{\nu_R=\frac{1}{2}}$	$\exp(-\frac{1}{2\pi i}S_0^\alpha) = \exp(\frac{1}{4\pi i}CS[\mathcal{A}^\alpha; M_3])$
Handle gluing operator $\mathcal{H}^\alpha_{\nu_R=\frac{1}{2}}$	$N\exp(-2S_1^\alpha) = N \times \mathbf{Tor}^{(\alpha)}_{M_3}(\tau_{\text{adj}}, N)$

- $\mathbf{Tor}^{(\alpha)}_{M_3}(\tau, N)$ is the analytic torsion (Ray-Singer torsion) for an associated vector bundle in a representation $\tau \in \mathrm{Hom}[PSL(N,\mathbb{C}) \to GL(V_\tau)]$ twisted by a flat connection $\mathcal{A}^\alpha$.
- The dictionary for the handle gluing operator works only for M_3 with vanishing $H_1(M_3, \mathbb{Z}_N)$.

Field Theory Answer: UV counting

$$\boxed{\begin{aligned} &|\mathcal{Z}^{\nu_R=\frac{1}{2}}_{g,p=0}(\mathcal{T}_N[M_3])| \\ &\xrightarrow{N\to\infty} 2\cos\big((1-g)\theta_{N,M_3}\big) \\ &\qquad\times\exp\Bigg((g-1)\Big(\frac{\mathrm{vol}(M_3)}{3\pi}(N^3-N) \\ &-a(M_3)(N-1)-b(M_3)+\log N-c(M_3;N)\Big)\Bigg) \\ &\qquad\times\left(1+e^{-(\ldots)}\right).\end{aligned}} \qquad (2)$$

- The $1/N$ expansion terminates at $o(N^0)$.
- Analytic result, no numerics involved.
- Logarithmic correction to the $\log\mathcal{Z}$ is $(g-1)\log N$.

The M-theory Solution

$$\begin{aligned} ds_{11}^2 &= \frac{1}{4}\lambda^{-1}\Big[4ds^2(\mathrm{AdS}_4) + \frac{dx^2+dy^2+dz^2}{z^2} \\ &\quad + \frac{8-\rho^2}{8+\rho^2}\mathrm{D}Y^a\mathrm{D}Y^a + \frac{1}{2}\left(\frac{d\rho^2}{8-\rho^2} + \frac{\rho^2}{8+\rho^2}d\psi^2\right)\Big], \\ \lambda^3 &= \frac{2}{8+\rho^2}, \quad \mathrm{D}Y^a = dY^a + \omega^a{}_bY^b, \quad \omega_{31} = dx/z, \omega_{32} = dy/z. \end{aligned}$$

- Y^a, $a = 1, 2, 3$, are constrained coordinates on an S^2 satisfying $Y^aY^a = 1$
- M_3 is expressed as a quotient of hyperbolic 3-space $\mathbb{H}^3$ by an discrete subgroup $\Gamma \subset PSL(2,\mathbb{C})$: $M_3 = \mathbb{H}^3/\Gamma$; in field theory M_3 is S^3 minus the tubular neighborhood of a knot.
- The coordinates Y^a together with ρ and ψ build up an S^4.

- Pernici-Sezgin, Acharya-Gautlett-Kim, Gauntlett-Kim-Waldram, Donos-Gauntlett-Kim-Varela.
- A universal black hole solution, universal sector of many sugras.

$$I = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{L^2}{4}F^2\right) . \tag{3}$$

$$\begin{aligned} \frac{ds^2}{L^2} &= -(\rho - \frac{1}{2\rho})^2 dt^2 + \frac{1}{(\rho - \frac{1}{2\rho})^2} d\rho^2 + \rho^2 ds^2(\Sigma_g) , \\ F &= \frac{1}{L^2}(\text{volume form on } \Sigma_g) . \end{aligned} \tag{4}$$

$$\begin{aligned} S_{\rm BH} = \frac{A_{\rm horizon}}{4G_4} &= \frac{2\pi(g-1)L^2}{4G_4} \\ \boxed{S = \frac{(g-1)\mathrm{vol}(M_3)N^3}{3\pi}\checkmark} \end{aligned} \tag{5}$$

Logarithmic correction: one-loop quantum gravity

[Gang-Kim-PZ]

$$\log Z_{1-loop} = (2-\beta_2) n_2^0 \log L = (2-7/2) 2(1-g) \log L = (g-1)\log N \checkmark$$

A challenge (prediction) for field theory for $b_1(M_3) \neq 0$:

$$\begin{aligned} \log Z\Big|_{C_3} &= (-1)^1 (\beta_{C_3} - 1) n_{C_3}^{(0)} \log L \\ &= -\left(\frac{5}{2} - 1\right) 2(1-g) b_1 \log L \\ &= 3(g-1) b_1 \log L \\ &= (g-1) b_1 \log N, \end{aligned} \tag{6}$$

The full answer:

$$\log Z_{1-loop} = (g-1)(1-b_1)\log N$$

- Pernici-Sezgin, Acharya-Gautlett-Kim, Gauntlett-Kim-Waldram, Donos-Gauntlett-Kim-Varela.
- A universal black hole solution, universal sector of many sugras.

$$I = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{L^2}{4}F^2\right) . \tag{3}$$

$$\begin{aligned} \frac{ds^2}{L^2} &= -(\rho - \frac{1}{2\rho})^2 dt^2 + \frac{1}{(\rho - \frac{1}{2\rho})^2} d\rho^2 + \rho^2 ds^2(\Sigma_g) , \\ F &= \frac{1}{L^2}(\text{volume form on } \Sigma_g) . \end{aligned} \tag{4}$$

$$\begin{gathered} S_{\rm BH} = \frac{A_{\rm horizon}}{4G_4} = \frac{2\pi(g-1)L^2}{4G_4} \\ \boxed{S = \frac{(g-1)\mathrm{vol}(M_3)N^3}{3\pi}\checkmark} \end{gathered} \tag{5}$$

Rotating black hole

$$
\begin{aligned}
ds^2 &= -\frac{\Delta_r}{W}\left(dt - \frac{a}{\Xi}\sin^2\theta\, d\phi\right)^2 + W\left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\right) \\
&\quad + \frac{\Delta_\theta \sin^2\theta}{W}\left(a\, dt - \frac{\rho^2 + a^2}{\Xi}\, d\phi\right)^2 \\
\rho &= r + 2m\sinh^2\delta \\
\Delta_r &= r^2 + a^2 - 2mr + g^2\rho^2(\rho^2 + a^2) \qquad \Delta_\theta = 1 - a^2 g^2 \cos^2\theta \\
W &= \rho^2 + a^2\cos^2\theta \qquad \Xi = 1 - a^2 g^2 \\
A &= \frac{2m\sinh 2\delta}{W}\rho\left(dt - \frac{a}{\Xi}\sin^2\theta\, d\phi\right) .
\end{aligned} \tag{7}
$$

Entropy of rotating black hole

- Due to supersymmetry and extremality, $J_{\rm BPS}$ and $Q_{\rm BPS}$ are non-linearly related.

$$S_{\rm BH} = \frac{\pi L}{4G_{(4)}}\,\frac{J_{\rm BPS}}{Q_{\rm BPS}} = \frac{\pi L^2}{2G_{(4)}}\left(\sqrt{1+64\frac{G_{(4)}^2}{L^2}Q_{\rm BPS}^2}\;-1\right)\,. \tag{8}$$

- The entropy function $\mathcal{S}$ is the "grand canonical partition function":

$$S_{\rm BH}(Q_{\rm BPS}) = \mathcal{S}(\omega) + \omega\,J_{\rm BPS} + (\omega+2\pi i)\,2LQ_{\rm BPS} \tag{9}$$

- The entropy function $\mathcal{S}$ also reproduces the non-linear constraint between $J_{\rm BPS}$ and $Q_{\rm BPS}$, from $S_{\rm BH}$ – real positive.

$$\mathcal{S} = i\,\frac{L^2}{8G_{(4)}}\,\frac{(\omega+2\pi i)^2}{\omega} = \frac{i\,N^3}{12\pi^2}\,{\rm vol}(M_3)\,\frac{(\omega+2\pi i)^2}{\omega}\,, \tag{10}$$

The Superconformal Index

[Benini-Gang-PZ]

$$\log \mathcal{I}_{\rm SCI}\big(q; \mathcal{T}_N[M_3]\big) = \frac{iN^3}{12\pi^2}\,\frac{(\omega+2\pi i)^2}{\omega} + \big(\text{subleading in } 1/N\big)$$
$$= \frac{iL^2}{8G_{(4)}}\,\frac{(\omega+2\pi i)^2}{\omega} + \big(\text{subleading in } G_{(4)}/L^2\ \big)\,. \quad (11)$$

- The above entropy function reproduces the entropy (Legendre transform).
- The logarithmic correction perfectly agrees with the macro, gravity result.

$$-(g-1)\log\frac{\big|\mathrm{Hom}[\pi_1(M_3)\to\mathbb{Z}_N]\big|}{N} \quad \xrightarrow{N\to\infty} \quad (g-1)\big(1-b_1(M_3)\big)\log N \quad (12)$$

The SCI of toric quiver gauge theories

- We analyze toric quiver gauge theories with gauge group: $G = \underbrace{SU(N) \times \cdots \times SU(N)}_{n_{\mathsf{v}} \text{ times}}$.
- n_χ chiral fields Φ_{ab} transforming in the bi-fundamental representation of a factor $SU(N_a) \times SU(N_b)$ in G, $(N_a = N_b = N)$.
- These theories can be represented with toric diagrams having d vertices each of which can be associated with $U(1)$ flavor symmetry with charges $Q_{i=1,\cdots,d-1}$ with chemical potentials Δ_i.
- These theories have a superpotential with n_F monomial terms each of which corresponds to a face of the graph associated to the quiver of the theory which can be drawn on a torus, which implies: $n_\chi - n_F - n_{\mathsf{v}} = 0$

$$\mathcal{I} = \quad \mathrm{Tr}_{\mathcal{H}(S^3 \times S^1)} (-1)^F e^{-\beta H} e^{2\pi i 2\tau\left(J + \frac{1}{d}\sum_{I=1}^{d} Q_d\right)} e^{2\pi i \sum_{i=1}^{d-1} \Delta_i (Q_i - Q_d)}$$

Integral representation and Bethe Ansätze (BA) solutions

- From [Romelsberger '07], the SCI of toric quiver gauge theories is:

$$\mathcal{I} = \kappa_G \oint_{\mathbb{T}^{\mathrm{rk}(G)}} \frac{\prod_{\Phi_{ab}} \prod_{i_a \neq j_b} \Gamma_e \left(u_{i_a} - u_{j_b} + \Delta_{ab}; \tau, \sigma\right)}{\prod_{\alpha \in \Delta} \Gamma_e \left(\alpha\left(u\right); \tau, \sigma\right)} \prod_{i=1}^{\mathrm{rk}(G)} \frac{dz_i}{2\pi i z_i}$$

$$\kappa_G = (q;q)_\infty^{2\mathrm{rk}(G)} / |\mathcal{W}_G|, \;\; z_i = e^{2\pi i u_i}$$

- Evaluating the integral [Benini-Milan, Closset-Kim-Willet]:

$$\mathcal{I} = \kappa_G \sum_{\hat{u} \in \mathfrak{M}_{BAE}} \mathcal{Z}\left(\hat{u}; \tau\right) H\left(\hat{u}; \tau\right)^{-1}; \; \mathfrak{M}_{BAE} = \{u : \mathcal{Q}_{i_a}\left(u; \tau\right) = 1\}$$

where $H = \det \left(\left[\frac{1}{2\pi i} \frac{\partial Q_{i_a}}{\partial u_{j_b}} \right]_{i_a j_b} \right)$ and $\mathcal{Q}_{i_a}$ is the BA operator.

- BA solutions: $u_{i_a} - u_{j_b} = \frac{\tau}{N}\left(i_a - j_b\right)$

Large-N leading behavior of $\log \mathcal{I}$ and Black Hole entropy

- Optimal obstruction of bosonic-fermionic cancellations is achieved by the shifting $\Delta_i \to \Delta_i - \frac{2\tau}{d}$ where $i = 1, \cdots, d-1$.
- Chemical potentials satisfying the constraint :$\sum_{I=1}^{d} \Delta_I - 2\tau = \pm 1$ and within the domain Im $\left(-\frac{1}{\tau}\right) >$ Im $\left(\frac{\sum_{i=1}^{d-1} [\Delta_i]_\tau}{\tau}\right) > 0$.
- The leading contribution is $\mathcal{O}(N^2)$:

$$\log \mathcal{I} = -\frac{i\pi N^2}{6\tau^2} C_{IJK} \Delta_I \Delta_J \Delta_K + \frac{i\pi N^2}{3\tau^2} \underbrace{(n_\chi - n_F - n_v)}_{\to 0} f(\tau).$$

- C_{IJK} are the Chern-Simons couplings of the holographic dual gravitational description.
- Upon Legendre transformation $\log \mathcal{I} = -\frac{i\pi N^2}{6\tau^2} C_{IJK} \Delta_I \Delta_J \Delta_K$ accounts for the Black Hole entropy.

$\log N$ contribution to $\log \mathcal{I}$: Preliminary

- The contribution from κ_G:
  $\log(n_\chi N|\mathcal{W}_G|\kappa_G) = \log N + \text{Constant} \times N + \mathcal{O}(1)$ where the factor $n_\chi N|\mathcal{W}_G|$ comes from the degeneracy of the BA solutions.
- The contribution from $\mathcal{Z}$:
  $\log \mathcal{Z} = -\frac{i\pi N^2}{6\tau^2} C_{IJK}\Delta_I\Delta_J\Delta_K + n_{\mathsf{v}} N \log N + \text{Constant} \times N + \mathcal{O}(1)$.
- The contribution from the Jacobian:
  $-\log H = -n_{\mathsf{v}} N \log N + \text{Constant} \times N + \mathcal{O}(1)$.
- The total contribution:
  $\log \mathcal{I} = -\frac{i\pi N^2}{6\tau^2} C_{IJK}\Delta_I\Delta_J\Delta_K + \log N + \text{Constant} \times N + \mathcal{O}(1)$
- For $\mathcal{N} = 4$ the same procedure leads to the same logarithmic correction: $\log \mathcal{I}_{\mathcal{N}=4} \sim N^2\Delta_1\Delta_2\Delta_3 + \log N + \ldots$
- Still to be checked by the gravity side ... work in progress.

Conclusions and Open Problems

- The degrees of freedom do not live locally at the horizon. Corrections to the Quantum Entropy Formula, extra hair in AdS [Sen]. Reconciling the near-horizon and the asymptotic region in AdS at the quantum level.
- Higher curvature corrections? MSW precedent.
- Electrically charged, rotating AdS_5 black hole entropy explained from the the $\mathcal{N}=4$ SYM index on $S^1 \times S^3$: Kim et al., Cabo-Bizet et al., Benini-Milan. [Dual gravity logarithmic police department!]
- A *precise setup* to attack important questions of black hole physics in the AdS/CFT: Information loss paradox.