Title: Echoes and Holography

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Echoes, Holography, and Chaos

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Overview

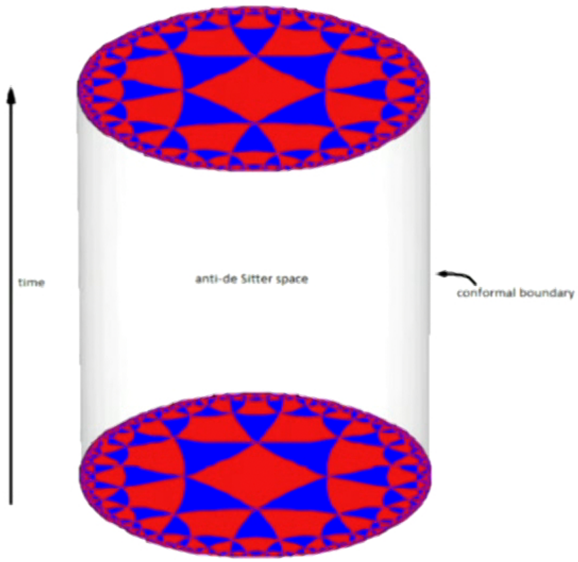
- Background and Context
- Pose a Question
- Progress (arXiv: 1906.02653)

Black Hole Evolution Unitary?

- QFT applied to BH background predicts BH evaporates into thermal radiation.
- BH evolution: pure state $\rightarrow$ mixed state
- Violates unitarity.
- Should a complete theory of quantum gravity violate unitarity?

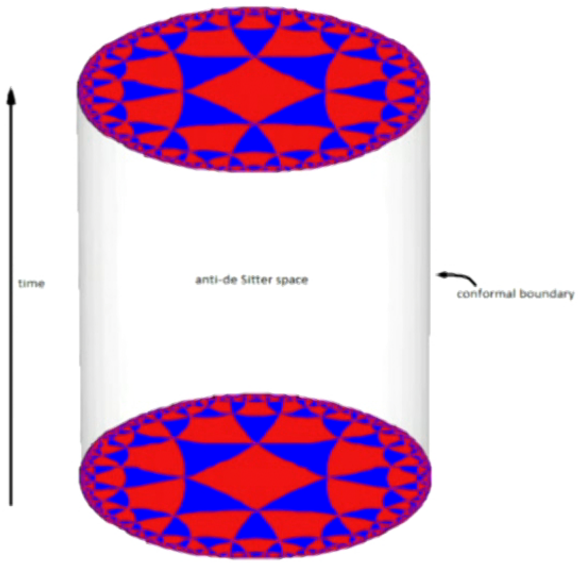
Holographic Principle

- Suggests gravitational systems in some subregion can be mapped to a system on the boundary of subregion.
- Most well understood realization of holographic principle comes from AdS/CFT
- Gravitational system in AdS can be understood in terms of conformal field theory on boundary of AdS.



Basic Idea behind AdS/CFT

- Different CFT states give rise to different AAdS bulk geometries E.g. $|0\rangle \to$ Empty AdS.
- Bulk field $\phi(z, x)$ dual to an operator in CFT $\mathcal{O}(x)$.
- Information of ϕ in bulk can be understood in terms of CFT correlators involving $\mathcal{O}$.



Does AdS/CFT Resolve Information Paradox?

- AdS BH can be mapped to CFT which respects unitarity $\Rightarrow$ AdS BH evolution respects unitarity.
- Where does Hawking's calculation go wrong?
- How is unitarity restored in bulk?
- Answering these questions requires a good understanding of how the bulk emerges from CFT dynamics.
- Current understanding of the how bulk arises from CFT is incomplete.
- Considerations from string theory and quantum information theory suggest that restoring unitarity requires modifications near the horizon (Fuzzball, Firewall, etc.).

Echoes in Holographic Quantum Systems?

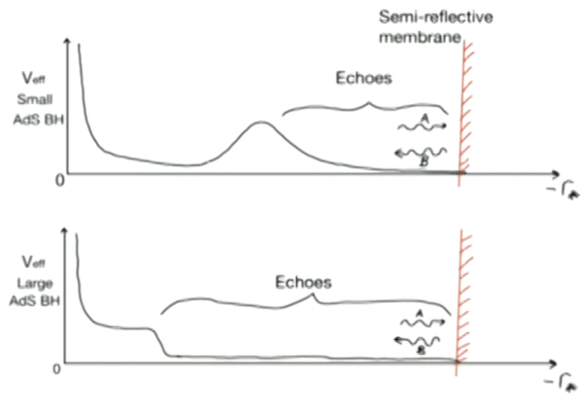
- If unitarity in bulk is restored via modification to horizon it is possible that such modifications give raise to echoes.
- Are their holographic CFT states that exhibit echoes?
- What exactly should we look for and calculate in CFT?

Echoes of AAdS BH

- Good place to start is to consider how echoes will be generated in bulk.
- Consider scalar waves in (d+1)-dimensional Schwarzschild AdS background.
- Radial part of solution will obey:

$$\frac{d^2\psi}{dr_*^2} + (\omega^2 - V)\psi = 0 \tag{1}$$

Effective Potential for AdS Schwarzschild BH



- Large AdS BH means $R_H \gg L$
- Small AdS BH means $R_H \ll L$

Echo Time in Geometric Optics Approx.

- Assume we have a BH metric of the form:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2 \tag{2}$$

- The echo time in geometric optic approx. given as:

$$t_{echo} = 2\int_{R_H+\delta R}^{r_t(\omega)} \frac{dr}{f(r)} \tag{3}$$

- δR is the coordinate radial distance perturbations get reflected (proper Planck length from horizon).

$$\ell_p = \int_{R_H}^{R_H+\delta R} \frac{dr}{\sqrt{f(r)}} \Rightarrow \delta R \sim \frac{\ell_p^2}{\beta} \tag{4}$$

- Where $\beta = T_H^{-1}$ is inverse of Hawking temprature.

Expansion of Echo time in ℓ_p

- Using ℓ_p as a perturbative parameter we do an expansion of echo time integral:

$$t_{echo} = \frac{\beta}{2\pi}\left[\ln\left(\frac{\beta R_H}{\ell_p^2}\right) - \ln\left(1 + \frac{f''(R_H)}{8}\beta R_H\right) + \chi_0(\omega) + \mathcal{O}(\ell_p)\right] \tag{5}$$

- Leading order term represents time spent in Rindler region.
- Second term important for near extremal BHs.
- Third term involving χ_0 represents time spent outside Rindler region.

Leading order contribution

- Can consider leading order term in expansion using the expression for β for (d+1) dimensional Schwarzschild AdS BH:

$$t_{echo} \simeq \frac{\beta}{2\pi} \ln\left[\frac{R_H}{\ell_p^2}\left(\frac{4\pi R_H L^2}{dR_H^2 + (d-2)L^2}\right)\right] \simeq \begin{cases} \frac{\beta}{2\pi} \ln\left(\frac{R_H^2}{\ell_p^2}\right) & R_H \ll L \\ \frac{\beta}{2\pi} \ln\left(\frac{L^2}{\ell_p^2}\right) & R_H \gg L \end{cases} \tag{6}$$

- For small AdS BH $t_{echo} \sim \frac{\beta}{2\pi} \ln(S_{BH})$
- For large AdS BH $t_{echo} \sim \frac{\beta}{2\pi} \ln(S_{cell})$

Holographic interpretation of echo time

- In AdS/CFT large AdS BHs in bulk is dual to strongly coupled large N gauge theory (at temperature T_H).
- In particular $N^2 \sim (L/\ell_p)^{d-1}$
- In this case the echo time is:

$$t_{echo} \sim \frac{\beta}{2\pi} \ln\left(N^2\right) \tag{7}$$

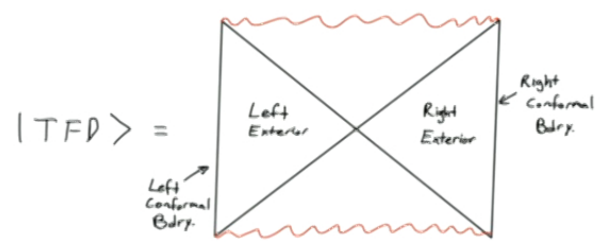
- Why is this interesting?

Entanglement Disruption for TFD

- Consider the Thermofield double state:

$$|TFD\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_L \otimes |n\rangle_R \tag{8}$$

- In AdS/CFT dual to a two sided Large AdS BH geometry:

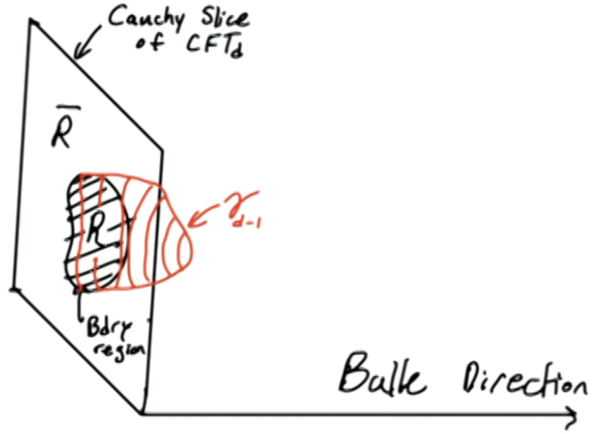


- Two exteriors connected by non-traversable wormhole.
- Entanglement of left and right is manifested geometrically by non-traversable wormhole.

- Consider mutual information between left and right CFT:

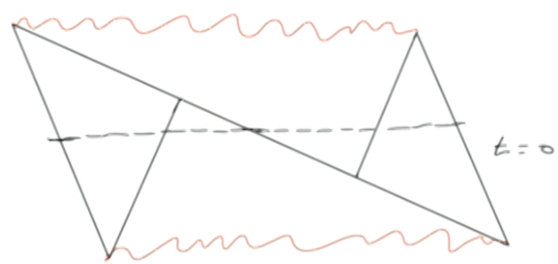
$$I(L,R) = S(L) + S(R) + S(L \cup R) \tag{9}$$

- Each term can be calculated geometrically using the area of co-dimension two extremal surface anchored to boundary (Ryu-Takayanagi Formula).



$$S(R) = \frac{A(\gamma_{d-1})}{4G_N} \tag{10}$$

- Now consider the perturbed state $\mathcal{O}_R(-\tau)\,|\,TFD\rangle$.
- In bulk can think of a particle that starts at boundary goes toward black hole.
- When t very far in past very large blue-shifted energy at $t = 0$ slice.
- Back-reacted geometry looks like shock-wave geometry:



- Use RT prescription in shockwave geometry.
- Mutual Info. is zero if $\tau \geq \frac{\beta}{2\pi} \ln\left(\frac{S}{\delta S}\right)$

Scrambling Time vs Echo Time

- What is the smallest choice of δS?
- If we want to interpret perturbation in terms of Hawking quanta $\delta S = 1 \Rightarrow \tau = \frac{\beta}{2\pi} \ln(S)$
- Another choice from geometric perspective.
- GR description of spacetime valid on proper lengths larger than Planck length.
- Suggests smallest semi-classical perturbation should shift the horizon by amount larger than proper Planck length.
- Can show $\tau \simeq t_{echo}$
- The echo time can be identified as the amount of time it takes to disrupt entanglement in *TFD*

Role of Entanglement in Horizon Crossing

- Close to horizon geometry can approximated by Rindler wedge.
- Same near horizon geometry can be "simulated" by constant accelerating observer in Minkowski space.
- Consider vacuum state in Minkowski space from perspective of accelerating observer.
- Vacuum state in flat spacetime in Rindler coordinates takes the form of TFD state.
- The TFD state contains specific correlations that leave vacuum regular at "horizon."
- But TFD is not unique purification of thermal state.
- General purifications will have divergent behaviour at horizon in Hamiltonian (presence of Firewall).

From Entanglement Disruption to Echoes?

- Start with black hole whose near horizon vacuum state is TFD.
- Introduce perturbation it will take $t_{echo}/2$ to get within Planck length of horizon.
- After t_{echo} information in perturbation gets scrambled and disrupts correlations near horizon.
- Perhaps these disruptions will generate modifications to near horizon geometry that partially reflects perturbations.
- Gives a hypothesis of a mechanism that generates echoes.
- How can we test it?

Quantum Chaos

- Chaos is generic feature thermal systems.
- Classically, chaos diagnoses sensitivity to initial conditions:

$$\{x(t), p(0)\} = \frac{\delta x(t)}{\delta x(0)} \sim e^{\lambda_L t} \tag{11}$$

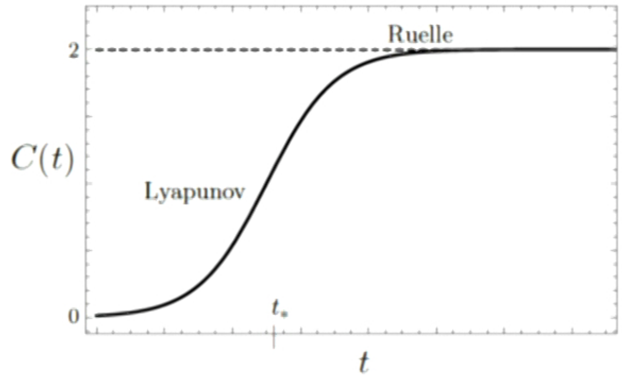
- λ_L called Lyapunov exponent.
- We say quantum system exhibits quantum chaos if:

$$C(t) = \langle |[\hat{W}(t), \hat{V}(0)]|^2 \rangle_\beta \sim e^{2\lambda_L t} \tag{12}$$

AdS BHs as Chaotic Systems

- AdS black holes dual to certain holographic thermal CFT states at $T = T_H$.
- Translate thermal field theory calculations to gravity calculation in AdS BH spacetime.
- Will find the commutator squared at times exhibits exponential growth in interval $t_d \ll t \ll t_*$.
- Lyapunov exponent equals $2\pi/\beta$.
- For thermal quantum systems with large number of d.o.f. it is conjectured that $\lambda_L \leq 2\pi/\beta$

C(t) for Large N Holographic Theories



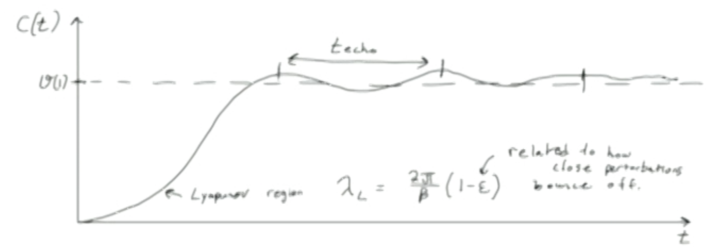
$$C(t) \sim \begin{cases} N^{-1} & t < t_d \\ N^{-1} e^{\lambda_L t} & t_d \ll t \ll t_* \\ \mathcal{O}(1) & t > t_* \end{cases} \tag{13}$$

- $C(t_*) \to \mathcal{O}(1)$ where $t_* \sim \beta \ln(N^2)$ is scrambling time.
- Ruelle region expected to be controlled by QNM of dual BH.

How Might Echoes Manifest in C(t)?

- Echoes are a symptom of near horizon modifications that allow for partial reflection of perturbations.
- $t_{echo} \sim t_{scr}$ if modifications are localized within proper Planck length of horizon.
- What should we look for in a calculation of $C(t)$ that indicates modifications that result in echoes?

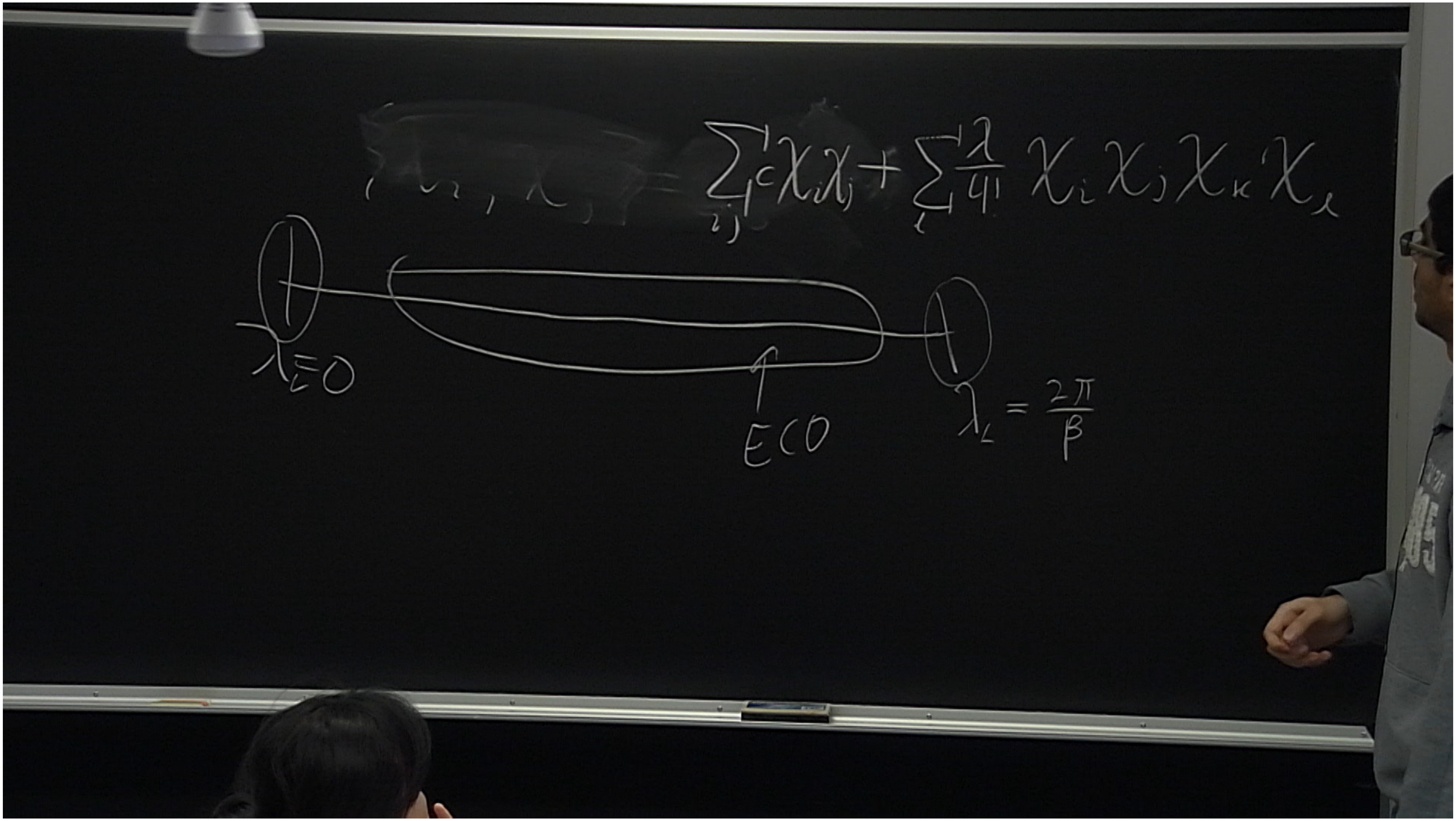
1) Lyapunov exponent should be smaller than $2\pi/\beta$.
2) $C(t > t_{scr})$ should exhibit periodic behaviour for a while before saturating to a constant.

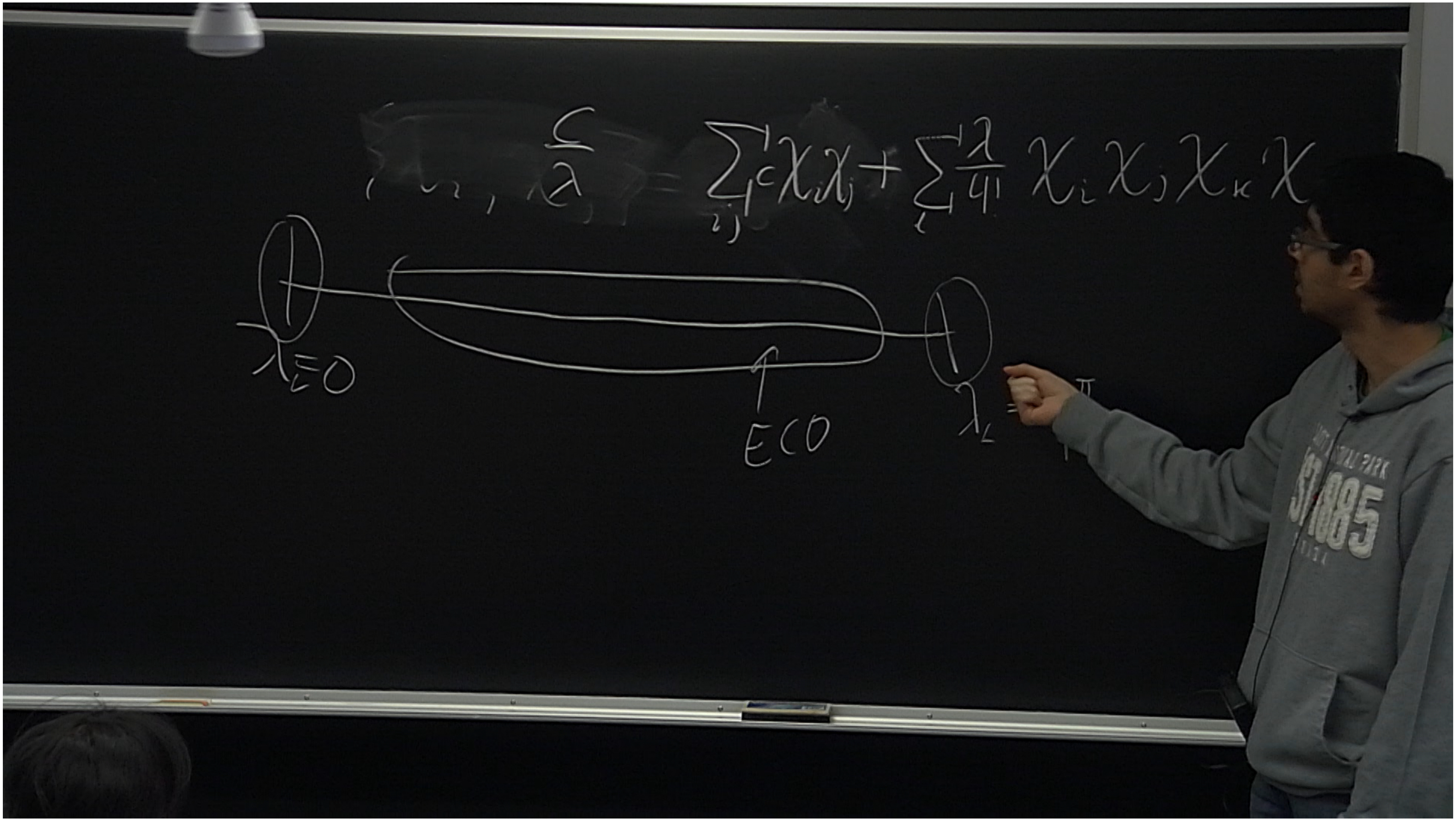


Summary

- Considered time scale of echoes for AAdS BHs.
- Showed that for perturbations that shift horizon by Planck length you can identify scrambling time scale with echo time scale.
- Suggest a bulk picture of how scrambling generates modified horizon structure which might generate echoes.
- Suggested to diagnose existence of echoes in Quantum system by calculating commutator square of Hermitian observables and check if it satisfy two criteria.

1) Lyapunov exponent should be smaller than $2\pi/\beta$.
2) $C(t > t_{scr})$ should exhibit periodic behaviour for a while before saturating to a constant.


$= \sum_{ij} c\, \chi_i \chi_j + \sum \frac{\lambda}{4!} \chi_i \chi_j \chi_k \chi_l$
$\lambda_L = 0$
ECO
$\lambda_L = \frac{2\pi}{\beta}$


ECO