Title: Spectral Search for Echoes

Speakers: Bob Holdom

Collection: Echoes in Southern Ontario

Date: February 25, 2020 - 12:00 PM

URL: http://pirsa.org/20020093

Spectral Search for Echoes

Bob Holdom

PI Feb 25 2020

26

obtain the spectrum

- gravitational waves on a truncated Kerr background
- a complex amplitude describing two polarizations

$$h(t,\Omega) = h_+(t,\Omega) - ih_\times(t,\Omega) \approx h(t)Y_{22}(\Omega) \qquad \ell = m = 2 \text{ dominates}$$

- $h(t)$ is related to ψ_ω, a quantity in frequency space
- ψ_ω can be obtained from a solution to the Sasaki-Nakamura (SN) equation

$$\frac{d^2\psi}{dx^2} - \mathcal{F}(\omega,x)\frac{d\psi}{dx} - \mathcal{U}(\omega,x)\psi = S(\omega,x)$$

- $\psi(\omega,x) \to \psi_\omega e^{i\omega x}$ for $x \to \infty$

obtain the spectrum

- gravitational waves on a truncated Kerr background
- a complex amplitude describing two polarizations

$$h(t,\Omega) = h_+(t,\Omega) - ih_\times(t,\Omega) \approx h(t)Y_{22}(\Omega) \qquad \ell = m = 2 \text{ dominates}$$

- $h(t)$ is related to ψ_ω, a quantity in frequency space
- ψ_ω can be obtained from a solution to the Sasaki-Nakamura (SN) equation

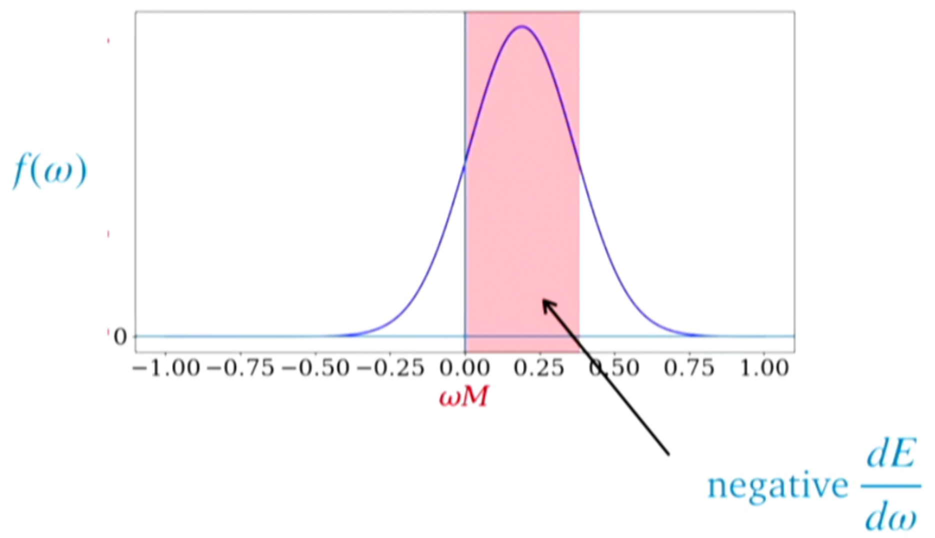
$$\frac{d^2\psi}{dx^2} - \mathcal{F}(\omega,x)\frac{d\psi}{dx} - \mathcal{U}(\omega,x)\psi = S(\omega,x)$$

- $\psi(\omega,x) \to \psi_\omega e^{i\omega x}$ for $x \to \infty$

initial condition from pulse starting on inside

- express initial condition in terms of an amplitude $f(\omega)$ that determines

spectral flux density $\dfrac{dE}{d\omega} = 8\omega(\omega - \omega_0)\,|\,f(\omega)\,|^2$

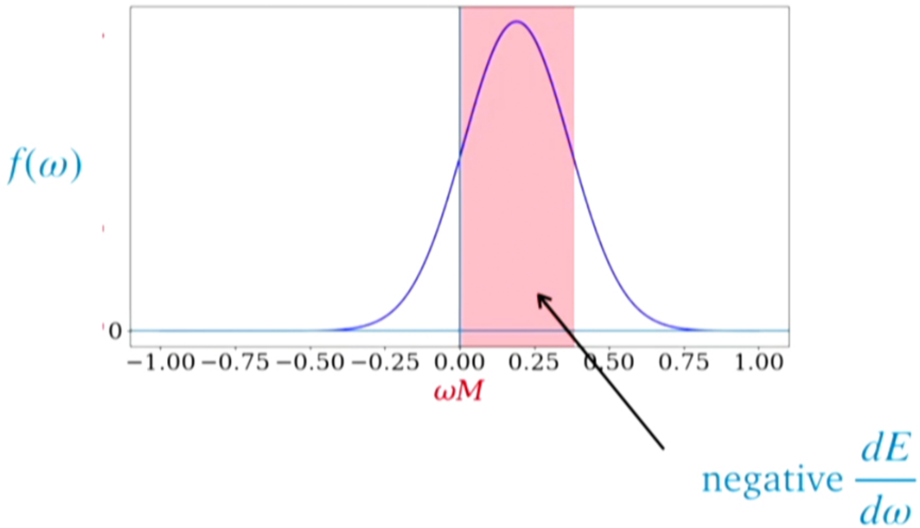


28

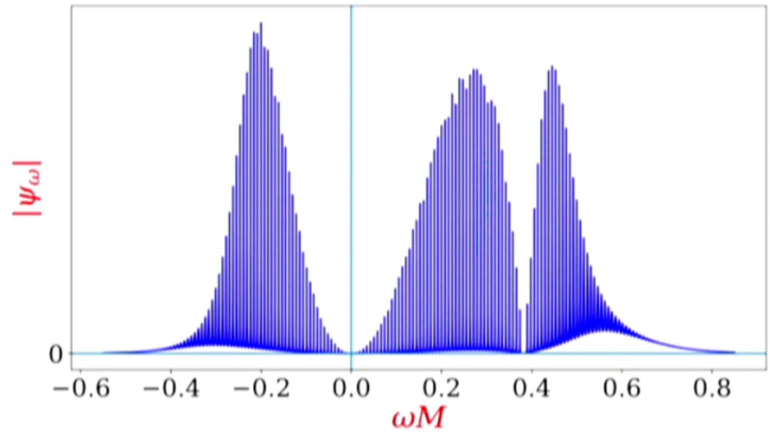
initial condition from pulse starting on inside

- express initial condition in terms of an amplitude $f(\omega)$ that determines

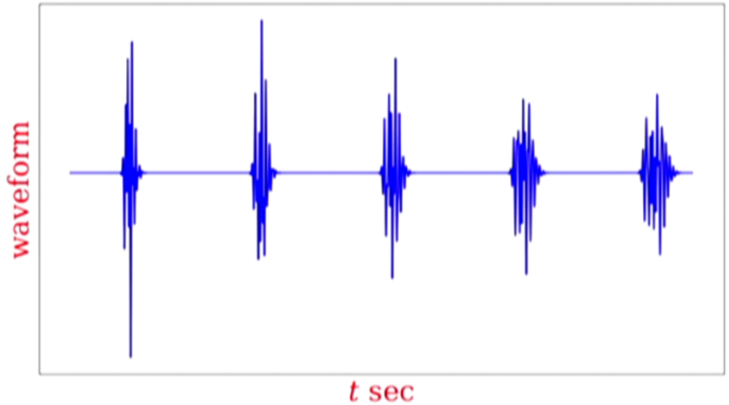
spectral flux density $\quad \dfrac{dE}{d\omega} = 8\omega(\omega - \omega_0)\,|\,f(\omega)\,|^2$



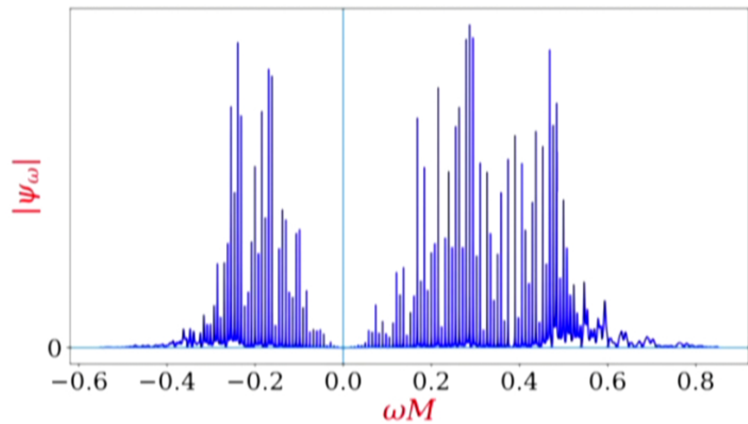
28



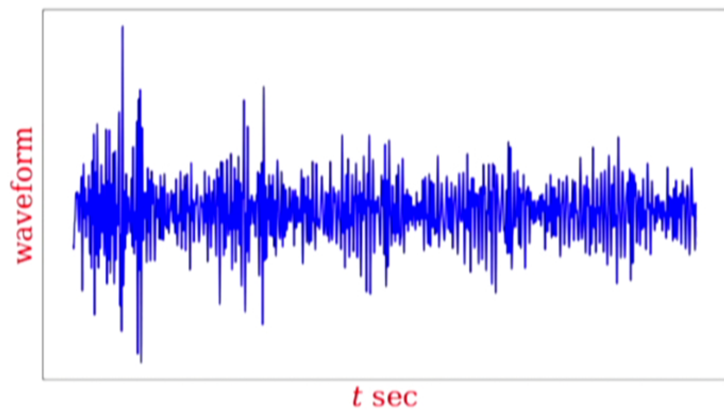
- single pulse in the cavity
- a zero is at $\omega_0 \propto \chi$ (the spin)



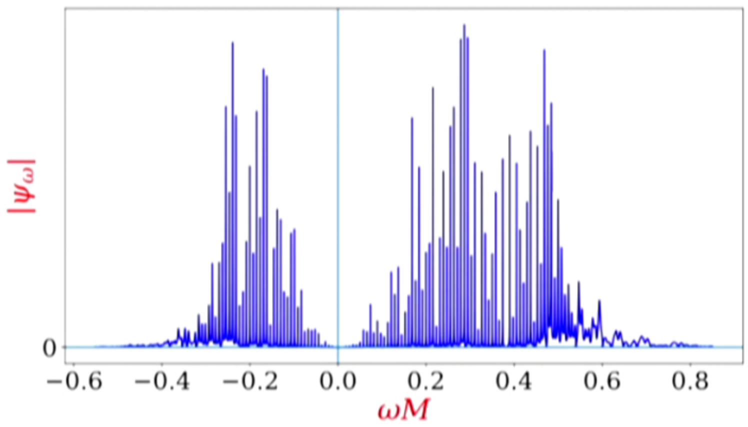
- irregular echoes



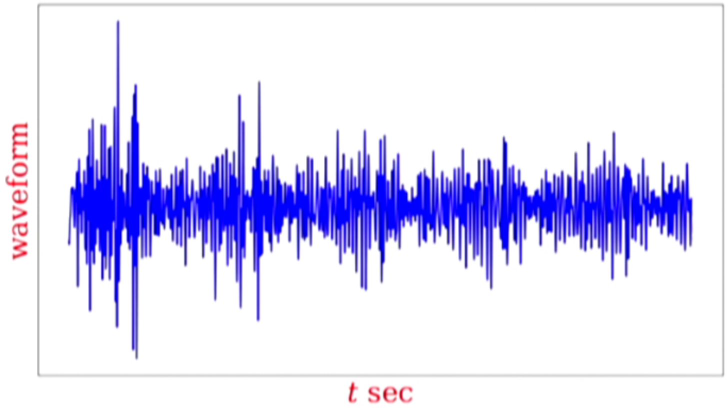
- 100 random pulses as an initial condition
- resonance structure is still present



- echo structure is gone



- 100 random pulses as an initial condition
- resonance structure is still present



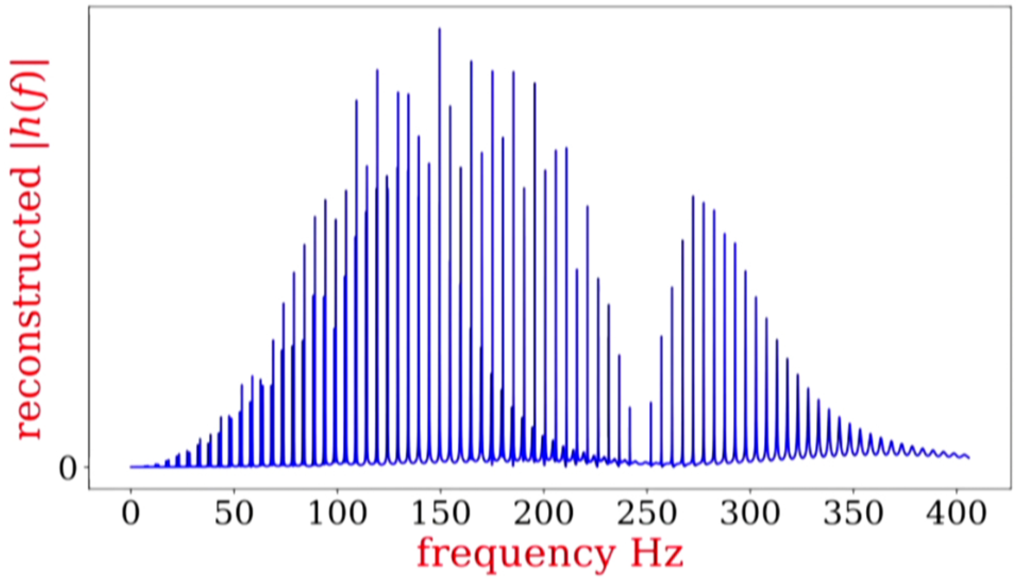
- echo structure is gone

the "reconstructed" spectrum

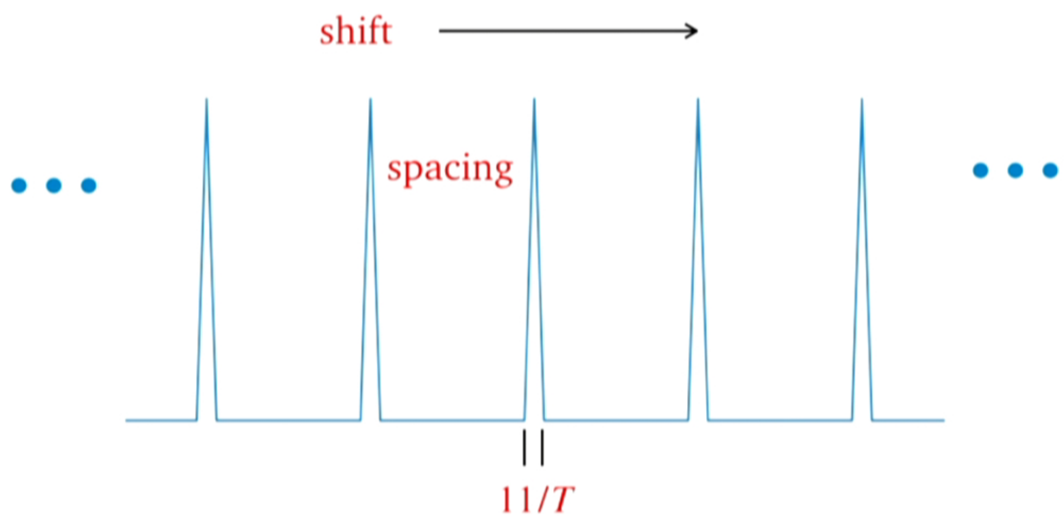
- a LIGO detector projects the complex waveform into a set of real numbers, the "strain data"
- we take the real part of the signal waveform $h(t)$ to model this projection
- to search for a resonance pattern:
  - choose the time duration $T = N_E \Delta t$ of the data to analyze
  - take the FFT
  - take the absolute value
- carrying out these steps on the real part of $h(t)$ gives a "reconstructed" $|h(f)|$

31

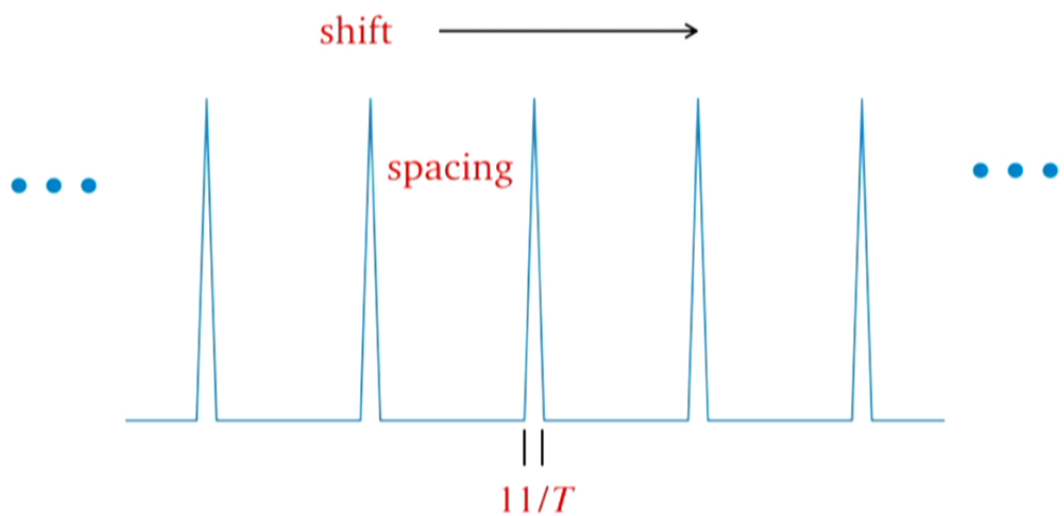
- an example for $N_E = 180, \chi = 2/3, M = 50M_\odot, \Delta t/M = 800$
- two component structure due to the real projection
- lower frequency structure comes from the negative frequency part of ψ_ω



- take |FFT(data of duration T after merger)| and choose a bandpass $f_{\min} < f < f_{\max}$
- multiply by a uniform comb structure characterized by a spacing between teeth and an overall shift
- take these amplitudes from the two detectors and search for a correlated enhancement for some spacing and shift



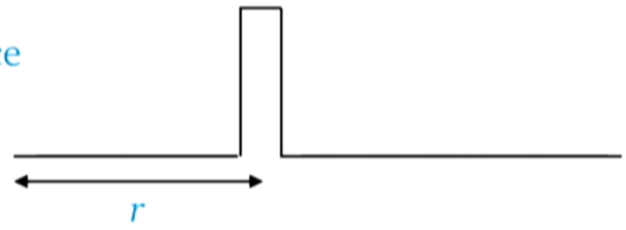
- take |FFT(data of duration T after merger)| and choose a bandpass $f_{\min} < f < f_{\max}$
- multiply by a uniform comb structure characterized by a spacing between teeth and an overall shift
- take these amplitudes from the two detectors and search for a correlated enhancement for some spacing and shift



in more detail ...

$$U = \text{mean}\left[\,|\,\text{FFT}(data)\,|\times\text{comb}(shift)\right]$$

- U is vector in shift space
- signal will produce larger values around some shift
- $V(r)$ is another vector in shift space
  - an idealized bump at shift r

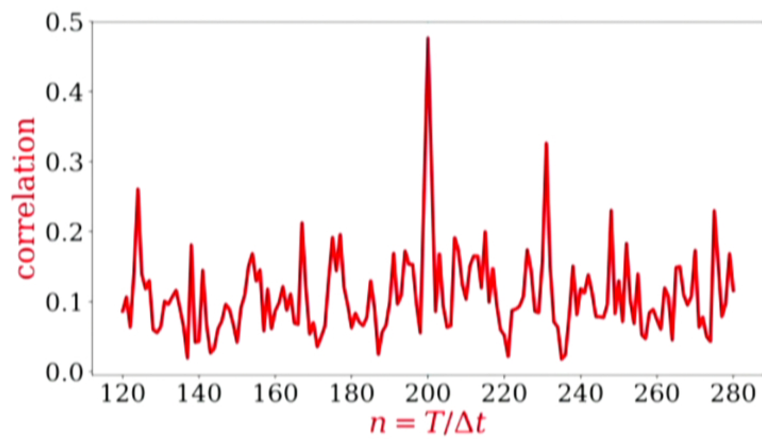


$$C = \max_{r} \text{Correlation}(U_H, V(r)) \times \text{Correlation}(U_L, V(r))$$

- C is our correlation between detectors, H and L, and it is still a function of the comb spacing $\Delta f = 1/\Delta t$

the “signal plot”

- consider discrete set of comb spacings Δf for integers $n = \Delta f/(1/T) = T/\Delta t$
- plot the correlation as a function of n and look for a peak
- can arrange for a peak to sit at $n = N_E$, some number of echoes
- peak should be enhanced by tuning Δt

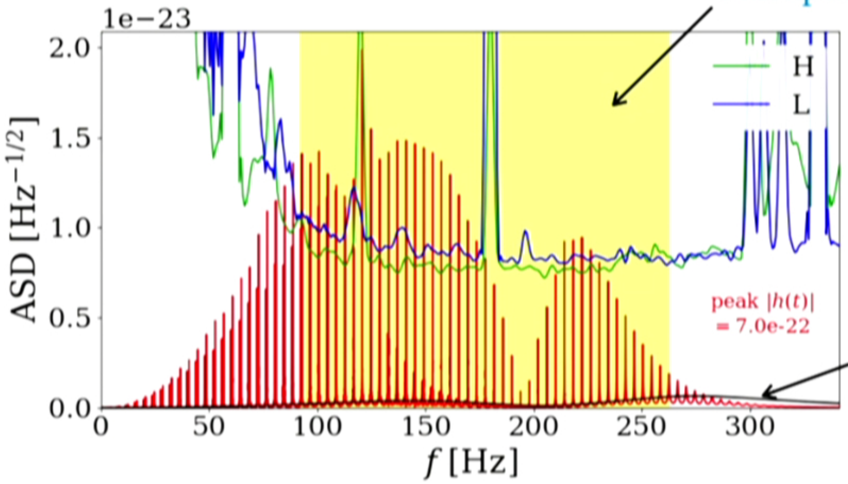


data driven bandpass

- optimize the signal (the peak height):
  - vary the number of echoes N_E (some multiple of 10)
  - vary the frequency bandpass $f_{\min} < f < f_{\max}$

- will compare this data driven bandpass to spectrum predicted by $M, \chi, \Delta t, N_E$
  - are they consistent with each other?

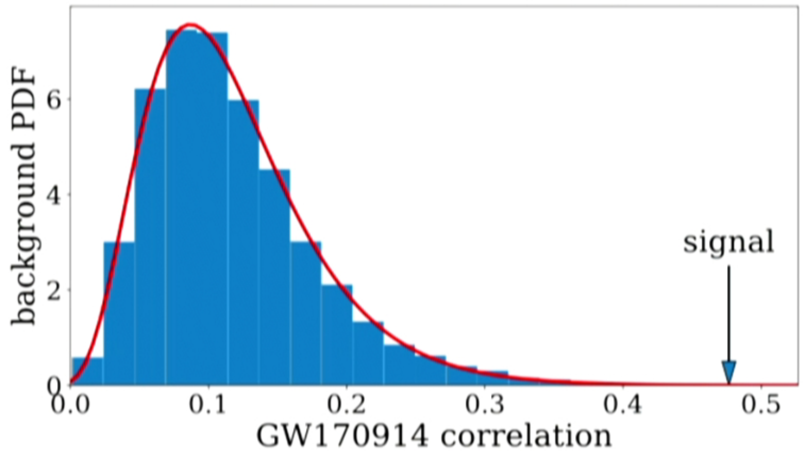
bandpass in yellow



first pulse in black

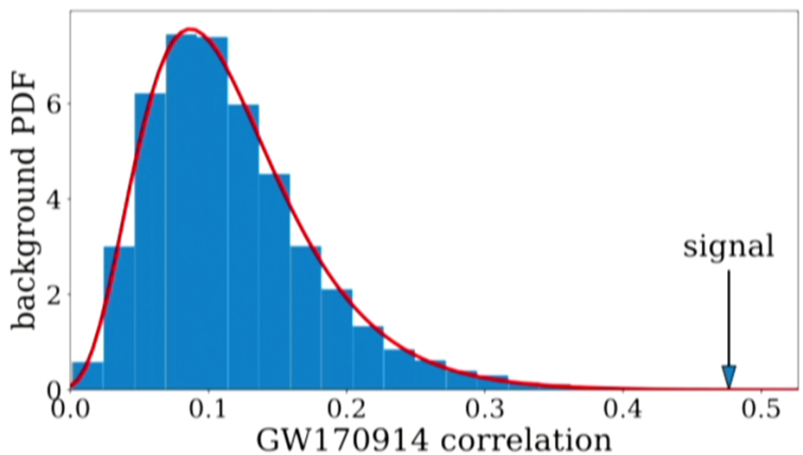
distribution of background correlations

- generate many background versions of the signal plot
  - can check they do not display excess correlation at the signal location
- collect all the correlation values and fit to a PDF (generalized gamma)
  - $P_{gg}(x) \propto x^{\alpha} e^{-\beta x}$ (4 parameters: α, β, scale parameter, location parameter)
  - maximum likelihood fit using $\sim$ 10000 to 20000 values
  - add histogram to show fit



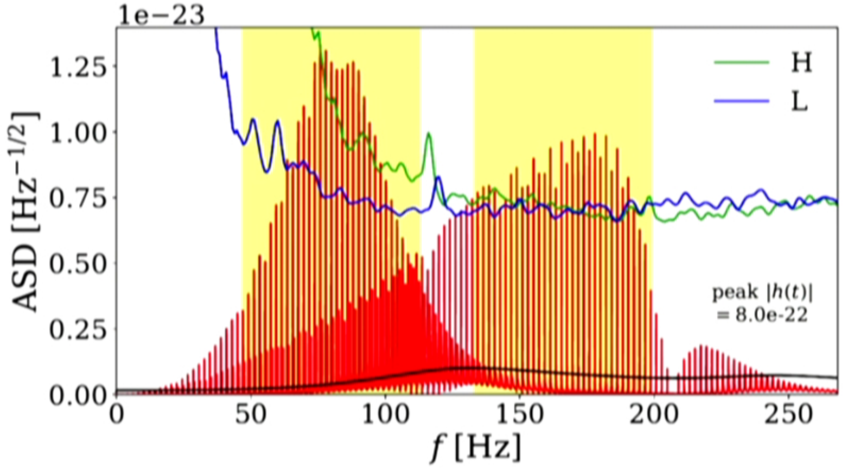
distribution of background correlations

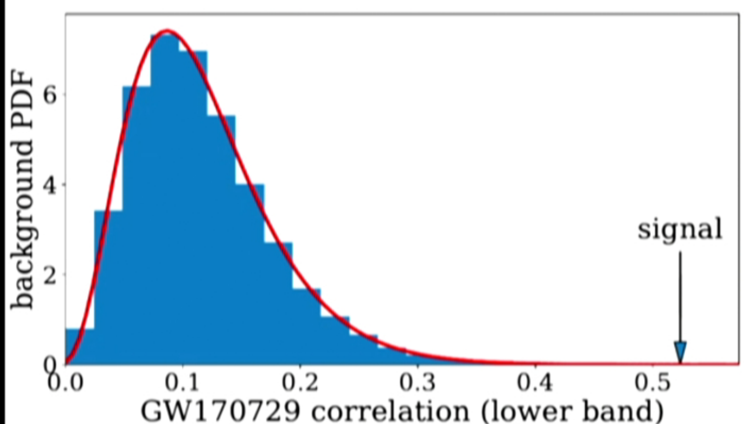
- generate many background versions of the signal plot
  - can check they do not display excess correlation at the signal location
- collect all the correlation values and fit to a PDF (generalized gamma)
  - $P_{gg}(x) \propto x^{\alpha} e^{-\beta x}$ (4 parameters: α, β, scale parameter, location parameter)
  - maximum likelihood fit using $\sim$ 10000 to 20000 values
  - add histogram to show fit

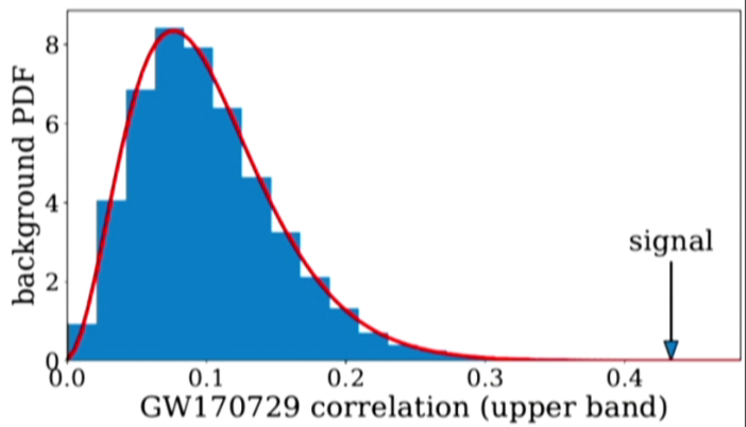


GW170729

N_E	180/170
Δf	2.0441/2.0443Hz
$\frac{\Delta t}{M}$	1240
χ	0.81







- Δt for a truncated Kerr black hole has known dependence on mass M, spin χ and redshift z

$$\frac{\Delta t}{M} = 4\,\eta\,\log(\frac{M}{\ell_{\rm Pl}})\,(\frac{1+(1-\chi^2)^{-\frac{1}{2}}}{2})\,(1+z)$$

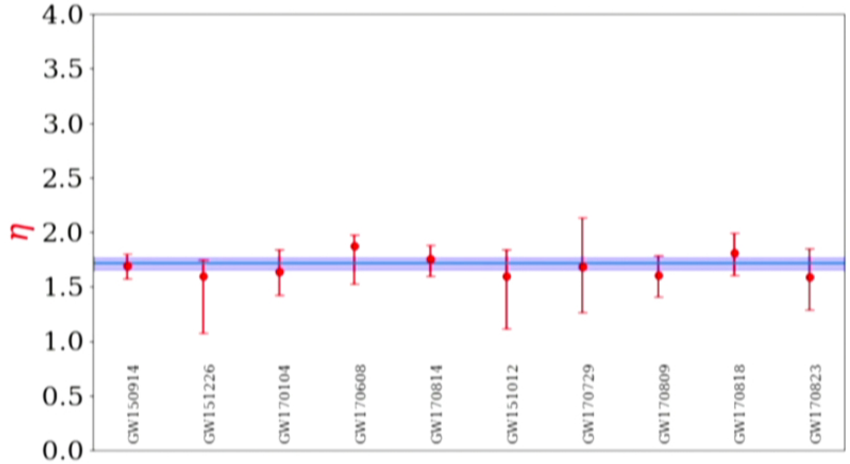
- a resonance signal determines Δt with negligible error
- LIGO measures M, χ and z of final BH with uncertainties
- thus determine an η for each event

- η determines δr, the distance from the would-be horizon to where the Schd metric applies

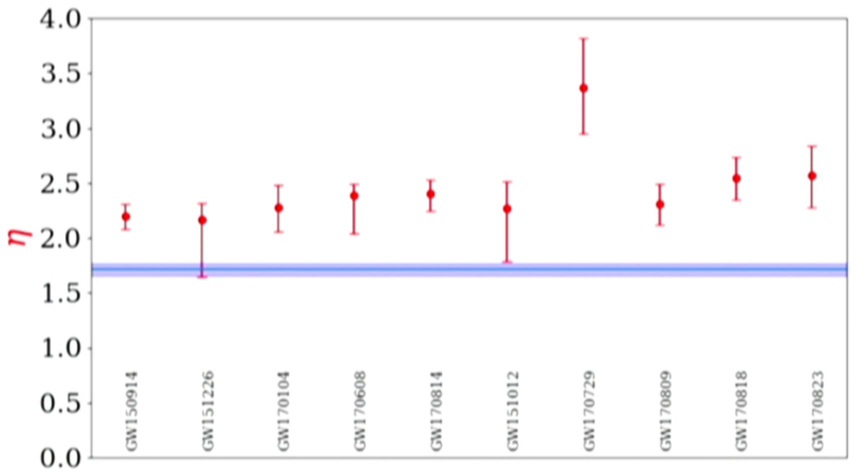
$$\delta r \approx \left(\frac{\ell_{\rm Pl}}{M}\right)^{(\eta-1)} \cdot \ell_{\rm Pl} \approx \left(\frac{M}{\ell_{\rm Pl}}\right)^{(2-\eta)} \cdot \bar{\ell}_{\rm Pl}$$

$\ell_{\rm Pl} \equiv$ Planck length

$\bar{\ell}_{\rm Pl} \equiv$ proper Planck length



- same a previous plot



- remove the spin and red-shift factors from the formula for η
- draws attention to GW170729

p-values

GW150914	0.011	GW151226	0.014
GW170104	0.33	GW170814	0.098
GW170608	0.038	GW170809	0.081
GW151012	0.0015	GW170823	0.026
GW170818	0.0094	GW170729	0.0010 & 0.0006

- there are two popular methods to combine independent p-values, Fisher and Stouffer
- they give 5×10^{-11} and 2×10^{-12} respectively
- even when adding 0.005 to all 11 p-values, the combined p-values are 1×10^{-8} and 2×10^{-10}

p-values

GW150914	0.011	GW151226	0.014
GW170104	0.33	GW170814	0.098
GW170608	0.038	GW170809	0.081
GW151012	0.0015	GW170823	0.026
GW170818	0.0094	GW170729	0.0010 & 0.0006

- there are two popular methods to combine independent p-values, Fisher and Stouffer
- they give 5×10^{-11} and 2×10^{-12} respectively
- even when adding 0.005 to all 11 p-values, the combined p-values are 1×10^{-8} and 2×10^{-10}

p-values

GW150914	0.011	GW151226	0.014
GW170104	0.33	GW170814	0.098
GW170608	0.038	GW170809	0.081
GW151012	0.0015	GW170823	0.026
GW170818	0.0094	GW170729	0.0010 & 0.0006

- there are two popular methods to combine independent p-values, Fisher and Stouffer
- they give 5×10^{-11} and 2×10^{-12} respectively
- even when adding 0.005 to all 11 p-values, the combined p-values are 1×10^{-8} and 2×10^{-10}

Conclusions

- LIGO is sensitive to not quite black holes
- this sensitivity to Planck scale physics is under appreciated

- signal waveform may be too variable and complicated for template analysis
- equally spaced resonance pattern is robust and relatively easy to search for
  - there is still a two component structure to explore

- evidence is building from all 10 events
  - consistency of $\Delta t/M$ values with spin and red-shift dependence
  - consistency of bandpasses with predicted spectrum
  - p-values and very small combined p-value